Xorshift* and Erlang/OTP: Searching for Better PRNGs

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Executive summary: do not try inventing your own random number generators!

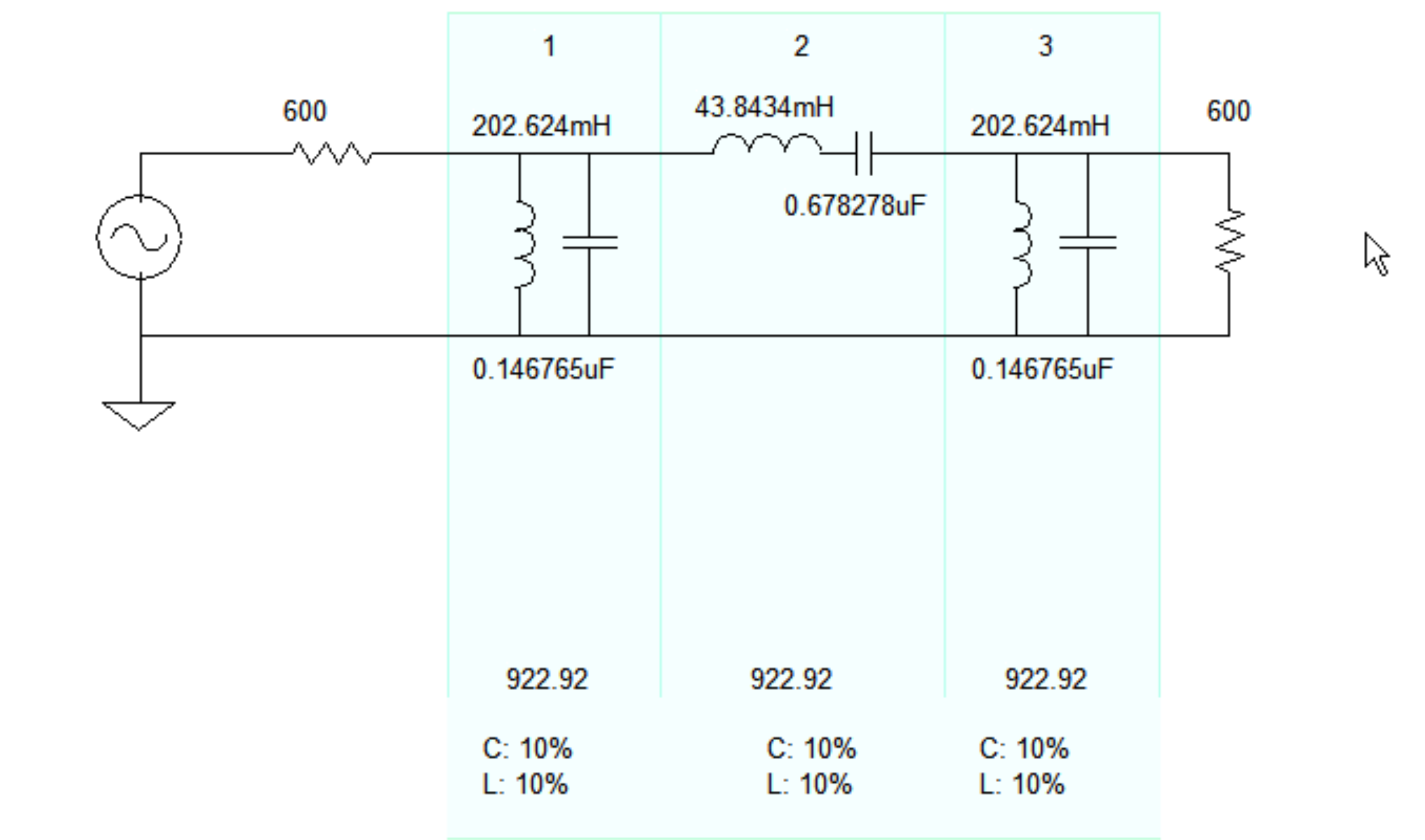
PRNGs matter

- The first talk of pseudo random number generators in Erlang Factory events was on 2011
- Now four years later, people are *still using* the good-old `random` module, already *fully exploited*. **We should stop using it!**
- So I decided to do the talk again with new algorithms, and the talk is accepted

