RSA CONFERENCE 2013

Security in knowledge

Proxy Re-Encryption in a Stronger Security Model Extended from CT-RSA2012

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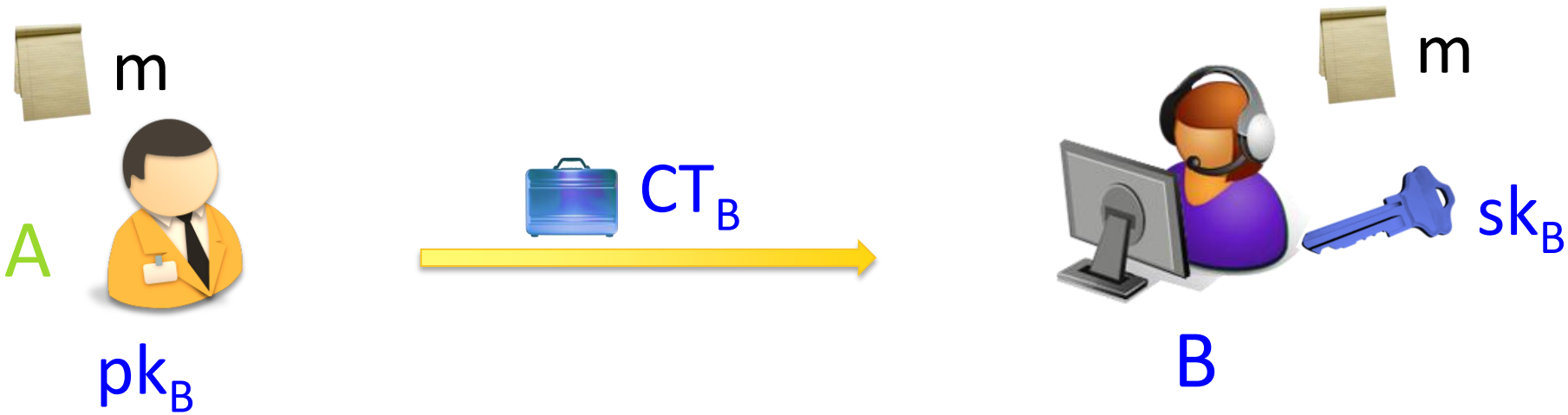
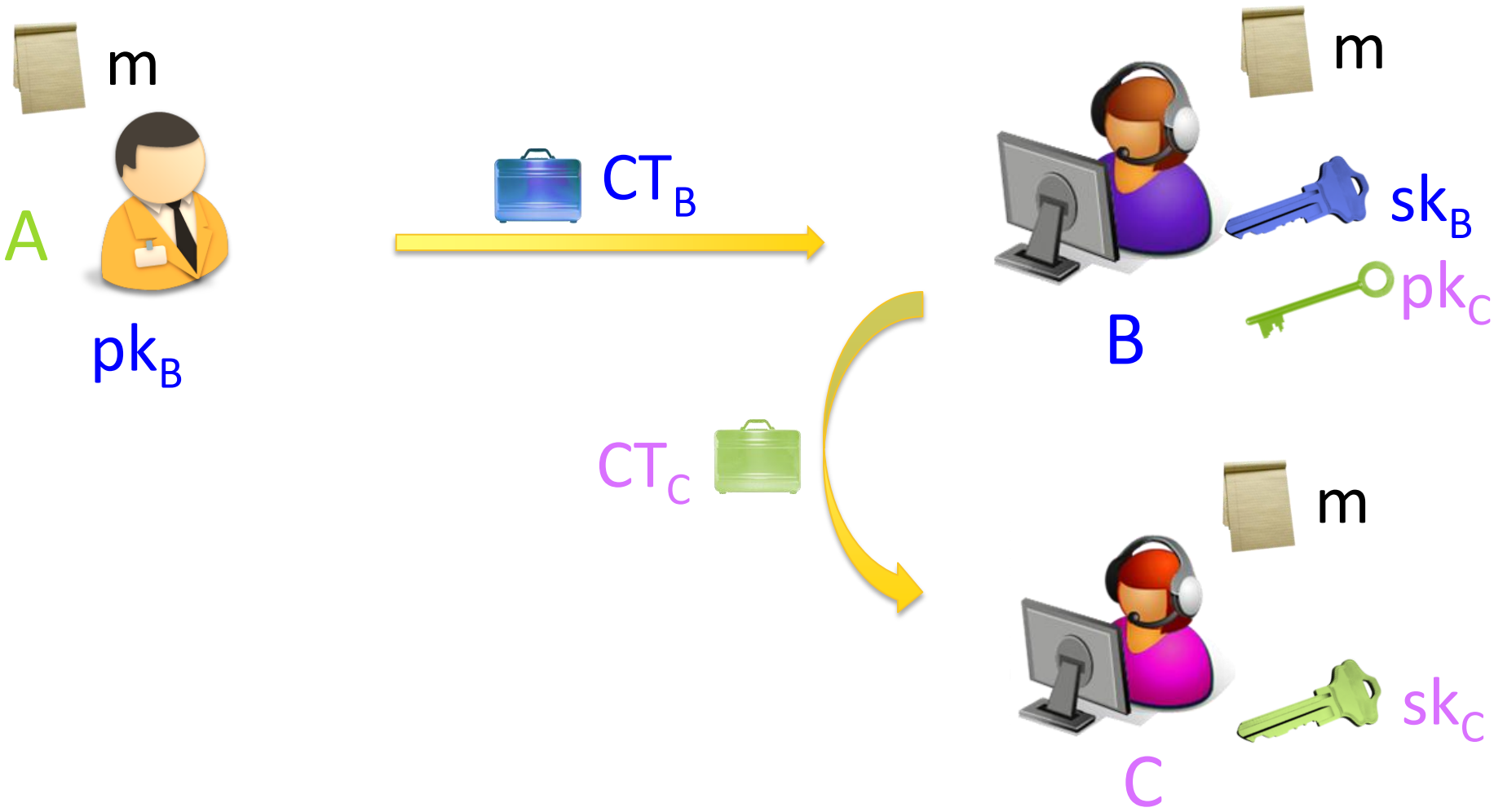
Session Classification: Advanced

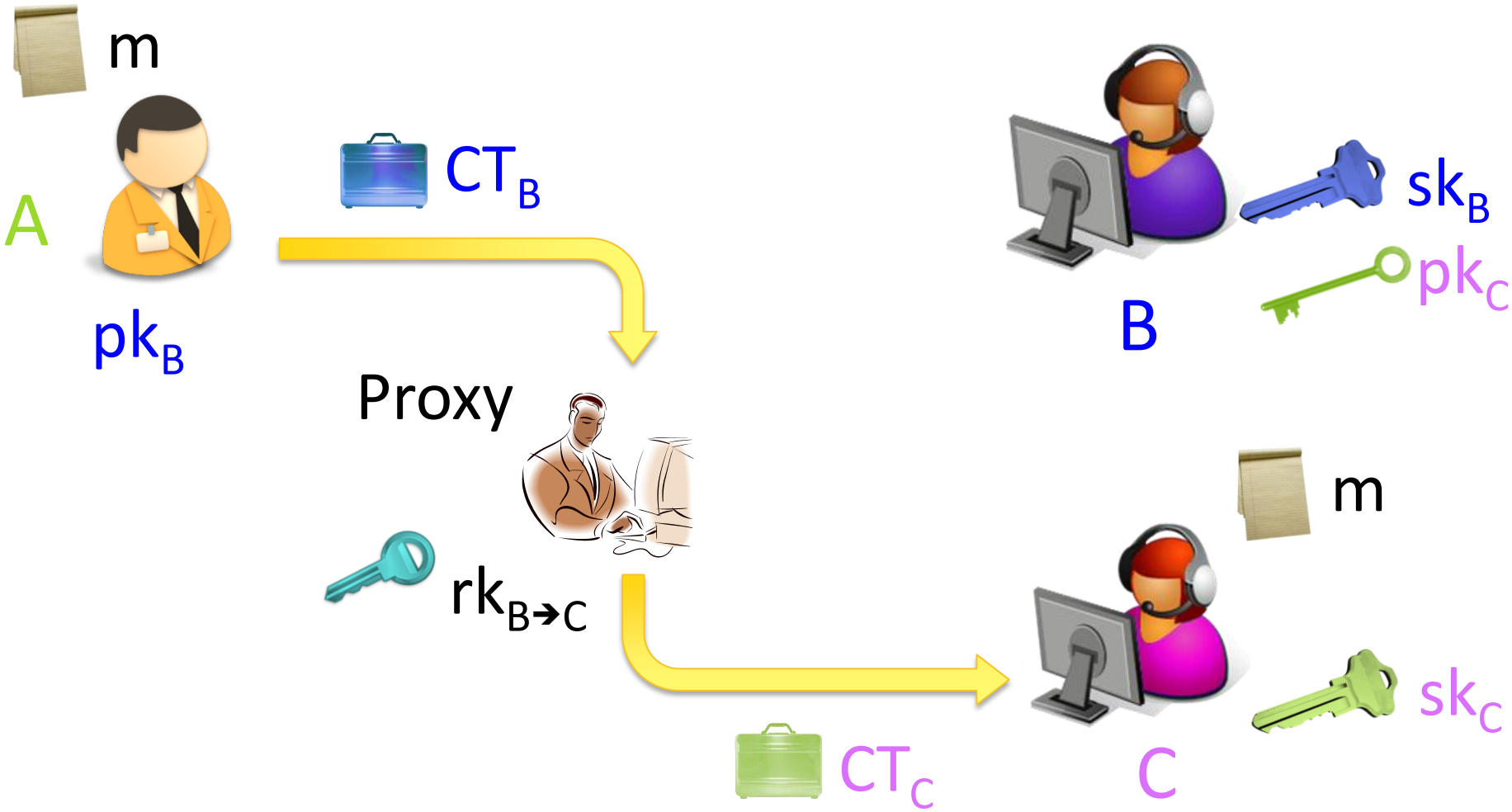
Public Key Encryption



Email Forwarding



Proxy Re-Encryption (PRE)



Proxy Re-Encryption (PRE)

Types

- Bidirectional or Unidirectional

- Multi-hop or Single-hop

This work: Unidirectional, Single-hop PRE

Our Results

- We present a definition of security against chosen-ciphertext attacks (CCA) for unidirectional, single-hop PRE
  - Extended from previous works
- New PRE scheme with this security

Agenda

- Model of PRE
- Security Notions of PRE
- Proposed PRE Scheme
- Concluding Remarks

Model of PRE

Model of PRE

- Setup$(1^k) \Rightarrow PP$
- KGen$(PP) \Rightarrow (pk,sk)$
- ReKey$(PP,sk_i,pk_j) \Rightarrow rk_{i->j}$
- Enc$_1(PP,pk_i,m) \Rightarrow 1^{st}\text{-}CT_i$ (first level)
- Enc$_2(PP,pk_i,m) \Rightarrow 2^{nd}\text{-}CT_i$ (second level)
- ReEnc$(PP, rk_{i->j}, 2^{nd}\text{-}CT_i) \Rightarrow 1^{st}\text{-}CT_j$ (first level)
- Dec$_1(PP,pk_i,1^{st}\text{-}CT_i) \Rightarrow m$
- Dec$_2(PP,pk_i,2^{nd}\text{-}CT_i) \Rightarrow m$

Model of PRE

(second level) 2nd-CT_i $\xrightarrow{\text{ReEnc}}$ (first level) 1st-CT_j

(first level) 1st-CT_i $\xrightarrow{\text{ReEnc}}$ ✗

- $\mathrm{Enc}_2(PP, pk_i, m) \Rightarrow 2^{nd}\text{-}CT_i$ (second level)
- $\mathrm{ReEnc}(PP, rk_{i\text{->}j}, 2^{nd}\text{-}CT_i) \Rightarrow 1^{st}\text{-}CT_j$ (first level)
- $\mathrm{Dec}_1(PP, pk_i, 1^{st}\text{-}CT_i) \Rightarrow m$
- $\mathrm{Dec}_2(PP, pk_i, 2^{nd}\text{-}CT_i) \Rightarrow m$

Summary of uni-PRE Schemes

Schemes	IND-CCA Security of ReEnc	RO-free	Security of Second Level Ciphertext
[AFGH06]	✗	○	CPA
[LV08]	✗	○	RCCA
[CWYD10]	✗	✗	CCA
[HMY+11]	✗	○	RCCA
[CDL11]	○	✗	CCA
[HKK+12]	✗	○	CCA
Ours	○	○	CCA

Security Notions of PRE

Security Notions of PRE

Previous works

- Security of second level ciphertext
- Security of first level ciphertext

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(E.g. [LV08], [CWYD10], [HKK+12], …)

Security Notions of PRE

<u>This work</u>

- Security of second level ciphertext
- Security of original first level ciphertext
- Security of re-encrypted ciphertext

Security Notions of PRE

<u>This work</u>

- Security of second level ciphertext

  (2nd-level-CCA security)

- Security of original first level ciphertext

  (1st-ori-CCA security)

- Security of re-encrypted ciphertext

  (1st-re-CCA security)

2nd-level-CCA Security

$\mathcal{O}_{pk}$, $\mathcal{O}_{sk}$, $\mathcal{O}_{rk}$, $\mathcal{O}_{re}$, $\mathcal{O}_{dec1}$, $\mathcal{O}_{dec2}$

With some conditions!!!

$\mathcal{O}_{racles}$

$pk_{i*}, (m_0, m_1)$

2nd-CT^*

$\mathcal{O}_{racles}$

Output b'

$b \in \{0,1\}$

2nd-$CT^* \leftarrow \mathrm{Enc}_2(pk_{i*}, m_b)$

2nd-level-CCA Security

Conditions:

- $\mathcal{O}_{\mathrm{sk}}(i^*)$ is not allowed

- $\mathcal{O}_{\mathrm{rk}}(i^*,j)$ is allowed if pk_j is uncorrupted key

- If *Adv* issues $\mathcal{O}_{\mathrm{re}}(i,j,CT_i)$ or $\mathcal{O}_{\mathrm{dec1}}(i,CT_i)$:

(pk_i, CT_i) can't be a derivative of $(pk_{i*}, 2^{nd}\text{-}CT^*)$

Extension from The Model of [HKK+12]

- Security of second level ciphertext
  - The gap w.r.t the restriction on the first level decryption queries

The example showing this gap in the proceedings is wrong, however, we can show the gap by other examples

- Security of first level ciphertext
  - …

1st-ori-CCA Security

[CDL11]

$\mathcal{O}_{pk}$, $\mathcal{O}_{sk}$, $\mathcal{O}_{rk}$, ~~$\mathcal{O}_{re}$~~, $\mathcal{O}_{dec1}$, $\mathcal{O}_{dec2}$

$\mathcal{O}racles$

$pk_{i*}, (m_0, m_1)$

1st-CT^*

$\mathcal{O}racles$

Output b'

$b \in \{0,1\}$

1st-$CT^* \leftarrow \mathrm{Enc}_1(pk_{i*}, m_b)$

1st-re-CCA Security

[CDL11]

$\mathcal{O}_{pk}$, $\mathcal{O}_{sk}$, $\mathcal{O}_{rk}$, ~~$\mathcal{O}_{re}$~~, $\mathcal{O}_{dec1}$, $\mathcal{O}_{dec2}$

$\mathcal{O}racles$

pk_i, pk_{i*} $\boxed{(sCT_0, sCT_1)}$

1st-CT^*

$sCT_0 \leftarrow \mathrm{Enc}_2(pk_i, m_0)$

$sCT_1 \leftarrow \mathrm{Enc}_2(pk_i, m_1)$

$\mathcal{O}racles$

$b \in \{0,1\}$

Output b'

1st-$CT^* \leftarrow \mathrm{ReEnc}_1(rk_{i->i*}, sCT_b)$

CCA-Security of
first level ciphertext

VS.

