Body Models I

Javier Romero

Max Planck Institute for Intelligent Systems

Perceiving Systems

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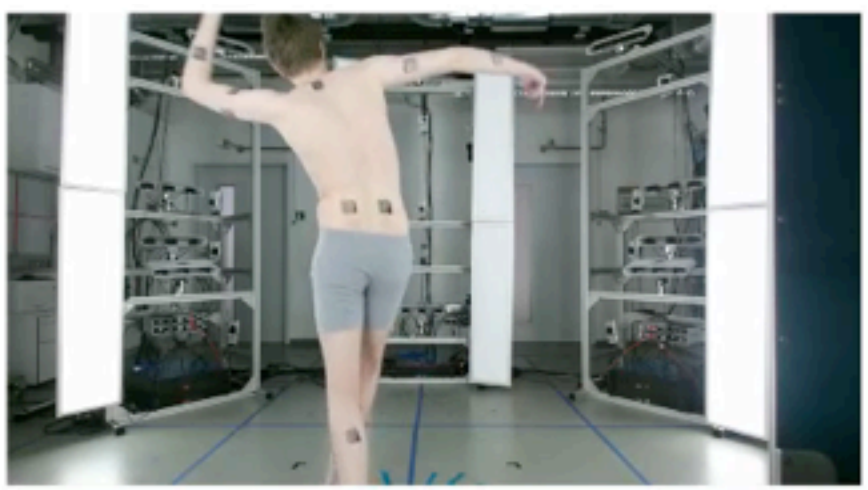
MAX-PLANCK-GESELLSCHAFT

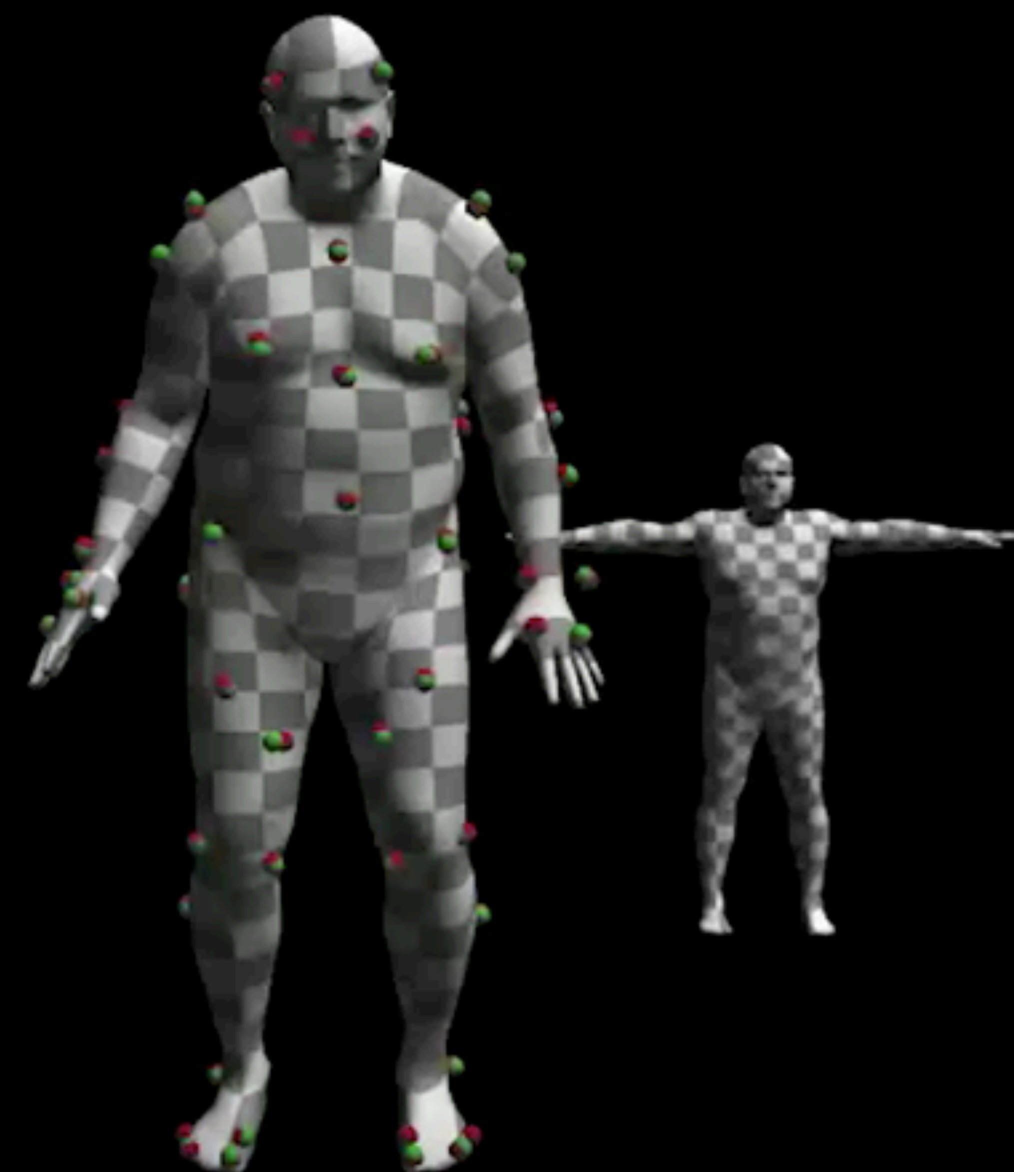
11.04.2016	Introduction
18.04.2016	Graphical Models 1
25.04.2016	Graphical Models 2 (Sand 6/7)
02.05.2016	Graphical Models 3
09.05.2016	Graphical Models 4
23.05.2016	**Body Models 1**
30.05.2016	Body Models 2
06.06.2016	Body Models 3
13.06.2016	Body Models 4
20.06.2016	Stereo
27.06.2016	Optical Flow
04.07.2016	Segmentation
11.07.2016	Object Detection 1
18.07.2016	Object Detection 2

What have we (you) learned so far?

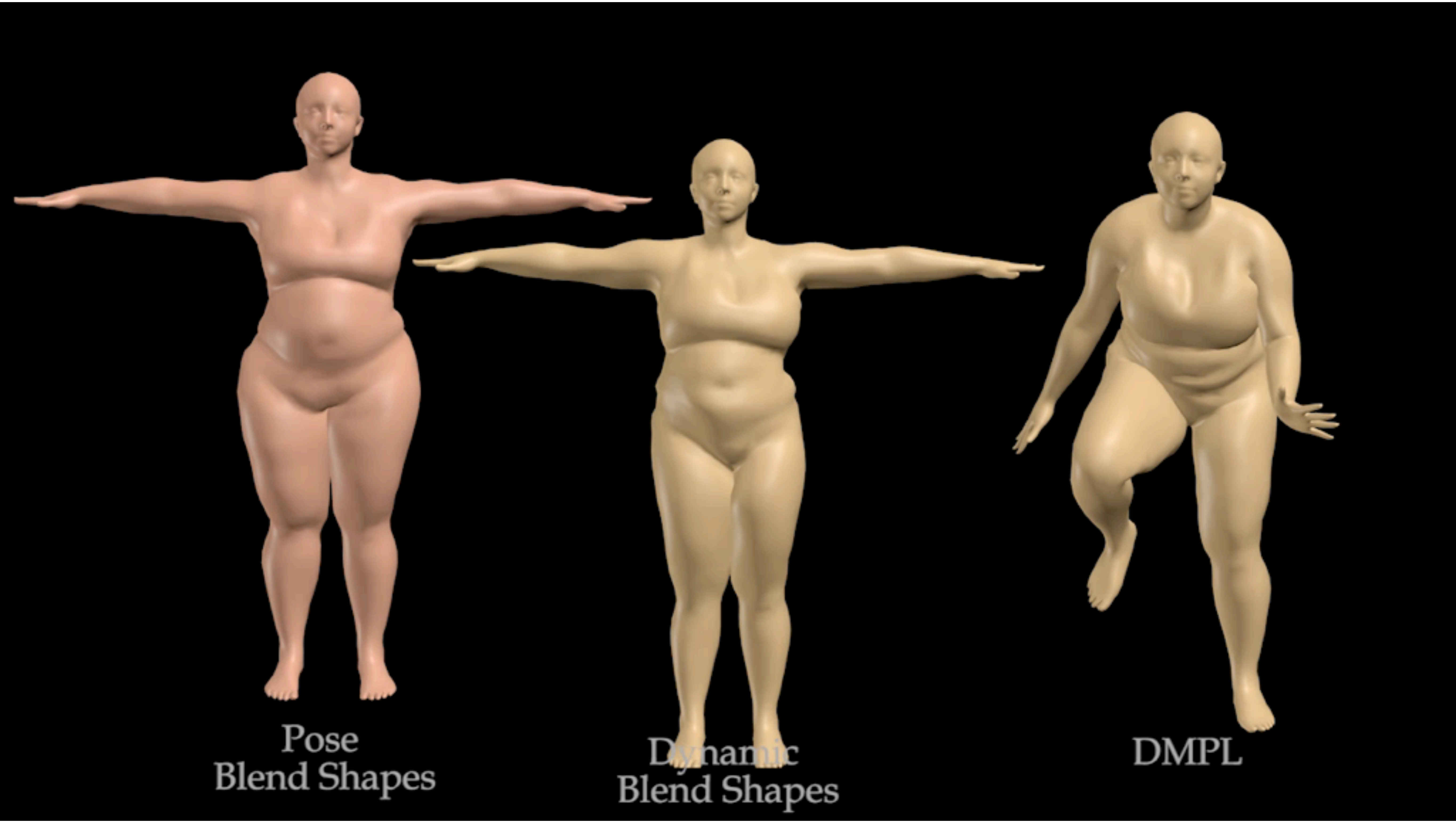
- Graphical Models
  - Belief Networks
  - Markov Networks
  - Factor Graphs
- Inference (Marginals, MAP)
  - Belief Propagation
  - Sampling
- Parameter Learning
  - Maximum-Likelihood
  - Max-Margin Methods

