Potential Future Exposure and Collateral Modelling of the Trading Book Using NVIDIA GPUs

19 March 2015
GPU Technology Conference 2015

Grigorios Papamanousakis
Quantitative Strategist, Investment Solutions
Aberdeen Asset Management

Jinzhe Yang
Quantitative Researcher, LDI
Aberdeen Asset Management

Grzegorz Kozikowski
Researcher
University of Manchester

Contents

Sketching the problem

Consider the problem of calculating the potential **Future Collateral Requirements** of a large derivative book for a global asset manager

- Purpose: Provide a probabilistic approach of what may be the collateral requirements, on various granularity levels (fund level, counterparty level, division level, company level, etc.) and different time horizons, of clients portfolio

Worse Case Scenario	Time Horizon			Collateral Requirement								
				Fund level			Division level			Company level		
90%	1w	1m	1y	…	…	£60mln	…	…	£45mln	…	…	£40mln
70%	1w	1m	1y	…	…	£30mln	…	…	£23mln	…	…	£20mln
Average case	1w	1m	1y	…	…	£18mln	…	…	£12mln	…	…	£10mln

- Additional information could be extracted as well. Collateral decomposition (Cash Collateral, Gov. Bonds, HY bonds, etc.), counterparty exposure, etc.
- Typically the portfolio will include interest rate swaps, swaptions, inflation swaps, cross currency swaps, equity options, CDSs, etc.

Aberdeen

Basic Definitions

- **Potential Future Exposure** (PFE) is defined as the **maximum** expected credit exposure over a specified period of time calculated at some level of confidence. PFE is a measure of counterparty credit risk.

- **Expected Exposure** (EE) is defined as the **average** exposure on a future date

- **Credit Valuation Adjustment** (CVA) is an adjustment to the price of a derivative to take into account counterparty credit risk.

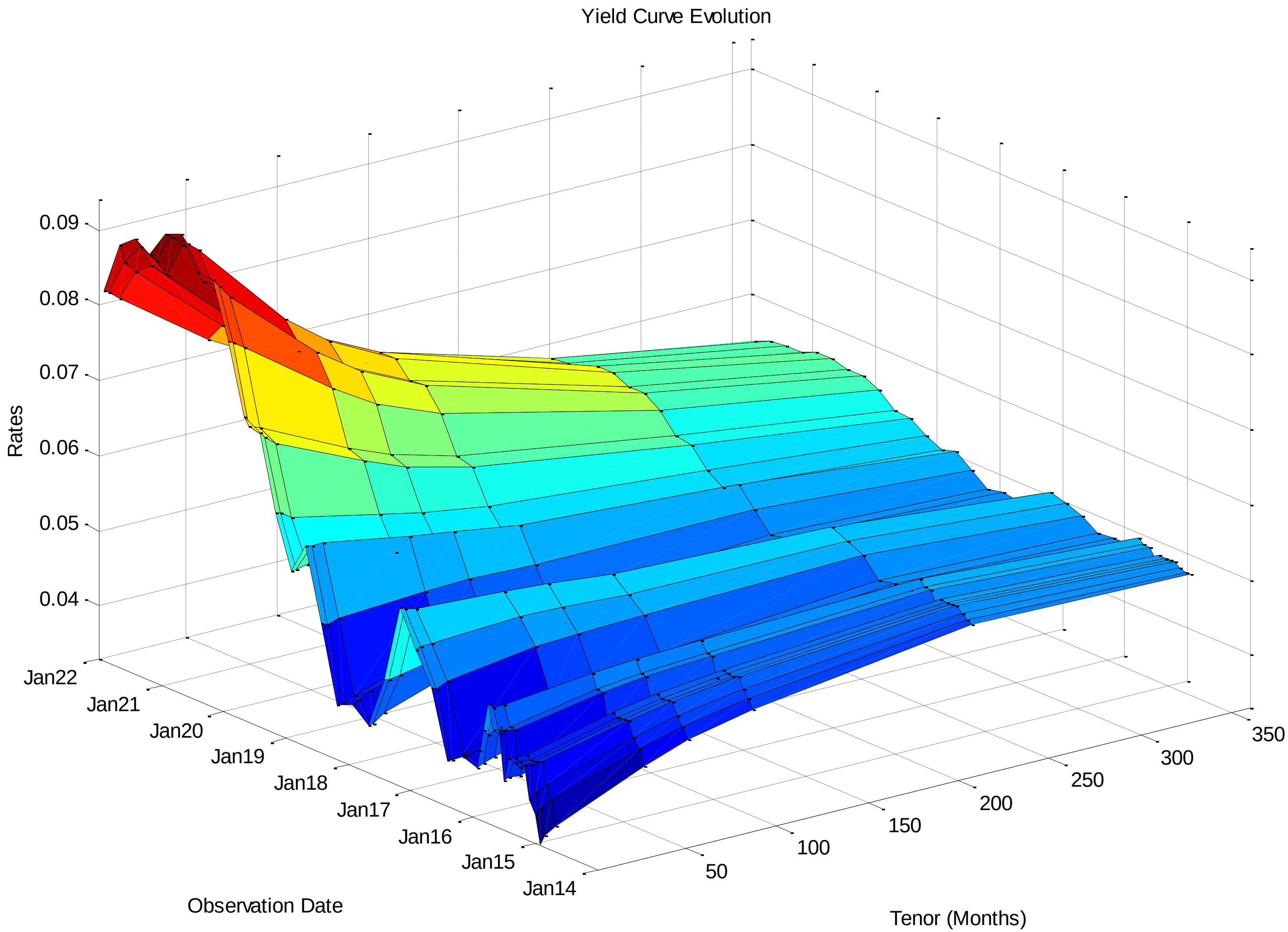
- **Collateral** can be considered any type of valuable liquid property that is pledged by the recipient as security against credit risk.