PRNGs are everywhere

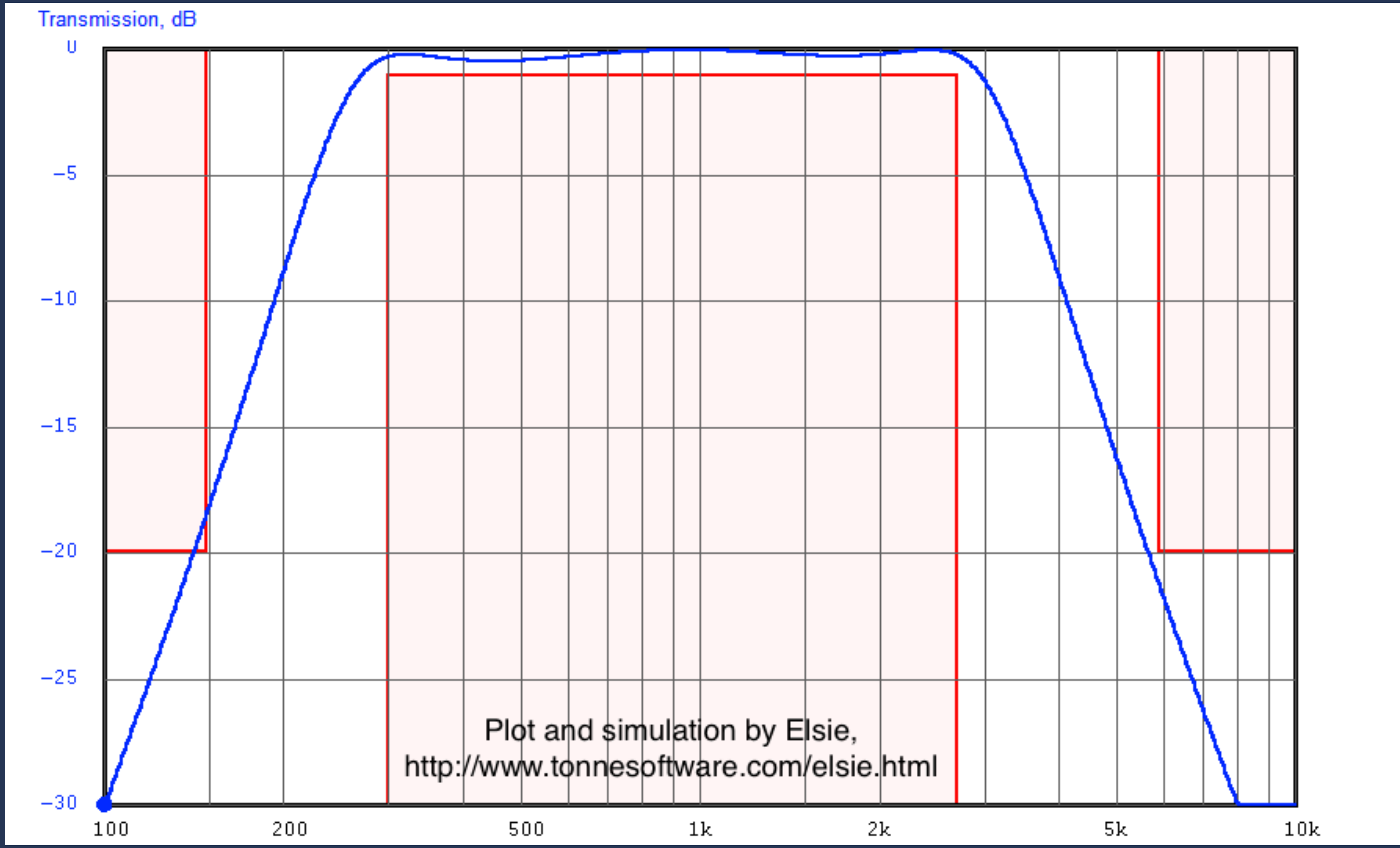
- Rolling dice (for games)
- (Property) testing (QuickCheck, ProPer, Triq)
- Variation analysis of electronic circuits
- Network congestion and delay analysis
- Risk analysis of project schedules
- Passwords (*Secure PRNGs* only!)