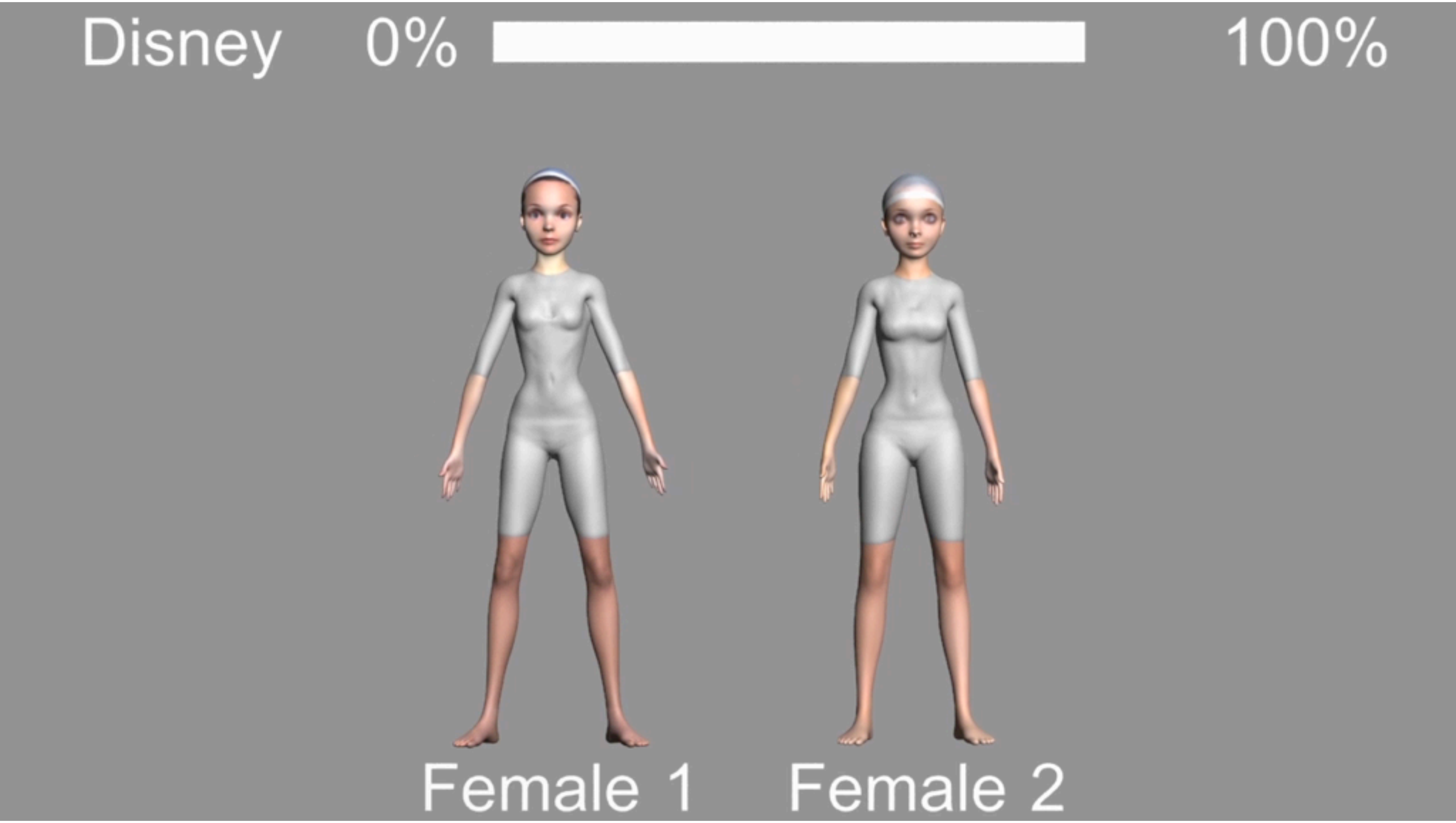


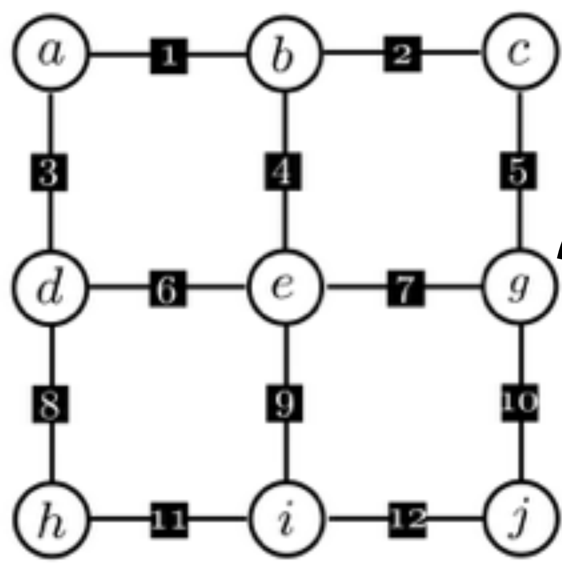
Reference Video



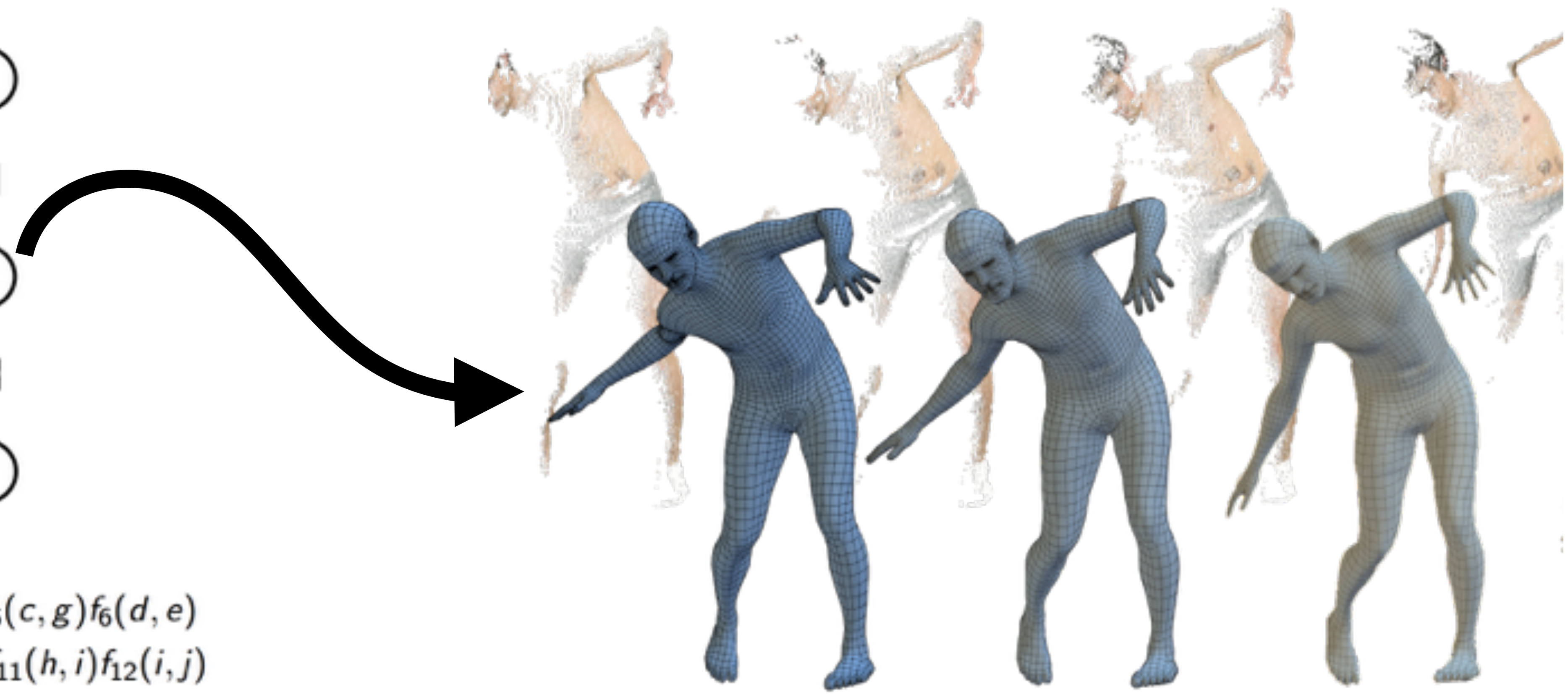
MoSh without Soft Tissue Motion


Pose
Blend Shapes
Dynamic
Blend Shapes
DMPL

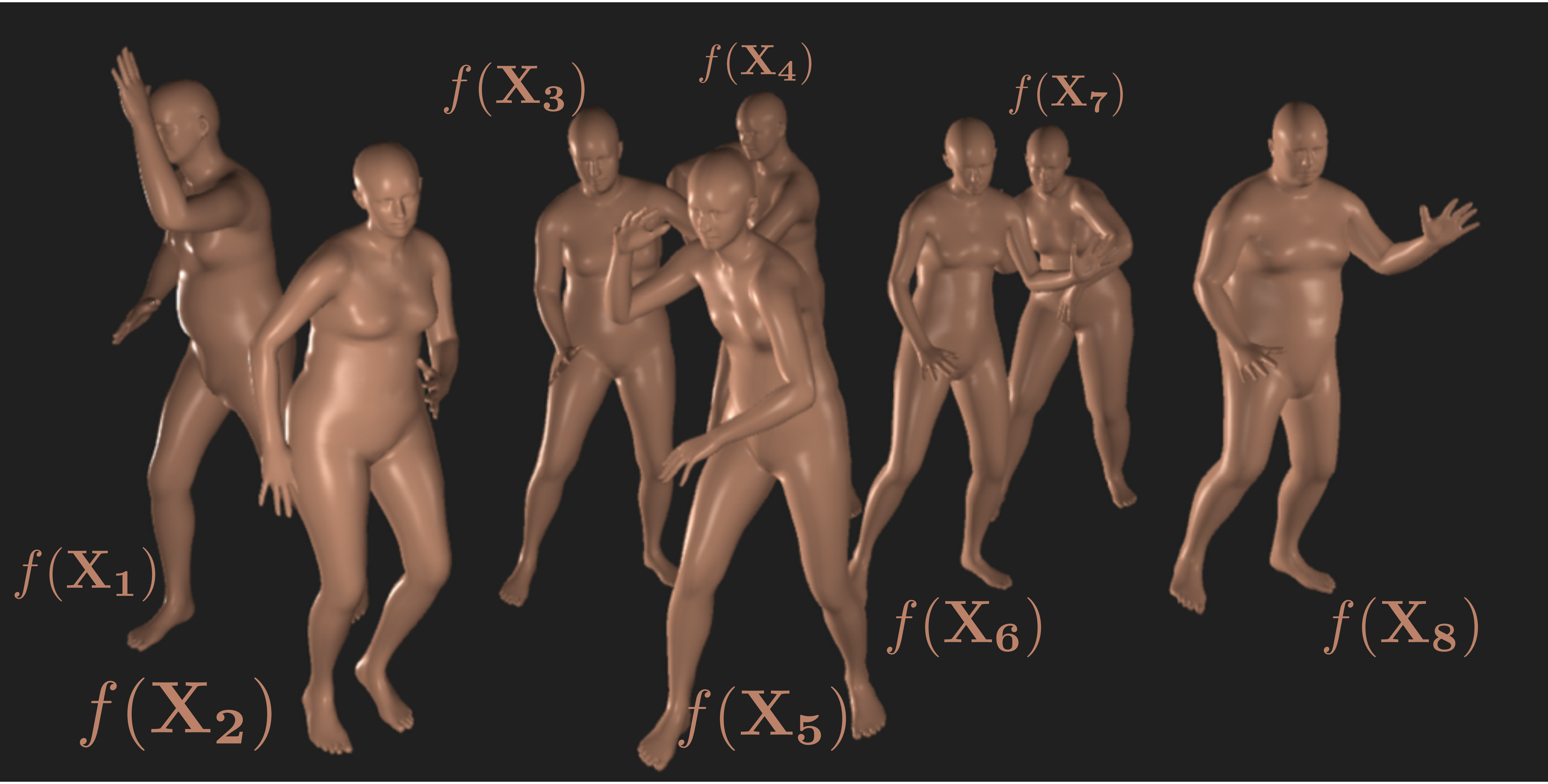

Disney
0%
100%
Female 1
Female 2



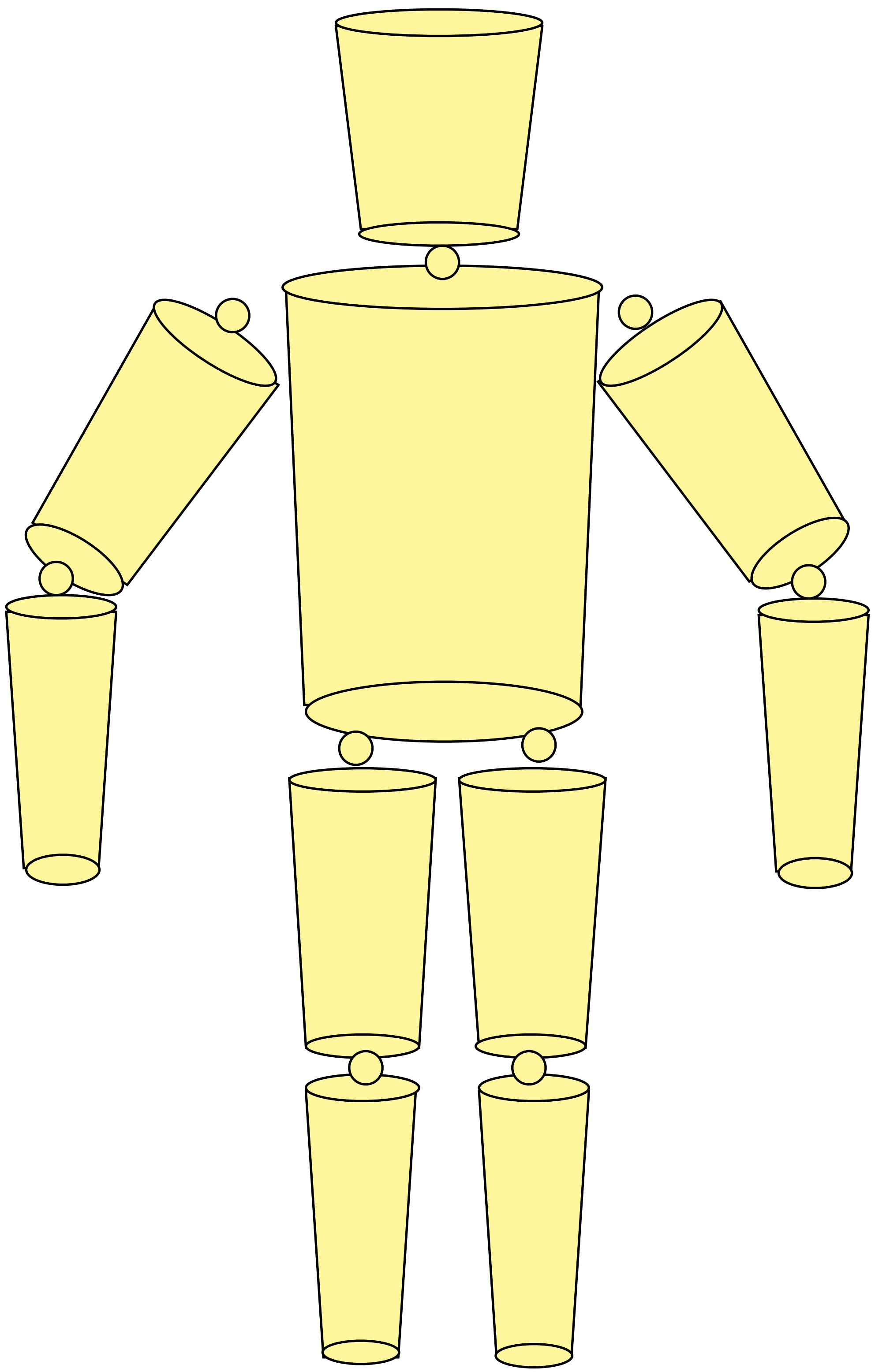
$$
\begin{aligned}
p(all) \;=\;& f_1(a,b)f_2(b,c)f_3(a,d)f_4(b,e)f_5(c,g)f_6(d,e) \\
& f_7(e,g)f_8(d,h)f_9(e,i)f_{10}(g,j)f_{11}(h,i)f_{12}(i,j)
\end{aligned}
$$

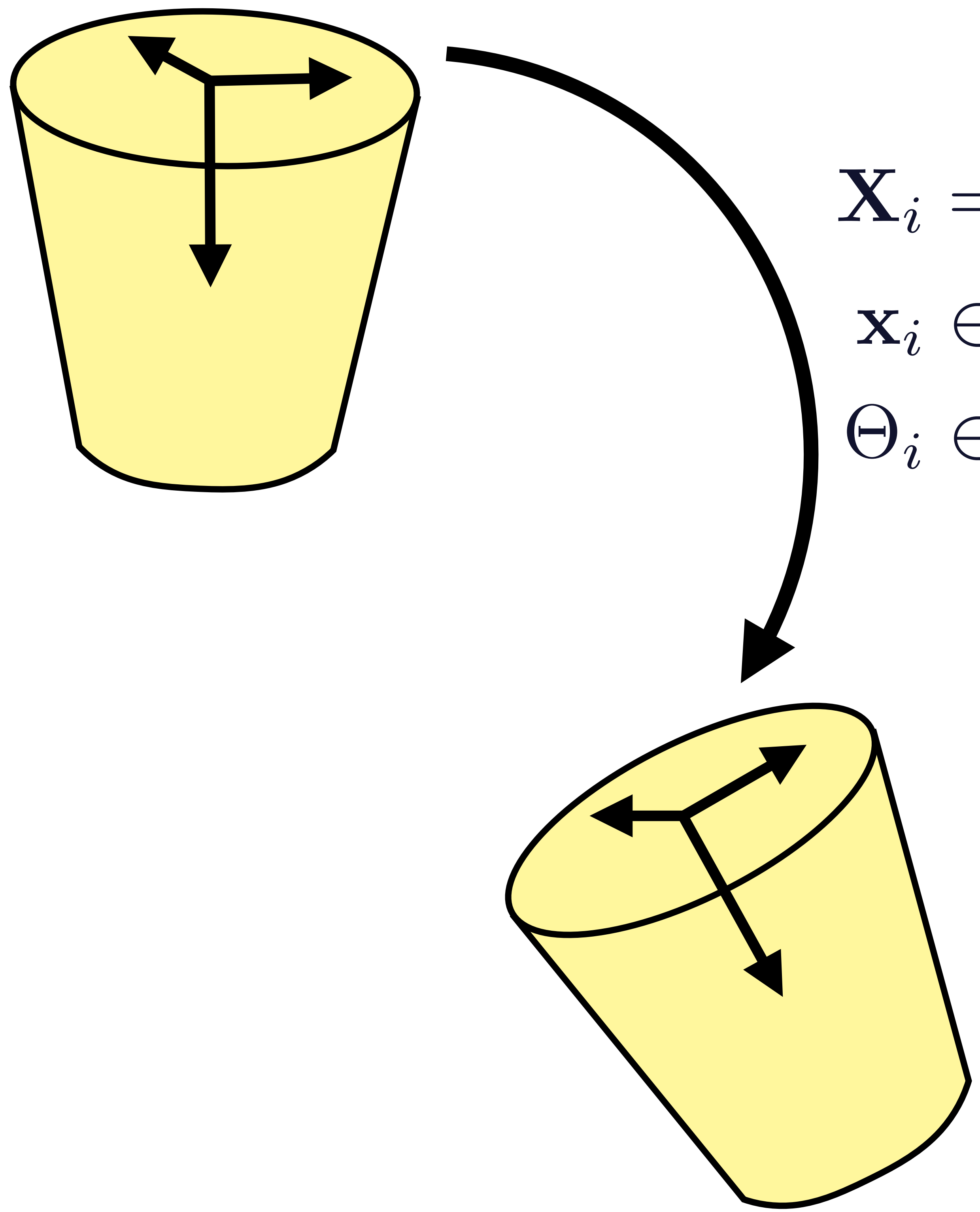


Wait, what?



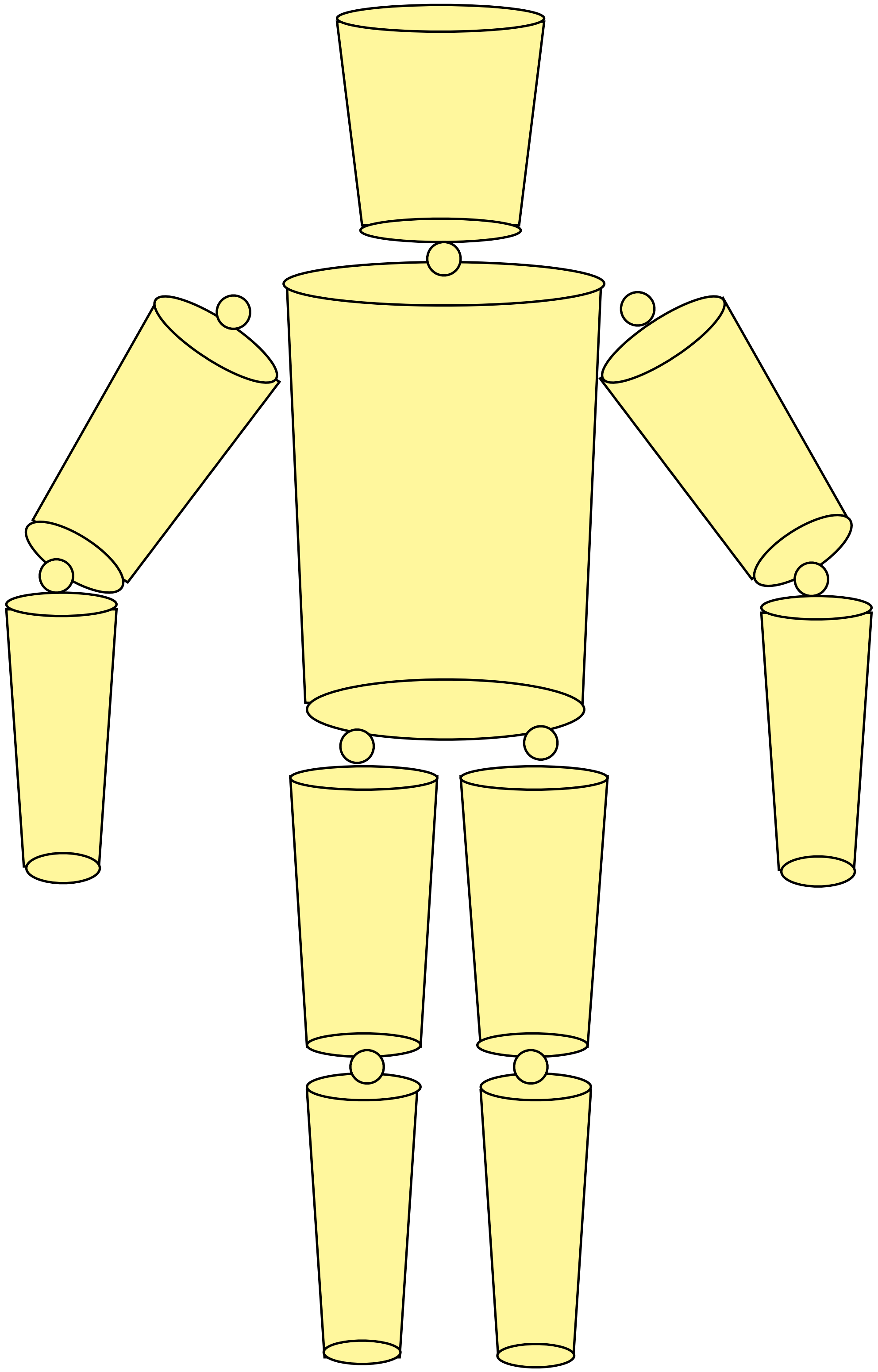
$f(\mathbf{X}_1)$
$f(\mathbf{X}_2)$
$f(\mathbf{X}_3)$
$f(\mathbf{X}_4)$
$f(\mathbf{X}_5)$
$f(\mathbf{X}_6)$
$f(\mathbf{X}_7)$
$f(\mathbf{X}_8)$



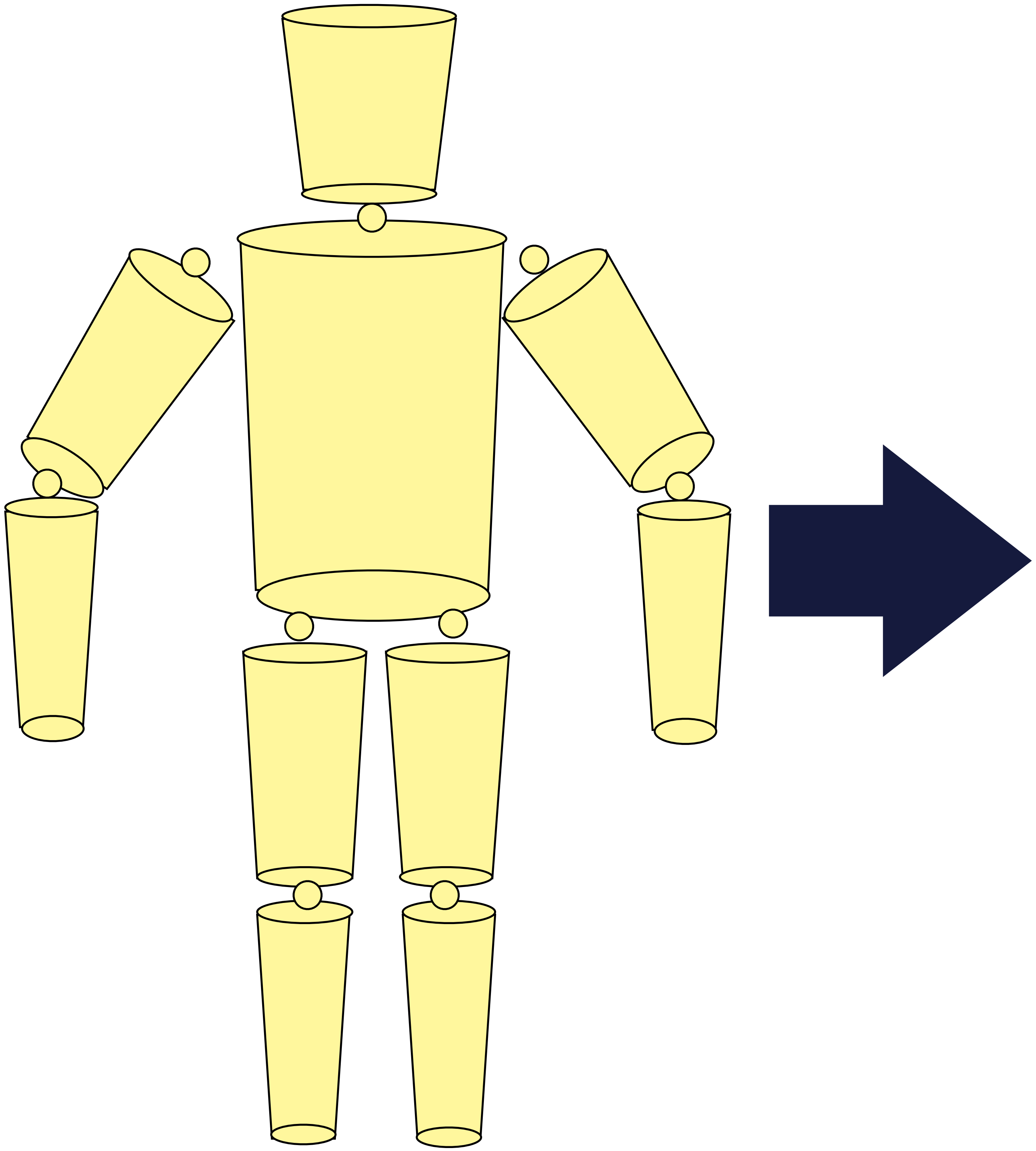


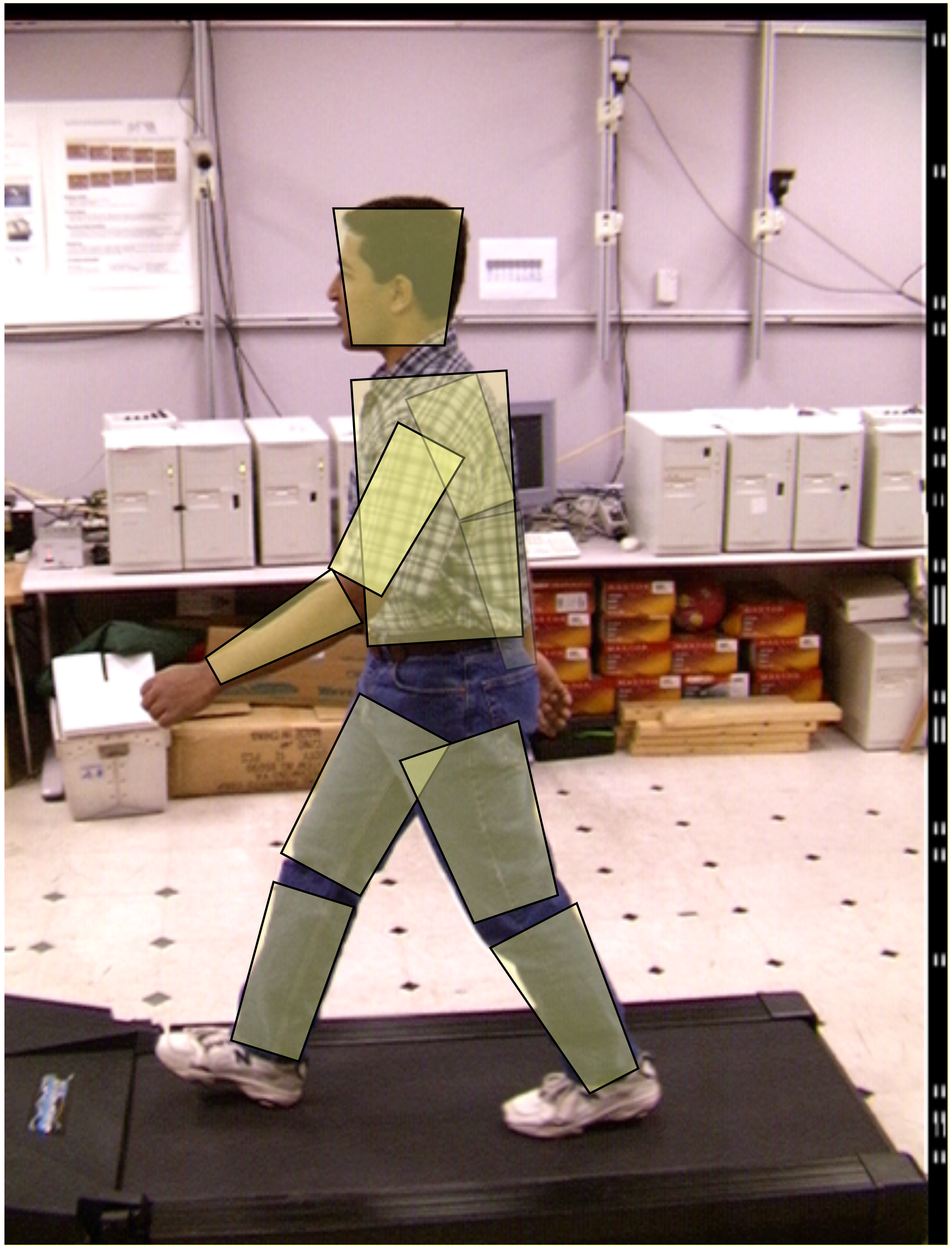
$$\mathbf{X}_i = (\mathbf{x}_i, \Theta_i)$$
$$\mathbf{x}_i \in \mathbb{R}^3$$
$$\Theta_i \in \mathbf{SO}(3)$$