PFE Modelling Explained

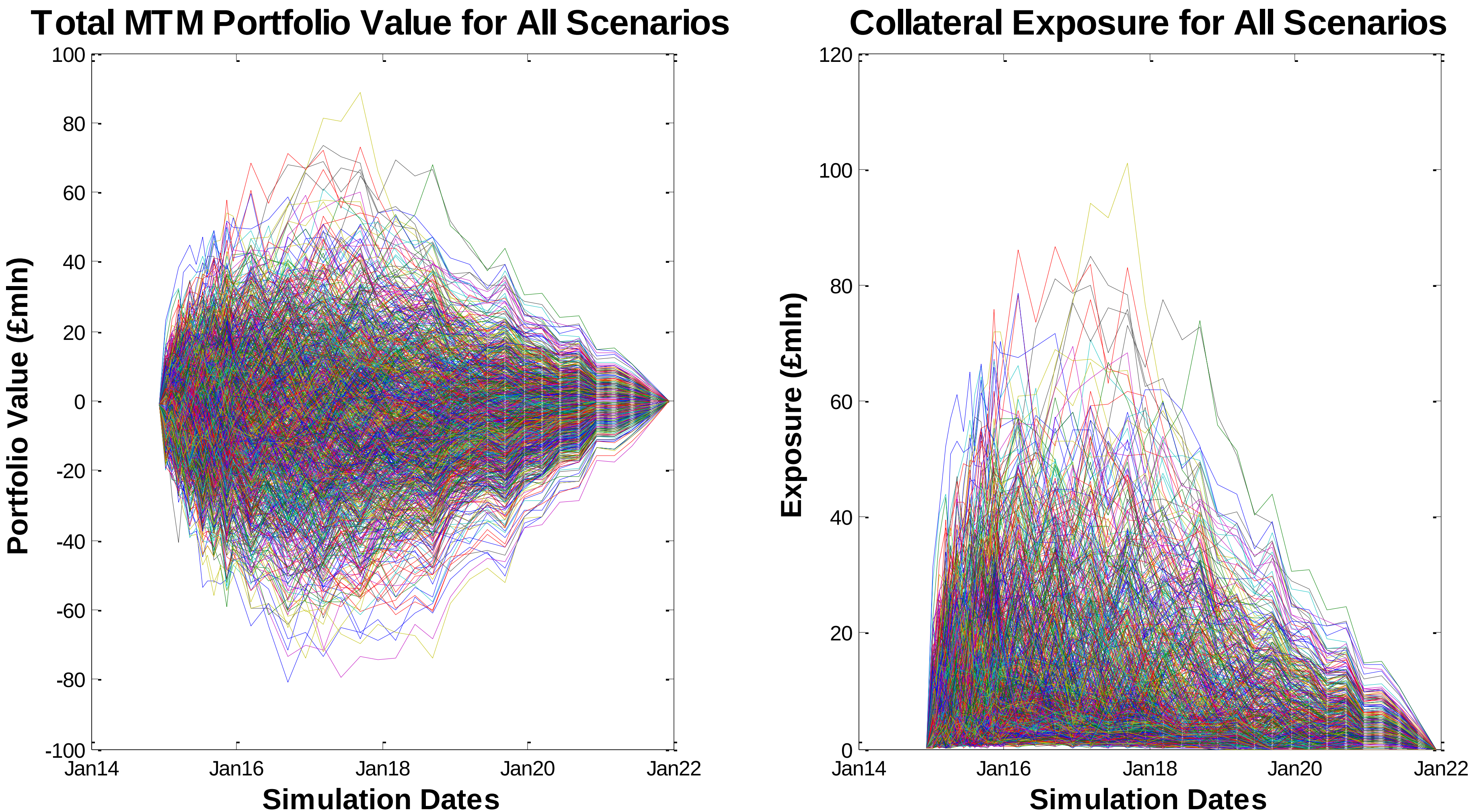
What do we need to built a ‘standard market’ PFE Model?

- A consistent simulation framework to project forward in time the interest rate curves. This will typically have **100-120 time-steps** and around **100k -1mln scenarios**
- **Multiple stochastic processes** to simulate the **FX exposure** for every different currency in the portfolio, aligned with the time steps and the number of simulations of the **interest rate scenarios**. The same idea applies for the inflation rates but with less time steps
- **Correlate** all the **Brownian motions** calibrating on the market data and use the Cholesky decomposition to form multivariate normal random variables and project forward in time on every scenario
- **Re-evaluate** every position, on every time step, for **every scenario** and aggregate per netting set to calculate the Expected Exposure - EE
- Calculate the collateral changes based on the CSA agreement with every counterparty and available collateral on our pool

