QUADRATIC PROGRAMMING SOLVER FOR NON-NEGATIVE MATRIX FACTORIZATION WITH SPARK

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ROADMAP

- Some overview on Matrix Factorization
- QP formulation of Non-negative Matrix Factorization (NMF)
- Algorithms to solve quadratic programming problems

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- Some overview on Matrix Factorization
- QP formulation of Non-negative Matrix Factorization (NMF)
- Algorithms to solve quadratic programming problems

- Some QP Applications (on MovieLens data)
  - unconstrained
  - linear constrained
- Results & Discussions

MATRIX FACTORIZATION

What is it- To decompose observed data (R rating matrix):

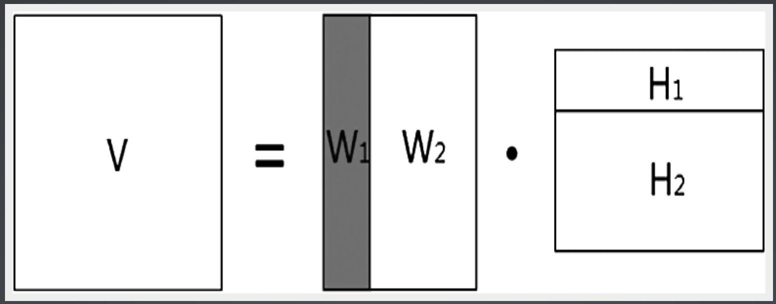
- User factors matrix (H)
- Movie factor matrix (W)

Solve for W & H

$$D(R|W,H) = \frac{1}{2}||R - WH||_F^2 +$$

$$\alpha_{l1w}||W|| + \alpha_{l2w}||W||_F^2 + \lambda_{l1h}|H||| + \lambda_{l2h}||H||_F^2$$

REGULARIZED ALTERNATING LEAST SQUARE (RALS)



Fixed-point RALS Algorithm: Equating gradient to zero, obtain iterative update scheme of W, H

- estimate H, given W (inner loop: tolerance based solutions are provided)
- enforce positivity (NMF)
- REPEAT until convergence (outer loop: factor matrices are updated until convergence)

Current effort aims to introduce flexibility to impose additional constraints (e.g. bounds on variables, sparsity, etc.)

NMF - ESSENTIALLY CLUSTERING

$$D(R|W, H) = \frac{1}{2} \sum_{i=1}^{n} ||r_i - Wh_i||_2^2$$

Solve n independent problems:

- $min_{h_i \geq 0} \frac{1}{2} ||r_i - Wh_i||_2^2$
- Aggregated solutions: $H = [h_1, h_2, h_3, \dots h_n]$

Our contribution: given W, we compute cluster possibilities (H) using a QP solver in Spark MLlib. (Note: This is inner loop)

QP TO ADMM/SOCP

- ADMM : solves by decomposing a hard problem into simpler yet efficiently solvable sub-problems and let them achieve consensus.

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- ADMM : solves by decomposing a hard problem into simpler yet efficiently solvable sub-problems and let them achieve consensus.
- ECOS: solves a specific class of problems that can be formulated as a second-order cone program (SOCP) using primal-dual interior-point method.

QP TO ADMM/SOCP

- Use: Our priliminary investigation shows that ADMM solves certain class of problems (e.g. bounds, l1 minimization) much faster than ECOS while the later proves effective in handling relatively complicated constraints (e.g. equality constraints).

QP: ADMM FORMULATION

Objective

$f(h) : 0.5||r - Wh||_2^2 => 0.5h^T(WW^T)h - (r^T W)h$

Constraints $g(z) : z >= 0$

ADMM formulation $f(h) + g(z)$

$$\text{s.t } h = z$$

ADMM Steps

- $h^{k+1} = argmin_h f(h + 0.5 \times \rho||h - z^k + u^k||_2^2)$
- $z^{k+1} = h^{k+1} + u^k \text{s.t } z^{k+1} \in g(z)$
- $u^{k+1} = u^k + h_{k+1} - z_{k+1}$