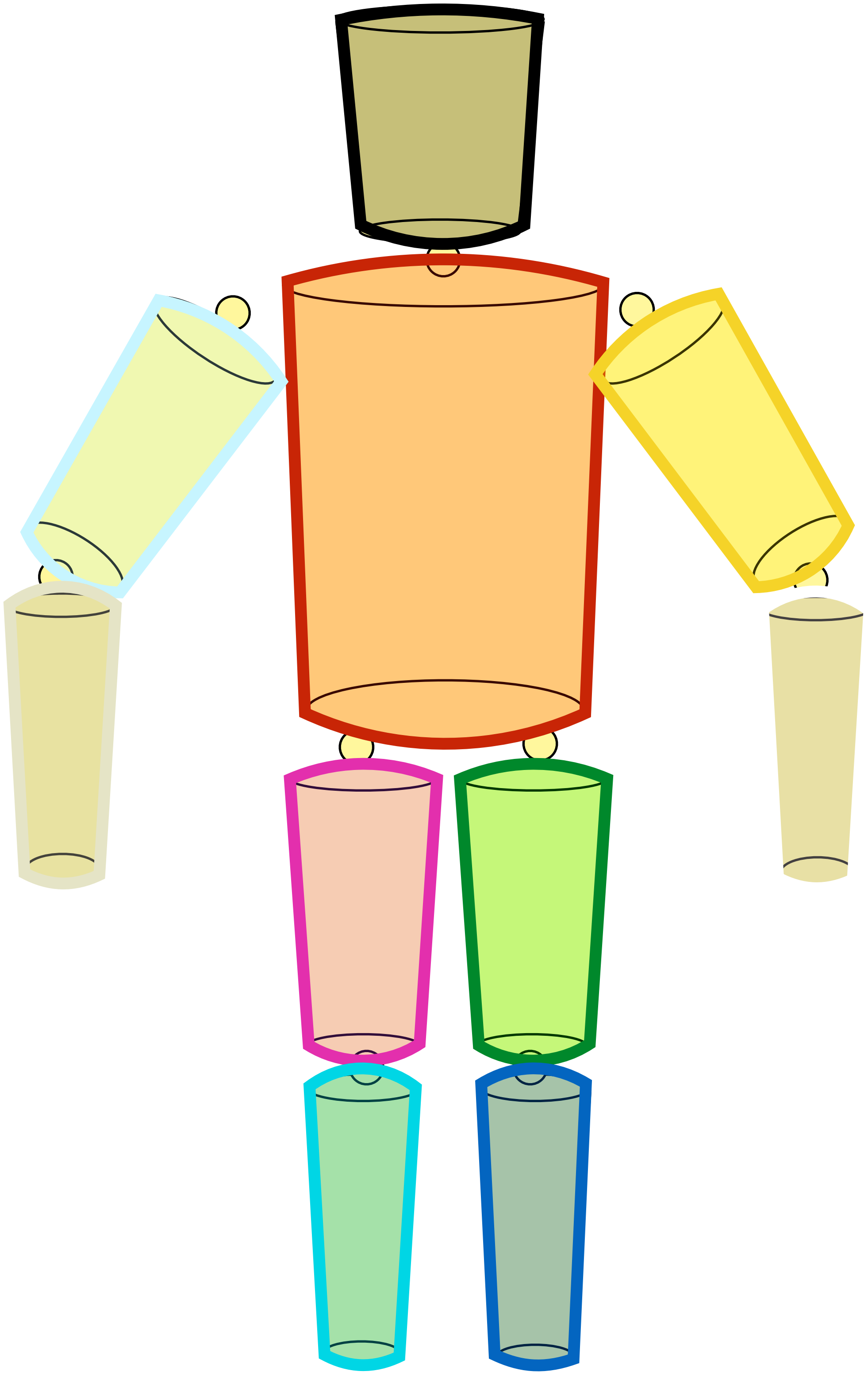
Attractive People: Assembling Loose-Limbed Models using Non-parametric Belief Propagation, Sigal et al.



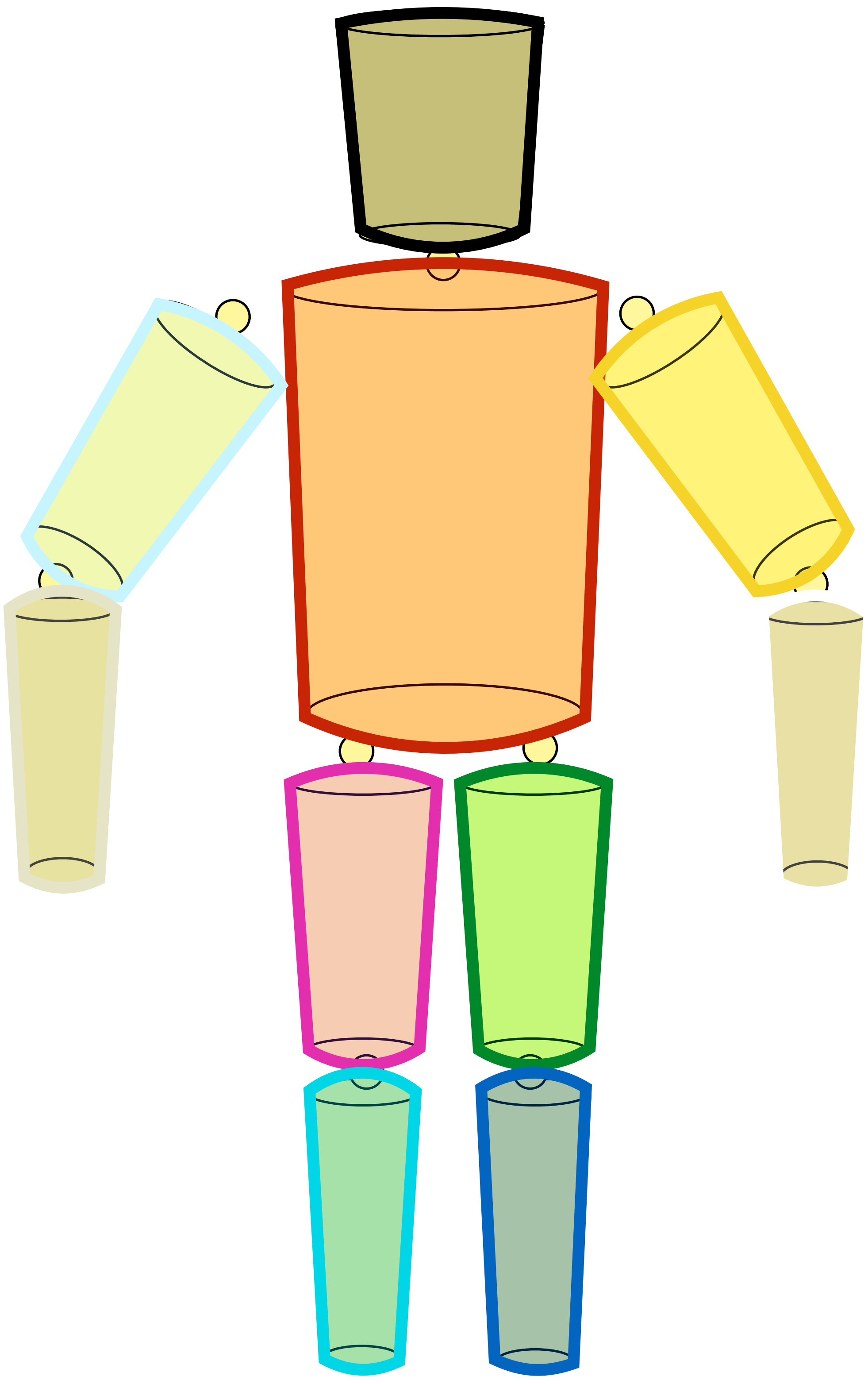
Goal







$$\mathbf{X}_i = \arg\min_{\mathbf{X}_i} \phi_i(I, \mathbf{X}_i)$$



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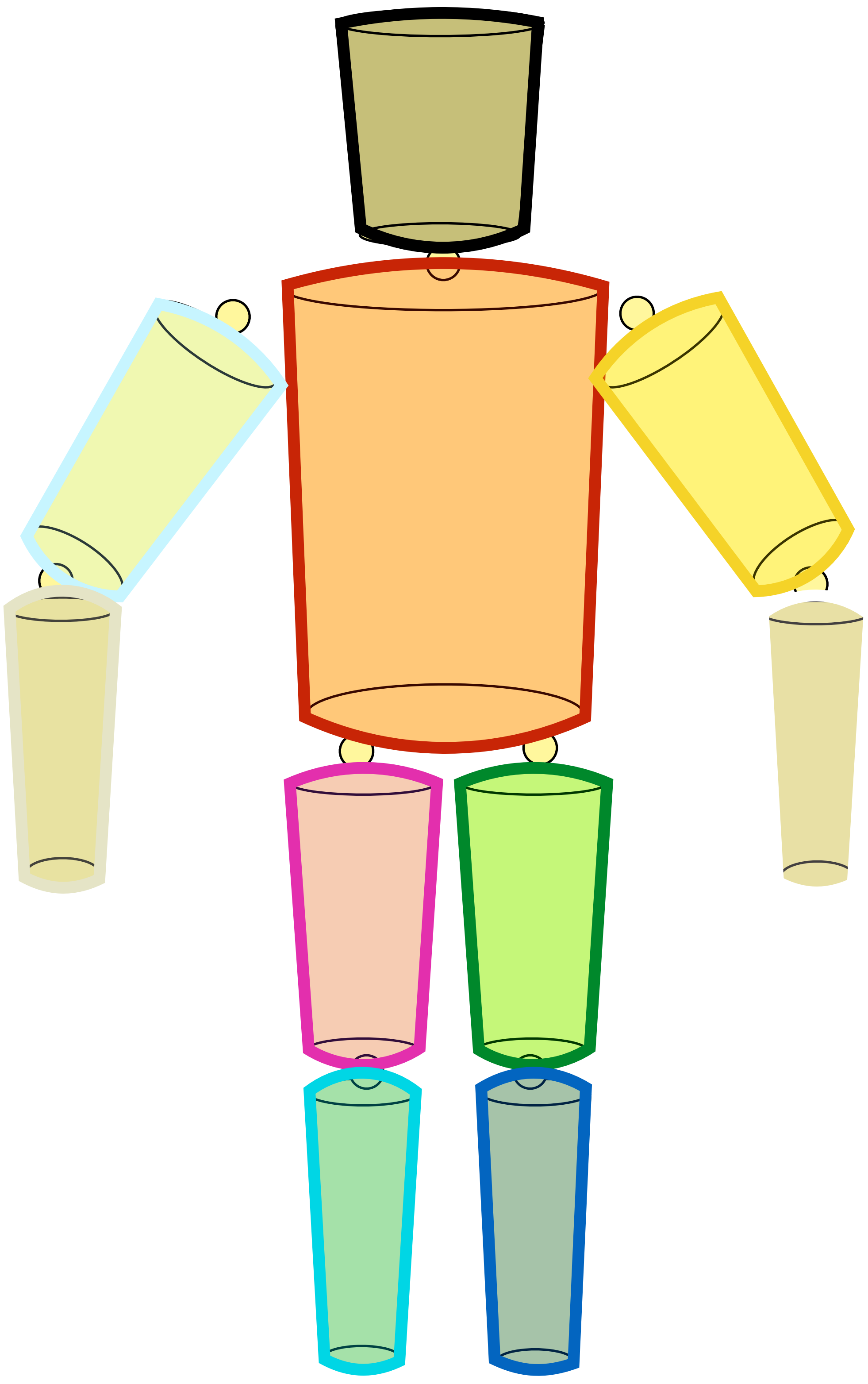
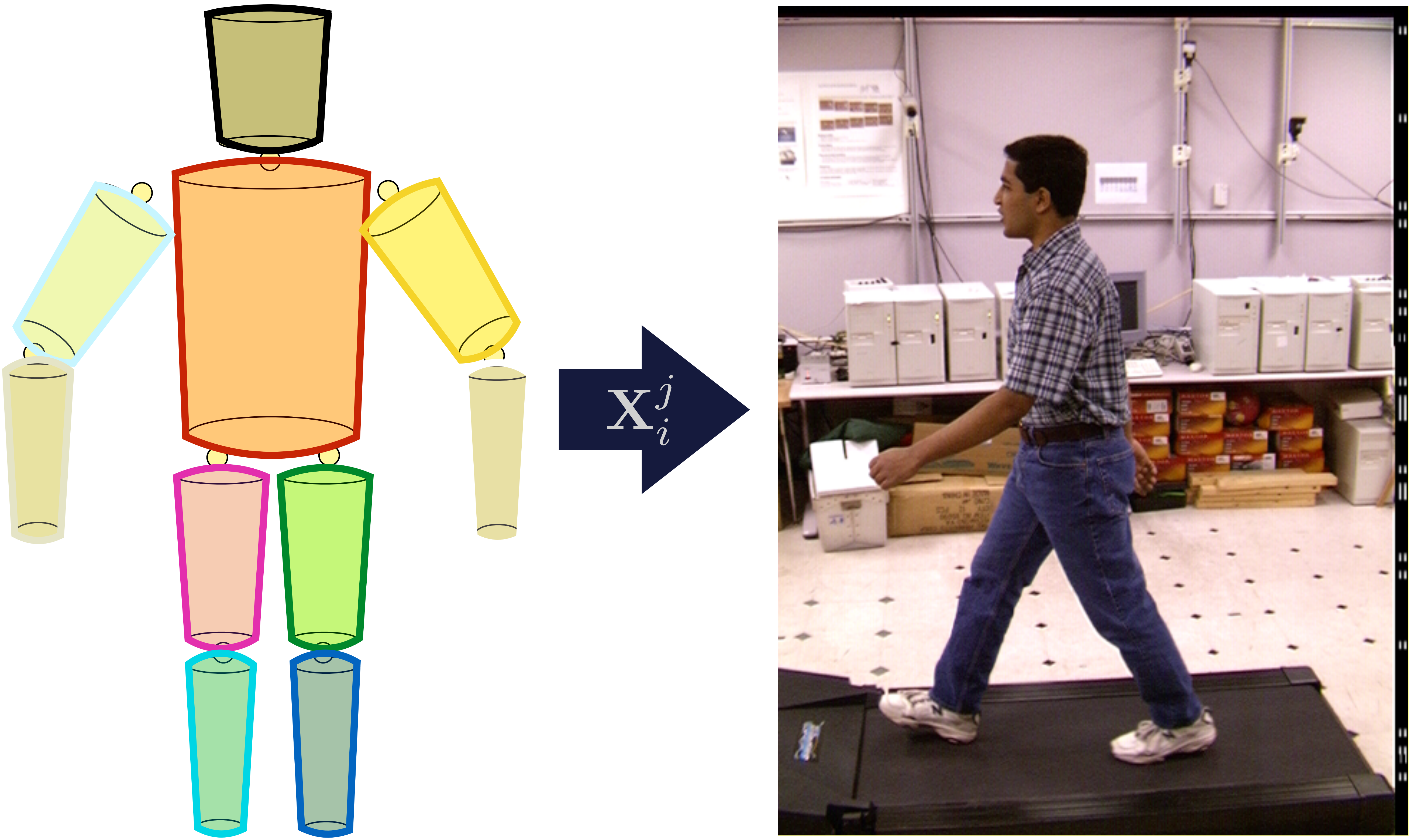
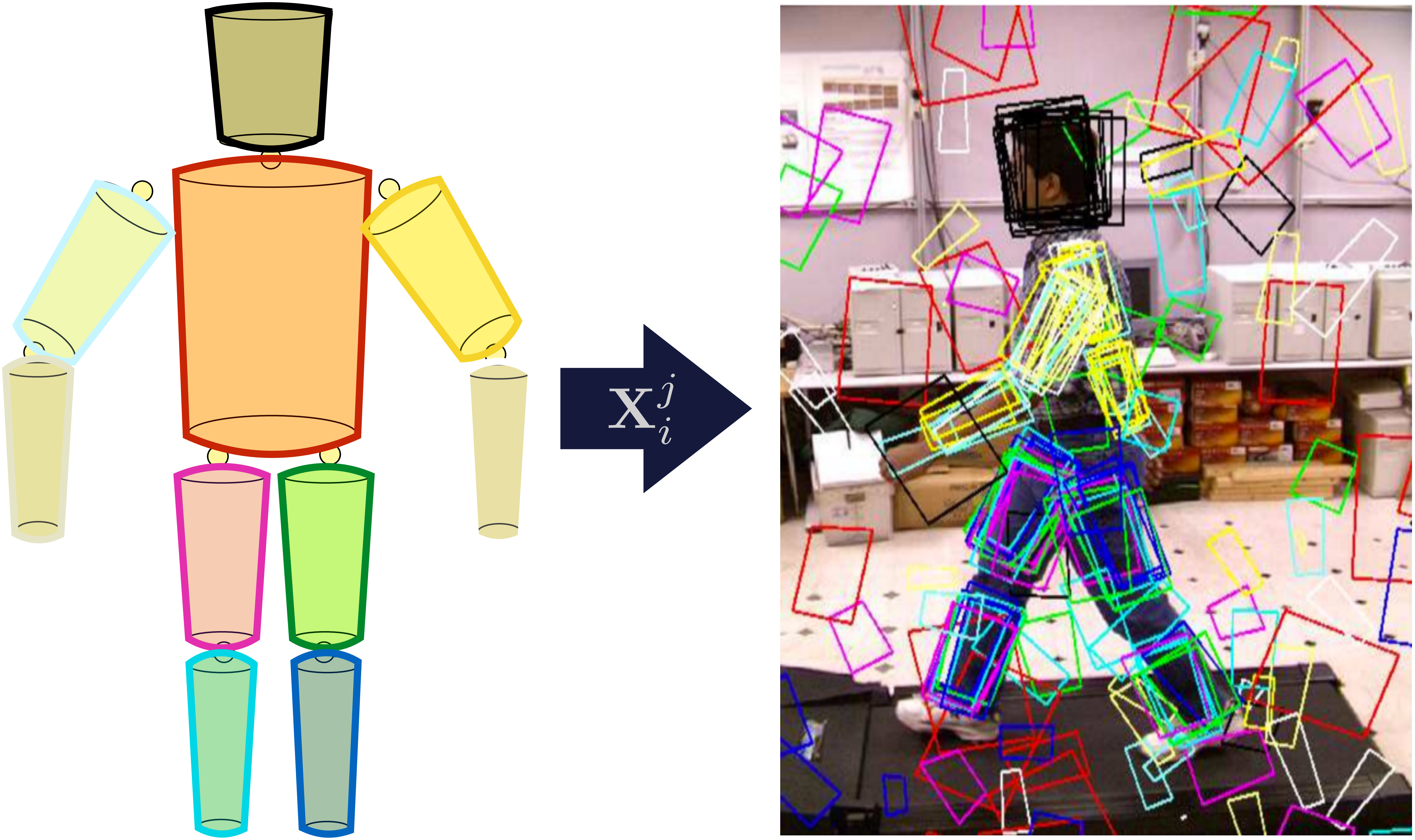