Example – Yield Curve Evolution

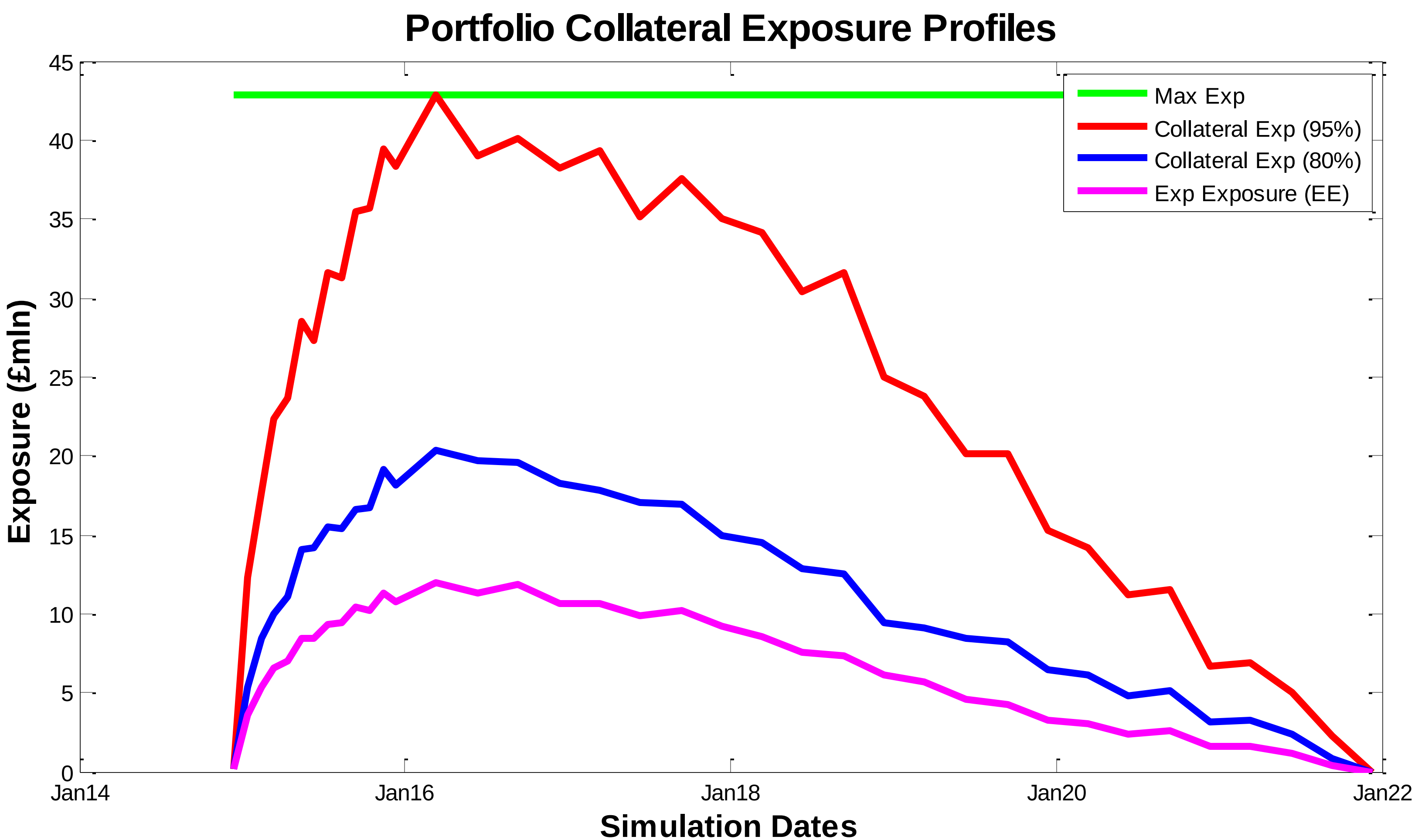


Example – Portfolio Value / Collateral Exposure over time



Example – Cash Collateral Exposure Distribution

95% & 80% Confidence level



Aberdeen

Dimensionality of the problem

The need for GPU acceleration

- For every interest rate curve we will typically have **120+ simulation steps** (50 year horizon)
- To construct the **full term structure** on every simulation step we need **20-25 tenor points**
- We simulate **different forward and discount curves** in order to capture the stochastic credit spreads between the two curves (e.g. Sonia and Libor)
- The same applies for every currency in our portfolio along with a stochastic process to simulate the FX exposure
- Depending the degree of convergence we want to achieve we simulate **100k-1mln scenarios**

So for the simulation the dimensionality of the problem is:

nSim x nSteps x nTenors x nCurves x nCurrencies

1mln x 120 x 25 x 15

For valuation purposes we have to take under consideration on top of that the size of the portfolio as well as the NO-Cash collateral

