## Functional Programming WS 2010/11

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Organization

## Lecture

- LV-Nr. 703017
- VO 2
- http://cl-informatik.uibk.ac.at/teaching/ws10/fp/
- slides are also available online
- office hours: Tuesday 12:00-14:00 in 3N01
- online registration required before 23:59 on October 30
- grading: written exam (closed book)


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## Exercises

- LV-Nr. 703018
- PS 1
- three groups: group 1 Friday 8:15-9:00 HS 11
group 2 Friday 9:15-10:00 HS 11
group 3 Friday 9:15-10:00 SR 12
- office hours: Monday 12:00-13:30 in 3M03 or by arrangement
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- grading: 2 tests + weekly exercises
- exercises start on October 15


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## Schedule

| week 1 | October | 6 | week 8 | November | 24 |
| :--- | :--- | ---: | :--- | :--- | ---: |
| week 2 | October | 13 | week 9 | December | 1 |
| week 3 | October | 20 | week 10 | December | 15 |
| week 4 | October | 27 | week 11 | January | 12 |
| week 5 | November | 3 | week 12 | January | 19 |
| week 6 | November | 10 | week 13 | January | 26 |
| week 7 | November | 17 | week 14 | February | 2 |

## Schedule

| week 1 | October | 6 | November 26: 1st test |  |  |
| :--- | :--- | ---: | :--- | :--- | :--- |
| week 2 | October | 13 | week 9 | December | 1 |
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| January | 26 |  |  |  |  |
| week 7 | November | 17 | February 2: 1st exam |  |  |

## Practical Topics

- lists
- strings
- trees
- sets
- combinator parsing
- lazy lists
- monads


## Theoretical Topics

- $\lambda$-calculus
- evaluation strategies
- induction
- reasoning about programs
- efficiency
- type checking/inference
- ...


## Today's Topics

- Historical Overview
- Notions
- A Taste of Haskell
- First Steps


## History



1936 Alonzo Church:
$\lambda$-calculus


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1924



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Haskell Curry:
combinatory logic


## 1924

## Moses

 Schönfinkel: combinatory logic$\lambda$-calculus
1941 23: 1st programmable, fully automatic computing machine

## 1937 <br> Alan Turing:

turing machines


Haskell Curry:
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Notions

State
a state is the "content" of a certain "region in memory"

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Example - Assignment
after $\mathrm{x}:=10$, the region called " x " has state 10

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## Example - $\sum_{i=0}^{n} i$

```
count := 0
total := 0
while count < n
    count := count + 1
    total := total + count
```


## Example - $\sum_{i=0}^{n} i$

the Haskell way of summing up the numbers from 0 to $n$ is
sum [1..n]

- [1..4] generates the list $[1,2,3,4]$
- sum is a predefined function, summing up the elements of a list


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## Self-Made Definitions

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## Self-Made Definitions

- [m..n] computes the range of numbers from $m$ to $n$

```
range m n = if m > n then []
    else m : range (m+1) n
```


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## Self-Made Definitions

- [m..n] computes the range of numbers from $m$ to $n$

```
range m n = if m > n then []
    else m : range (m+1) n
```

- sum xs computes the sum of all elements in xs
mySum [] = 0
mySum ( $x$ :xs) $=x+$ mySum $x s$

Pure Functions
a function is pure if it

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- does not have side effects


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## Example - Random Numbers

The function rand (producing random numbers) is not pure

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\begin{aligned}
\operatorname{rand}() & =0 \\
\operatorname{rand}() & =10 \\
\operatorname{rand}() & =42
\end{aligned}
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data whose state does not change after the initial creation

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## Example - Linked Lists

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after concatenation

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\mathrm{zs}=\mathrm{xs}++\mathrm{ys}
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$$


zs
copied

## Recursion

a function is recursive if it is used in its own definition

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## Example - Factorial Numbers

```
factorial n =
    if n < 2
    then 1
    else n * factorial (n - 1)
```

Evaluating Functions by Hand (aka Equational Reasoning)

- functions are defined by equations and pattern matching
- general idea: "replace equals by equals"


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## Example - mySum

given the following two equations for mySum

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\begin{align*}
\text { mySum }[] & =0  \tag{1}\\
\operatorname{mySum}(x: x s) & =x+\operatorname{mySum} x s \tag{2}
\end{align*}
$$

we evaluate mySum [1, 2, 3] like

$$
\begin{aligned}
\text { mySum }[1,2,3] & =1+\text { mySum }[2,3] & & \text { using }(2) \\
& =1+(2+\text { mySum }[3]) & & \text { using }(2) \\
& =1+(2+(3+\text { mySum }[])) & & \text { using }(2) \\
& =1+(2+(3+0)) & & \text { using }(1) \\
& =6 & & \text { by def. of }+
\end{aligned}
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## empty list

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\end{align*}
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- functions are defined by equations and pattern matching
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## Example - mySum

given the following two equations for mySum

$$
\begin{align*}
& \text { list with "head" } x \text { and "tail" } x s \\
& \text { mySum }[]=0  \tag{1}\\
& \operatorname{mySum}(x: \times s)=x+\text { mySum } x s \tag{2}
\end{align*}
$$