$$\mathbf{X}_i = \arg\min_{\mathbf{X}_i} \phi_i(I, \mathbf{X}_i)$$

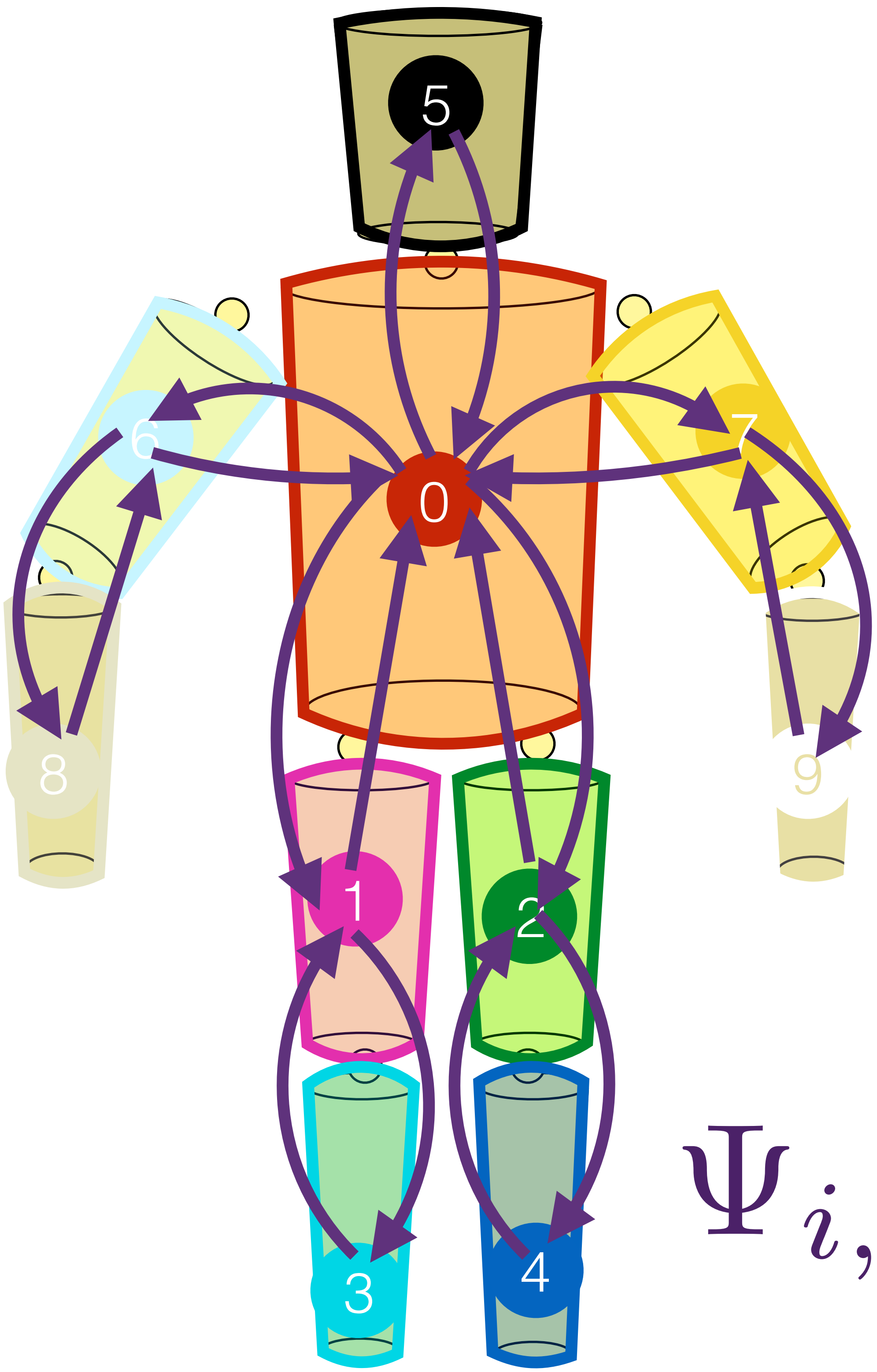

$\mathbf{x}_i^j$


$\mathbf{X}_i^j$

Can we do something better?

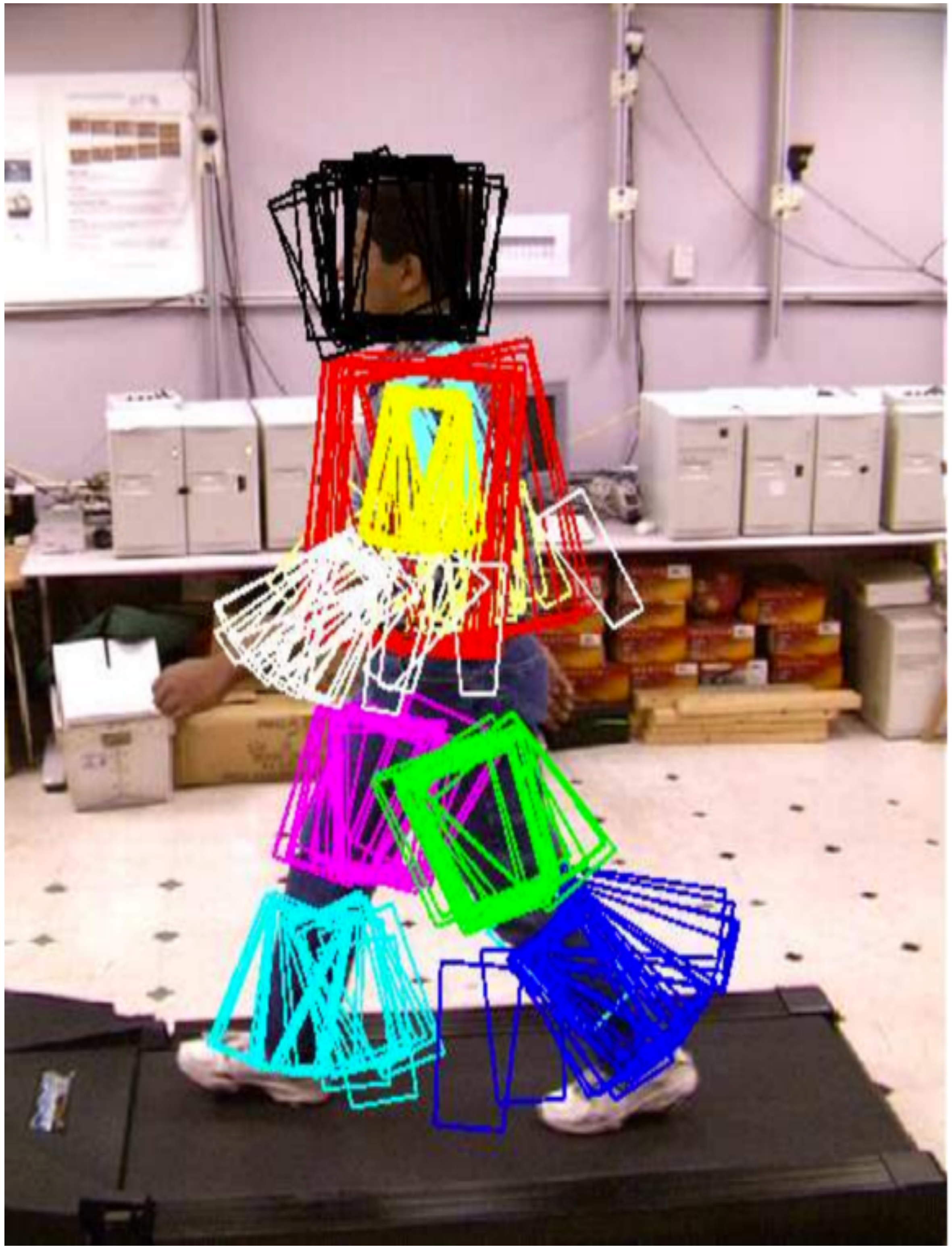
Can we do something better?

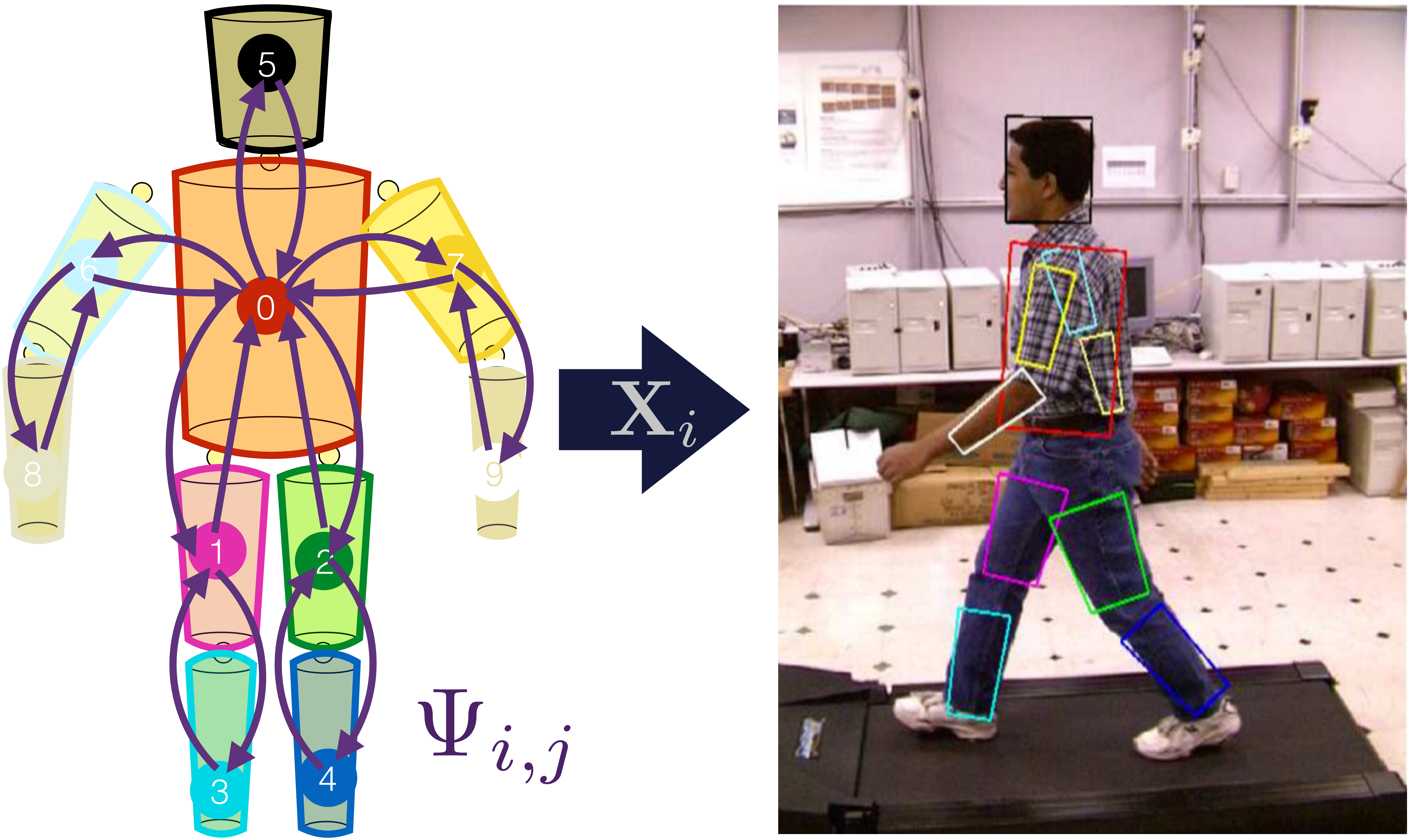
… with something that you already know?

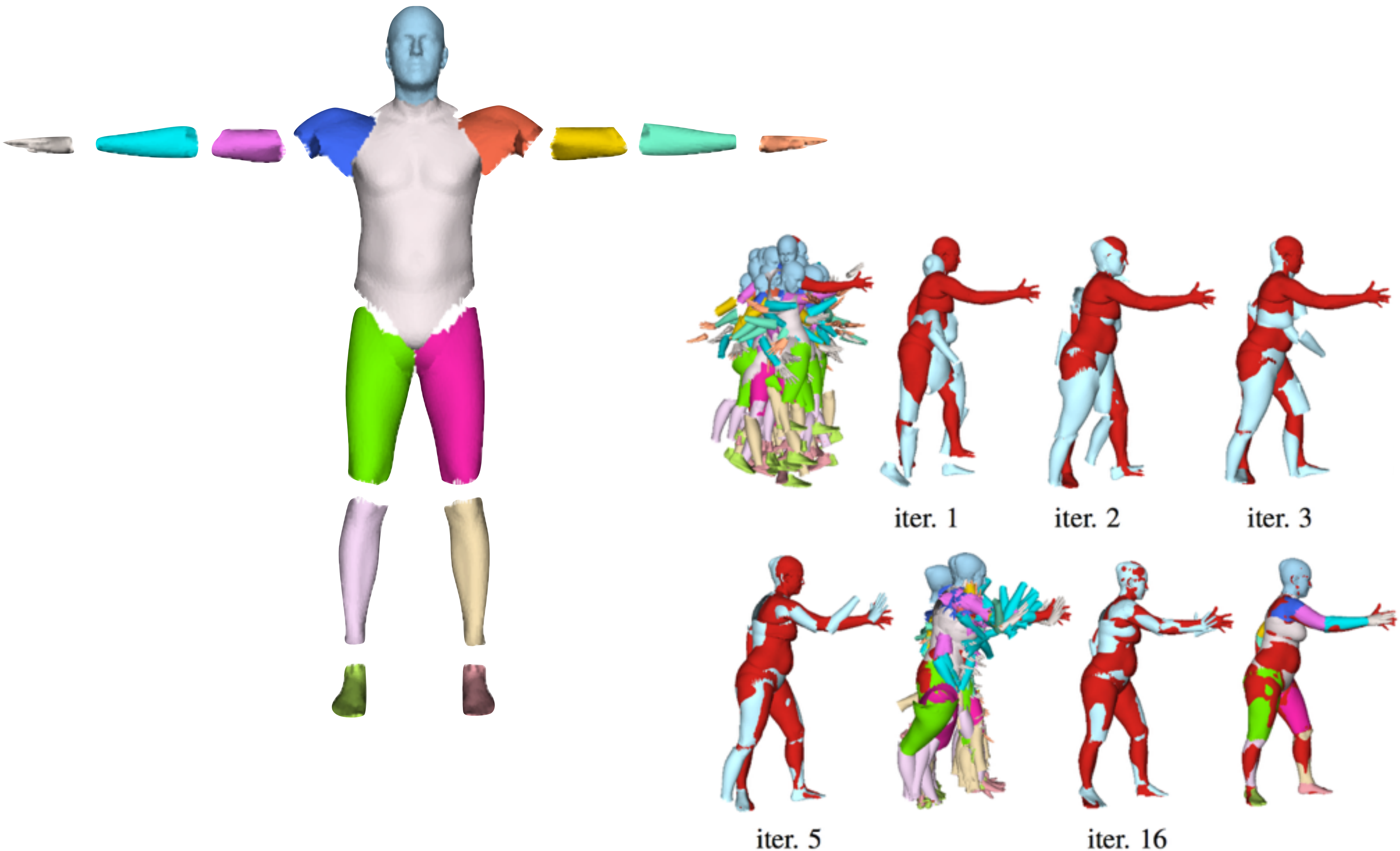


$\Psi_{i,j}$

$\mathbf{x}_i^j$




5
6
0
7
8
9
1
2
3
4
$\mathbf{x}_i$
$\Psi_{i,j}$



The Stitched Puppet: A Graphical Model of 3D Human Shape and Pose, Zuffi and Black

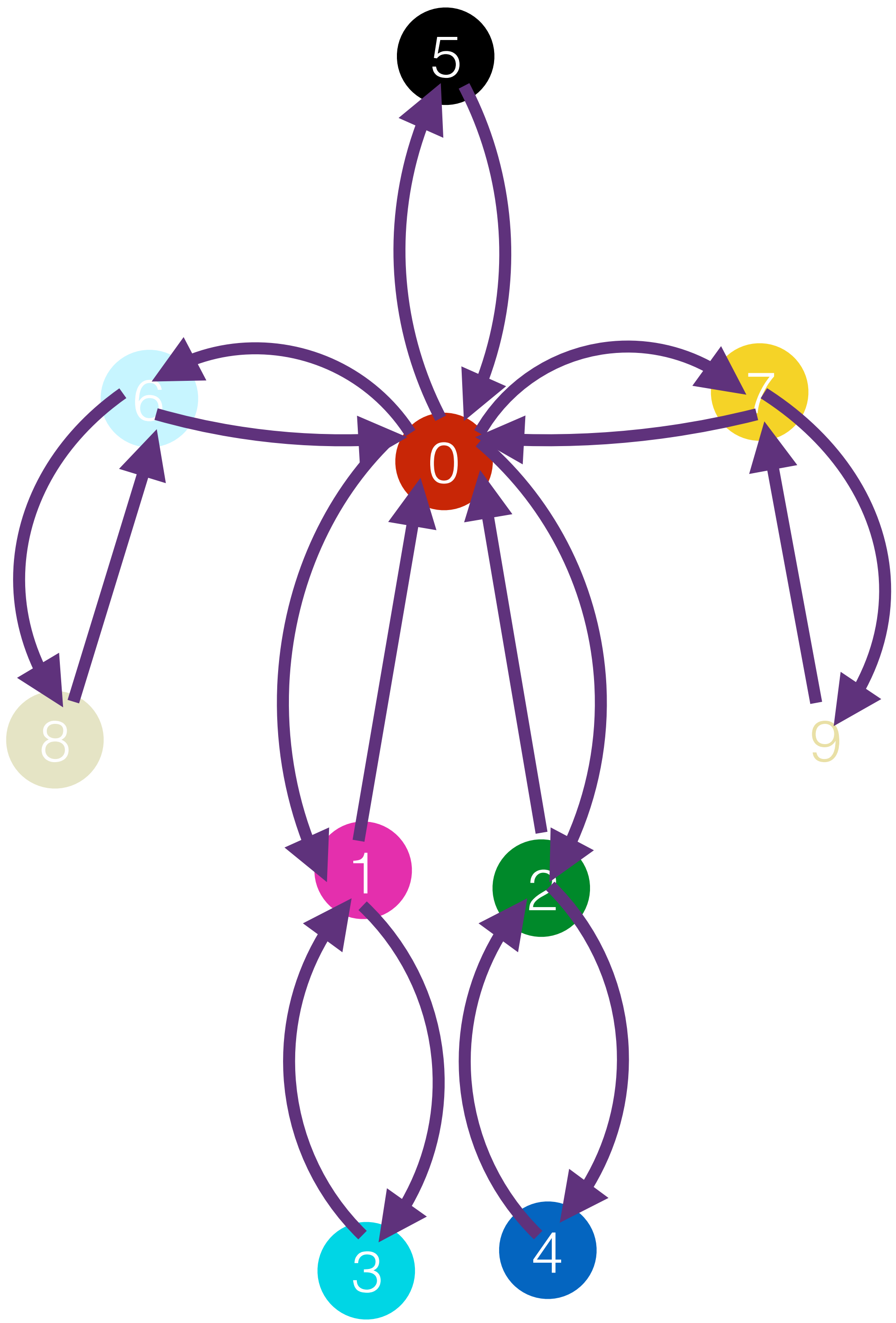
Questions before we start

- What is a good unary ϕ_i for the pose estimation problem?
- What is a good pair-wise term $\Psi_{i,j}$ for the same problem?