Contents

CSA-dependent Valuation

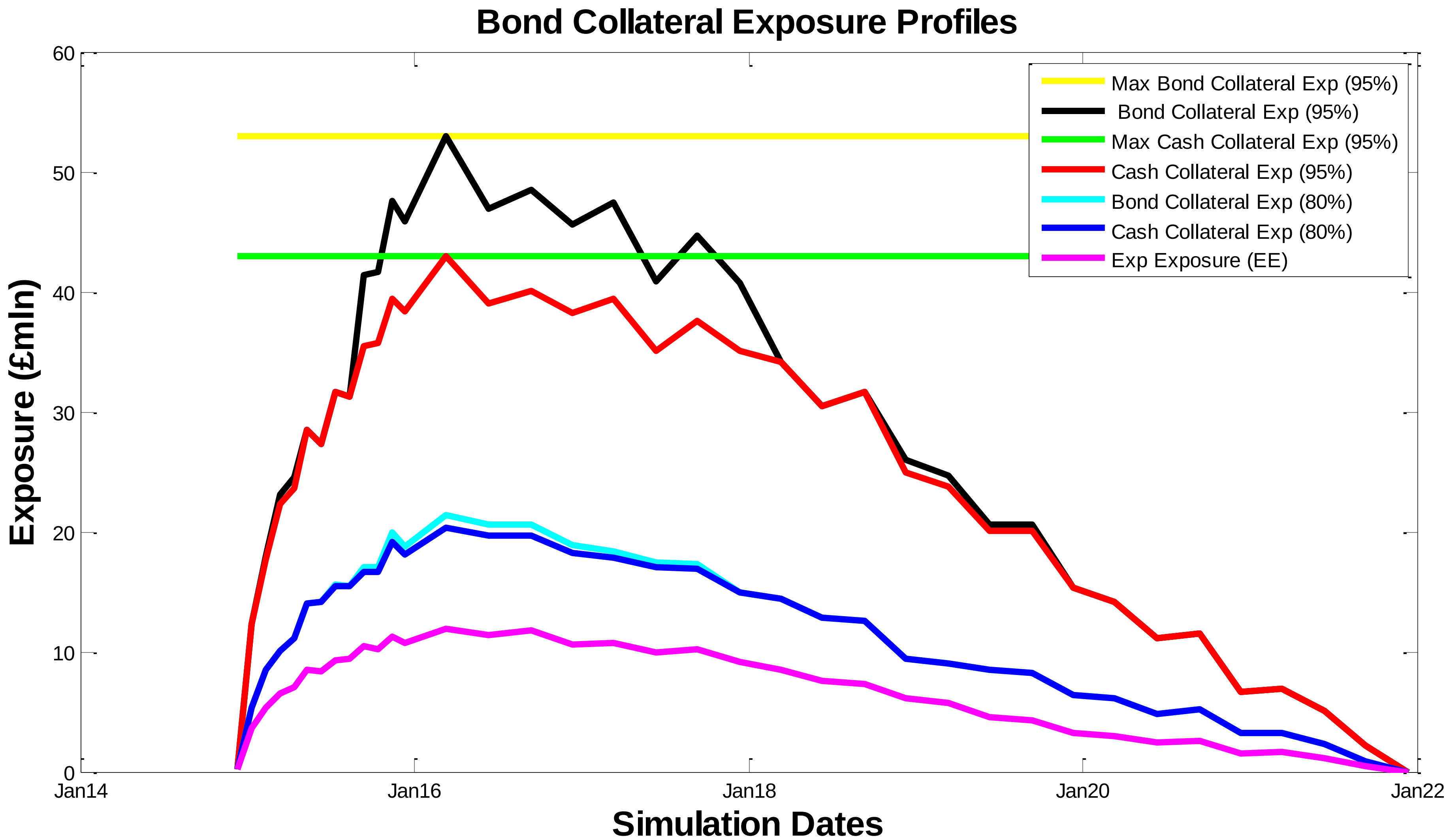
- Trading with **broker/fund-specific CSA**
  - Credit-risky collateral requires a **haircut** to be applied on the notional amount (10%-30% depending on the credit quality, the broker and the fund)
  - For Bond Collateral we have to **re-finance** our position frequently, on every coupon payment
    - Additional management cost and **liquidity** issues
    - **Repo** markets to generate cash involves extra operations and costs and an agreed legal framework known as **GMRA** (global master repurchase agreement)
  - For more complex collateral products (CDSs / MBSs / Equities) the daily valuation is more complicated and more difficult to monitor
  - Foreign currency collateral implies cross-currency bootstrapping, incorporating **cross-currency basis**