1st-ori-CCA Security

1st-re-CCA Security

CCA-Security of
first level ciphertext

$1^{st}\text{-}CT^{*} = \mathrm{Enc}_1(pk_{i^*}, m_b)$ or
$\mathrm{ReEnc}(rk_{i\text{->}i^*}, \mathrm{Enc}_2(pk_i, m_b))$

CCA-Security of
first level ciphertext

$$1^{st}\text{-}CT^{*}=\mathrm{Enc}_1(pk_{i^*},m_b) \text{ or } \mathrm{ReEnc}(rk_{i\text{->}i^*},\mathrm{Enc}_2(pk_i,m_b))$$

1st-ori-CCA
Security

1st-re-CCA
Security

CCA-Security of
first level ciphertext

CCA-Security of
first level ciphertext

$1^{st}\text{-}CT^* = \mathrm{Enc}_1(pk_{i^*}, m_b)$ or
$\mathrm{ReEnc}(rk_{i\text{->}i^*}, \mathrm{Enc}_2(pk_i, m_b))$

If *Adv both compromised*
delegator (sk_i) and proxy ($rk_{i\text{->}i^*}$)

then *Adv* may have obtained the original ciphertext
$2^{nd}\text{-}CT_b \leftarrow \mathrm{Enc}_2(pk_i, m_b)$ and use sk_i to decrypt trivially

CCA-Security of first level ciphertext ⇏ Secure

1st-re-CCA Security ⇒ Secure

If *Adv both compromised*
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CCA-Security of
first level ciphertext

1st-ori-CCA
Security

1st-re-CCA
Security

Proposed PRE Scheme

Introduction of Our Scheme

- Construction based on
  - IND-CCA secure PKE scheme from CT-RSA'10 (Lai, Deng, Liu, and Kou)
  - IND-CCA secure PKE scheme from PKC'07 (Kiltz)
- Properties
  - 2nd-level-CCA
  - 1st-ori-CCA
  - 1st-re-CCA
  - w/o Random Oracle
  - Using bilinear maps

The Proposed Scheme (1/2)

- Setup(1^λ): PP=(p,G,G_T,g,g_1,h,u,v,d,u_1,v_1,d_1,e,H, TCR, TCR').

- KGen: $pk = (g^x, g_1^{x^2}, g^y), sk = (x, y)$

- ReKey(sk_i,pk_j): $rk_{i \to j} = g_1^{x_j^2 / x_i}$

The Proposed Scheme (2/2)

- $\mathrm{Enc}_2(pk_i, m)$: $CT_i = (C_1, C_2, C_3, C_4, C_5), where$

$$C_1 = pk_{i1}^r, C_2 = h^r, C_3 = e(g, g_1)^r \times m, t = H(C_2, C_3),$$

$$C_4 = (u^t v^s d), C_5 = s.$$

- $\mathrm{ReEnc}(rk_{i\to j}, pk_i, CT_i)$:
  - Check the validity of CT_i
  - Compute $CT_j = (A, B, C), where$

$$C_6 = C_1^R, C_7 = pk_{i1}^R, C_8 = rk_{i\to j}^{1/R}, CT' = C_2 \| C_3 \| \ldots \| C_8$$

$$A = g^{r'}, t' = TCR(A), B = (pk_{i,3}^{t'} \cdot h)^{r'},$$

$$C \leftarrow SYM.Enc(H(pk_{i,3}^{r'}), CT')$$

Security of Our Scheme

Theorem 1

Our scheme meets the 2nd-level-CCA security, assuming the hash function H is target collision resistant, the 6-AmDBDH assumption holds in groups (G, G_T), *and the 2-AmCDH problem is hard.*

Theorem 2

Our scheme meets the 1st-ori-CCA and the 1st-re-CCA security, assuming the hash function TCR' is target collision resistant, the GHDH problem is hard, and **SYM** *is CCA-secure.*

Concluding Remarks

Concluding Remarks

- We have present a CCA security definition for unidirectional, single-hop PRE
  - Extended from previous works
- New PRE scheme with this security

Solving BDD by Enumeration: An Update

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[2]INRIA and Tsinghua University

CT-RSA 2013

Outline

Motivation

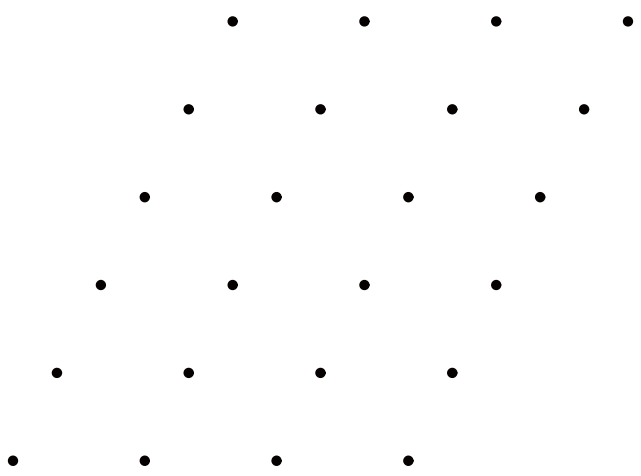
Algorithm for BDD
- BDD Problem
- Babai NearestPlane Algorithm
- Lindner-Peikert's NearestPlanes Algorithm Revisited

Solving BDD by (GNR) Pruned Enumeration
- Randomizing the NearestPlanes Algorithm
- GNR Pruned Enumeration to BDD

Conclusion

BDD: Why Is Important

- Lattice is a discrete additive subgroup of $\mathbb{R}^m$
- BDD: Bounded Distance Decoding, a basic lattice problem, a special case of CVP (closest vector problem), the target vector very close to lattice



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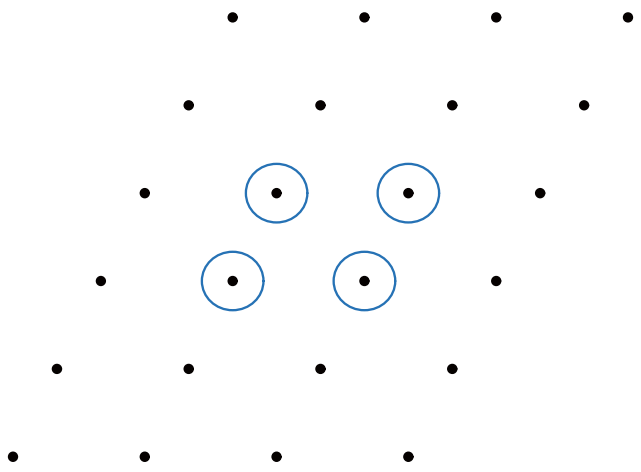

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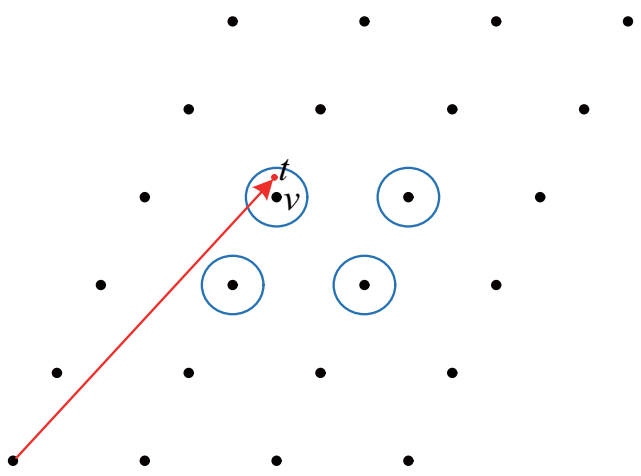
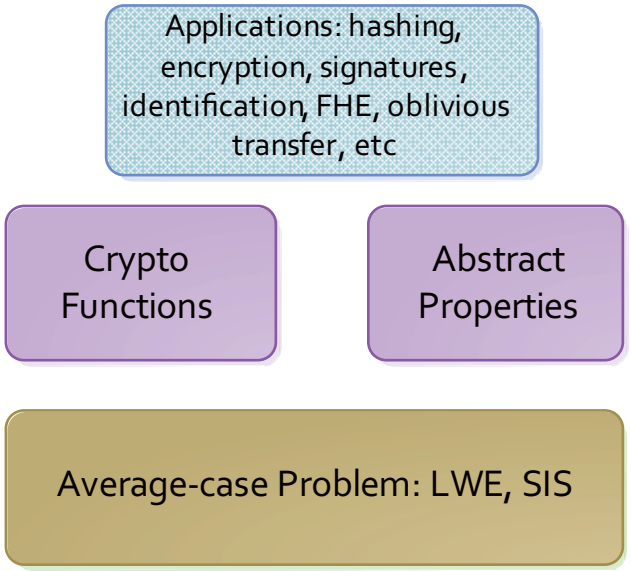


Lattice in Crypto

- Applications to cryptanalysis
  - Lattice based schemes:NTRU, GGH etc
  - Non-lattice based schemes: knapsack, RSA, DSA, etc

- Cryptography



BDD: Why Is Important

- The security of most lattice-based encryption schemes relies on the hardness of BDD, such as
  - LWE (Learning with error): one of the two average-case problems most of the provably-secure lattice-based constructions are based on
  - Hidden number problem(HNP)– DSA with partially known nonces, chosen ciphertext attack on RSA-PKCS#1 encryption version 1.5,

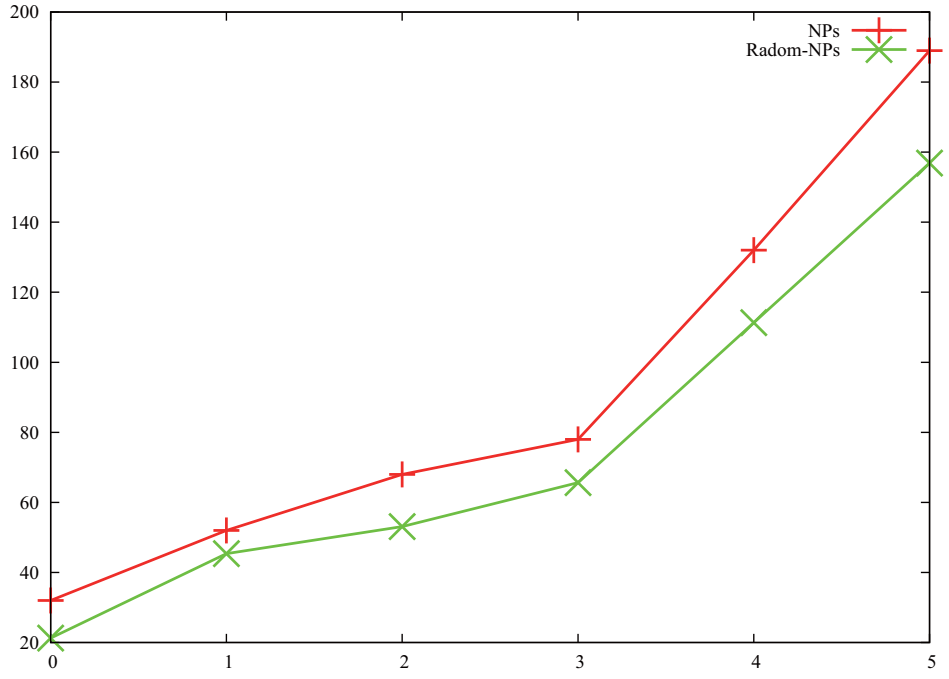
  What is the best algorithm for BDD ?

Previous Work

- Babai's NearestPlane algorithm, Babai, 1985
- NearestPlanes algorithm, Lindner and Peikert, 2011
  - Claimed that this was the best attack known on Search-LWE
- Embedding method, Kannan 1987
  - Heuristically reduces BDD to the unique-shortest vector problem
  - The largest BDD cryptanalytic instances ever solved in practice using this method

Our Work

- Rephrasing NearestPlanes algorithm in the pruned-enumeration framework of Gama-Nguyen-Regev (GNR)
  - Present a simple randomized variant.
  - In the case of LWE, significantly better than the Lindner-Peikert attack the speedup can be as big as 2^{32}



Our Work

- Consider GNR pruned-enumeration algorithms to solve BDD:
  - Provides even better attacks on Search-LWE
  - The method of choice in practice for the general BDD case.
    - Recover the DSA secret key from $2 \leftarrow 3$ least significant bits with 100 instances
    - Lattice version of Bleichenbacher's celebrated chosen ciphertext attack [Ble98] on RSA-PKCS#1.5 encryption version 1.5, 1024bit, 80 dimension (Babai algorithm)$\longrightarrow$ 65 dimesion (Enumeration with pruning)
    - GGH: the first partial secret-key recovery in dimensions 200-300 and re-solved the 350-dimensional message-recovery challenge using much weaker lattice reduction

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Conclusion

BDD Problem

- Bounded Distance Decoding (BDD): Given a lattice and a target vector unusually close to the lattice, to find the closest lattice vector to the target
- What is the best algorithm for solving BDD in practice?
- Several parameters impact the answer, *e.g.* the dimension, the size and shape of the error.