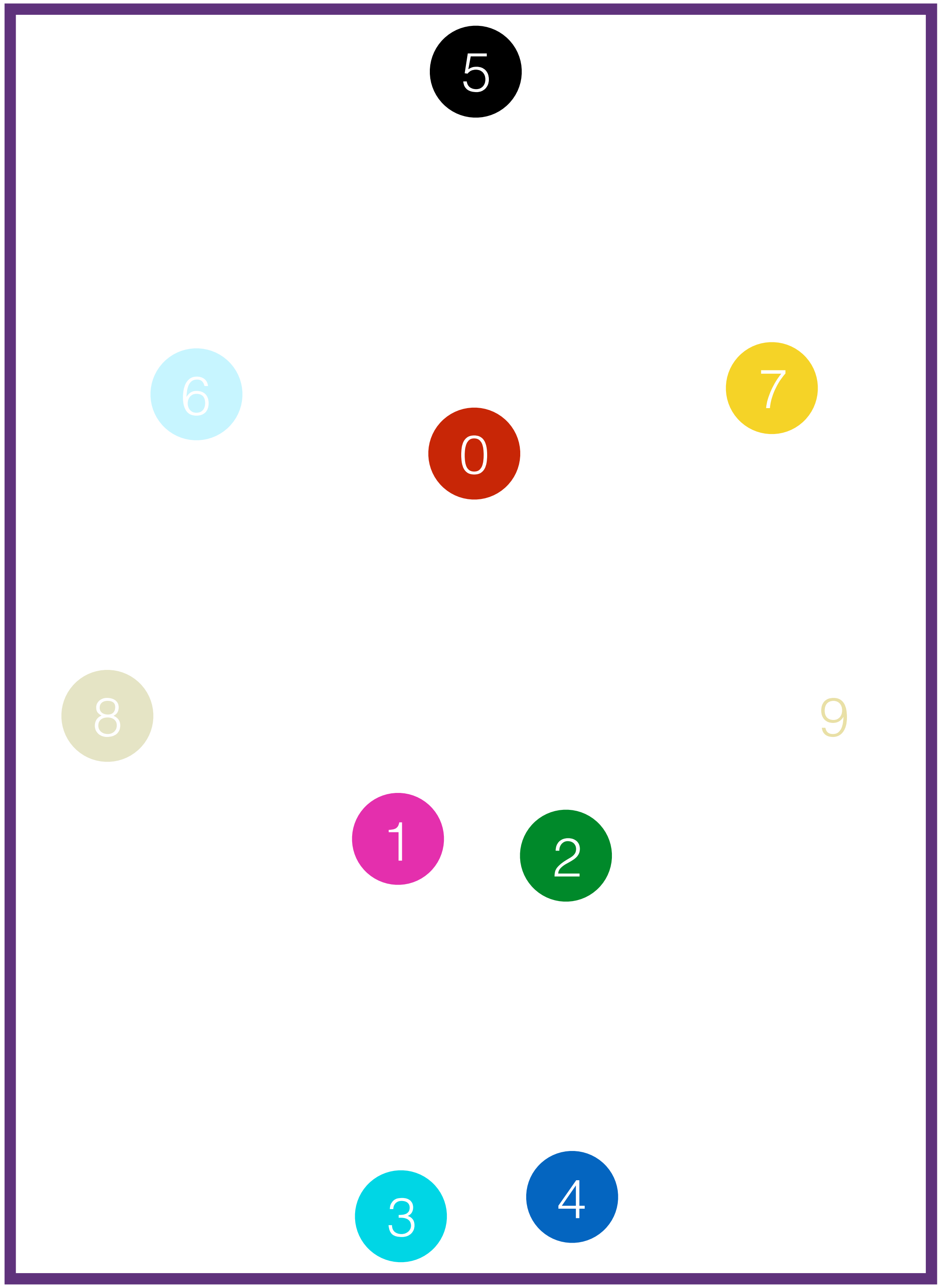
But wait a second

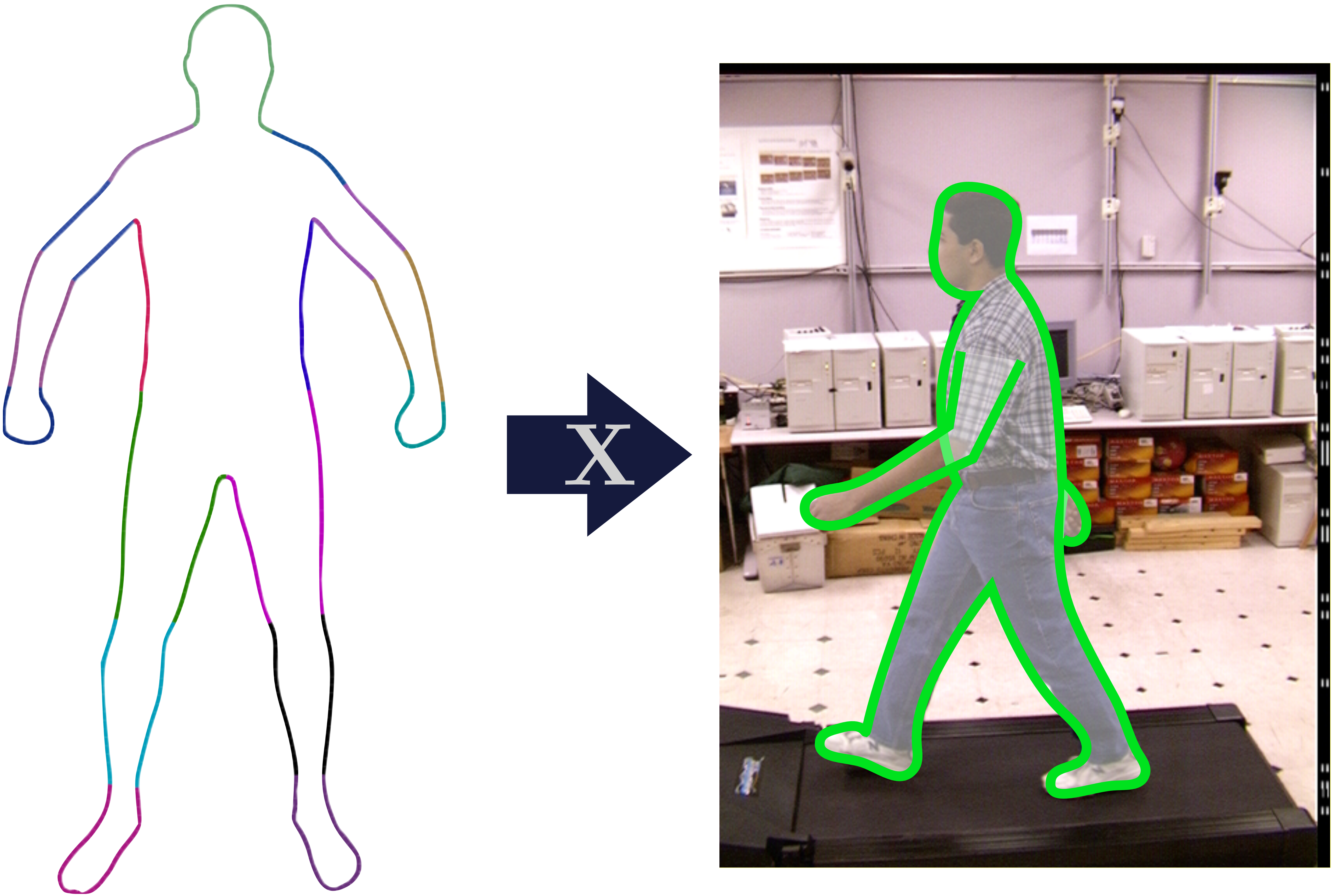
- With these ingredients we could attack the problem of pose estimation, but…
- What is the advantage of using a graphical model?
- Can we use another representation?

But wait a second



Why not all at once?

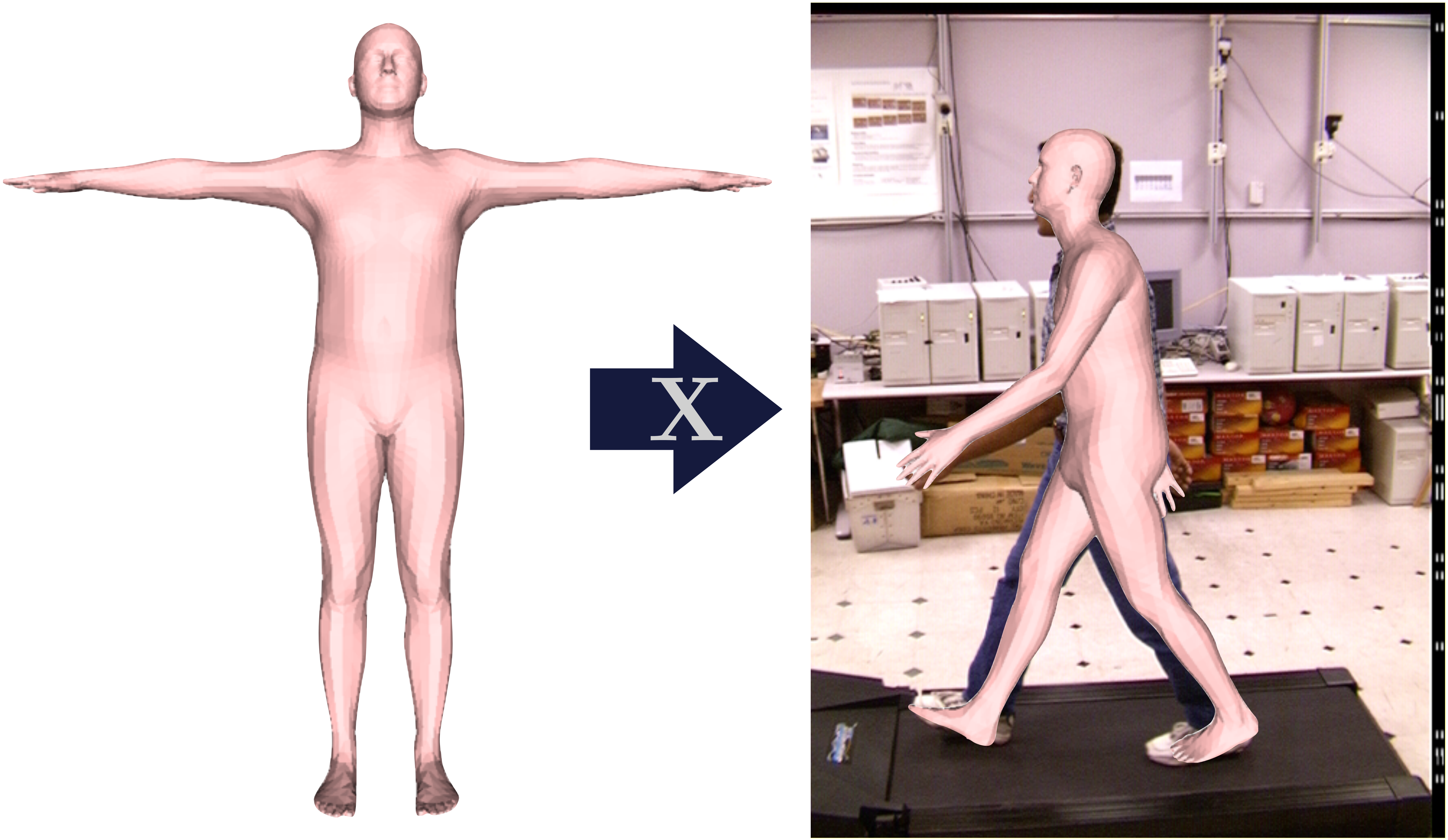




Contour People: A Parameterized Model of 2D Articulated Human Shape, Freifeld et al.

Could we do something better?

In which space does the entity that we want model (human bodies) live?

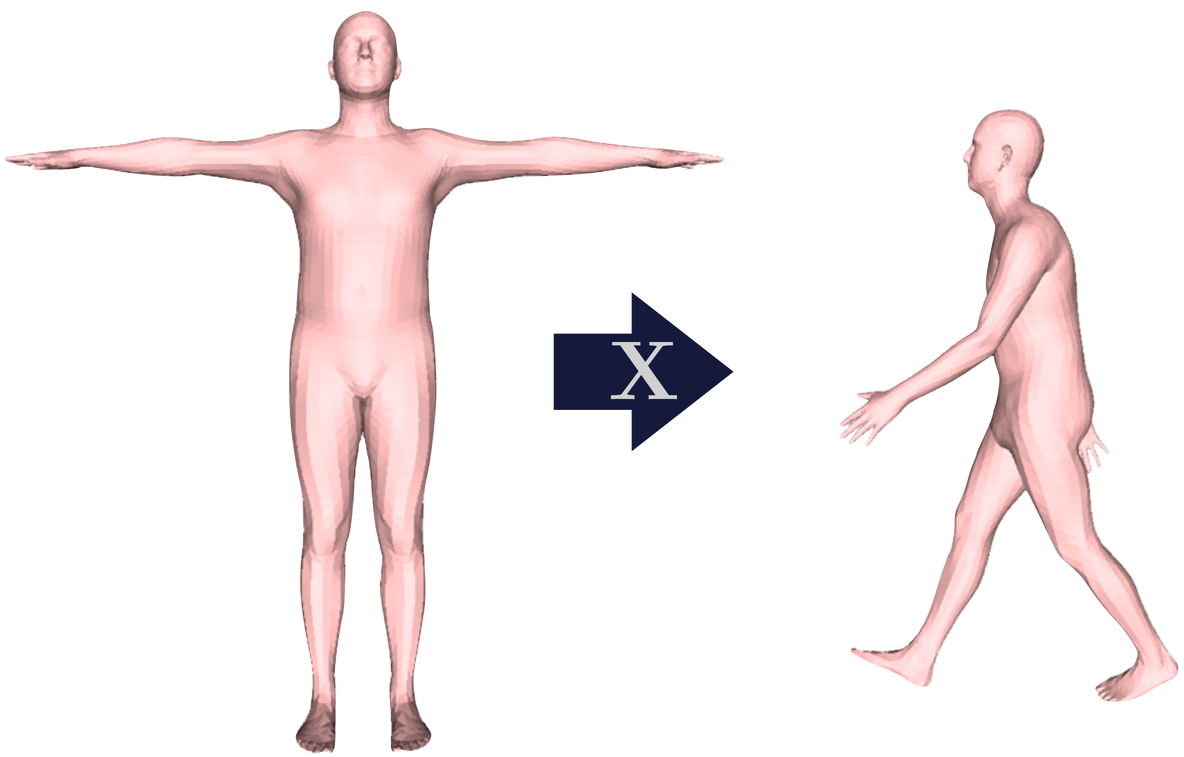


SMPL: A Skinned Multi-Person Linear Model, Loper et al.

Part-based models VS holistic models

- Part-based
  - Efficient
  - Fast exploration of parameter space
- Holistic
  - Part relation **imposed** by model, not optimised
  - Realistic solutions (up to the model's realism)
  - Simple optimisation

Part-based models VS holistic models



- Holistic
  - Part relation **imposed** by model, not optimised
  - Realistic solutions (up to the model's realism)
  - Simple optimisation

SMPL: A Skinned Multi-Person Linear Model, Loper et al.

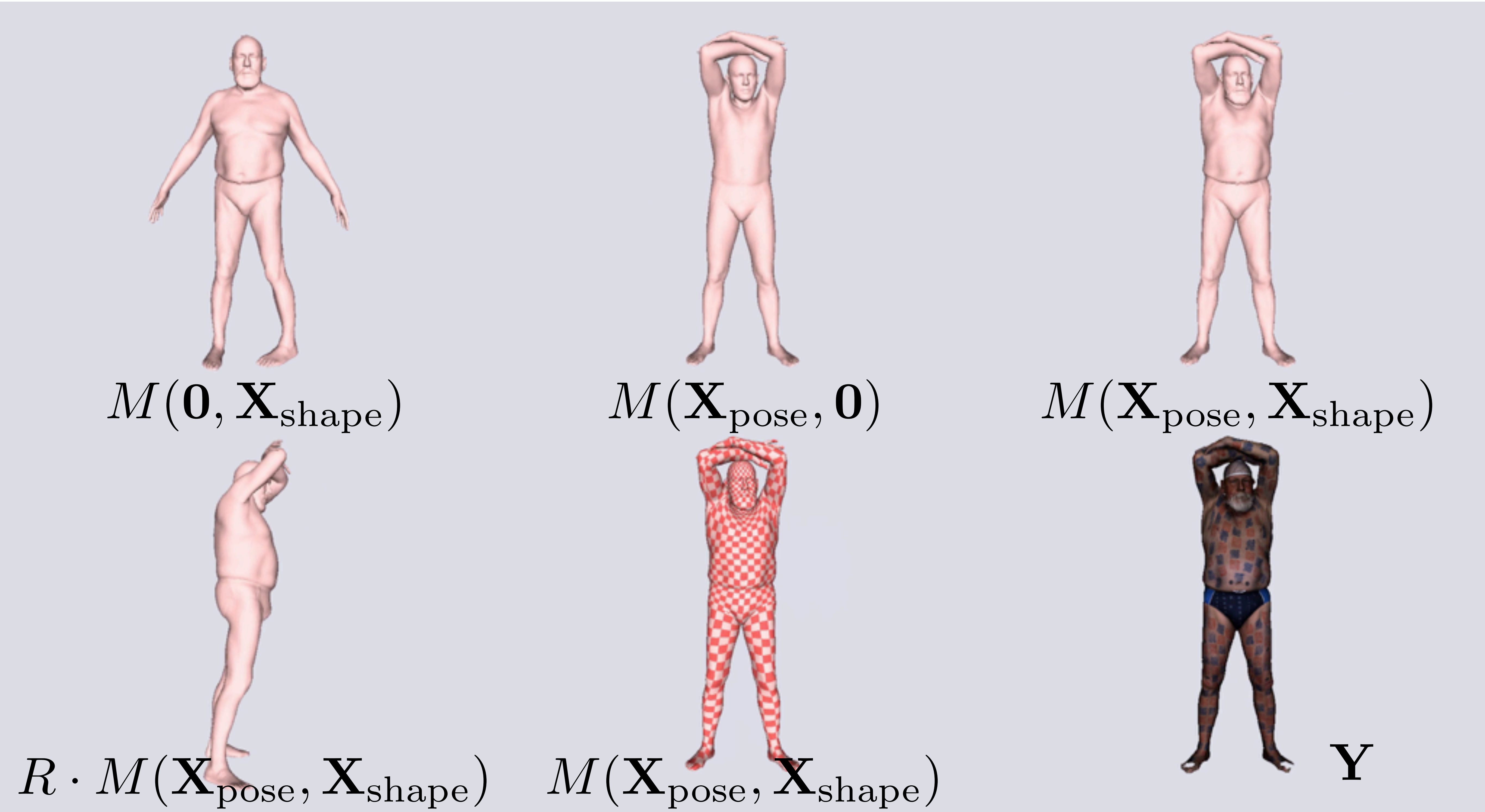
Body models: the gritty details

- Today: Geometry
  - Points, triangles and triangulated meshes
  - Similarity transformations
  - Procrustes Method