Variation analysis of a band pass filter

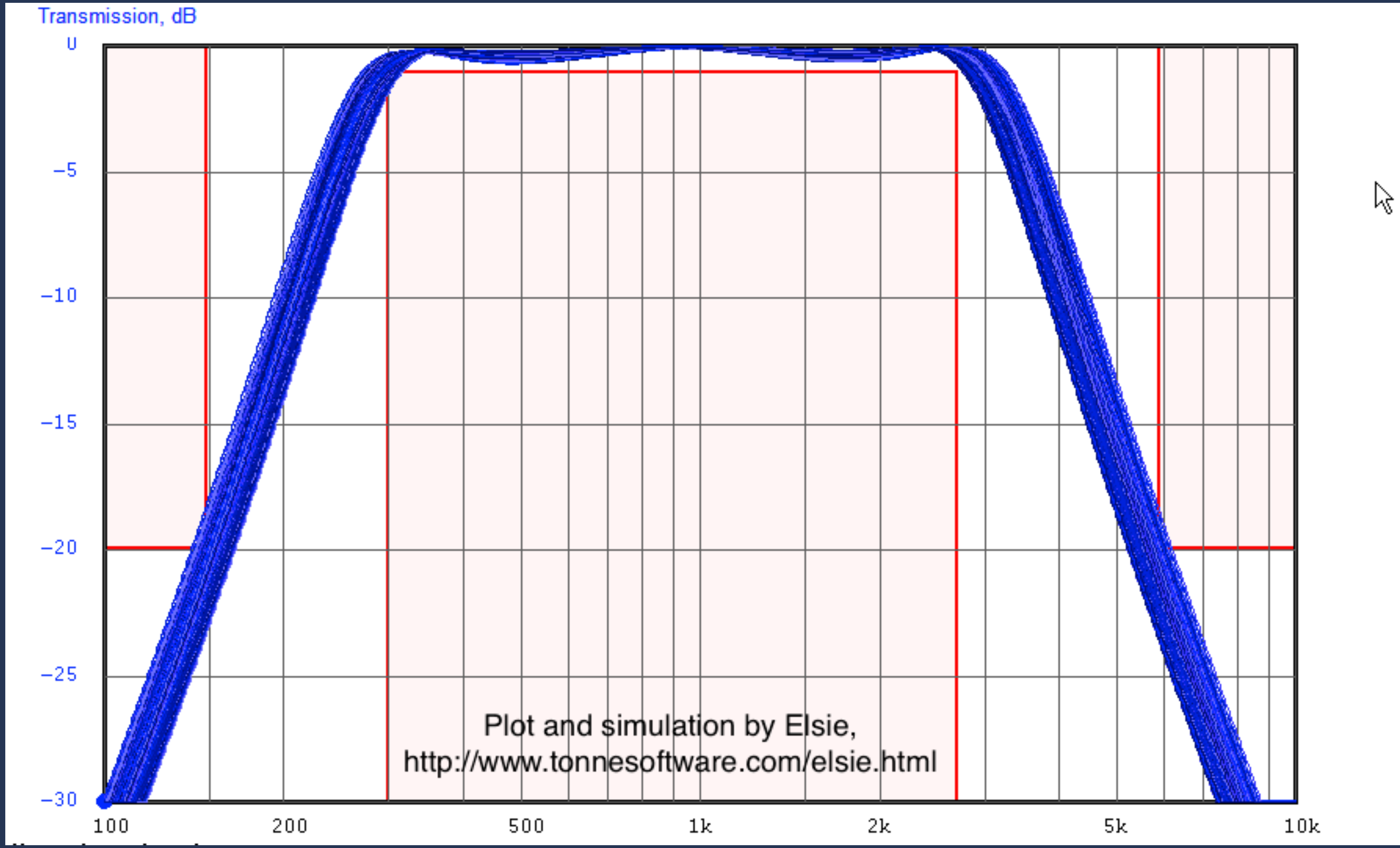


A circuit example of Elsie, http://www.tonnesoftware.com/elsie.html

Without variance



With 10% variance



How PRNG works

- Sequential iterative process
- For multiple processes, seeds and other parameters should be chosen carefully to prevent sequence overlapping

```
% Give a seed S1
{Result1, S2} = prng(S1),
{Result2, S3} = prng(S2),
% ... and on and on
```