LWE problem

- Search: find $\mathbf{s} \in \mathbb{Z}_q^n$, given 'noisy random inner product'

$$\begin{aligned} a_1 \quad &, \quad b_1 = \langle a_1, s \rangle + e_1 \mod q \\ a_2 \quad &, \quad b_2 = \langle a_2, s \rangle + e_2 \mod q \\ &\vdots \end{aligned}$$

- Uniform $\mathbf{a}_i \in \mathbb{Z}_q^n$, Gaussian error $\mathbf{e}_i$
- LWE is a BDD instance for the lattice $\Lambda_q(A) = \{\mathbf{y} \in \mathbb{Z}^m : \mathbf{y} = \mathbf{s}A \mod q \text{ for } \mathbf{s} \in \mathbb{Z}_q^n\}$

LWE problem

- Search: find $\mathbf{s} \in \mathbb{Z}_q^n$, given 'noisy random inner product'

$$\begin{array}{ccc} a_1 & , & b_1 \\ a_2 & , & b_2 \\ a_3 & , & b_3 \end{array}$$

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LWE problem

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$$m\left\{\begin{pmatrix} \vdots \\ A^t \\ \vdots \end{pmatrix}, \begin{pmatrix} \vdots \\ b \\ \vdots \end{pmatrix}\right. = A^t s + e$$

- Uniform $\mathbf{a}_i \in \mathbb{Z}_q^n$, Gaussian error $\mathbf{e}_i$
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HNP Problem: Hidden Number Problem

- Recover $\alpha \in \mathbb{Z}_q$, given $u_i = APP_{\ell,q}(\alpha t_i)$, $1 \leq i \leq d$ where $APP_{\ell,q}(n)$ denotes any rational number r satisfying $|n - r|_q \leq \frac{q}{2^{\ell+1}}$, $|z|_q = \min_{b \in \mathbb{Z}} |z - bq|$
- Example: DSA with partially known nonces, chosen ciphertext attack on RSA-PKCS#1 encryption version 1.5

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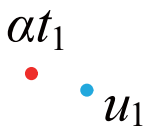

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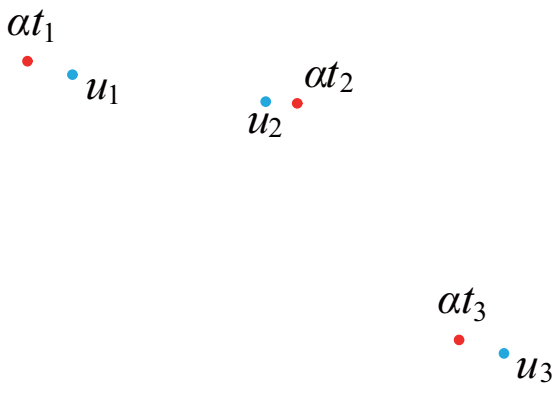

HNP Problem: Hidden Number Problem

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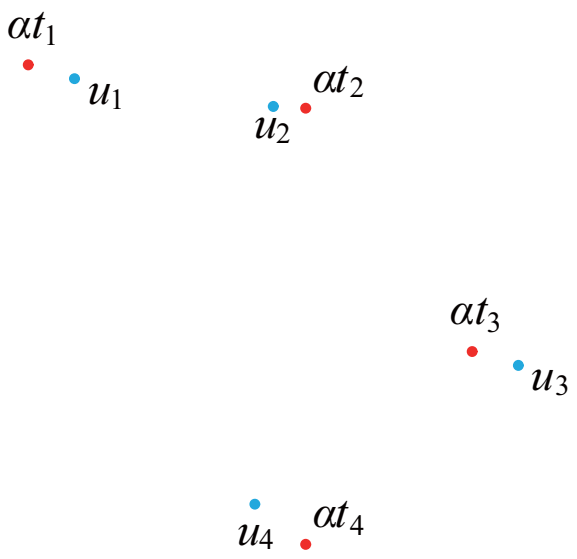
- Example: DSA with partially known nonces, chosen ciphertext attack on RSA-PKCS#1 encryption version 1.5



HNP Problem: Hidden Number Problem

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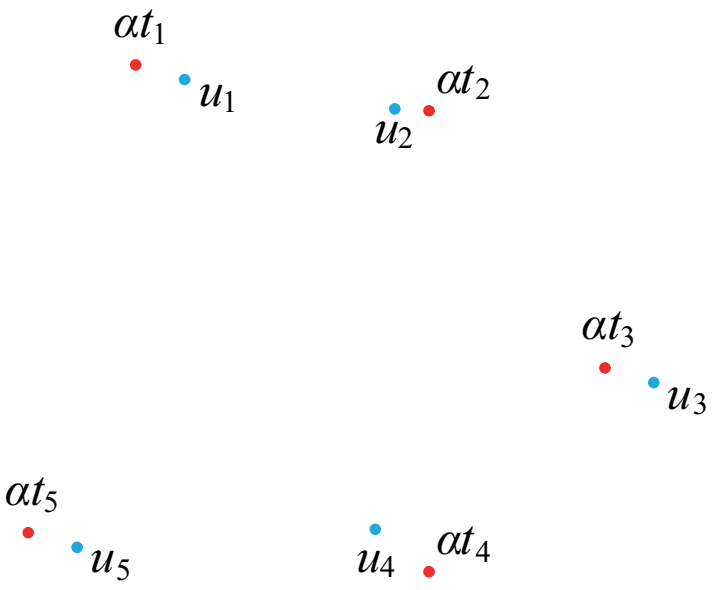

HNP Problem: Hidden Number Problem

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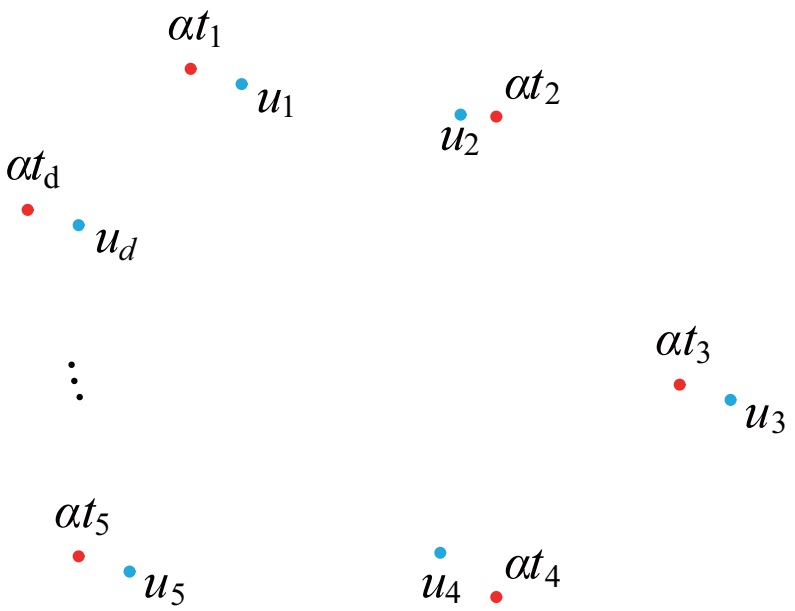
- Example: DSA with partially known nonces, chosen ciphertext attack on RSA-PKCS#1 encryption version 1.5



HNP Problem: Hidden Number Problem

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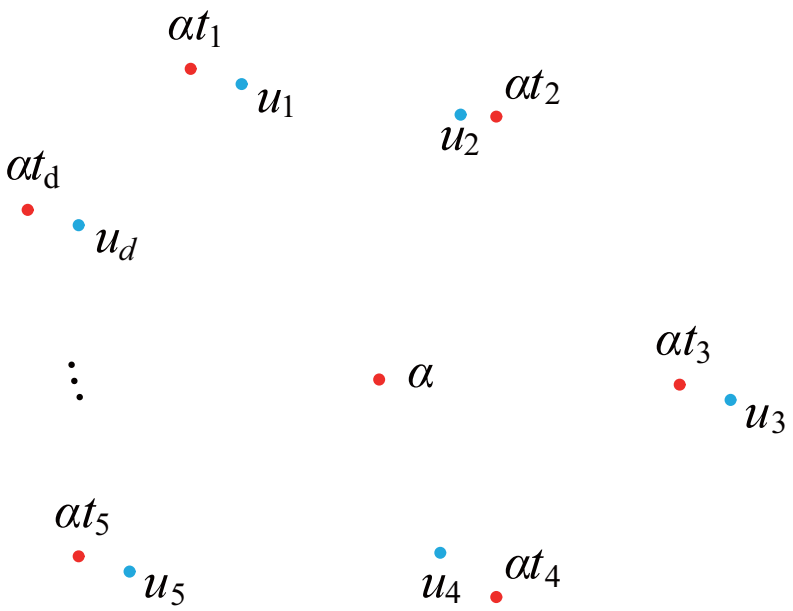

HNP Problem

- Can be reduced to :BDD problem in a $d+1$-dimensional lattice， target vector： $\mathbf{u} = (u_1, u_2, \ldots, u_d, 0)$

$$\begin{pmatrix} q & 0 & \cdots & 0 & 0 \\ 0 & q & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & q & 0 \\ t_1 & \cdots & \cdots & t_d & \frac{1}{2^{l+1}} \end{pmatrix} \tag{1}$$

- There exists a lattice vector $\mathbf{h} = (\alpha t_1 + qh_1, \ldots, \alpha t_d + qh_d, \frac{\alpha}{2^{\ell+1}})$, such that $\| \mathbf{h} - \mathbf{u} \| \leq \sqrt{d+1}\frac{q}{2^{\ell+1}}$

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Conclusion

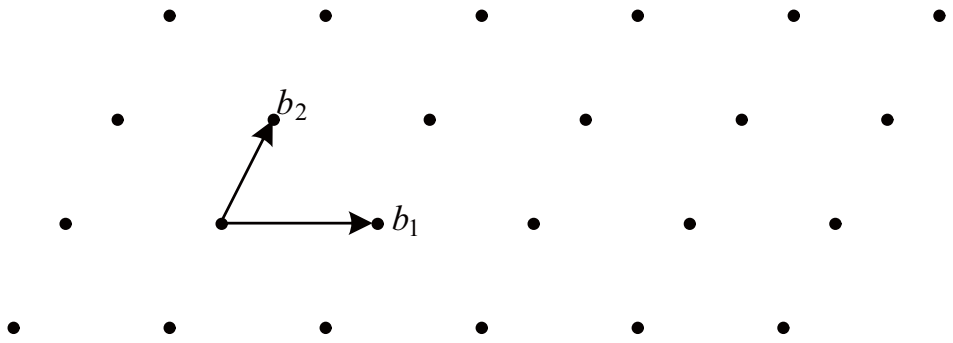
Babai's NearestPlane Algorithm

- Gram-schmidt orthogonalization: Give n linearly independent vetors, GS constructs n orthogonal vectors span the same subspaces. $\mathbf{b}_i^* = \pi_i(\mathbf{b}_i)$, where π_i: orthogonal projection over span $(\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_{i-1})^\perp$

$$
\begin{array}{c}
\left(\begin{array}{ccccc}
\|b_1^*\| & 0 & 0 & \cdots & 0 \\
\mu_{21}\|b_1^*\| & \|b_2^*\| & 0 & \cdots & 0 \\
\mu_{31}\|b_1^*\| & \mu_{32}\|b_2^*\| & \|b_{m-k+1}^*\| & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\mu_{m1}\|b_1^*\| & \mu_{m2}\|b_2^*\| & \mu_{m\ m-k+1}\|b_{m-k+1}^*\| & \cdots & \|b_m^*\|
\end{array}\right) \\
\begin{array}{ccccc}
\pi_1(B) & \pi_2(B) & \pi_{m-k+1}(B) & & \pi_m(B)
\end{array}
\end{array}
$$

Babai's NearestPlane Algorithm

- Babai's algorithm outputs a lattice vector $\mathbf{v}$ relatively close to the input target vector $\mathbf{t}$. $\mathbf{v}$ is the unique lattice vector such that $\mathbf{v}-\mathbf{t} \in \mathcal{P}_{1/2}(B^*) = \{\sum_{i=1}^{m} x_i b_i^* : -\frac{1}{2} < x_i \leq \frac{1}{2}\} \Longleftrightarrow \|\xi_i(\mathbf{v}-\mathbf{t})\| \leq \|\mathbf{b}_i^*\|/2$ where $\xi_i(\mathbf{x})$ denotes the i-th coordinate of x in the normalized Gram-Schmidt basis $(\mathbf{b}_1^*/\|\mathbf{b}_1^*\|, \ldots, \mathbf{b}_m^*/\|\mathbf{b}_m^*\|)$.



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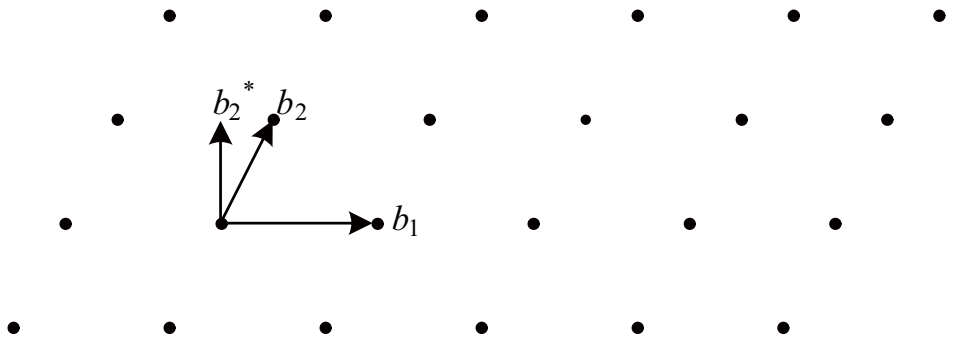

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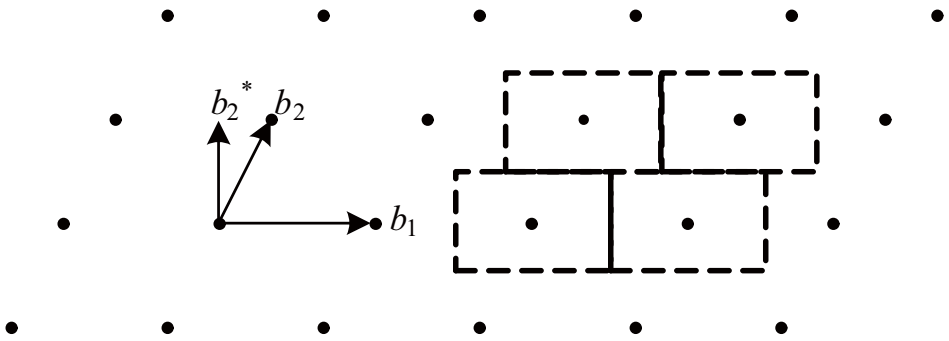

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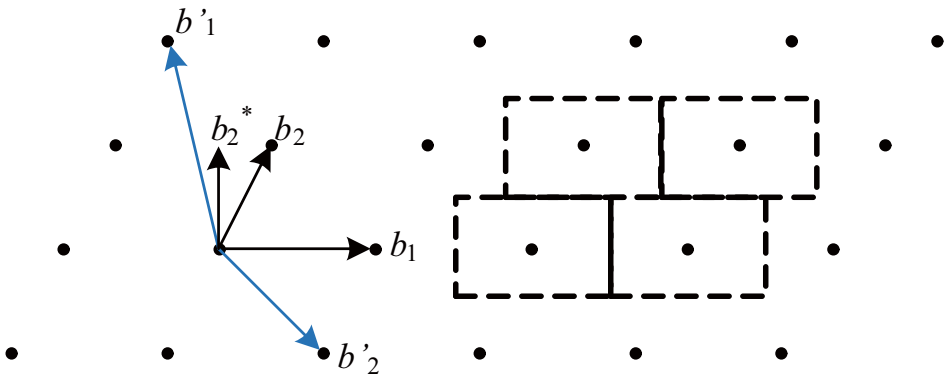