3D Holistic body models

Transform parameters into geometry in 3D

$$M(\mathbf{X}) \rightarrow \mathbb{R}^3$$



$$\mathbf{X} = \{\mathbf{X}_{\text{pose}}, \mathbf{X}_{\text{shape}}\}$$

3D Holistic body models

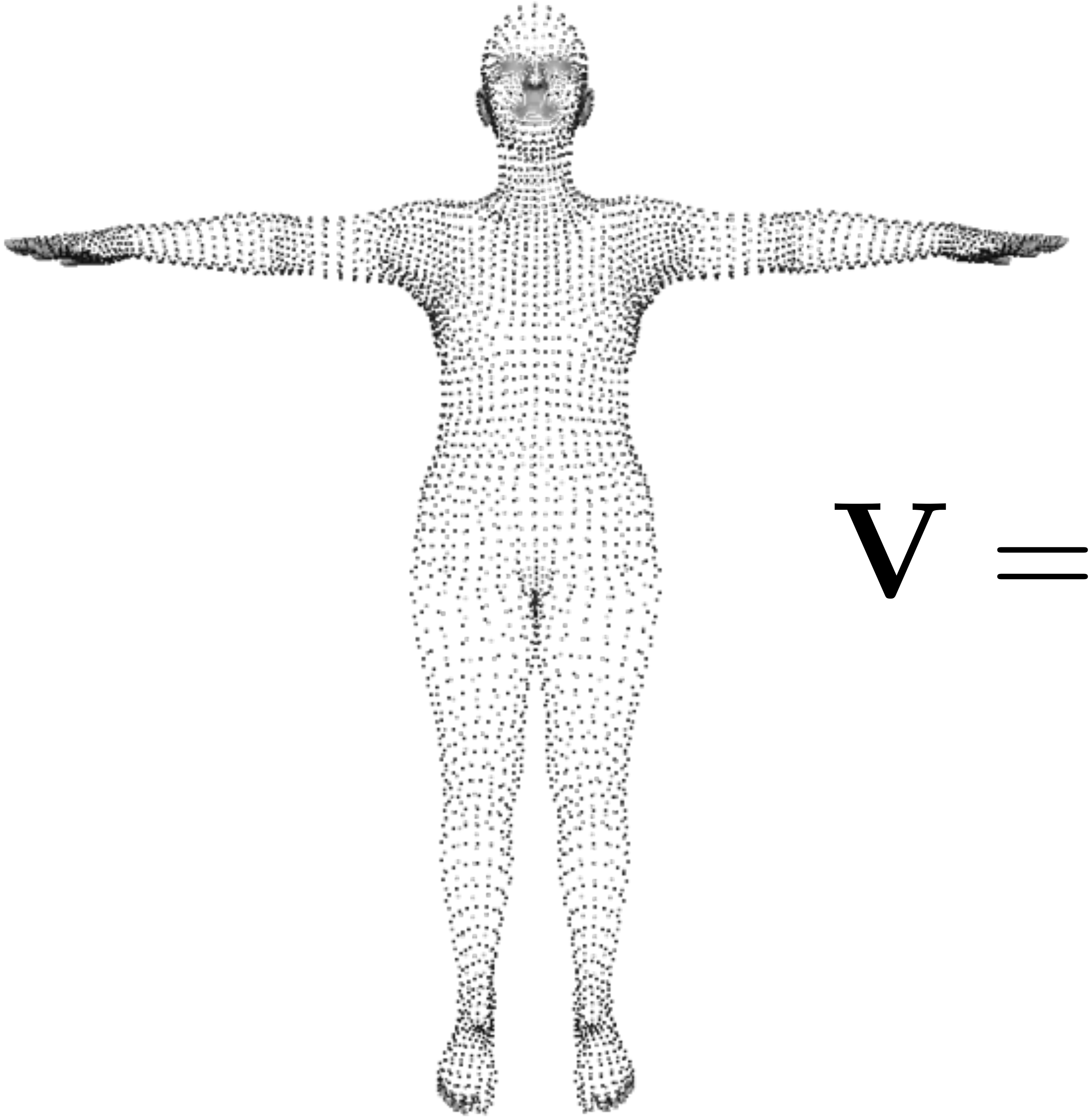
Transform parameters into geometry in 3D

$$M(\mathbf{X}) \rightarrow \mathbb{R}^3$$

Simplest $\mathbb{R}^3$ entity

- $\mathbf{v}$

Simplest $\mathbb{R}^3$ entity



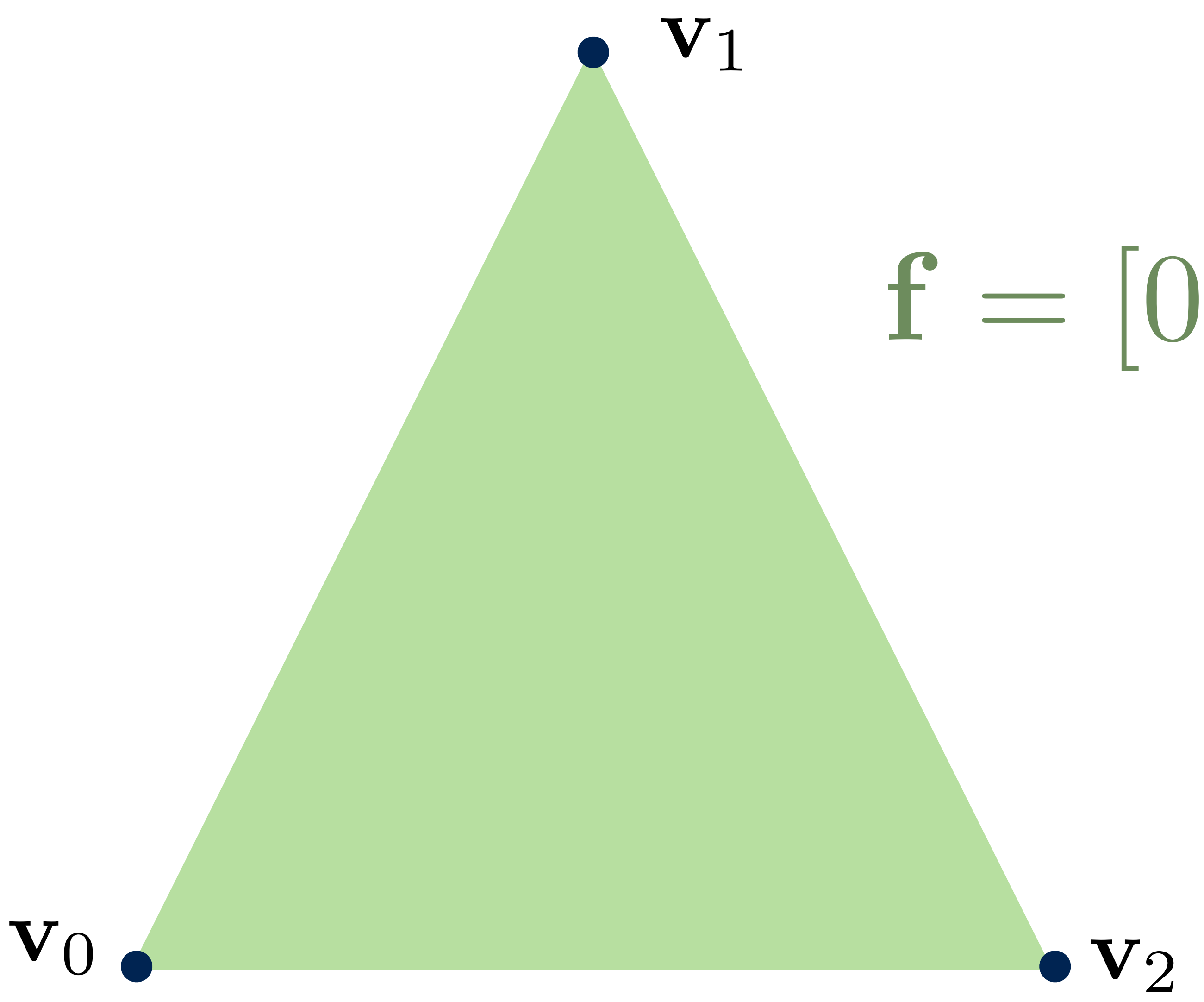
$$\mathbf{V} = \{\mathbf{v}_i\}$$

Problem

How does a pointcloud $\mathbf{V} = \{\mathbf{v}_i\}$ rendered in e.g. OpenGL?

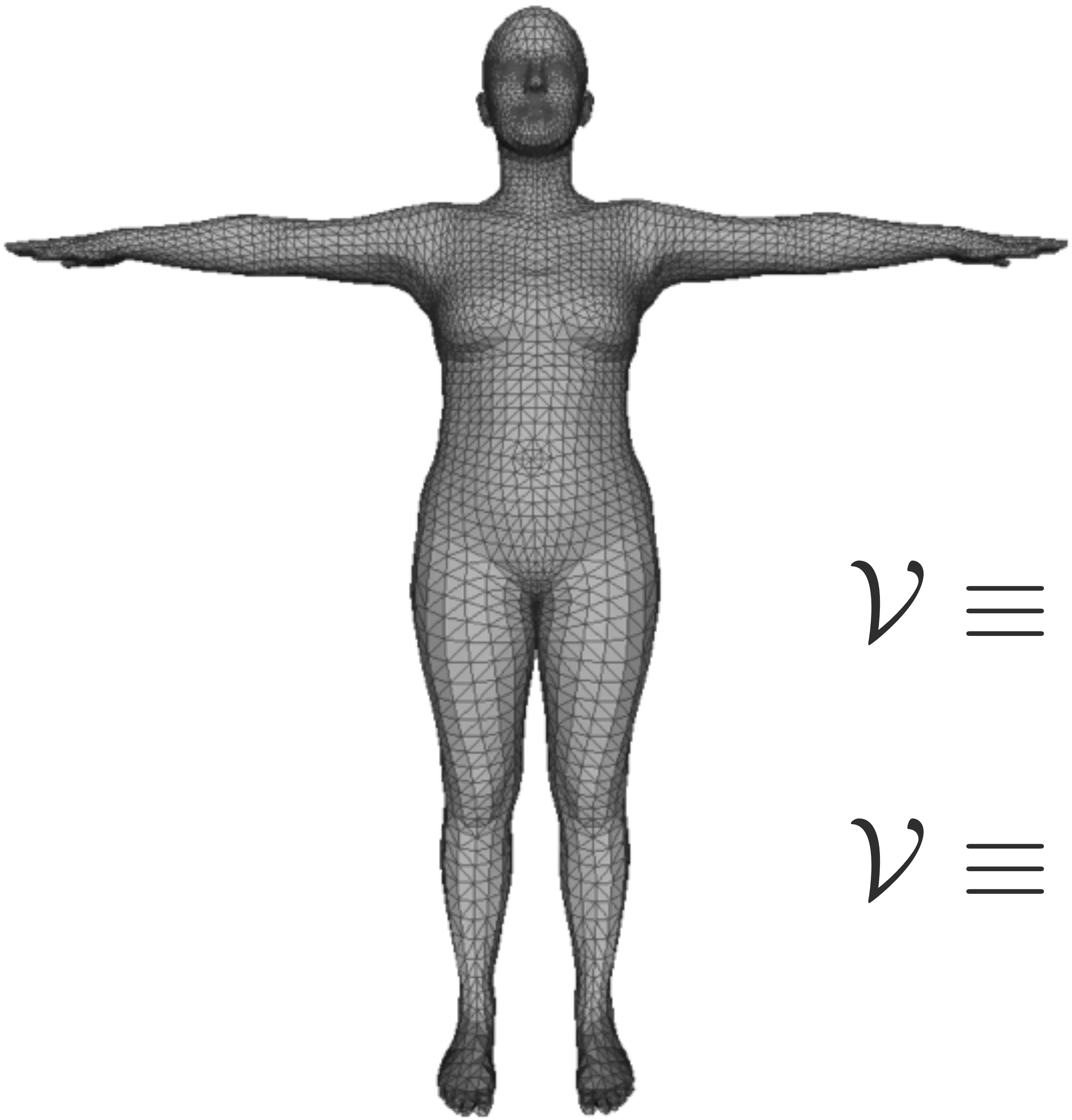
Like this

Simplest $\mathbb{R}^3$ surface representation



$$\mathbf{f} = [0, 1, 2] \in \mathbb{N}^3$$

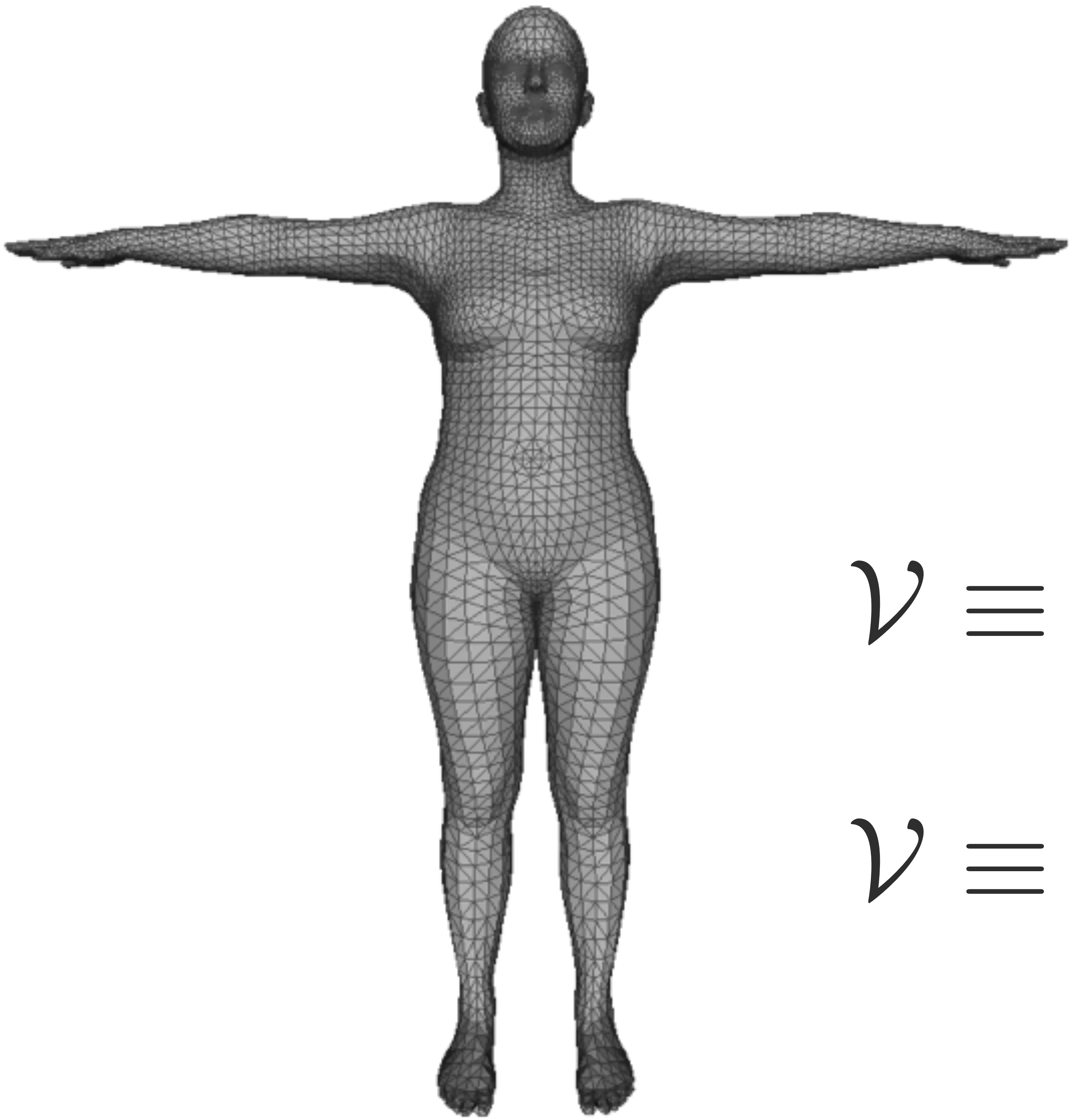
Triangulated Mesh



$$\mathcal{V} \equiv \begin{cases} \mathbf{F} \in \mathbb{N}^{M \times 3} \\ \mathbf{V} \in \mathbb{R}^{N \times 3} \end{cases}$$

$$\mathcal{V} \equiv \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

Triangulated Mesh



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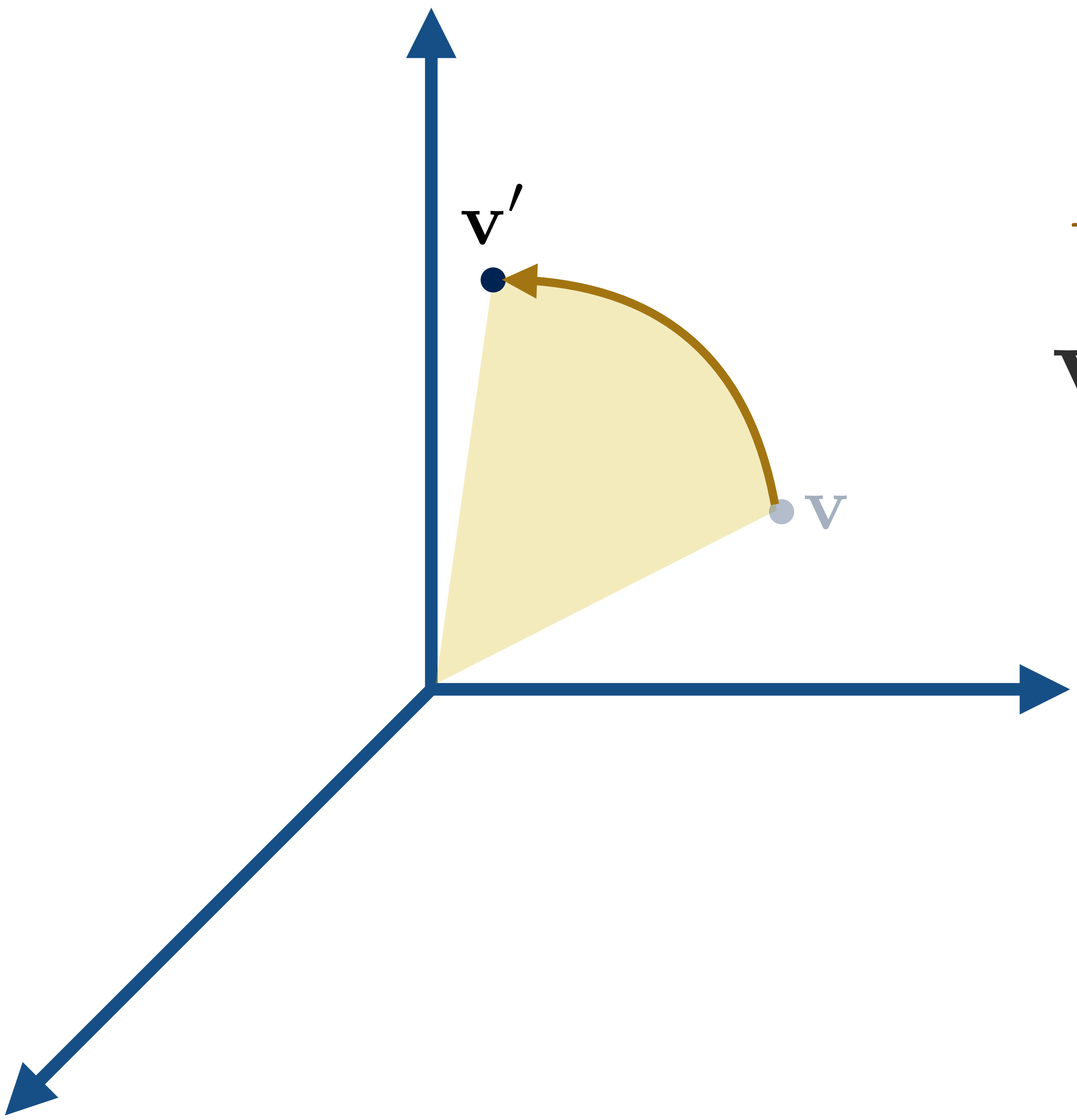
$$\mathcal{V} \equiv \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

Similarity deformations

- What can we rigidly do with a set of points)?
  - Translate it $$\mathbf{V}' = \mathbf{V} + \mathbf{t}, \mathbf{t} \in \mathbb{R}^3$$
  - Rotate it $$\mathbf{V}'^{\mathrm{T}} = \mathbf{R} \cdot \mathbf{V}^{\mathrm{T}}, \mathbf{R} \in \mathbf{SO}(3)$$
- Today we'll consider also
  - Scale it $$\mathbf{V}' = s \cdot \mathbf{V}^{\mathrm{T}}, s \in \mathbb{R}$$