Cash vs Bond Collateral

Introducing the collateral pool on a multi-curve framework

- We assume that the fund on the previous example has a collateral pool with
  - **£15mln Cash**
  - **£10mln of Gilts** with 10% haircut
  - **£10mln of AAA Bonds** with 20% haircut
  - **£20mln of HY Bonds** with 30% haircut
- We now model a **dynamic** collateral portfolio that changes over time
- We **re-evaluate** the collateral on every time step, assuming there is no credit migration on the collateral posted
- **Reconstruct** the new collateral portfolio and simulate forward (additional computational complexity)

Bond Collateral Exposure Profiles - Results



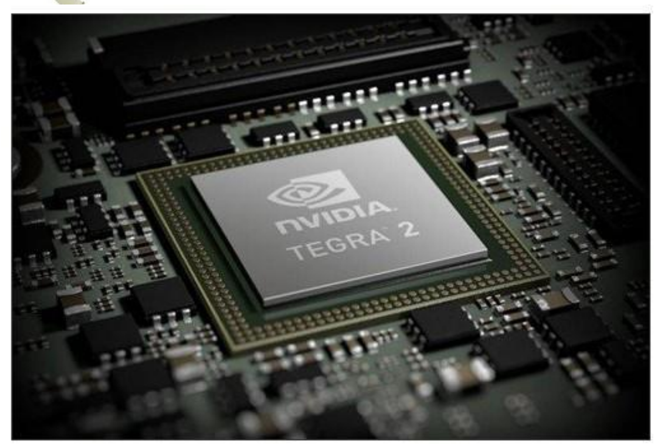
Computational Impact

- When we include No-Cash collateral instruments in our collateral pool the **complexity** of problem increases **exponentially**
- We now have a **dynamic portfolio** that changes continuously, adding an **optimization depending collateral decision** on every step we re-evaluate the portfolio
- The moment that the first No-Cash Collateral is been posted either from us or our counterparty, we need to start simulating the **credit migration risk** associated with that bond

This is clearly an computational intensive exercise, ideal for parallelization and acceleration using GPUs

Contents

NVIDIA GPU Computing: Graphical Processing Units



Aberdeen

Decomposing the problem

- Preprocessing the portfolio
  - Fetch all the interest rate curve / FX data live from Bloomberg / Thomson Reuters
  - Read the portfolio and construct all the asset classes and projected cash-flows
  - Preprocess the portfolio for different calendars, payment/receive/reset frequencies

Mainly a task we use only CPUs for portfolios with less than 10000 derivatives. Banks with larger portfolios may consider parallelize this step as well.

- Monte Carlo simulation
  - Perform the simulation for the various stochastic processes and construct the full term structure for every simulation step

- Post process the portfolio
  - Re-Evaluate the portfolio on every scenario and simulation step adjusting for the collateral posted from either us or our counterparties

Acceleration using GPUs is essential!

GPU Implementation

Four GPU kernels (parallelized dimensions)

- Interest rate path generation (1mln paths)
- Rates conversion (number of paths x number of tenors)
- Zero rates (number of paths x cash flow dates)
- Forward rates (number of paths x cash flow dates)