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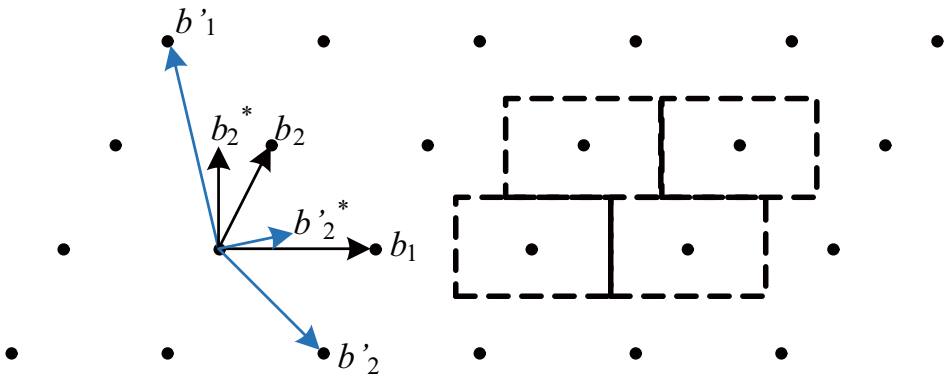

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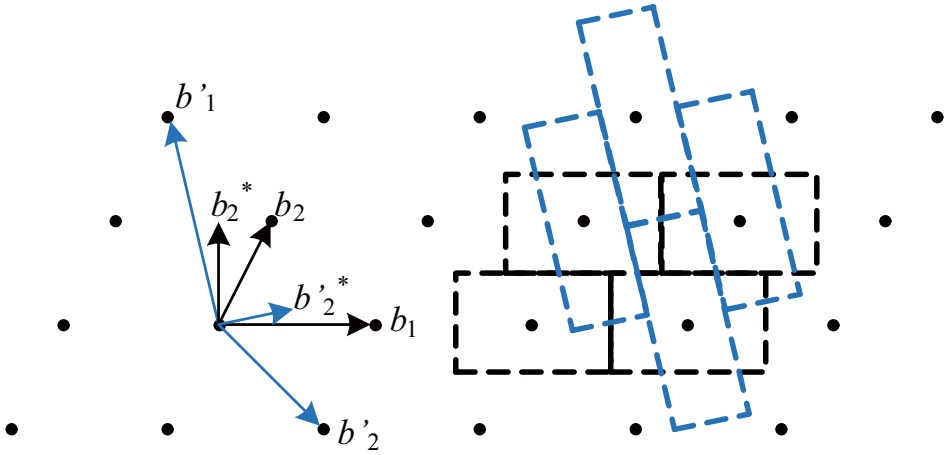

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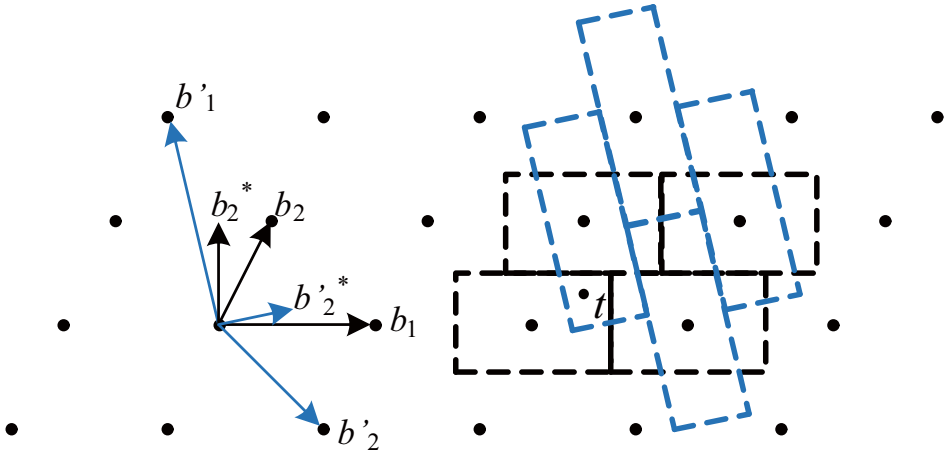

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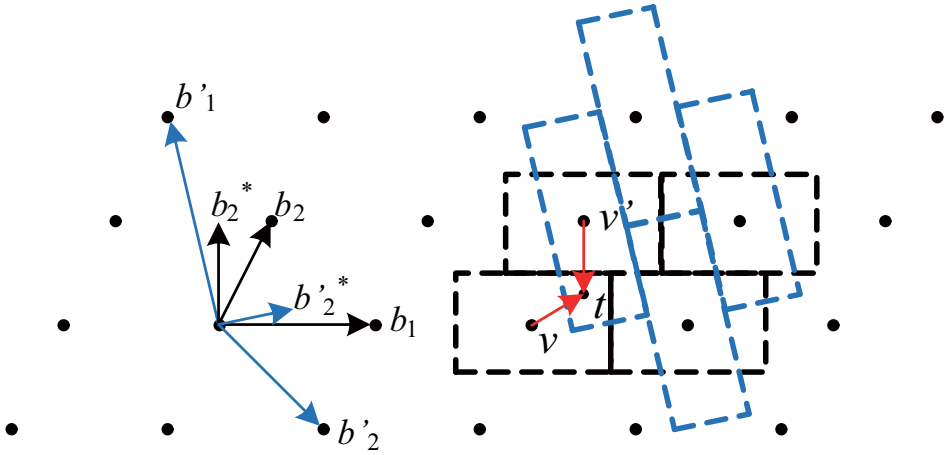
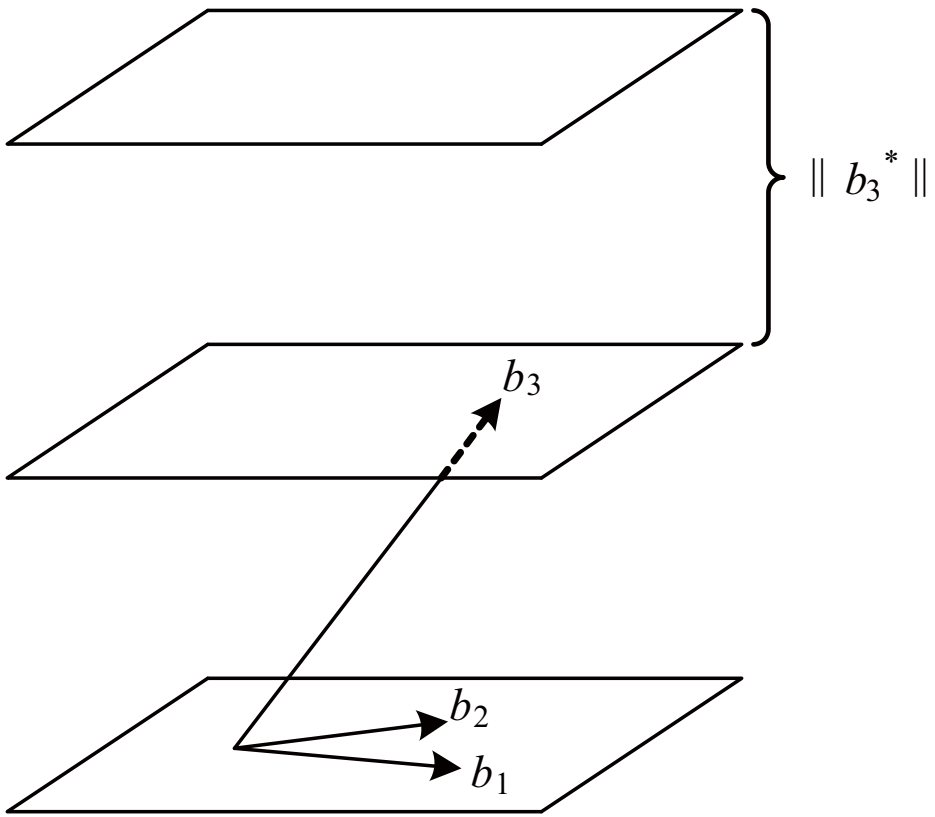


Babai's Nearest Plane Algorithm

- The algorithm success $\Longleftrightarrow$ error vector in $\mathcal{P}_{1/2}(B^*)$
- When the error distribution is Gaussian with standard deviation $s/\sqrt{2\pi}$, the success probability $\prod_{i=1}^{m} erf\left(\frac{\|\mathbf{b}_i^*\|\sqrt{\pi}}{2s}\right)$

Babai's NearestPlane Algorithm

- Split the m-dimensional lattice as $m-1$-dimensional hyperplane $\Sigma_{i=1}^{m-1} x_i \mathbf{b}_i + k\mathbf{b}_m$, project the target vector to the nearest one. Then solve the $(m-1)$-dimensional case.



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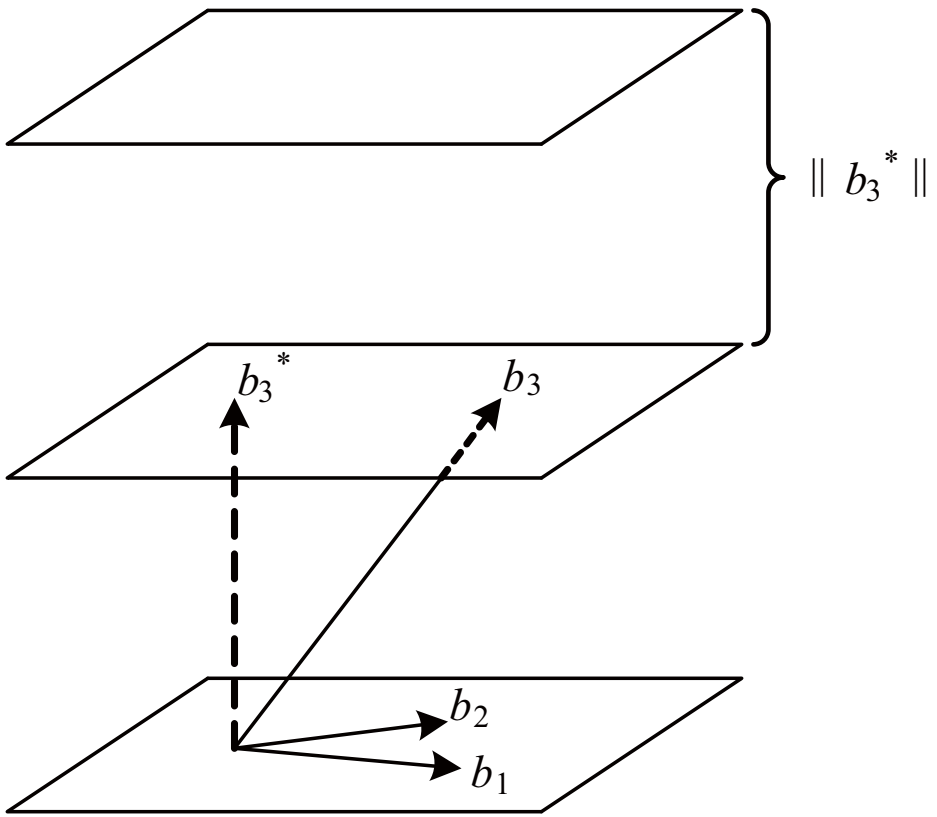

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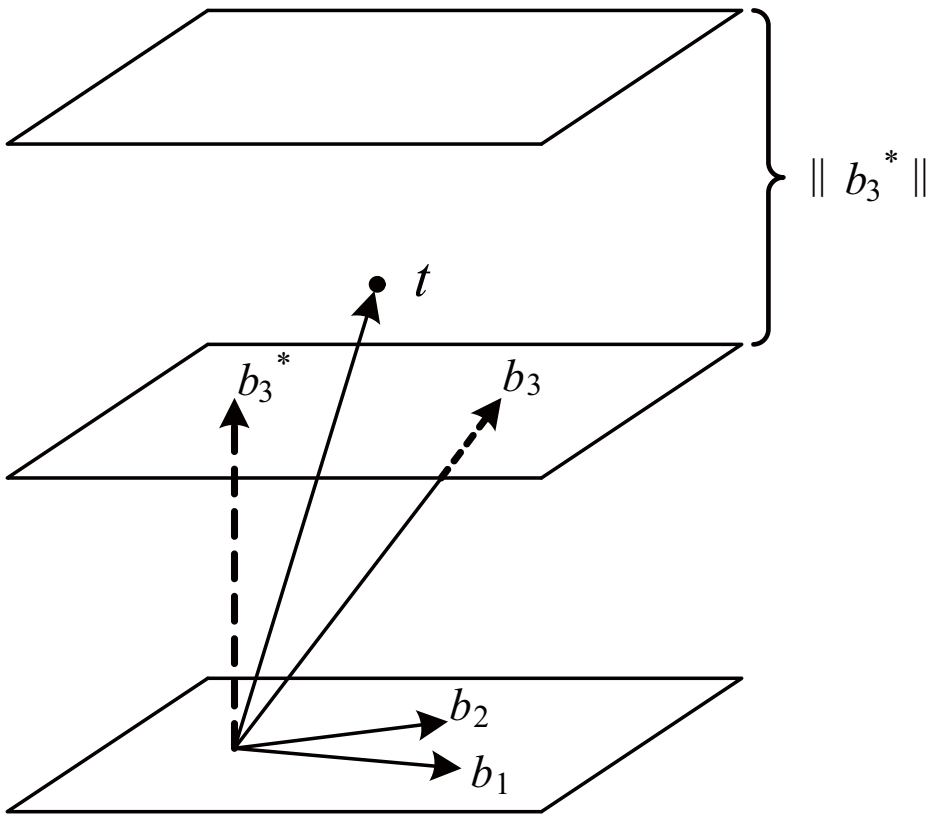