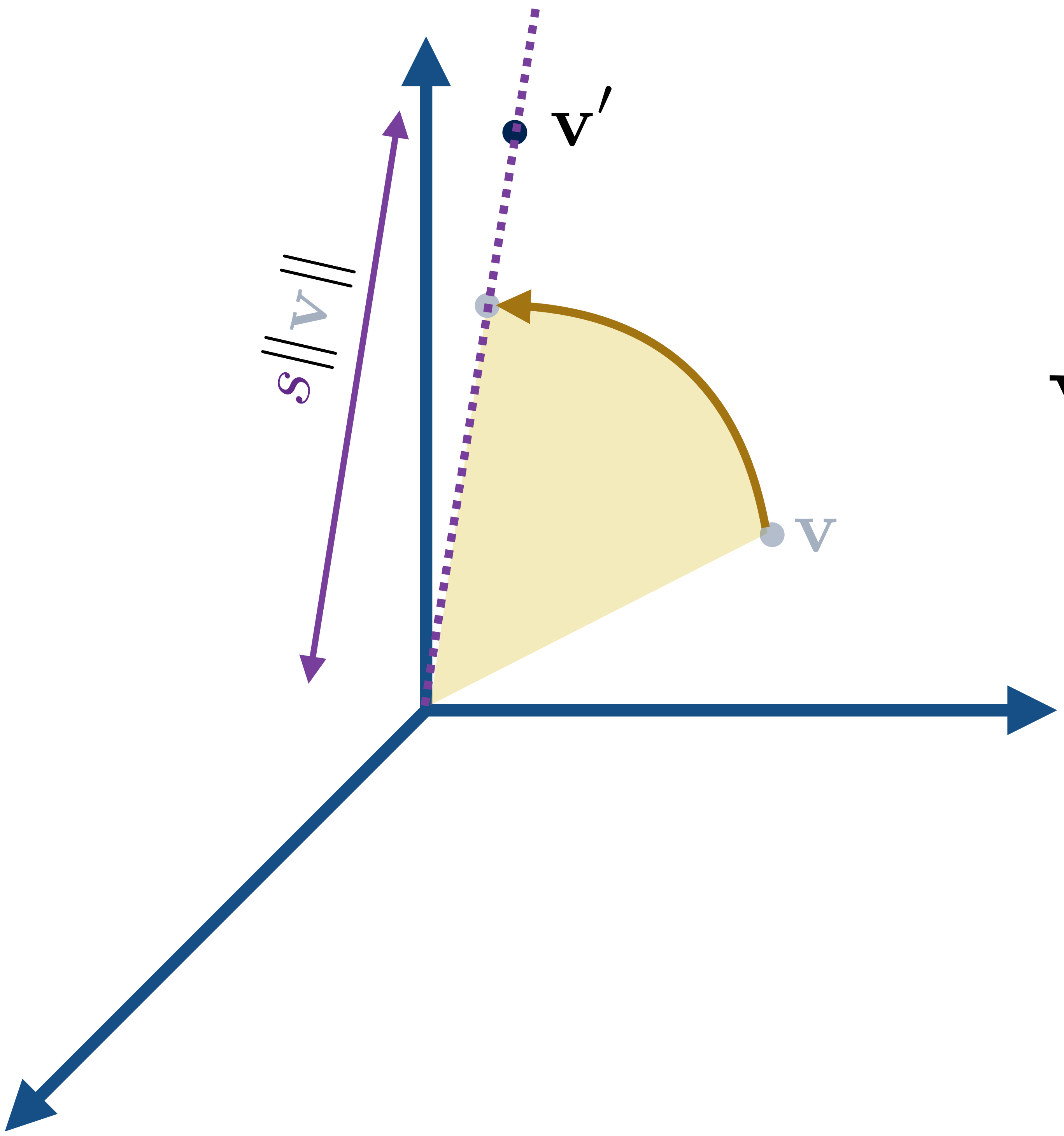
Rotation



$$R : \mathbb{R}^3 \to \mathbb{R}^3$$

$$\mathbf{v}' = R\mathbf{v}$$

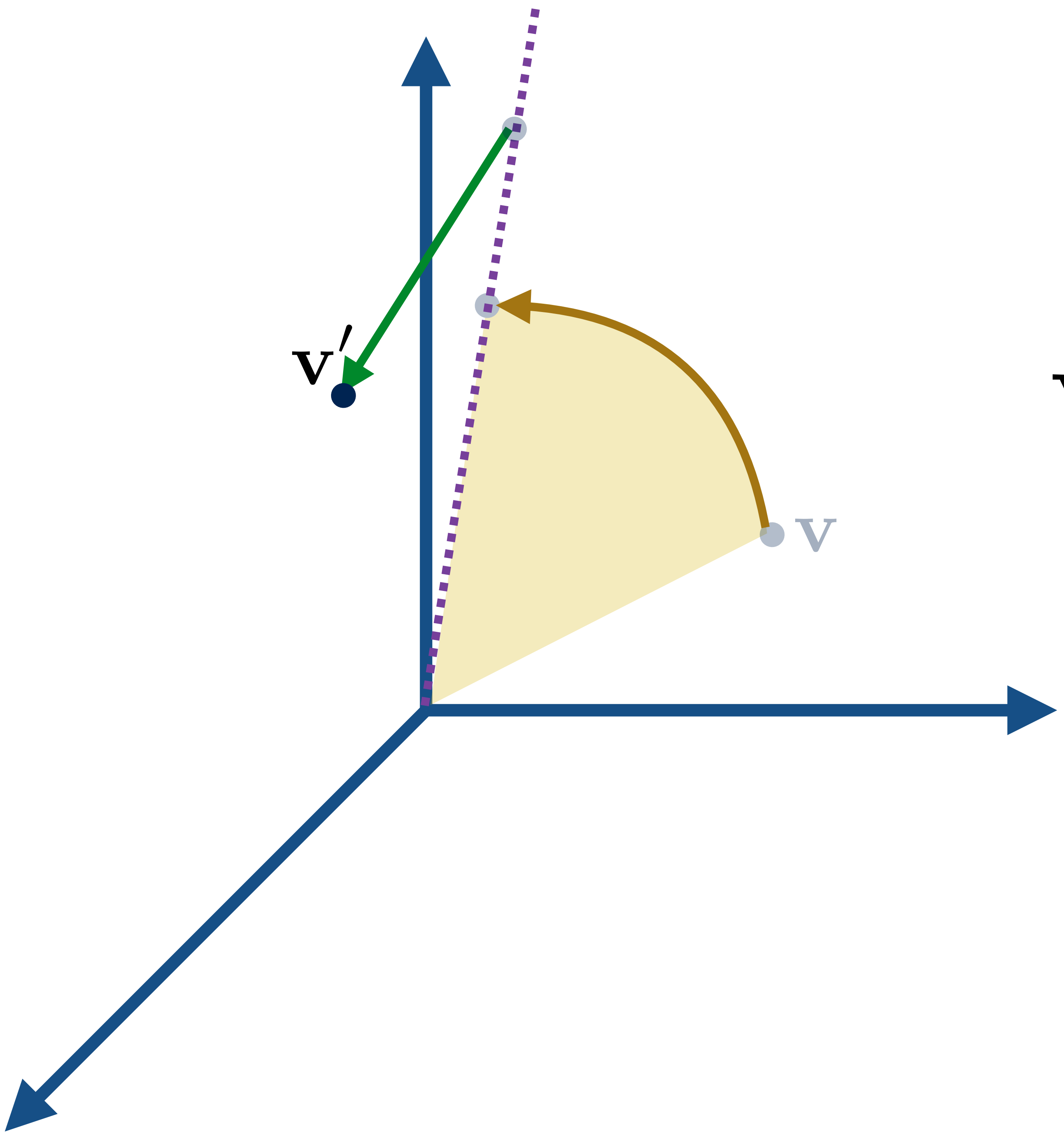
Rotation + Scale



$$R : \mathbb{R}^3 \to \mathbb{R}^3$$

$$\mathbf{v}' = sR\mathbf{v}$$

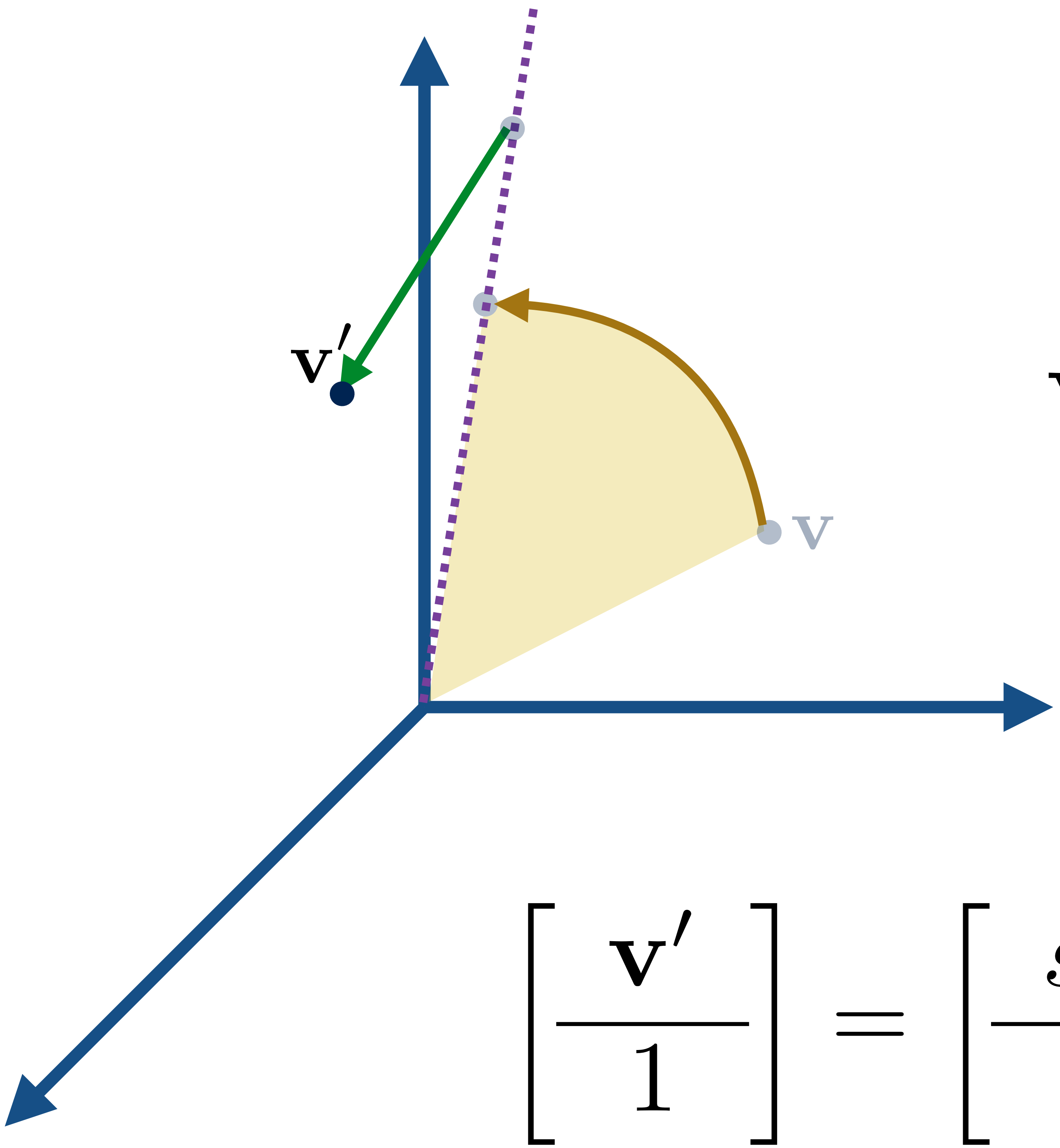
Rotation + Scale + Translation



$$R : \mathbb{R}^3 \to \mathbb{R}^3$$

$$\mathbf{v}' = sR\mathbf{v} + \mathbf{t}$$

Rotation + Scale + Translation



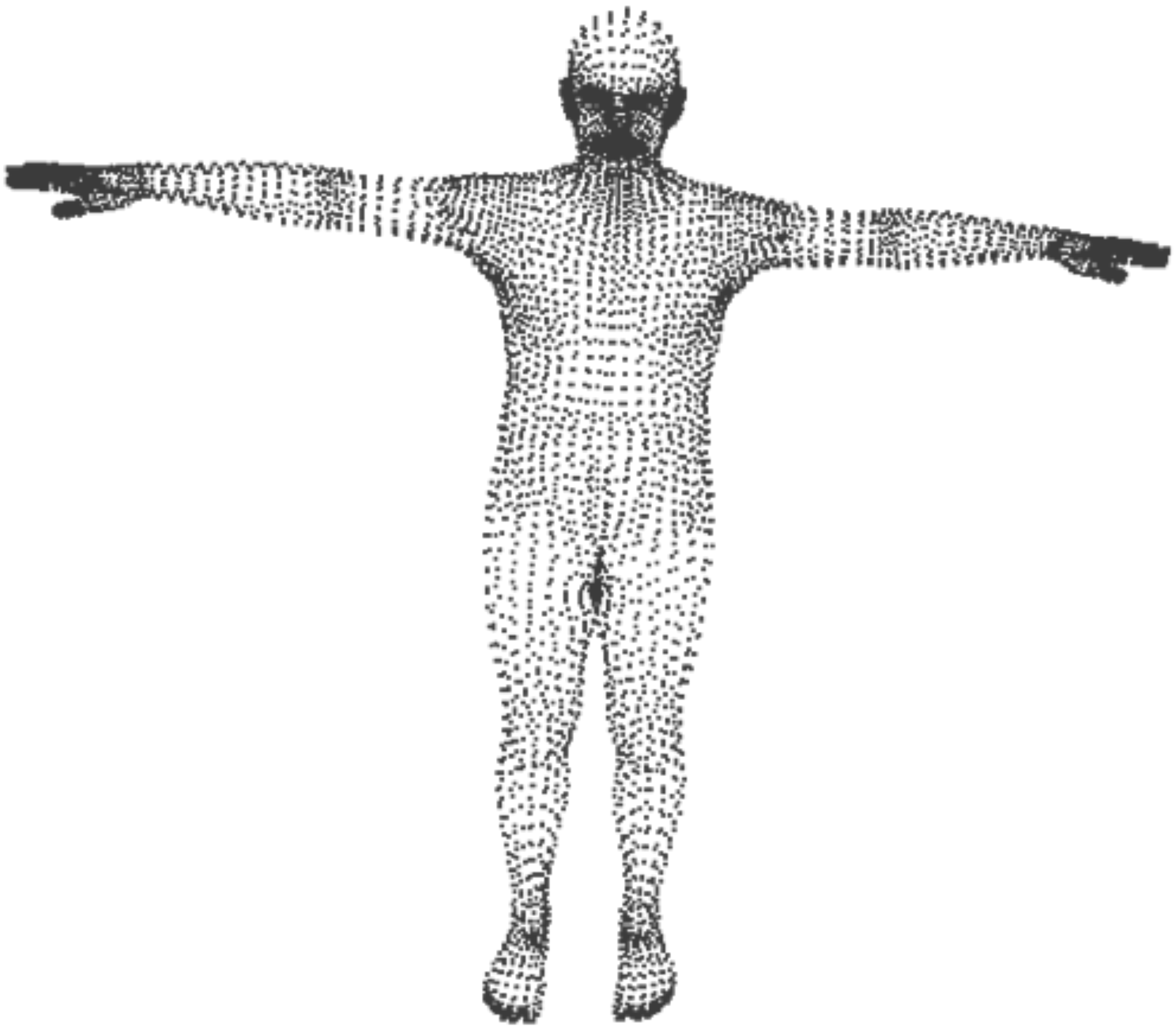
$$R : \mathbb{R}^3 \to \mathbb{R}^3$$

$$\mathbf{v}' = sR\mathbf{v} + \mathbf{t}$$

$$\left[\begin{array}{c} \mathbf{v}' \\ \hline 1 \end{array}\right] = \left[\begin{array}{c|c} s\mathbf{R} & \mathbf{t} \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c} \mathbf{v} \\ \hline 1 \end{array}\right]$$

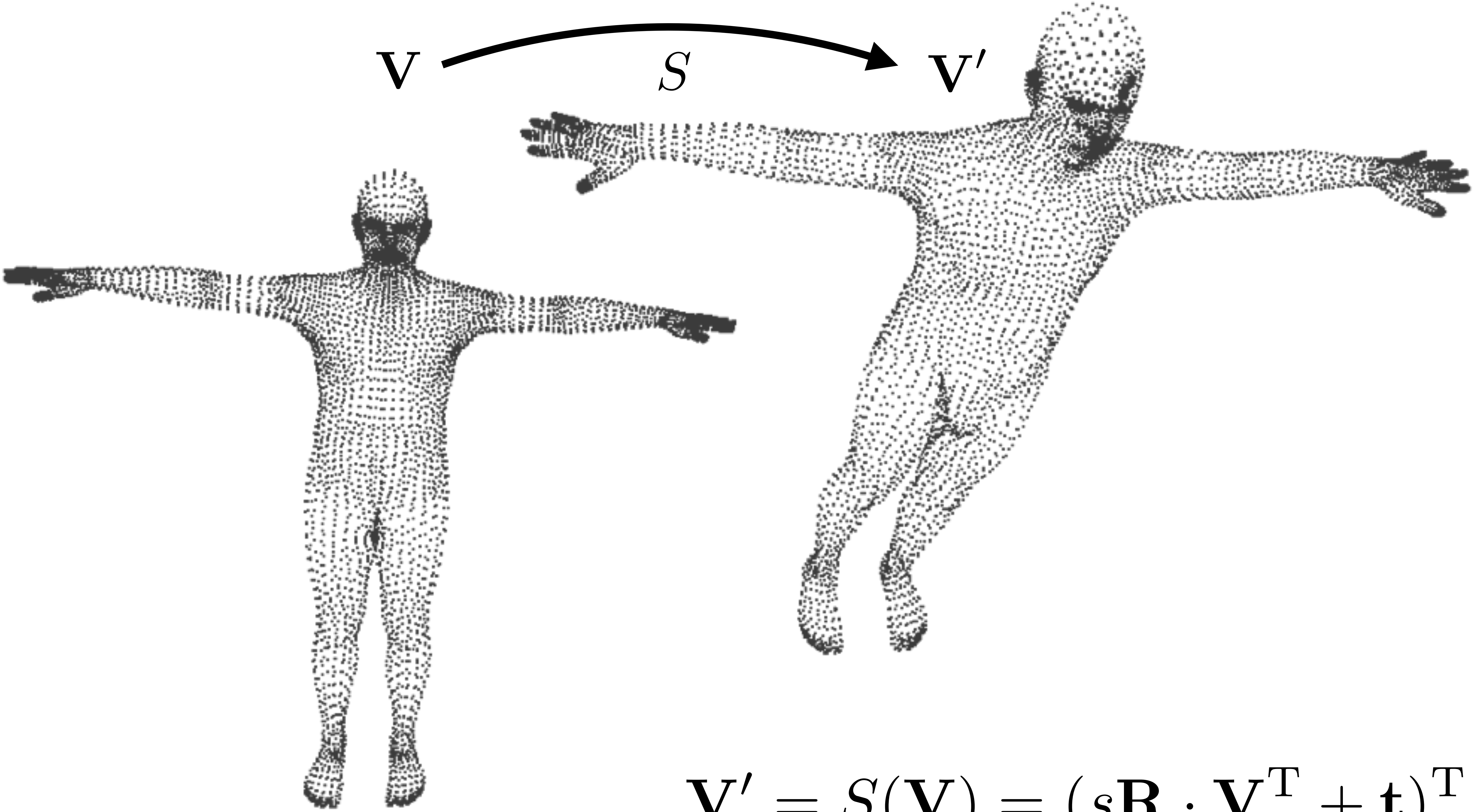
Similarity transform

$$\mathbf{v} \xrightarrow{S} \mathbf{v}'$$



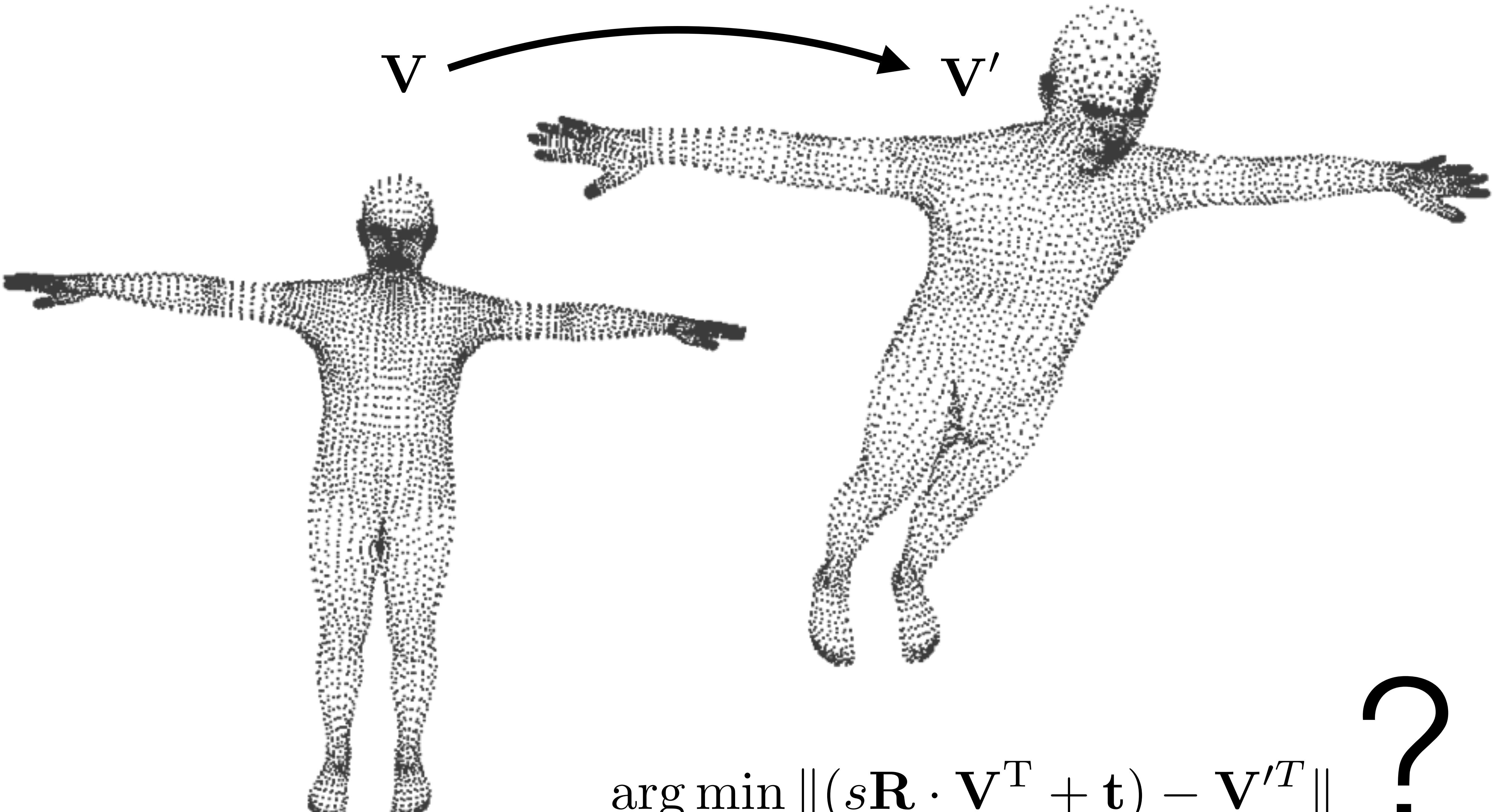
?