Number of paths per thread block: 1024 / number of tenors

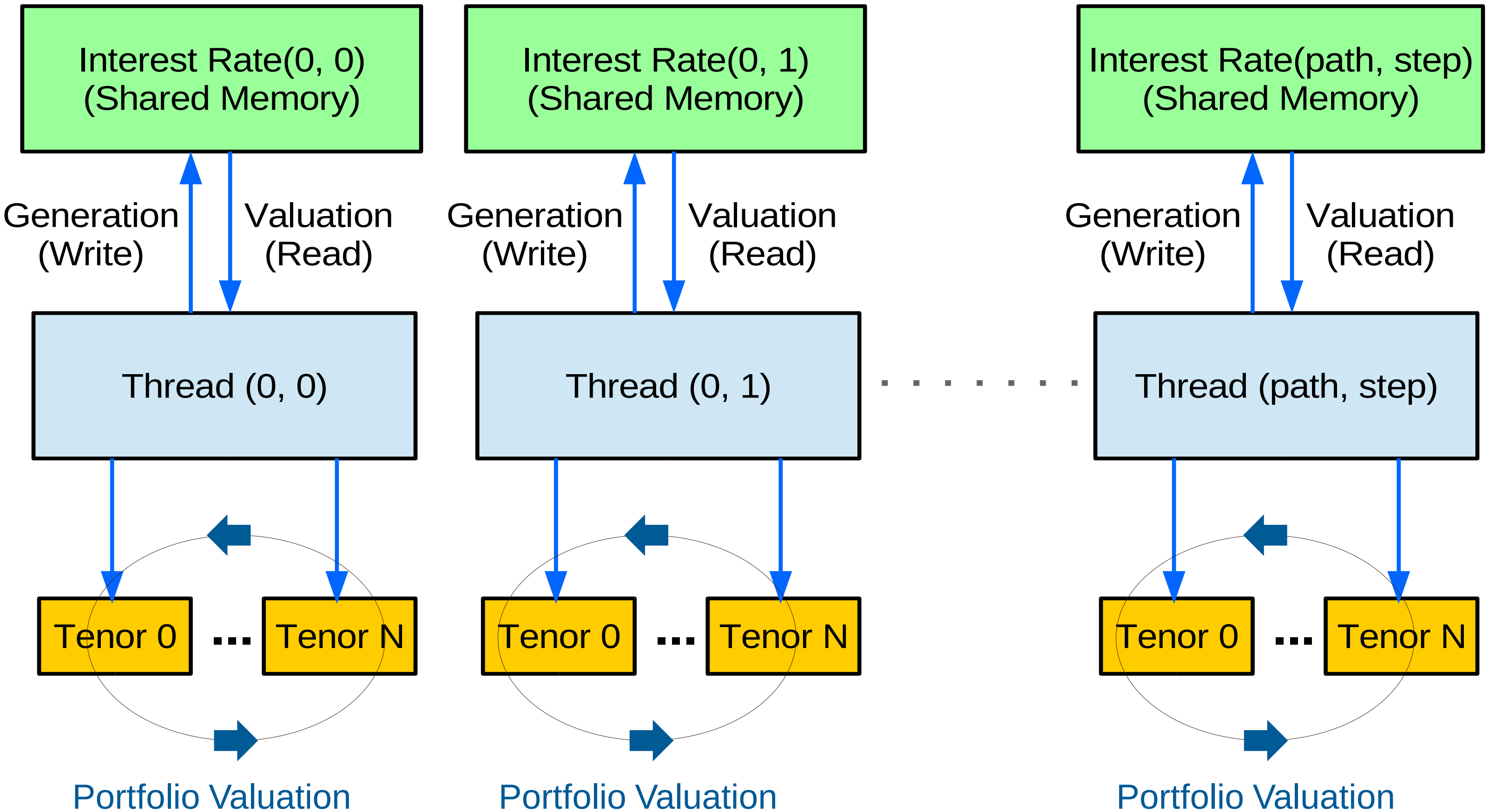
Number of thread blocks per thread grid: paths x tenors / 1024