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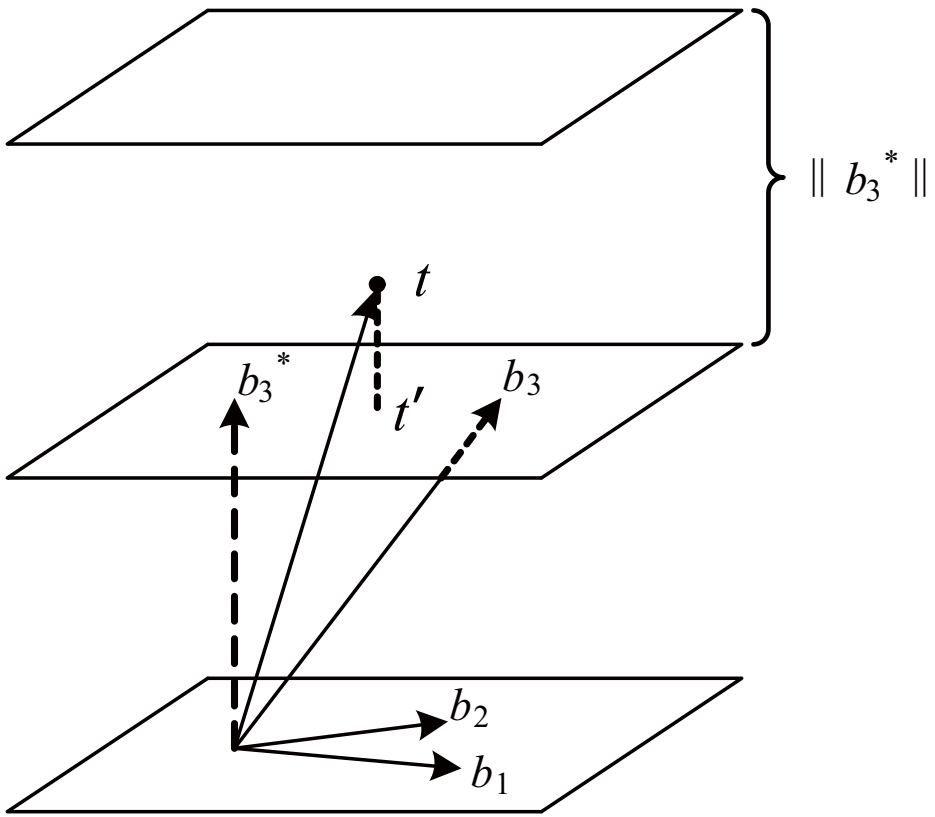


Outline

Motivation

Algorithm for BDD
- BDD Problem
- Babai NearestPlane Algorithm
- Lindner-Peikert's NearestPlanes Algorithm Revisited

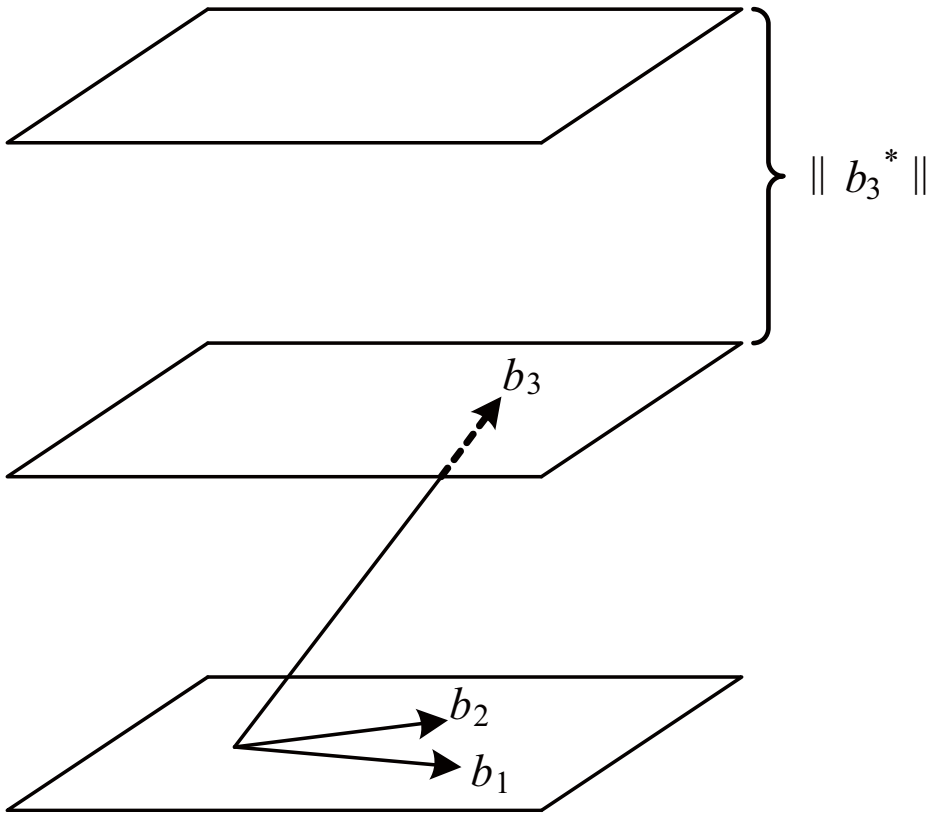
Solving BDD by (GNR) Pruned Enumeration
- Randomizing the NearestPlanes Algorithm
- GNR Pruned Enumeration to BDD

Conclusion

The NearestPlanes Algorithm

- Choose $d_i \geq 1$ distinct planes in the i-th level
- These generalizations make $\mathcal{P}_{1/2}(B^*)$ wider in the direction of $\mathbf{b}^*{}_i$ by a factor of d_i
- Output a set of Πd_i distinct lattice vectors in $\mathcal{L}(\mathbf{B})$, $\mathbf{v} - \mathbf{t} \in \mathcal{P}_{1/2}(DB^*)$ where D is diagonal matrix with $(d_1, d_2, ..., d_m)$. $\|\xi_i(\mathbf{v} - \mathbf{t})\| \leq \|d_i \mathbf{b}_i^*\|/2$
- The success probability is $\prod_{i=1}^{m} erf\left(\frac{d_i \|\mathbf{b}_i^*\| \sqrt{\pi}}{2s}\right)$.
- Lindner and Peikert[LP11]: their algorithm is better for most parameters and success probability than the distinguisher of Micciancio and Regev[MR08] for LWE.

The NearestPlanes Algorithm



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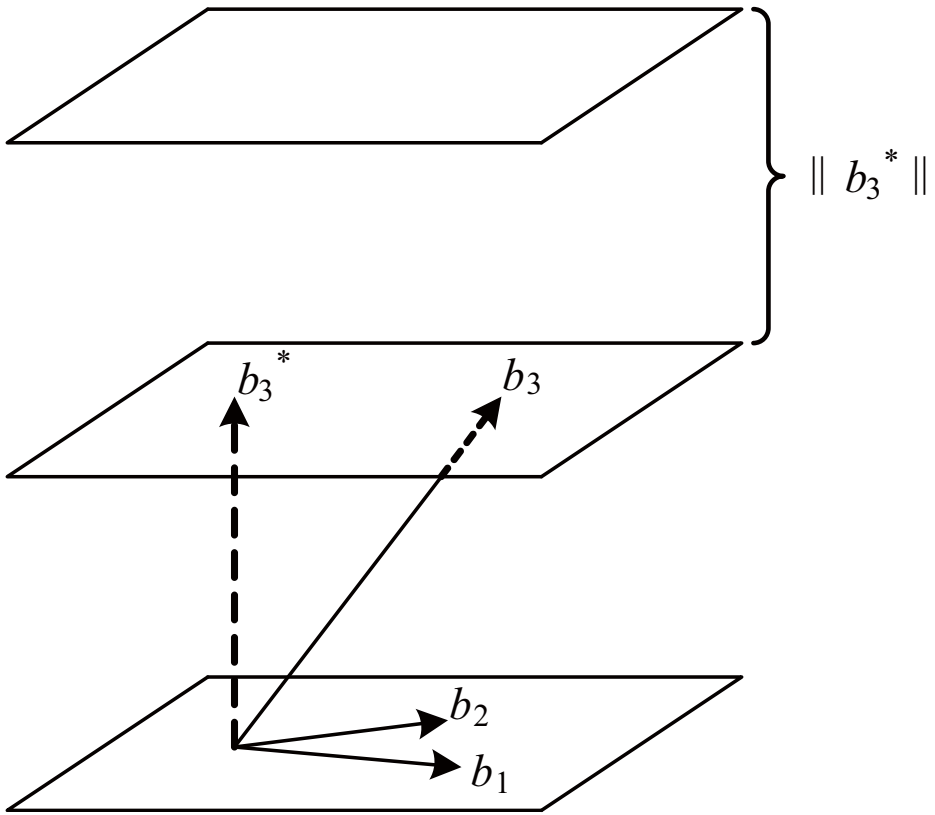

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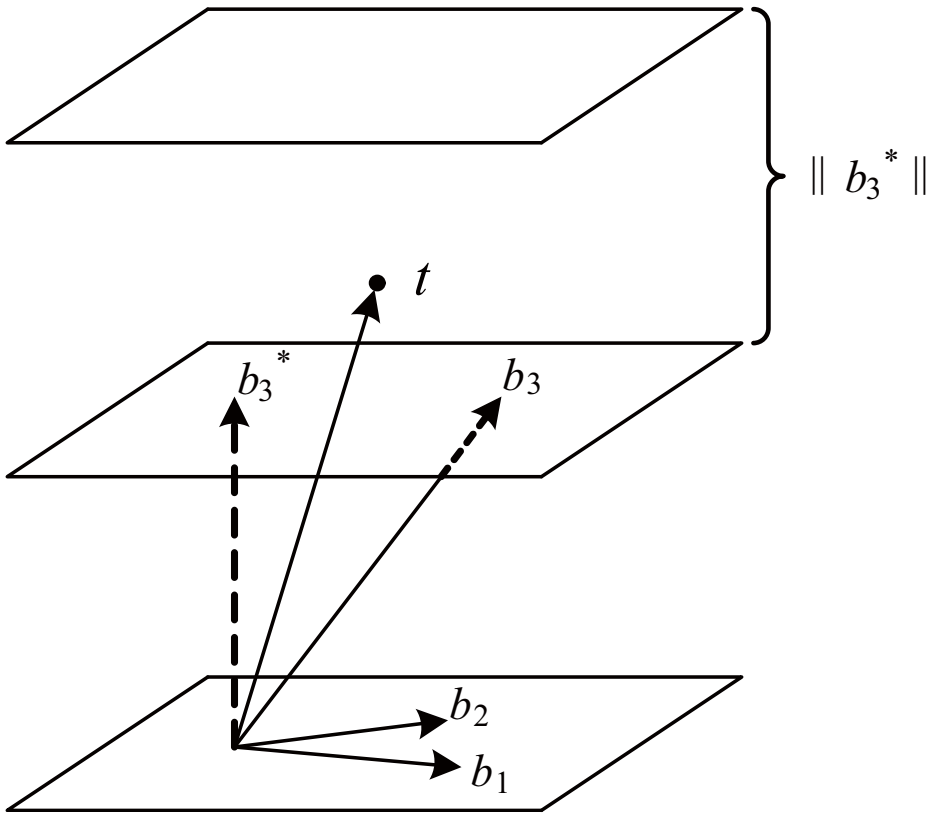

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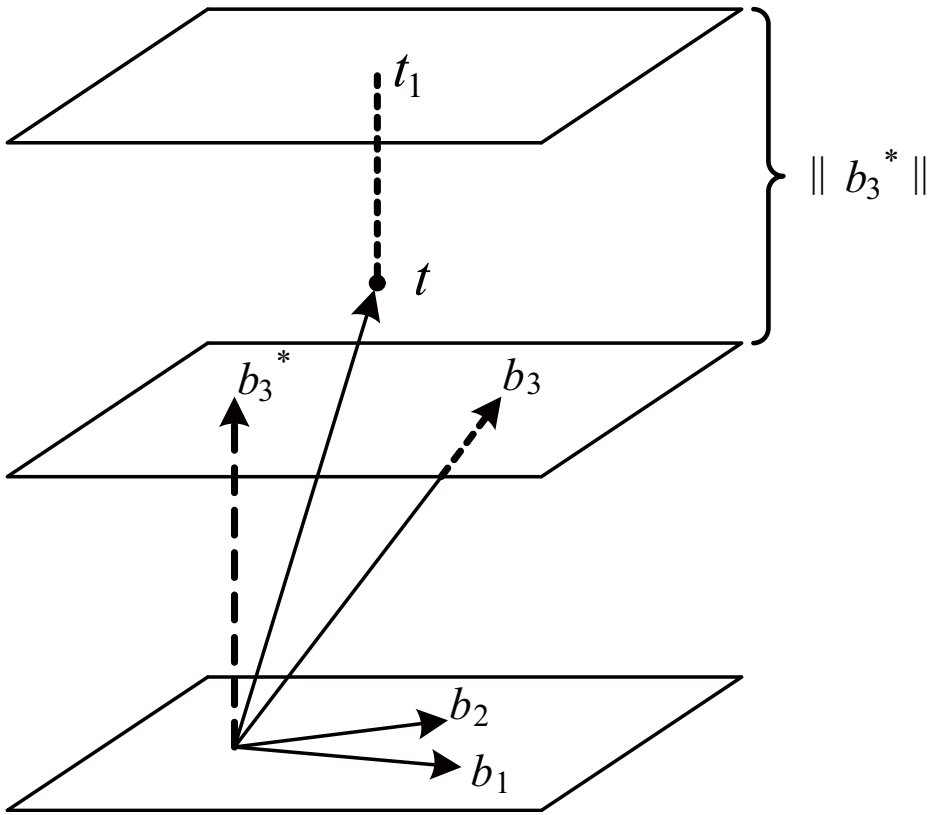

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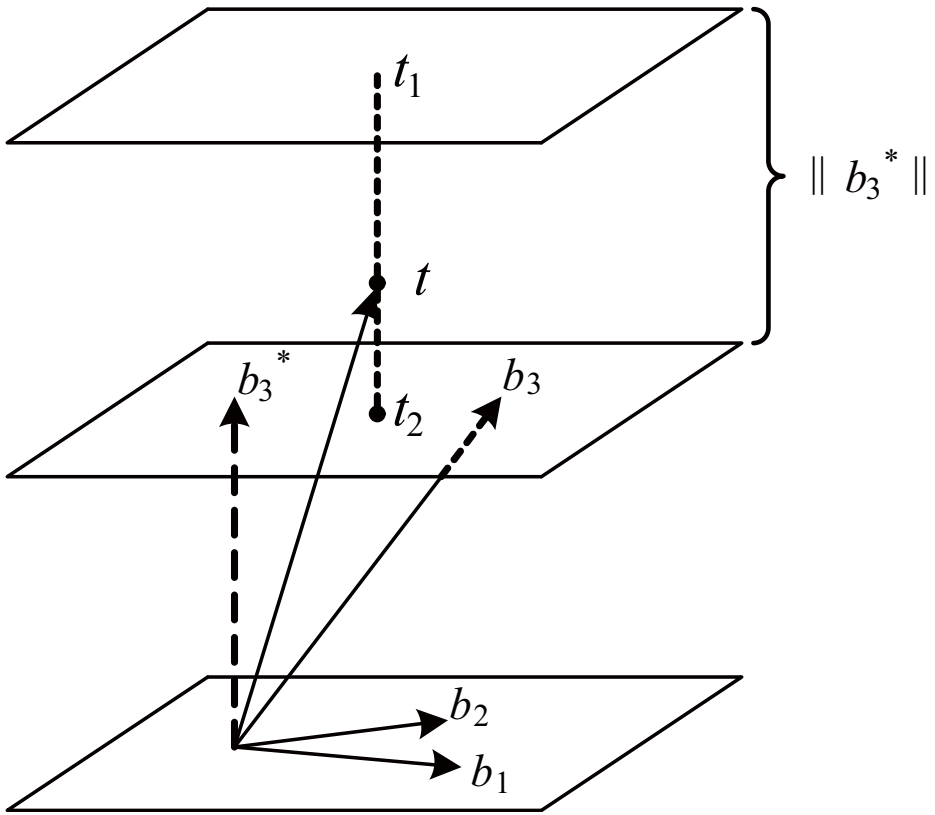