NOT in this talk: Secure PRNGs

- For password and cryptographic key generation with strong security
- Use `crypto:strong_rand_bytes/1`
- Remember entropy gathering takes time
- This is *cryptography* - use and *only use* proven algorithms! *Do not invent yours!*

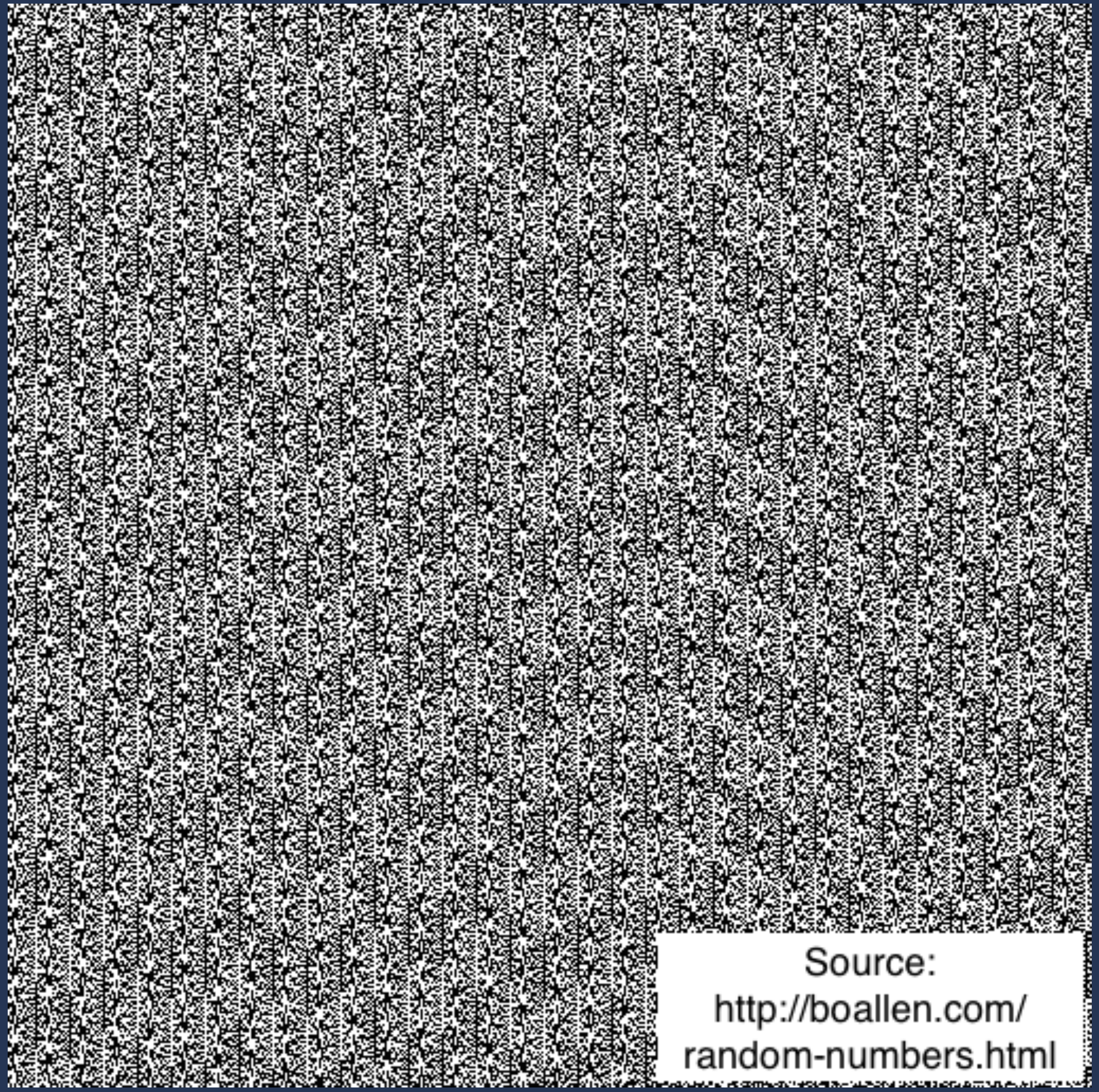
In this talk: non-secure PRNGs

- May be *vulnerable to cryptographic attacks*
- (Uniform) distribution guaranteed
- *Predictive*: same seed = same result
- Lots of seed (internal state) choices
- Long period: no intelligible patterns