QP: ADMM IMPLEMENTATION(I)

```
class DirectQpSolver(nGram: Int,
  lb: Option[DoubleMatrix] = None, ub: Option[DoubleMatrix] = None,
  Aeq: Option[DoubleMatrix] = None,
  alpha: Double = 0.0, rho: Double = 0.0,
  addEqualityToGram: Boolean = false) = {

def solve(H: DoubleMatrix, q: DoubleMatrix,
            beq: Option[DoubleMatrix] = None): DoubleMatrix = {
  wsH = H + rho*I
  solve(q, beq)
}

def solve(q: DoubleMatrix, beq: Option[DoubleMatrix]): DoubleMatrix = {
  //Dense cholesky factorization
  val R = Decompose.cholesky(wsH)
  ADMM(R)
}
```

QP : ADMM IMPLEMENTATION(II)

```
def ADMM(R: DoubleMatrix): DoubleMatrix = {
  rho = 1.0
  alpha = 1.0
  while (k ≤ MAX_ITERS) {
    scale = rho*(z - u) - q
    // x = R \ (R' \ scale)
    solveTriangular(R, scale)
    //z-update with relaxation
    zold = (1-alpha)*z
    x_hat = alpha*x + zold
    z = xHat + u

    Proximal(z)
    if(converged(x, z, u)) return x
    k = k + 1
  }
}
```

QP: SOCP FORMULATION

Objective transformation $\text{minimize } t$

$$\text{s.t } 0.5h^T(WW^T)h - (r^TW)h \leq t$$

Constraints $h >= 0$

$$Aeq \times h = Beq$$

$$A \times h \leq B$$

Quadratic constraint transformation

$$\left| \begin{matrix} Q_{chol}h \\ c \end{matrix} \right| \leq d$$

QP: SOCP IMPLEMENTATION

```
class QpSolver(nGram: Int, nLinear: Int = 0, diagonal: Boolean = false,
  Equalities: Option[CSCMatrix[Double]] = None,
  Inequalities: Option[CSCMatrix[Double]] = None,
  lbFlag: Boolean = false, ubFlag: Boolean = false) = {

  NativeECOS.loadLibraryAndCheckErrors()

  def solve(H: DoubleMatrix, f: Array[Double]): (Int, Array[Double]) = {
    updateHessian(H)
    updateLinearObjective(f)
    val status = NativeECOS.solveSocp(c, G, h, Aeq, beq, linear, cones, x)
    (status, x.slice(0, n))
  }
}
```

USE CASE: POSITIVITY

Application: Image feature extraction / energy spectrum where negative coefficients or factors are counter intuitive

A test to compute Negative Coefficients (< -1e-4) & RMSE

	OCTAVE	**ALS**	**ECOS**	**ADMM**
Negative Coefficients:	0	2	0	**1**
RMSE:	N.A.	2.3e-2	3e-4	**2.7e-4**

USE CASE: POSITIVITY

```
//Spark Driver
val lb = DoubleMatrix.zeros(rank, 1)
val ub = DoubleMatrix.zeros(rank, 1).addi(1.0)
val directQpSolver = new DirectQpSolver(rank, Some(lb), Some(ub)).setProximal

val factors = Array.range(0, numUsers).map { index =>
  // Compute the full XtX matrix from the lower-triangular part we got above
  fillFullMatrix(userXtX(index), fullXtX)
  val H = fullXtX.add(YtY.get.value)
  val f = userXy(index).mul(-1)
  val directQpResult = directQpSolver.solve(H, f)
  directQpResult
}
```

USE CASE: POSITIVITY

```
//DirectQpSolver Projection Operator
def projectBox(z: Array[Double], l: Array[Double], u: Array[Double]) {
  for(i <- 0 until z.length) z.update(i, max(l(i), min(x(i), u(i))))
}
```

USE CASE: SPARSITY

Application: signal recovery problems in which the original signal is known to have a sparse representation

A test to compute Non-Sparse Coefficients (> 1e-4) & RMSE

	OCTAVE	ALS	ECOS	ADMM
Non-Sparse Coefficients:	16	20	18	**18**
RMSE:	N.A.	9e-2	2e-2	**2e-2**