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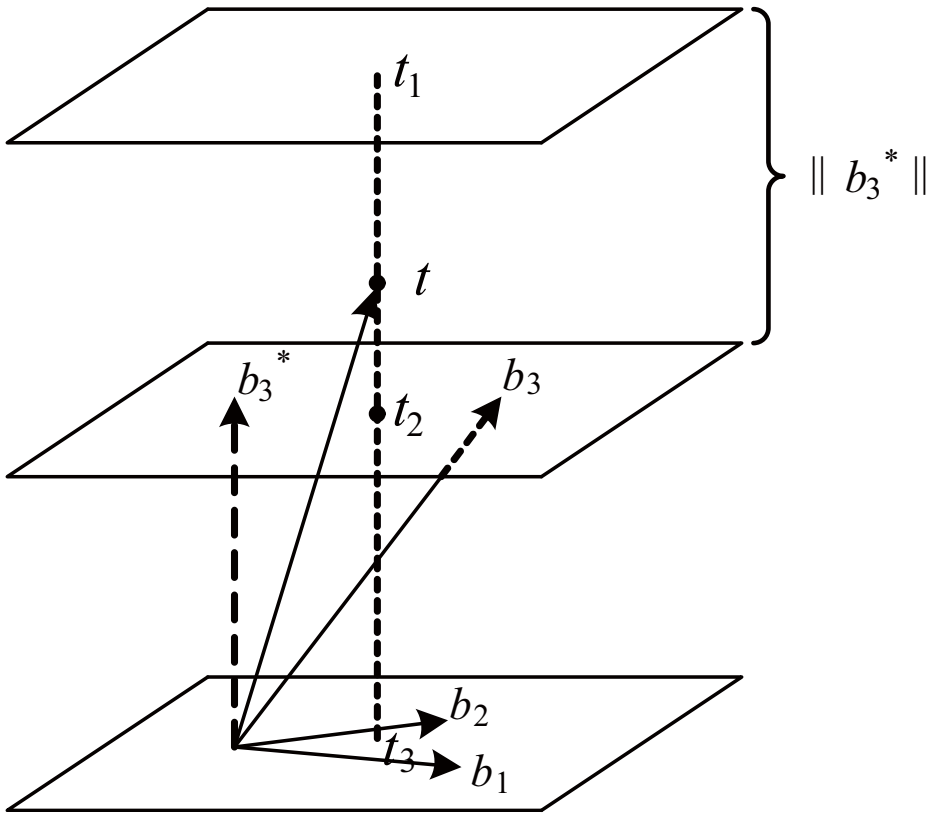

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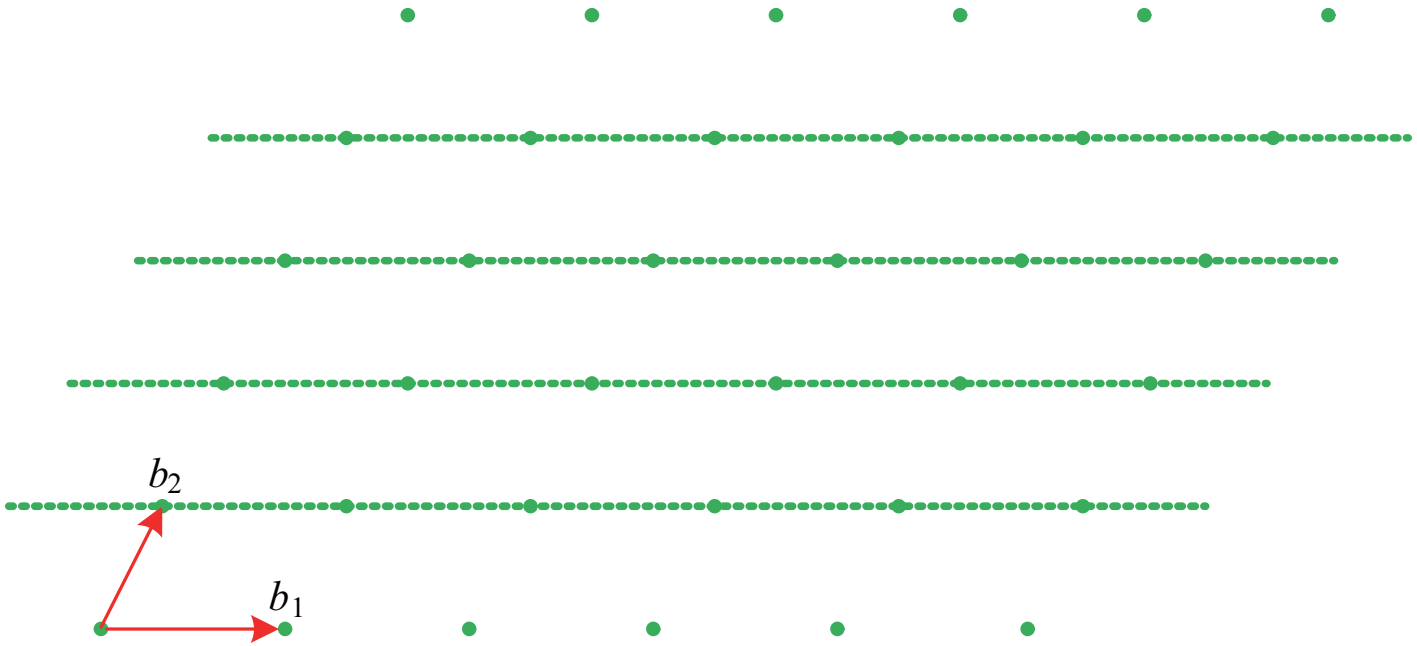

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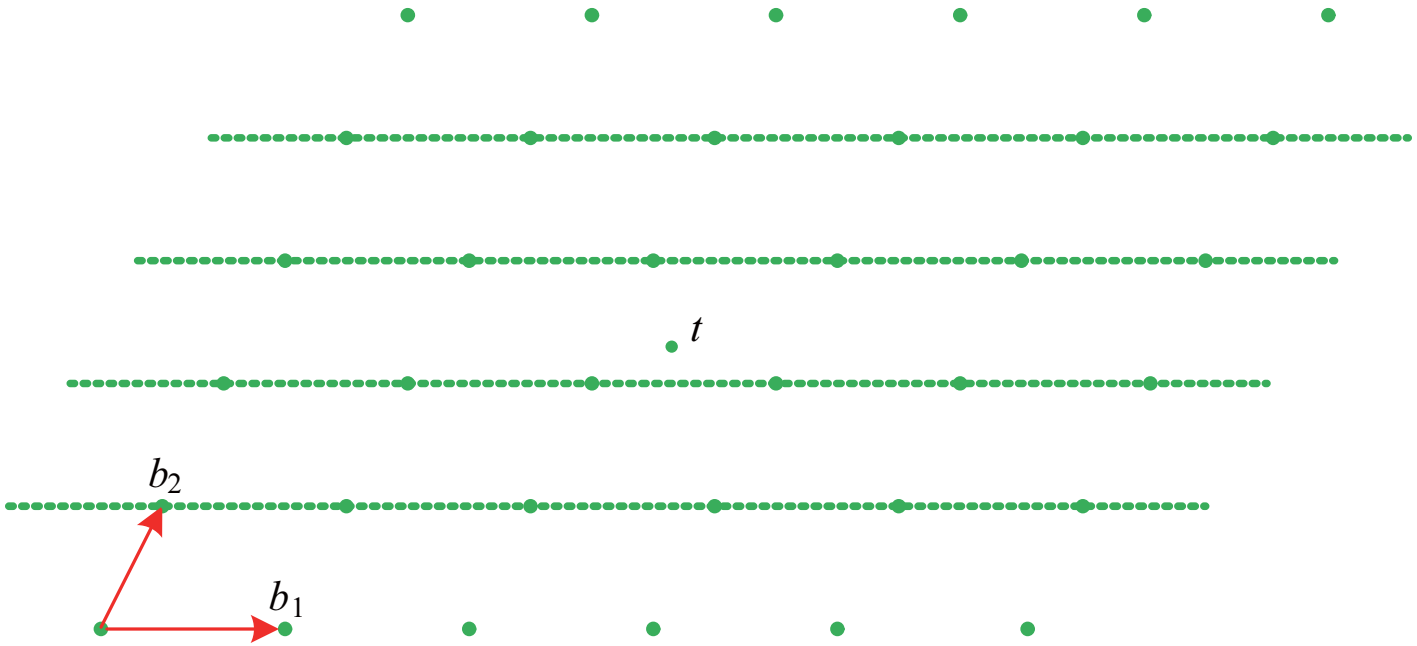

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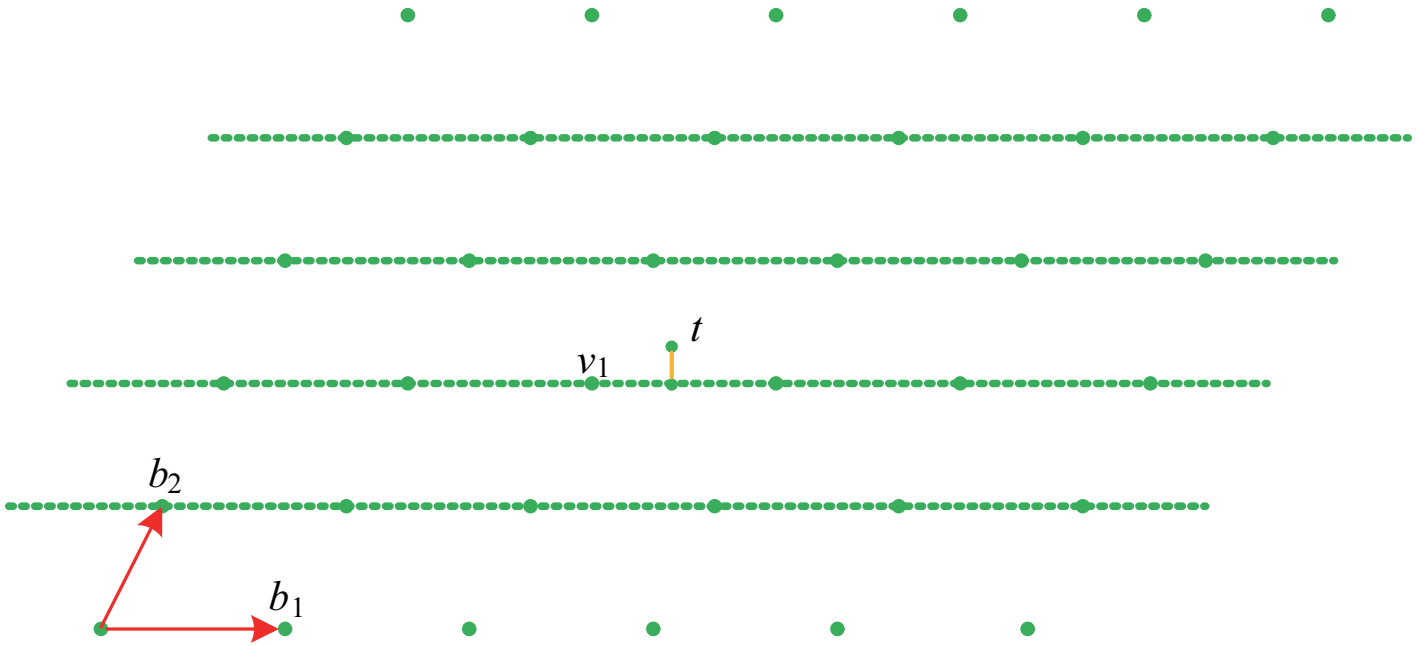

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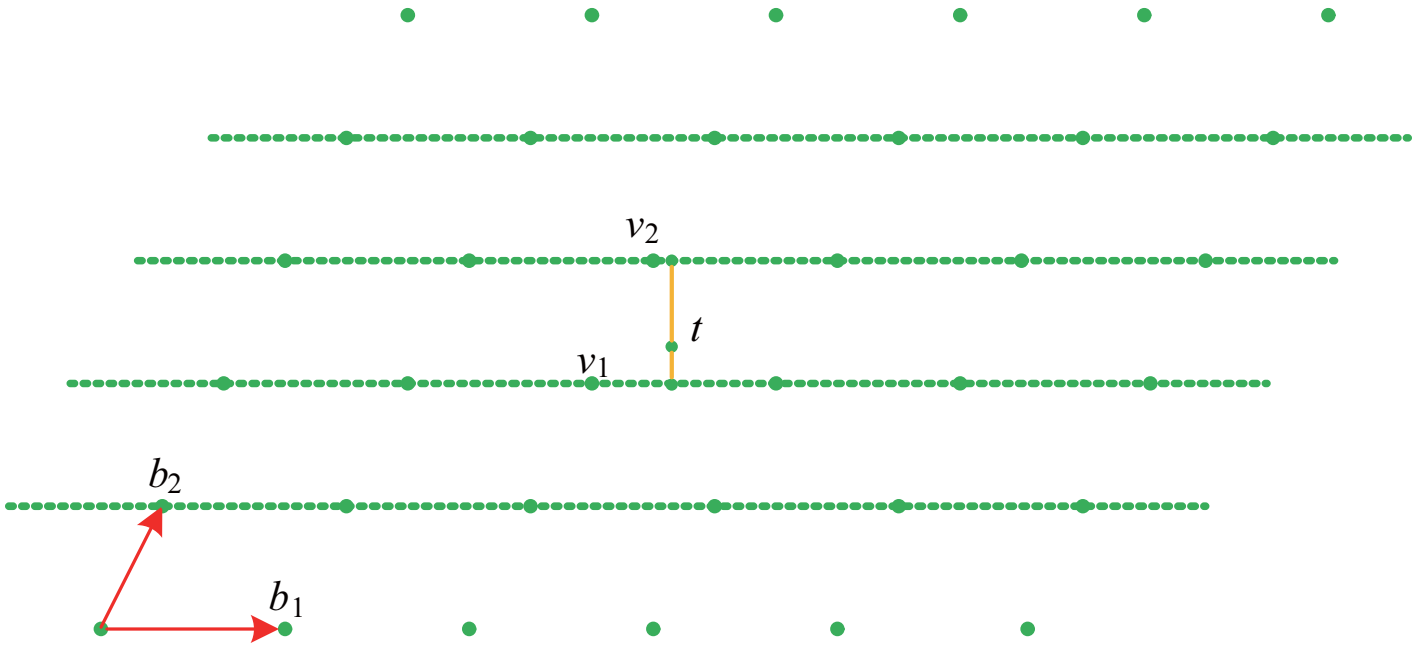