Even non-secure PRNGs fail

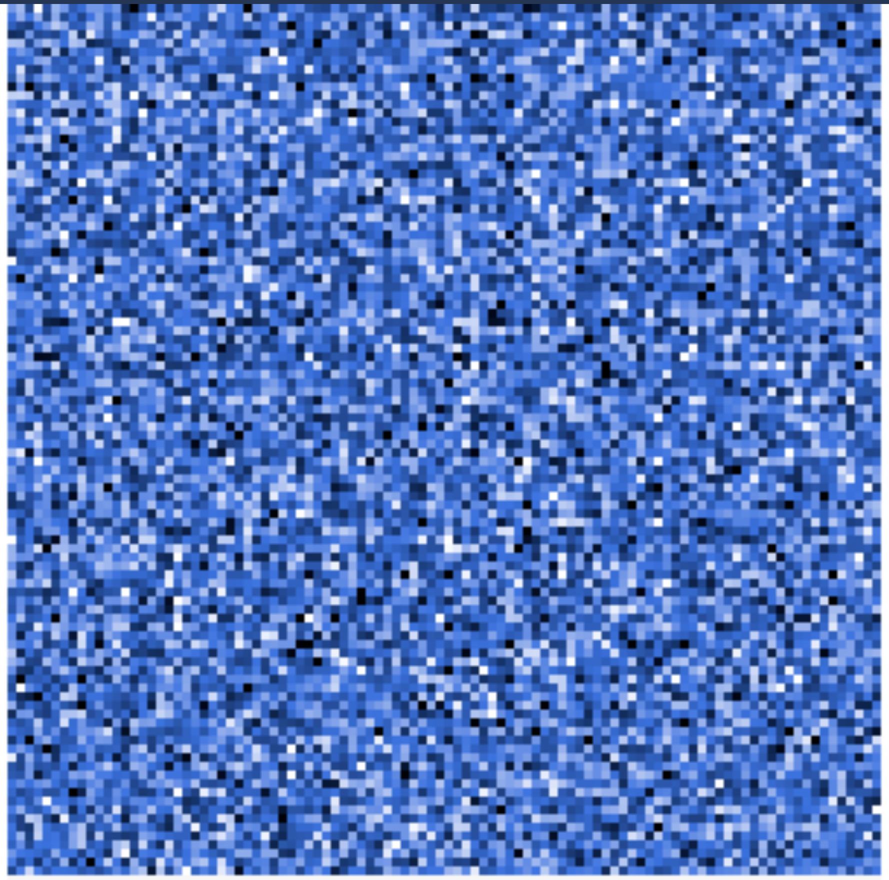
- Found from the observable patterns by making a graphical representation
- *Very short period* of showing up the same number sequence again
- Even a fairly long sequence of numbers can be *fully exploited* and made predictable

PHP5 on Windows (2012)