Monte Carlo simulation evolution – N time steps

Term structure
k – Tenor Points

T_1	T_2	T_3	…	T_{N-2}	T_{N-1}	T_N
$r_1(T_1)$						
$r_2(T_1)$	$r_2(T_2)$					
$r_3(T_1)$	$r_3(T_2)$					
.						
.						
$r_{k-1}(T_1)$	$r_{k-1}(T_2)$	$r_{k-1}(T_3)$	…	$r_{k-1}(T_{N-2})$	$r_{k-1}(T_{N-1})$	$r_{k-1}(T_N)$
$r_k(T_1)$	$r_k(T_2)$	$r_k(T_3)$	…	$r_k(T_{N-2})$	$r_k(T_{N-1})$	$r_k(T_N)$

Parallel Architecture



Acceleration Results

- CPU execution times & Speedup
- Test Environment:
  - CPU: an Intel Xeon Sandybridge X5650 2.66 Ghz (16 cores)
  - an NVIDIA Tesla K40

Number of paths	Total CPU time (seconds)	Total GPU time (seconds)	Speed-up (Total)
10000	5.1	22.8	-
20000	8.8	19.6	-
30000	13.1	21.4	-
40000	17.2	21.7	-
50000	21.5	20.2	1x
100000	43.3	22.8	2x
500000	251.8	20.2	**12.5x**
1000000	503.6	22.2	**25x**

Conclusions

- Multi-curve modelling is here to stay
  - Impact varies, but dual curve discounting is essential for accurate pricing

- No-Cash Collateral management is not trivial
  - **Multiple** broker/fund specific **CSA agreements**
  - **Haircuts** do have a significant impact on collateral and fund's **liquidity**
  - The **dynamic** portfolio grows the computational complexity exponentially
  - Monte Carlo framework provides a sensible solution

- High Performance Computing using GPUs is essential
  - Significant increases the performance of Monte-Carlo simulation
    - Memory allocation and data transfer problems have to be taken under consideration
  - **Total cost of ownership**
  - Solution depends on the business case
  - In-house development has significant **Person Risk**
    - **Outsourcing?**

Thank you / Any questions?

References

1. Amin, A. (2010). Calibration, simulation and hedging in a Heston Libor market model with stochastic basis
2. Andersen, L. and Piterbarg, V. (2010). Interest rate modelling, in three volumes. *Atlantic Financial Press*
3. Brigo, D. and Mercurio, F. (2006). *Interest Rate Models, Theory and Practice*. Springer Finance.
4. Crepey, S. Grbac, Z. Ngor, N. Skovmand, D. A Levy HJM multiple-curve model with application to CVA computation
5. Henrad, M. (2007) The irony in derivatives discounting. *Wilmott Magazine*
6. Henrad, M. (2013). Multi-curves Framework with Stochastic Spread: A Coherent Approach to STIR Futures and Their Options. *OpenGamma Quant Research*
7. Kenyon, C. (2010). Post-shock short-rate pricing. *Risk*
8. Mercurio, F. and Xie, Z. (2012). *The basis goes stochastic basis. Risk, Dec 2012*
9. Mercurio, F. (2010). A LIBOR market model with stochastic basis
10. Moreni, N. and Pallavicini, A. (2010). Parsimonious HJM modelling for multiple yield-curve dynamics. *Working paper*

Past performance is not a guide to future returns. The value of investments, and the income from them, can go down as well as up and your clients may get back less than the amount invested.

The views expressed in this presentation should not be construed as advice on how to construct a portfolio or whether to buy, retain or sell a particular investment. The information contained in the presentation is for exclusive use by professional customers/eligible counterparties (ECPs) and not the general public. The information is being given only to those persons who have received this document directly from Aberdeen Asset Management (AAM) and must not be acted or relied upon by persons receiving a copy of this document other than directly from AAM. No part of this document may be copied or duplicated in any form or by any means or redistributed without the written consent of AAM.

The information contained herein including any expressions of opinion or forecast have been obtained from or is based upon sources believed by us to be reliable but is not guaranteed as to the accuracy or completeness.

Issued by Aberdeen Asset Managers Limited which is authorised and regulated by the Financial Conduct Authority in the United Kingdom.