USE CASE: SPARSITY

```
//Spark Driver
val directQpSolverL1 = new DirectQpSolver(rank).setProximal(ProximalL1)
directQpSolverL1.setLambda(lambdaL1)

val factors = Array.range(0, numUsers).map { index =>
  // Compute the full XtX matrix from the lower-triangular part we got above
  fillFullMatrix(userXtX(index), fullXtX)
  val H = fullXtX.add(YtY.get.value)
  val f = userXy(index).mul(-1)
  val directQpL1Result = directQpSolverL1.solve(H, f)
  directQpL1Result
}
```

USE CASE: SPARSITY

```
//DirectQpSolver Proximal Operator
def proximalL1(z: Array[Double], scale: Double) {
  for(i <- 0 until z.length)
    z.update(i, max(0, z(i) - scale) - max(0, -z(i) - scale))
}
```

USE CASE: EQUALITY WITH BOUND

Application: Support Vector Machines based models with sparse weight representation or portfolio optimization

A test to compute Sum, Non-Sparse Coefficients (> 1e-4) & RMSE

	OCTAVE	ALS	ECOS	ADMM
Sum of Coefficients:	1	-	0.99	**1**
Non-Sparse Coefficients:	4	-	4	**4**
RMSE:	N.A.	1.1	2e-4	**5.5e-5**

USE CASE: EQUALITY WITH BOUNDS

```
//Equality constraint x1 + x2 + ... + xr = 1
//Bound constraint is 0 ≤ x ≤ 1
val equalityBuilder = new CSCMatrix.Builder[Double](1, rank)
for (i <- until rank) equalityBuilder.add(0, i, 1)
val qpSolverEquality =
          new QpSolver(rank, 0, false,
            Some(equalityBuilder.result), None, true, true)
qpSolverEquality.updateUb(Array.fill[Double](rank)(1.0))
qpSolverEquality.updateEquality(Array[Double](1.0))
```

USE CASE: EQUALITY WITH BOUNDS

```
val factors = Array.range(0, numUsers).map { index =>
  fillFullMatrix(userXtX(index), fullXtX)
  val H = fullXtX.add(YtY.get.value)
  val f = userXy(index).mul(-1)
  val qpEqualityResult = qpSolverEquality.solve(H, f)
  qpEqualityResult
}
```

RUNTIME EXPERIMENTS (IN MS)

Dataset: Movielens 1M ratings from 6040 users on 3706 movies

Example run

MASTER=local[1] run-example mllib.MovieLensALS --rank 25 --numIterations 1 --kryo --qpProblem 1 ratings.dat
Algorithms variants

Quadratic Minimization(QP), with Positivity(QpPos), bounds(QpBounds), Sparsity(QpL1), Equality and Bounds(QpEquality)

1 ALS iteration: (userUpdate) + (movieUpdate)

	LS	ECOS	ADMM
Qp	(30)+(57)	(3826)+(6943)	(99)+(143)
QpPos	(98)+(320)	(6288)+(11975)	(265) + (2135)
QpBounds	(39)+(55)	(6709)+(11951)	(1556)+(1329)
QpL1	(54)+(80)	(32171)+(58766)	(352)+(1593)
QpEquality	(63)+(133)	(5231)+(7912)	(14681)+(65893)

FUTURE WORK

- Release QpSolver-Spark after rigorous testing
- Runtime optimizations for QpSolver-Spark integration
- Matrix Factorization using Gram Matrix broadcast
- Release ECOS based LP and SOCP solvers based on community feedback
- BFGS/CG based IterativeQpSolver for large ranks with application to Kernel-SVMs
- QpSolver-Spark applications to Verizon datasets

QUESTIONS

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References

- ECOS by Domahidi et al. https://github.com/ifa-ethz/ecos
- ADMM by Boyd et al. http://web.stanford.edu/~boyd/papers/admm/
- Proximal Algorithms by Neal Parikh and Professor Boyd