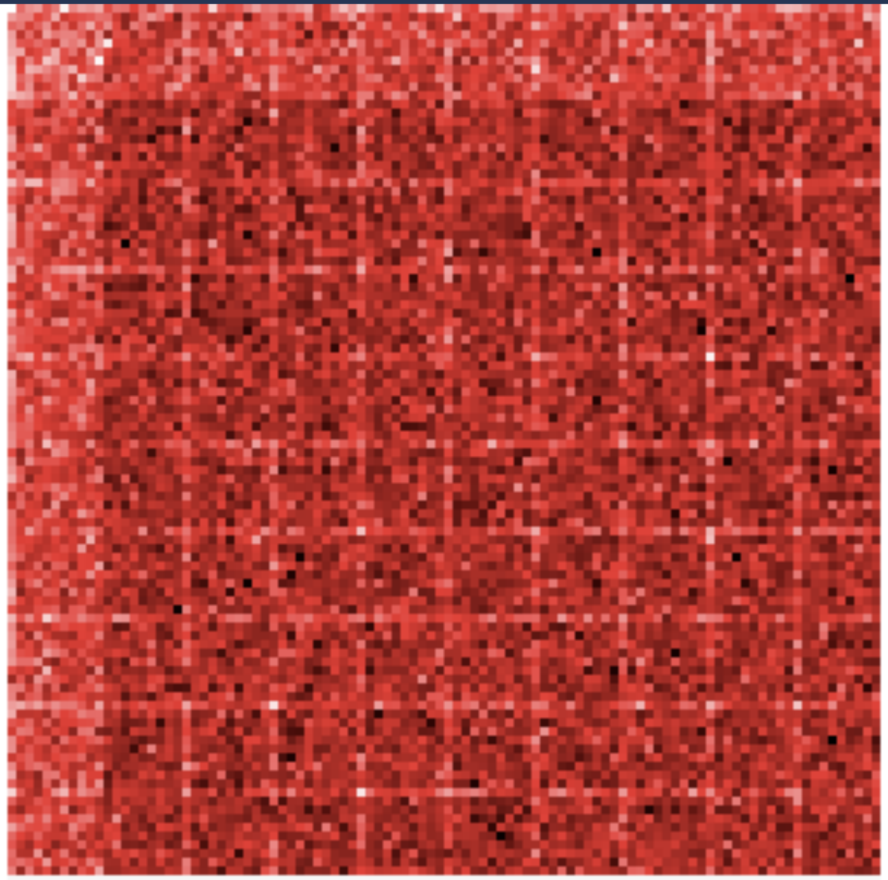


Other PRNG failures

- Cryptocat 2013 (blue: OK, red: bad)



Colourmap of 20,000,000 Cryptocat floats (derived from /dev/urandom values 0..249)



Colourmap of 20,000,000 old-school Cryptocat floats (derived from PRNG values 0..250)

Erlang/OTP's first ever security advisory

- ... was about PRNG! (R14B02, 2011)
- US CERT VU#178990: Erlang/OTP SSH library uses a weak random number generator (CVE-2011-0766)
- Used `random` non-secure PRNG for the SSH session RNG seed, easily exploitable

Erlang `random`'s problem

- The algorithm AS183 is too old (designed in 1980s for 16-bit computers)
- Period: 6953607871644 ~= 2^(42.661), too short for modern computer exploits
- Fully exploited in < 9 hours on Core i5 (single core) (my C source) - Richard O'Keefe told me this was *nothing new in either academic and engineering perspectives* (he is right!)

Alternative Erlang PRNGs

- sfmt-erlang (SFMT, 2^19937-1, 32-bit)
- tinymt-erlang (TinyMT, 2^127-1, ~2^56 orthogonal sequences, 32-bit)
- exs64 (XorShift*64, 2^64-1, 64-bit)
- exsplus (Xorshift+128, 2^128-1, 64-bit)
- exs1024 (Xorshift*1024, 2^1024-1, 64-bit)

SFMT

- Mersenne Twister: default PRNG on Python, MATLAB, C++11, R, etc.
- Internal state: 624 32-bit integers (2496 bytes)
- SIMD-oriented Fast Mersenne Twister (SFMT) = MT improved
- Extremely long period (2^19937-1, longer variants available)

sfmt-erlang: on NIFs

sfmt-erlang gains a lot by NIFs because:

- It needs bulk state initialization (624 x 32-bit)
- NIFnizing it makes total execution time ~16 times faster (on FreeBSD, OTP 17.4.1)
- Execution time of state initialization: ~100 times faster (~1600 -> ~15 microseconds)

TinyMT

- Tiny Mersenne Twister for restricted resources
- Shorter but sufficient period (2^127-1)
- 127-bit state + three 32-bit words for the polynomial parameters
- ~2^56 choice of orthogonal polynomials, suitable for parallelism
- On Erlang: non-NIF only

tinymt-erlang: on NIFs

tinymt-erlang did not gain much from NIFs presumably because:

- No bulk initialization, state calculation complexity is small
- Most of execution time: function calling overhead
- In NIFs, sfmt-erlang was *faster* for generating a large sequence

So are NIFs effective?