Similarity transform



$$\mathbf{V}' = S(\mathbf{V}) = (s\mathbf{R} \cdot \mathbf{V}^{\mathrm{T}} + \mathbf{t})^{\mathrm{T}}$$

How can we estimate S ?



$$\arg\min_{S} \|(s\mathbf{R} \cdot \mathbf{V}^{\mathrm{T}} + \mathbf{t}) - \mathbf{V}'^{T}\|$$?

Procrustes a.k.a. he who stretches

Procrustes

$$s, \mathbf{R}, \mathbf{t} = \arg\min_{s,\mathbf{R},\mathbf{t}} \sum_i \|s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i\|^2$$

warning: change of variable names V->X, V'->Y

SVD

in general, applied to a real matrix:

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathsf{T}}$$

$$\mathbf{A} \equiv (M \times N) \text{ real}$$

$$\mathbf{U} \equiv (M \times M) \text{ orthogonal, unit norm}$$

$$\mathbf{V} \equiv (N \times N) \text{ orthogonal, unit norm}$$

$$\mathbf{\Sigma} \equiv (M \times N) \text{ diagonal}$$

warning: this is not the vertex matrix!

SVD

applied to a 3D matrix

$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^{\mathsf{T}}$$

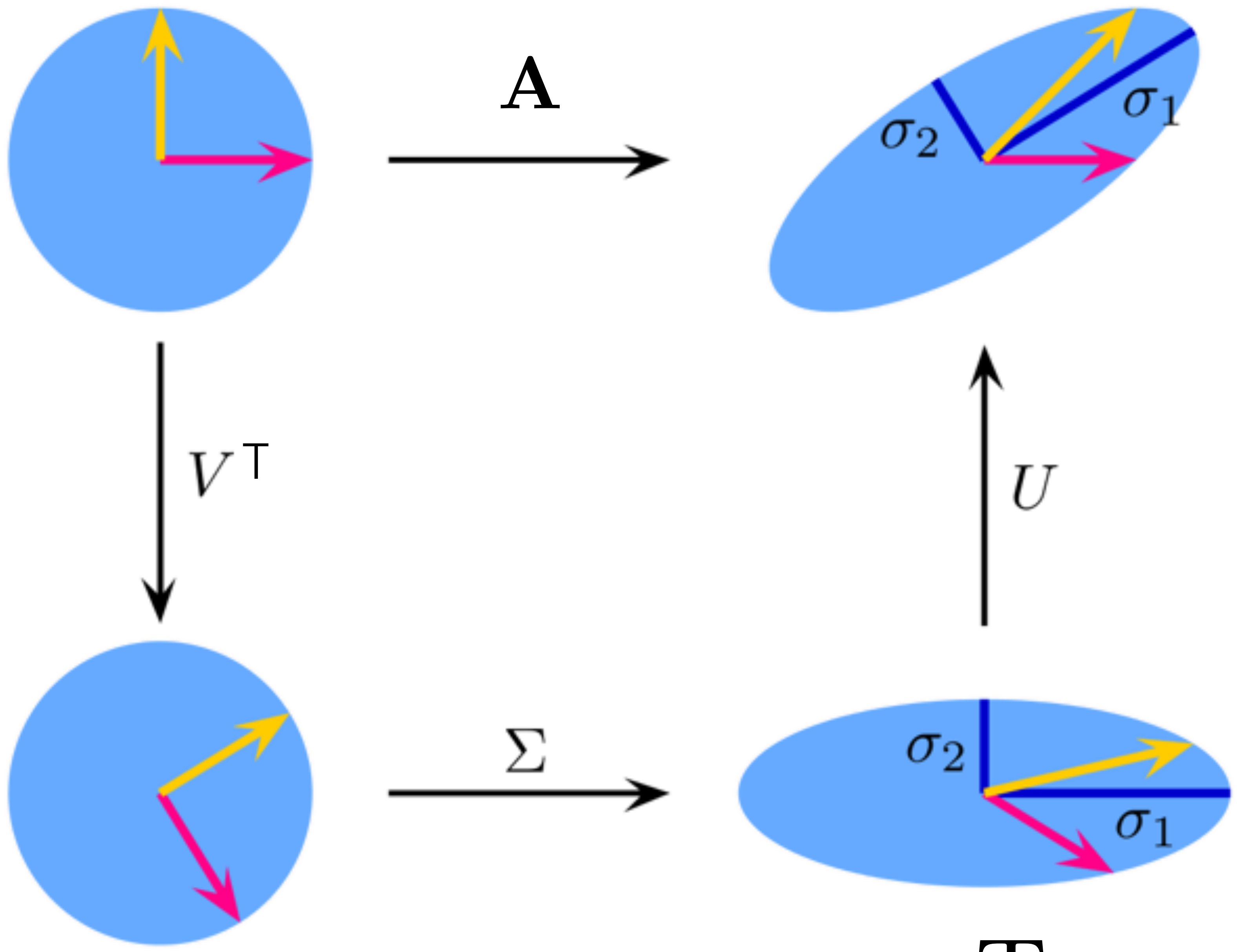
$$\mathbf{A} \equiv (3 \times 3) \text{ real}$$

$$\mathbf{U} \equiv (3 \times 3) \text{ rotation } + \text{ mirroring matrix}$$

$$\mathbf{V} \equiv (3 \times 3) \text{ rotation } + \text{ mirroring matrix}$$

$$\Sigma \equiv (3 \times 3) \text{ 3D scaling}$$

SVD



$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{W}^{\mathrm{T}}$$

Procrustes

Let's call V and V', X and Y

$$\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{(N \times 3)}$$

$$\mathbf{x}_i, \mathbf{y}_i \in \mathbb{R}^{(3 \times 1)}$$

$$s, \mathbf{R}, \mathbf{t} = \arg \min_{s, \mathbf{R}, \mathbf{t}} E$$

$$E \equiv \sum_i \| s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i \|^2$$

minimize the L2 distance between transformed source points and target points

$$= \sum_i (s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i)^\mathsf{T} (s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i)$$

$$= \sum_i s^2 \mathbf{x}_i^\mathsf{T} \mathbf{x}_i + \mathbf{t}^\mathsf{T}\mathbf{t} + \mathbf{y}_i^\mathsf{T}\mathbf{y}_i + 2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{t} - 2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{y}_i - 2\mathbf{t}^\mathsf{T}\mathbf{y}_i$$

Procrustes

If we remove the elements that do not depend on the translation and solve for t

$$\mathbf{t} = \arg\min_{\mathbf{t}} E = \arg\min_{\mathbf{t}} \sum_i \cancel{s^2 \mathbf{x}_i^\mathsf{T} \mathbf{x}_i} + \mathbf{t}^\mathsf{T}\mathbf{t} + \cancel{\mathbf{y}_i^\mathsf{T}\mathbf{y}_i} + 2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{t} - \cancel{2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{y}_i} - 2\mathbf{t}^\mathsf{T}\mathbf{y}_i$$

$$= \arg\min_{\mathbf{t}} \sum_i \mathbf{t}^\mathsf{T}\mathbf{t} + 2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{t} - 2\mathbf{t}^\mathsf{T}\mathbf{y}_i$$

$$\bar{\mathbf{x}} \equiv \frac{\sum_i \mathbf{x}_i}{N}, \bar{\mathbf{y}} \equiv \frac{\sum_i \mathbf{y}_i}{N}$$

compute the centroid of the point clouds

$$\mathbf{t} = \arg\min_{\mathbf{t}} E = \arg\min_{\mathbf{t}} (\mathbf{t}^\mathsf{T}(2s\mathbf{R}\bar{\mathbf{x}} + \mathbf{t} - 2\bar{\mathbf{y}})) = \bar{\mathbf{y}} - s\mathbf{R}\bar{\mathbf{x}}$$

So given s and R, we can compute the translation t

Procrustes

subtract the centroid from the points to obtain a simpler expression for E

$$\bar{\mathbf{x}}_i \equiv \mathbf{x}_i - \bar{\mathbf{x}}, \bar{\mathbf{y}}_i \equiv \mathbf{y}_i - \bar{\mathbf{y}}$$

$$E = \sum_i \|s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i\|^2 = \sum_i \|s\mathbf{R}\bar{\mathbf{x}}_i - \bar{\mathbf{y}}_i\|^2 = \sum_i s^2 \bar{\mathbf{x}}_i{}^\mathsf{T}\bar{\mathbf{x}}_i + \bar{\mathbf{y}}_i{}^\mathsf{T}\bar{\mathbf{y}}_i - 2s\bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i$$

$$\mathbf{A} \equiv \bar{\mathbf{X}}^\mathsf{T}\bar{\mathbf{Y}} = \sum_i \bar{\mathbf{x}}\bar{\mathbf{y}}^\mathsf{T}$$

define the cross-covariance of X and Y

$$\mathbf{R} = \arg\min_{\mathbf{R}} E = \arg\min_{\mathbf{R}} (\sum_i \cancel{s^2 \bar{\mathbf{x}}_i{}^\mathsf{T}\bar{\mathbf{x}}_i + \bar{\mathbf{y}}_i{}^\mathsf{T}\bar{\mathbf{y}}_i} - 2s\bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i)$$

$$= \arg\max_{\mathbf{R}} (\sum_i \bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i) = \arg\max_{\mathbf{R}} (\sum_i \mathrm{tr}(\bar{\mathbf{x}}_i\bar{\mathbf{y}}_i{}^\mathsf{T}\mathbf{R})) = \arg\max_{\mathbf{R}} (\mathrm{tr}(\mathbf{A}\mathbf{R}))$$

Procrustes

$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^\mathsf{T}, \text{ through SVD}$$

Decompose the cross-covariance with SVD

$$\mathbf{R} = \mathbf{V}\mathbf{U}^\mathsf{T} \text{ because}$$

$$\mathbf{AR} = (\mathbf{U}\Sigma\mathbf{V}^\mathsf{T})(\mathbf{V}\mathbf{U}^\mathsf{T}) = \mathbf{U}\Sigma\mathbf{U}^\mathsf{T} \text{ symmetric positive semidefinite, so } \mathbf{R} \text{ maximizes } \mathrm{tr}(\mathbf{AR})$$

Theorem 1: The trace of a square matrix is smaller than the sum of its eigenvalues, equal if an only if the matrix is symmetric and positive semi-definite

Theorem 2: For a square matrix $\mathbf{B}$, there is an orthogonal matrix $\mathbf{C}$ such that $\mathbf{BC}$ is symmetric and positive semi-definite. If $\mathbf{D}$ is any other orthogonal matrix, $\mathrm{tr}(\mathbf{BD}) \leq \mathrm{tr}(\mathbf{BC})$, equal if and only if $\mathbf{BD}$ is symmetric and positive semi-definite

formulation thanks to Dr. Dan Curtis

Procrustes

Optimize scale given the rotation

$$s = \arg\min_s E = \arg\min_s (\sum_i s^2 \bar{\mathbf{x}}_i{}^\top \bar{\mathbf{x}}_i + \cancel{\bar{\mathbf{y}}_i{}^\top \bar{\mathbf{y}}_i} - 2s\bar{\mathbf{x}}_i{}^\top \mathbf{R}^\top \bar{\mathbf{y}}_i)$$

$$= \arg\min_s (s^2 \sum_i (\bar{\mathbf{x}}_i{}^\top \bar{\mathbf{x}}_i) - 2s \sum_i (\bar{\mathbf{x}}_i{}^\top \mathbf{R}^\top \bar{\mathbf{y}}_i))$$

$$= \arg\min_s (s^2 a - 2sb) = \frac{b}{a} = \frac{\bar{\mathbf{x}}_i{}^\top \mathbf{R}^\top \bar{\mathbf{y}}_i}{\bar{\mathbf{x}}_i{}^\top \bar{\mathbf{x}}_i}$$

$$= \frac{\mathrm{tr}(\mathbf{AR})}{\|\bar{\mathbf{X}}\|^2} = \frac{\mathrm{tr}(\Sigma)}{\|\bar{\mathbf{X}}\|^2}$$

Procrustes

whole algorithm

$$\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{(N\times 3)}$$
$$\mathbf{x}_i, \mathbf{y}_i \in \mathbb{R}^{(3\times 1)}$$
$$s, \mathbf{R}, \mathbf{t} = \arg\min_{s,\mathbf{R},\mathbf{t}} E$$
$$E \equiv \sum_i \|s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i\|^2$$
$$= \sum_i (s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i)^\mathsf{T}(s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i)$$
$$= \sum_i s^2\mathbf{x}_i^\mathsf{T}\mathbf{x}_i + \mathbf{t}^\mathsf{T}\mathbf{t} + \mathbf{y}_i^\mathsf{T}\mathbf{y}_i + 2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{t} - 2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{y}_i - 2\mathbf{t}^\mathsf{T}\mathbf{y}_i$$
$$\mathbf{t} = \arg\min_{\mathbf{t}} E = \arg\min_{\mathbf{t}} \sum_i \cancel{s^2\mathbf{x}_i^\mathsf{T}\mathbf{x}_i} + \mathbf{t}^\mathsf{T}\mathbf{t} + \cancel{\mathbf{y}_i^\mathsf{T}\mathbf{y}_i} + 2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{t} - \cancel{2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{y}_i} - 2\mathbf{t}^\mathsf{T}\mathbf{y}_i$$
$$= \arg\min_{\mathbf{t}} \sum_i \mathbf{t}^\mathsf{T}\mathbf{t} + 2s\mathbf{x}_i^\mathsf{T}\mathbf{R}^\mathsf{T}\mathbf{t} - 2\mathbf{t}^\mathsf{T}\mathbf{y}_i$$
$$\bar{\mathbf{x}} \equiv \frac{\sum_i \mathbf{x}_i}{N}, \bar{\mathbf{y}} \equiv \frac{\sum_i \mathbf{y}_i}{N}$$
$$\mathbf{t} = \arg\min_{\mathbf{t}} E = \arg\min_{\mathbf{t}}(\mathbf{t}^\mathsf{T}(2s\mathbf{R}\bar{\mathbf{x}} + \mathbf{t} - 2\bar{\mathbf{y}})) = \bar{\mathbf{y}} - s\mathbf{R}\bar{\mathbf{x}}$$
$$\bar{\mathbf{x}}_i \equiv \mathbf{x}_i - \bar{\mathbf{x}}, \bar{\mathbf{y}}_i \equiv \mathbf{y}_i - \bar{\mathbf{y}}$$
$$E = \sum_i \|s\mathbf{R}\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i\|^2 = \sum_i \|s\mathbf{R}\bar{\mathbf{x}}_i - \bar{\mathbf{y}}_i\|^2 = \sum_i s^2\bar{\mathbf{x}}_i{}^\mathsf{T}\bar{\mathbf{x}}_i + \bar{\mathbf{y}}_i{}^\mathsf{T}\bar{\mathbf{y}}_i - 2s\bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i$$
$$\mathbf{A} \equiv \bar{\mathbf{X}}^\mathsf{T}\bar{\mathbf{Y}} = \sum_i \bar{\mathbf{x}}\bar{\mathbf{y}}^\mathsf{T}$$
$$\mathbf{R} = \arg\min_{\mathbf{R}} E = \arg\min_{\mathbf{R}}(\sum_i \cancel{s^2\bar{\mathbf{x}}_i{}^\mathsf{T}\bar{\mathbf{x}}_i + \bar{\mathbf{y}}_i{}^\mathsf{T}\bar{\mathbf{y}}_i} - 2s\bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i)$$
$$= \arg\max_{\mathbf{R}}(\sum_i \bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i) = \arg\max_{\mathbf{R}}(\sum_i \mathrm{tr}(\bar{\mathbf{x}}_i\bar{\mathbf{y}}_i{}^\mathsf{T}\mathbf{R})) = \arg\max_{\mathbf{R}}(\mathrm{tr}(\mathbf{A}\mathbf{R}))$$
$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^\mathsf{T}, \text{ through SVD}$$
$$\mathbf{R} = \mathbf{V}\mathbf{U}^\mathsf{T} \text{ because}$$
$$\mathbf{A}\mathbf{R} = (\mathbf{U}\Sigma\mathbf{V}^\mathsf{T})(\mathbf{V}\mathbf{U}^\mathsf{T}) = \mathbf{U}\Sigma\mathbf{U}^\mathsf{T} \text{ symmetric positive semidefinite, so } \mathbf{R} \text{ maximizes } \mathrm{tr}(\mathbf{A}\mathbf{R})$$
$$s = \arg\min_{s} E = \arg\min_{s}(\sum_i s^2\bar{\mathbf{x}}_i{}^\mathsf{T}\bar{\mathbf{x}}_i + \cancel{\bar{\mathbf{y}}_i{}^\mathsf{T}\bar{\mathbf{y}}_i} - 2s\bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i)$$
$$= \arg\min_{s}(s^2\sum_i(\bar{\mathbf{x}}_i{}^\mathsf{T}\bar{\mathbf{x}}_i) - 2s\sum_i(\bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i))$$
$$= \arg\min_{s}(s^2a - 2sb) = \frac{b}{a} = \frac{\bar{\mathbf{x}}_i{}^\mathsf{T}\mathbf{R}^\mathsf{T}\bar{\mathbf{y}}_i}{\bar{\mathbf{x}}_i{}^\mathsf{T}\bar{\mathbf{x}}_i}$$
$$= \frac{\mathrm{tr}(\mathbf{A}\mathbf{R})}{\|\mathbf{X}\|^2} = \frac{\mathrm{tr}(\Sigma)}{\|\mathbf{X}\|^2}$$

Applications

As-Rigid-As-Possible Surface Modeling, Sorkine and Alexa

Take-home message

- A human body can be expressed as a graphical model with pairwise terms modelling the relation between parts
- However, we will study, a holistic model which is more realistic
- Translations and rotations are basic building blocks in geometric transformations
- The optimal similarity transformation between two point sets can be obtained with the Procrustes algorithm