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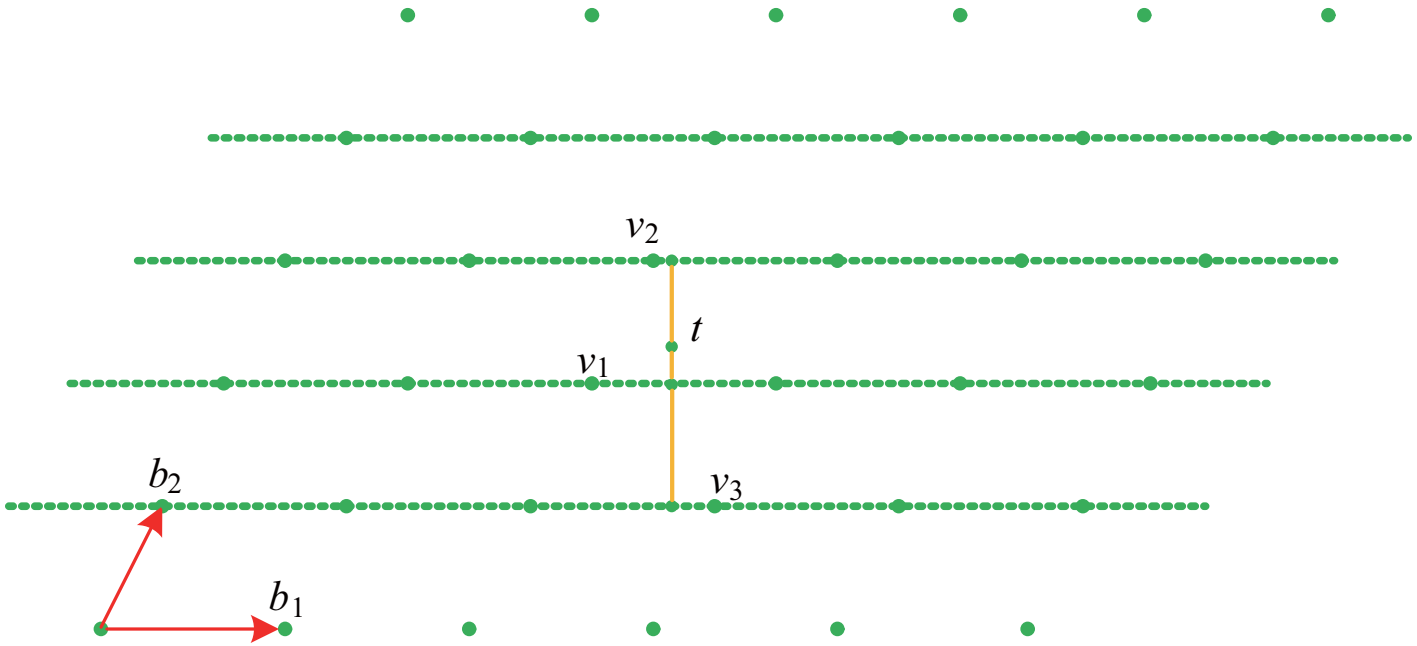

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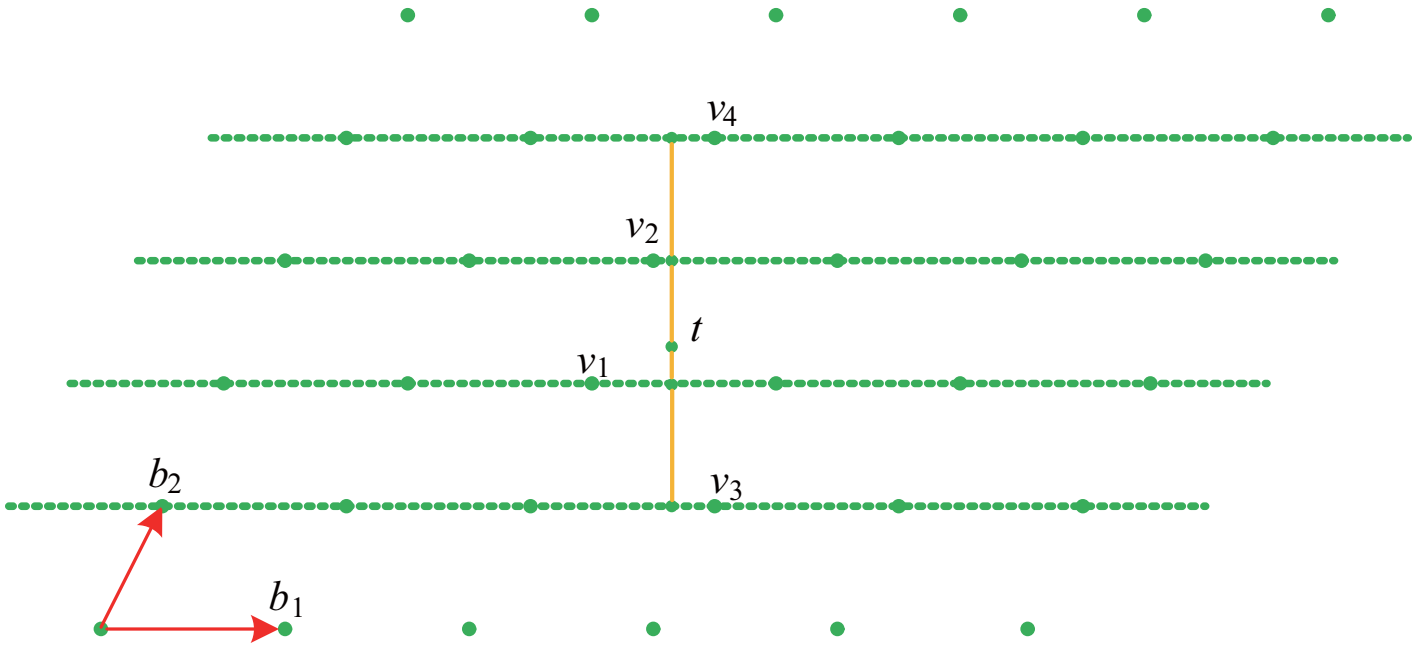

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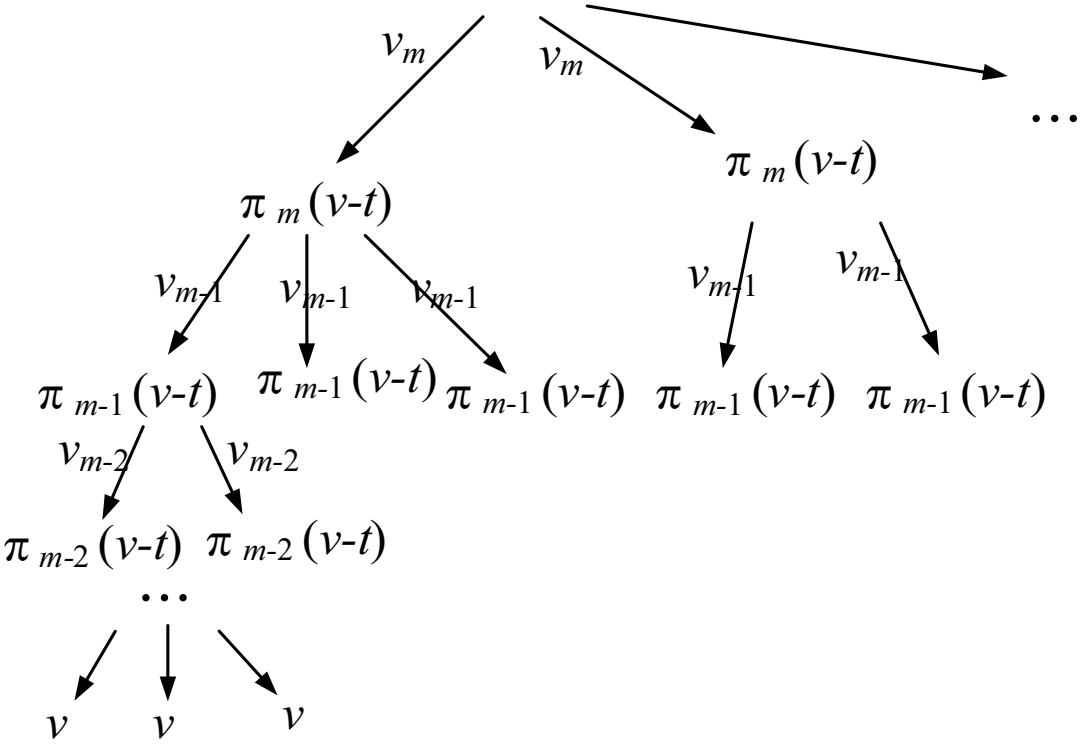
Conclusion

Lattice Enumeration for CVP

- Both Babai's algorithm and NearestPlanes algorithm can be viewed as a pruned enumeration:
  $\|\xi_i(\mathbf{v}-\mathbf{t})\| \leq \|\mathbf{b}_i^*\|/2$ Vs $\|\xi_i(\mathbf{v}-\mathbf{t})\| \leq \|d_i\mathbf{b}_i^*\|/2$
- Outline of enumeration
  - Step 1: Basis reduction
  - Step 2: Enumerate all lattice vector $\mathbf{v}$, $\|\mathbf{v}-\mathbf{t}\| \leq R$ by project lattice. R is the expected length of error vector.
- Target vector $\mathbf{t} = \Sigma_{i=1}^{m} t_i\mathbf{b}_i$, $\mathbf{v} = \Sigma_{i=1}^{m} v_i\mathbf{b}_i$. $\|\pi_i(\mathbf{v}-\mathbf{t})\| \leq R$, $i = 1, ..., m$
- $\|\pi_m(\mathbf{v}-\mathbf{t})\| \leq R \Longrightarrow v_m \in (t_m - \frac{R}{\|\mathbf{b}_m^*\|}, t_m + \frac{R}{\|\mathbf{b}_m^*\|})$
- For each value of v_m, $\|\pi_{m-1}(\mathbf{v}-\mathbf{t})\| \leq R$ implies that the integer v_{m-1} belongs to an interval of "small" length

Lattice Enumeration for CVP



Prunned Technique

- First proposed by Schnorr and Euchner 1994
- First rigorous analysis: Gama, Nguyen, Regev, 2010
- Cutting down branches to decrease the complexity at the cost of losing the solution. The success probability should be considered.

Prunned Technique

- Replace each inequality $\|\pi_{m-k+1}(\mathbf{v}-\mathbf{t})\| \leq R$ by $\|\pi_{m-k+1}(\mathbf{v}-\mathbf{t})\| \leq R_k$ for each index k , where $0 < R_1 \leq R_2 \leq \cdots R_m \leq R$.
- Linear pruning $R_k = \sqrt{\frac{k}{m}}R$.
- Choosing the optimal bounding function is very helpful for finding the solution.
- Take dimension 3 as a example, the search range from a ball to a cylinder $\begin{cases} x_1^2 \leq R_3^2 \\ x_1^2 + x_2^2 \leq R_2^2 \\ x_1^2 + x_2^2 + x_3^2 \leq R_1^2 \end{cases}$

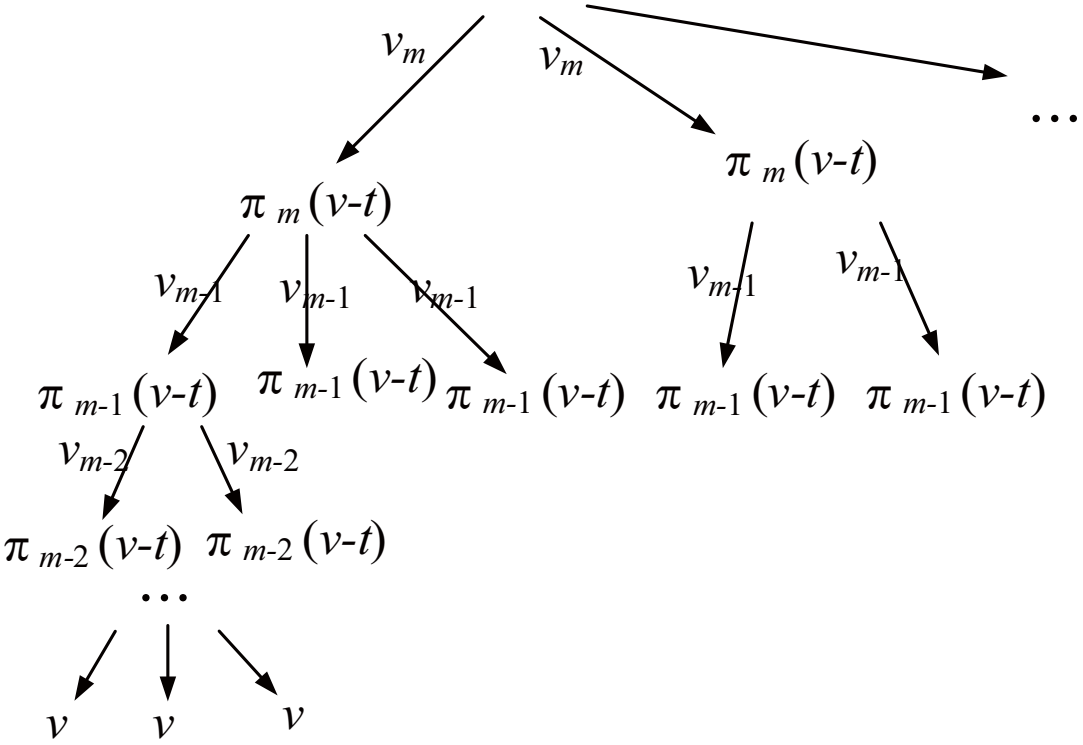
Algorithm

Repeat the following:

1. Generate a reduced basis

2. Do pruned enumeration

Prunned Technique



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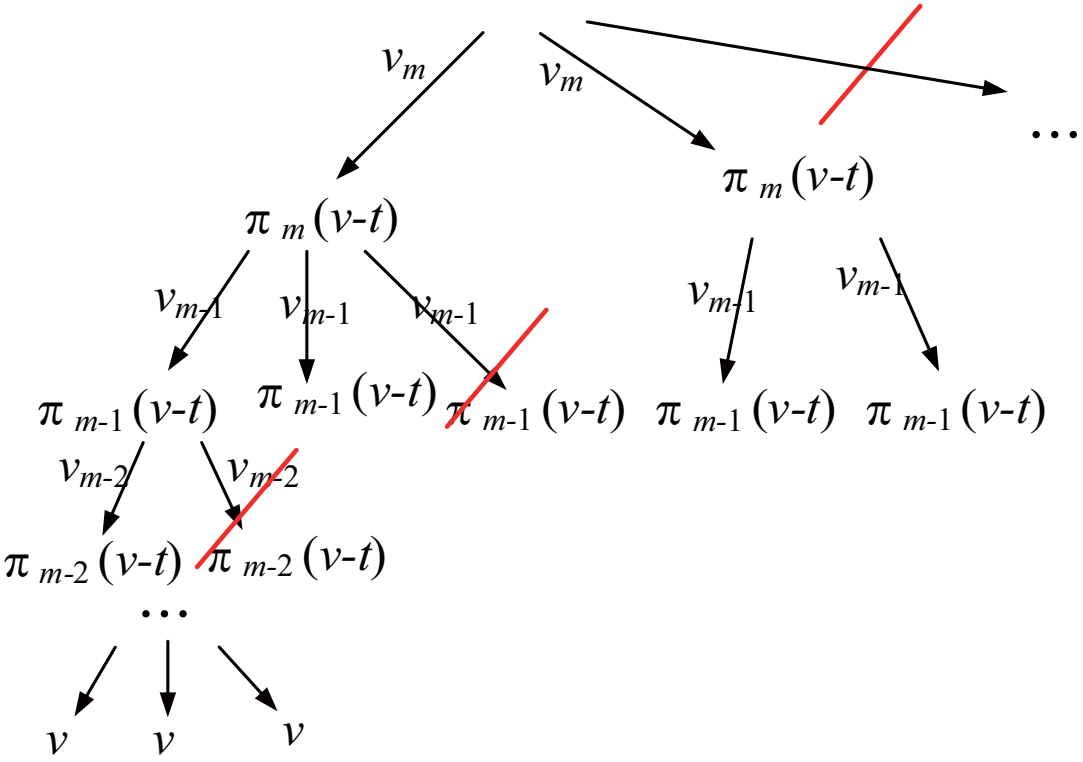
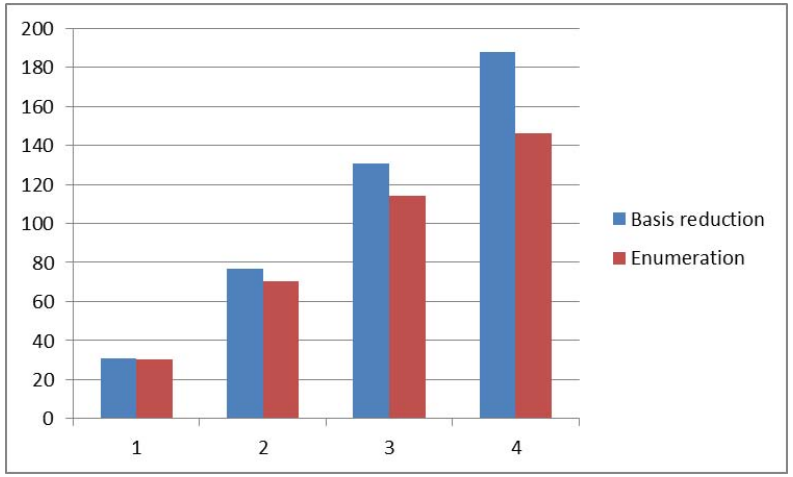


Complexity of Enumeration

- The complexity of enumeration is, up to a polynomial factor, the number of lattice points in all projected lattices inside the enumeration range.
- By the Gaussian heuristic, this number should be
  - Full enumeration : $\approx \Sigma_{1\leq k\leq m} v_k(R)/vol(\pi_{m-k+1}(L))$, where $v_k(R)$ is the volume of the k-dim ball of radius R.
  - Pruning technique: $\approx \Sigma_{1\leq k\leq m} vol(C_{R_1,R_2,\ldots,R_k})/vol(\pi_{m-k+1}(L))$, where $C_{R_1,R_2,\ldots,R_k} = \{(x_1, x_2, \ldots, x_k) \in \mathbb{R}^k, \forall j \leq k, \sum_{l=1}^{j} x_l^2 \leq R_j^2\}$.
- Total cost = $\frac{T_{redu}+T_{Enum}}{p_{succ}(R_1,\ldots,R_m)}$ where T_{Redu} is the basis reduction time, T_{Enum} is the time for enumerating

Randomizing the NearestPlanes Algorithm

- Similar to pruning, we randomize the Nearestplanes algorithm by repeating several times the basic algorithm with different randomized reduced bases
- The numerical data from [LP11] is far from optimal:the running times of basis reduction and enumeration are not totally balanced



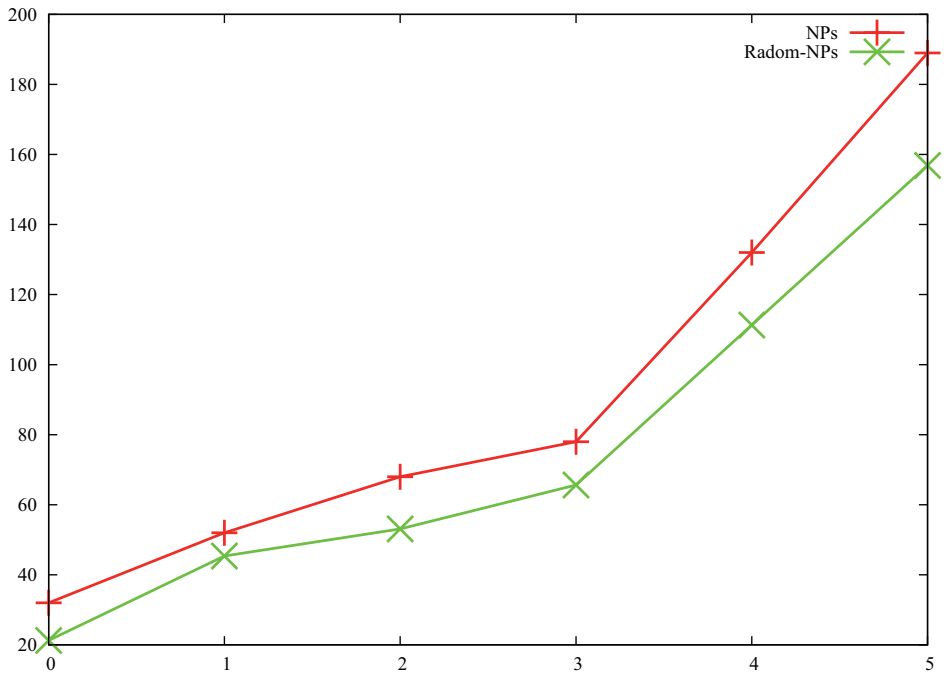
Numerical Comparison

- Our algorithm can get a better trade off.
- In the LWE parameters [LP11] consider, the comparison is following

			NearestPlanes				Randomized-NP					Log
n	q	s	δ	red	enum	Cost	δ	red	enum	pro	Cost	Speedup
128	2053	6.67	1.0089	30.8	47	32	1.0104	10.59	27	−9.65	21.24	10.76
192	4093	8.87	1.0067	76.84	87	78	1.0077	52.65	68.87	−11.99	65.64	12.36
256	4093	8.35	1.0052	130.55	131	132	1.006	98.56	114.96	−11.76	111.32	20.68
320	4093	8	1.0042	187.69	163	189	1.005	140.16	156.88	−15.7	156.86	32.14
136	2003	13.01	1.0071	66.35	82	68	1.0083	40.94	56.94	−11.17	53.11	14.89
214	16381	7.37	1.0078	50.58	66	52	1.0085	37.41	53.77	−6.94	45.35	6.65

Numerical Comparison

- Our algorithm can get a better trade off.
- In the LWE parameters [LP11] consider, the comparison is following



Outline

Motivation

Algorithm for BDD
- BDD Problem
- Babai NearestPlane Algorithm
- Lindner-Peikert's NearestPlanes Algorithm Revisited

Solving BDD by (GNR) Pruned Enumeration
- Randomizing the NearestPlanes Algorithm
- GNR Pruned Enumeration to BDD

Conclusion

Adapt Enumeration with Pruning to BDD

- Further improvement can be obtained by Pruned enumeration
- Previous: best algorithm for BDD is embedding method, our experiments show pruned enumeration performs better
- For SVP, the shortest vector is assumed uniformly distribute in the ball with radius R-expected length. For BDD, the shape of error affects the success probability, which make a theoretical analysis difficult. So compute this probability experimentally (by sampling) is essential

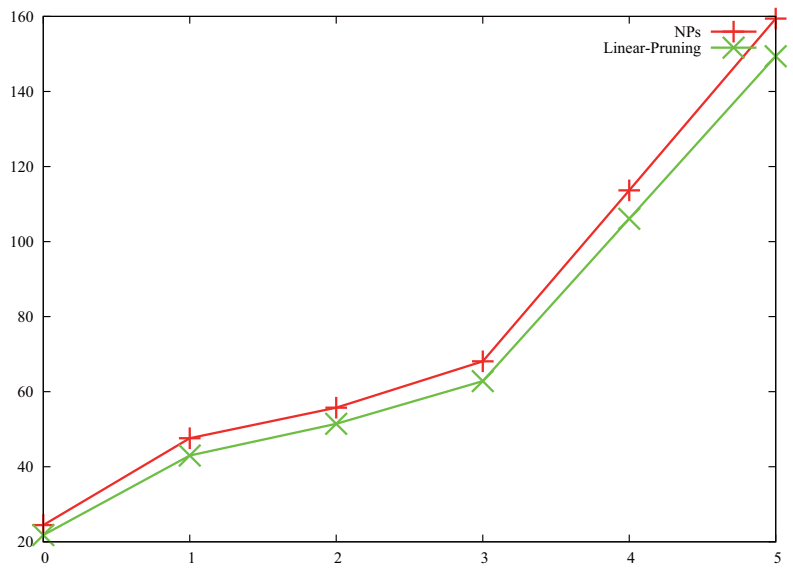
Application to LWE

- Bounding function: Linear, search $R^2 = c \cdot ||\text{expected length of error}||^2$

			Randomized-NP					Linear Pruning						Log
n	q	s	δ	red	enum	pro	Cost	δ	red	enum	pro	f-redius	Cost	Speedup
128	2053	6.67	1.0101	14.15	31.18	−9.28	24.46	1.0098	17.93	33.97	−2.98	1.08	21.8	2.66
192	4093	8.87	1.0075	56.98	74.04	−10.10	68.10	1.0074	59.23	74.88	−2.99	1.08	62.8	5.3
256	4093	8.35	1.006	98.56	115.56	−14.09	113.66	1.0059	102.09	116.8	−3.58	1.04	106.1	7.56
320	4093	8	1.005	140.16	157.17	−18.24	159.42	1.0049	145.25	159.76	−3.92	1.03	149.4	10.02
136	2003	13.01	1.0082	42.78	59.73	−11.95	55.73	1.008	46.58	63	−3.92	1.01	51.44	4.29
214	16381	7.37	1.0084	39.15	56.11	−7.43	47.59	1.0084	39.15	54.77	−3.24	1.06	43	4.59

Application to LWE

- Bounding function: Linear, search $R^2 = c \cdot ||\text{expected length of error}||^2$



Application to GGH

- Key recovery challenge: m BDD-instance with error vector chosen uniformly at random from $[-4, ..., +3]^m$

Dimension	200	250	300	300
BKZ blocksize	60	60	90	90
Bounding function	linear	linear	linear	optimized
Estimated Nb of Nodes	800	5.84×10^8	7.58×10^{10}	1.33×10^9
Average Nb of Nodes	666	5.93×10^8	-	1.35×10^9
Success probability	0.0418	0.0409	0.0371	0.0185
Nb of Success	4	11	-	2

Table 1: Key-recovery for GGH Challenges

Application to GGH

- Message recovery challenge: In 1999, Nguyen showed it can be reduce to a BDD instance with error in $\{-\frac{1}{2}, \frac{1}{2}\}^m$. Using the embedding method and BKZ 20 basis , he solved dimension 200,250 and 300 instances. For dimension 350, pruned-BKZ with blocksize 60 is required.
- For dimension 350, we do enumerate with pruning to recover 350-dimension message
  - A bounding function according to the error distribution. $R_k^2 = \min\{E(X_k) + 3D(X_k), 1\}R_m^2$, $E(X_k) = \frac{k}{m}$, $D(X_k) = \sqrt{\frac{k(m-k)}{m^2(\frac{m}{2}+1)}}$.
  - Success probability 92%
  - BKZ-20 basis
  - Time: 2935.421s in a single 3-Ghz Intel-Core2 core

Application to DSA

- Each DSA signature generation require the use of a one-time key k modulo q, where q is usually a 160-bit prime number.
- In 2002, Nguyen and Shparlinski showed that disclosing ℓ bits of each one-time key k for several (message,signature) pair to recover the DSA secret key is a HNP problem $\Longrightarrow$ BDD instance
- They used embedding method to recover the DSA secret key in a few hours, given the $\ell = 3$ least significant bits of each one-time key for about 100 signatures, but the attack failed for $\ell = 2$.
- Enumeration with pruning: using BKZ-90 reduction[CN11] and linear pruning, $\ell = 2$ case, given about 100 signatures, 4185 seconds, success probability 23%

Application to DSA

- Choose the enumeration radius
- Here, the x-coordinate is the ratio between the (squared) enumeration radius and the (squared) expected length the error vector. The y-coordinate is $\log_2$(Number of enumeration nodes/Success probability)

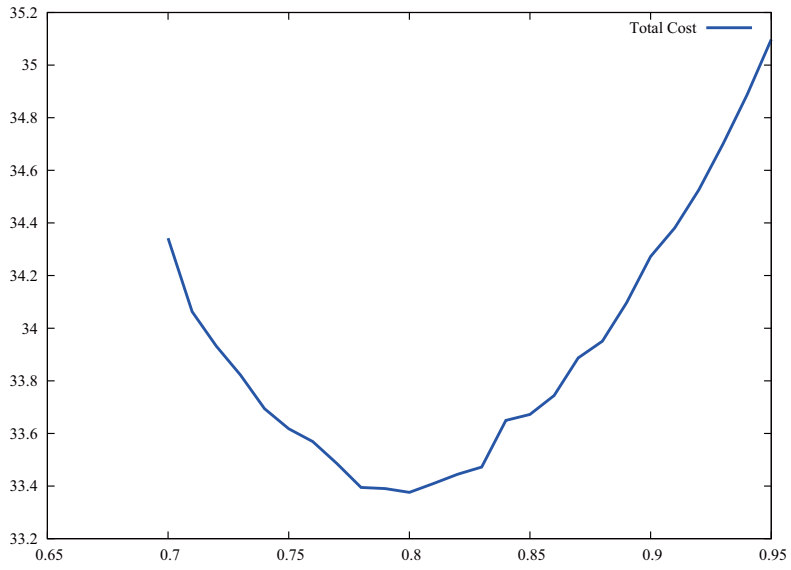


Figure 1: Total cost for DSA

Conclusion

- Our work shows that any security estimate of BDD-based cryptosystems must take into account enumeration attacks
- NearestPlanes BDD algorithm [LP11] does not seem to offer any practical advantage over GNR pruning [GNR10], despite having appeared later.
- BDD enumeration can be practical even in high dimension like 350

Thank you for your attention!