- Not really, unless processing a bulk generation/computation
- Remember NIFs *block* the scheduler
- If NIFs are not needed, don't use them
- If NIFs are really needed, tuning the scheduler is *inevitable* - ask the gurus for the details

Xorshift*/+ algorithms

- Marsaglia's Xorshift, output scrambled by the algorithm of Sebastiano Vigna for the best result against TestU01 strength test
- Xorshift64*, Xorshift128+, Xorshift1024* are so far the most practical three choices
- C code in public domain
- Deceptively simple

Xorshift64*

```
% See https://github.com/jj1bdx/exs64
-type uint64() :: 0..16#ffffffffffffffff.
-opaque state() :: uint64().
-define(UINT64MASK, 16#ffffffffffffffff).
-spec next(state()) -> {uint64(), state()}.
next(R) ->
    R1 = R bxor (R bsr 12),
    R2 = R1 bxor ((R1 bsl 25) band ?UINT64MASK),
    R3 = R2 bxor (R2 bsr 27),
    {(R3 * 2685821657736338717) band ?UINT64MASK, R3}.
```

Xorshift1024* (1/2)

```
% See https://github.com/jj1bdx/exs1024
-type uint64() :: 0..16#ffffffffffffffff.
-opaque seedval() :: list(uint64()). % 16 64-bit integers
-opaque state() :: {list(uint64()), list(uint64())}.
-define(UINT64MASK, 16#ffffffffffffffff).
%% calc(S0, S1) -> {X, NS1} / X: random number output
-spec calc(uint64(), uint64()) -> {uint64(), uint64()}.
calc(S0, S1) ->
    S11 = S1 bxor ((S1 bsl 31) band ?UINT64MASK),
    S12 = S11 bxor (S11 bsr 11),
    S01 = S0 bxor (S0 bsr 30),
    NS1 = S01 bxor S12,
    {(NS1 * 1181783497276652981) band ?UINT64MASK, NS1}.
```

Xorshift1024* (2/2)

```erlang
-spec next(state()) -> {uint64(), state()}.
% with a ring buffer using a pair of lists
next({[H], RL}) ->
    next({[H|lists:reverse(RL)], []});
next({L, RL}) ->
    [S0|L2] = L,
    [S1|L3] = L2,
    {X, NS1} = calc(S0, S1),
    {X, {[NS1|L3], [S0|RL]}}.
```

Performance implications

- HiPE highly recommended
- Handling full 64-bit numbers means handling BIGNUMs and slow; short integers are up to (2^59)
- `exs64`: < x2 execution time of `random`
- `exs1024`: slower, but ~ x2 of `random`
- Speed penalty: worth being paid for

Suggested purposes for the alternative PRNGs

- sfmt-erlang: proven, can be chosen in ProPer
- tinymt-erlang: proven, has ~268 million polynomial parameters available at tinymtdc-longbatch
- exs64: replacement of AS183
- exsplus: an alternative to exs64
- exs1024: good choice for simulation

Merging to OTP (1/2)

- Dan Gudmundsson (of OTP Team) offered me to help writing a multi-algorithm successor of `random` module
- exs64/plus/1024: MIT licensed (by me)
- sfmt-erlang/tinymt-erlang: BSD licensed
- All pieces of code had to be relicensed in *Erlang Public License* to be included in OTP

Merging to OTP (2/2)

- It was expected to be called as new `random`, but the OTP team didn't want it (presumably due to backward compatibility issues), so it's called `rand`
- Project name: emprng
- `random`-compatible functions currently available for the six algorithms: as183, exs64 (default), exsplus, exs1024, sfmt, tinymt

Future directions

- Keep promoting banning/deprecating the good-old `random` module and use *something else that is much better* (try exs64)
- Merge emprng to OTP: more algorithms, user-supplied functions, tests
- Analyze performance implication on large-scale applications

Thanks
Questions?