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A Taste of Haskell

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## Example - qsort

- sort list of elements smaller than or equal to $x$
- sort list of elements larger than $x$
- insert $x$ in between

$$
\begin{aligned}
& \text { qsort [] = [] } \\
& \text { qsort (x:xs) = qsort leq ++ [x] ++ qsort gt } \\
& \text { where } \\
& \text { leq }=[\mathrm{a} \mid \mathrm{a}<-\mathrm{xs}, \mathrm{a}<=\mathrm{x}] \\
& \text { gt }=[b \mid \mathrm{b}<-\mathrm{xs}, \mathrm{~b}>\mathrm{x}]
\end{aligned}
$$

First Steps

## The Haskell Platform

- available from http://hackage.haskell.org/platform/
- ships with the most widely used Haskell compiler: GHC
- and its interpreter GHCi


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## Starting the Interpreter (GHCi)

## \$ ghci

GHCi, version 6.12.1: http://www.haskell.org/ghc/
:? for help

Prelude>

- on startup GHCi loads the file Prelude.hs, importing many standard functions


## The Standard Prelude

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## Examples

- arithmetic: +, -, *, /, ^, mod, div
- lists

| drop | drop the first $n$ elements from a list |
| :--- | :--- |
| head | extract the first element from a list |
| length | number of elements in a list |
| reverse | reverse the order of elements in a list |
| sum | sum up the elements of a list |
| tail | obtain the list without its first element <br> take |

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| sum | sum up the elements of a list |
| tail | obtain the list without its first element |
| take | take the first $n$ elements from a list |

- note: in code examples Prelude functions are denoted like this and others like this


## Function Application

- in mathematics: function application is denoted by enclosing the arguments in parenthesis, whereas multiplication of two arguments is often implicit (by juxtaposition)
- in Haskell: reflecting its primary status, function application is denoted silently (by juxtaposition), whereas multiplication is denoted explicitly by *


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- in Haskell: reflecting its primary status, function application is denoted silently (by juxtaposition), whereas multiplication is denoted explicitly by *


## Examples

| Mathematics | Haskell |
| :--- | :--- |
| $f(x)$ | $f \times x$ |
| $f(x, y)$ | $f \times \mathrm{y}$ |
| $f(g(x))$ | $f(\mathrm{~g} \mathrm{x})$ |
| $f(x, g(y))$ | $\mathrm{fx}(\mathrm{g} \mathrm{y})$ |
| $f(x) g(y)$ | $\mathrm{fx} * \mathrm{~g} \mathrm{y}$ |
| $f(a, b)+c d$ | $\mathrm{f} \mathrm{a} \mathrm{b}+\mathrm{c} * \mathrm{~d}$ |

## Haskell Scripts

- define new functions inside scripts
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## My First Script - test.hs

- set editor from inside GHCi : set editor vim
- start editor : edit test.hs and type

```
double x = x + x
quadruple x = double (double x)
```

- load script

```
Prelude> :load test.hs
    [1 of 1] Compiling Main ( test.hs, interpreted )
Ok, modules loaded: Main.
*Main>
```


## Interpreter Commands

| Command | Meaning |
| :--- | :--- |
| :load $\langle$ name $\rangle$ | load script $\langle$ name $\rangle$ |
| :reload | reload current script |
| : edit $\langle$ name $\rangle$ | edit script $\langle$ name $\rangle$ |
| : edit | edit current script |
| :type $\langle$ expr | show type of $\langle$ expr $\rangle$ |
| :set $\langle$ prop $\rangle$ | change various settings |
| : show $\langle$ info $\rangle$ | show various information |
| :! $\langle c m d\rangle$ | execute $\langle c m d\rangle$ in shell |
| :? | show help text |
| :quit | bye-bye! |

## Example Session

> :load test.hs
> quadruple 10
40
> take (double 2) $[1,2,3,4,5,6]$
[1, 2, 3, 4]
> :edit test.hs
factorial $\mathrm{n}=$ product [1..n]
average ns = sum ns `div` length ns
> :reload
> factorial 10
3628800
> average [1, 2, 3, 4, 5]
3

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## Naming Requirements

names of functions and their arguments have to conform to the following syntax

$$
\begin{aligned}
\langle\text { lower }\rangle & \stackrel{\text { def }}{=} \mathrm{a}|\ldots| \mathrm{z} \mid \\
\langle\text { upper }\rangle & \stackrel{\text { def }}{=} \mathrm{A}|\ldots| \mathrm{Z} \\
\langle\text { digit }\rangle & \stackrel{\text { def }}{=} 0|\ldots| 9 \\
\langle\text { name }\rangle & \stackrel{\text { def }}{=}\langle\text { lower }\rangle\left(\langle\text { lower }\rangle \mid\langle\text { upper }\rangle \mid\left.\langle\text { digit }\rangle\right|^{\prime}\right)^{*}
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\langle\text { upper }\rangle & \stackrel{\text { def }}{=} \mathrm{A}|\ldots| \mathrm{Z} & \text { zero ore } \mathrm{m} \\
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## Reserved Names

case class data default deriving do else foreign if import in infix infixl infixr instance let module newtype of then type where _

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## Examples

myFun fun1 arg_2 $x^{\prime}$

## The Layout Rule

- items that start in the same column are grouped together
- by increasing indentation, items may span multiple lines
- groups end at EOF or when indentation decreases
- ignore layout: enclosing groups in braces $(\{\}$,$) and separating$ items by semicolons (;)
- the content of a script is a group, nested groups are started by one of where, let, do, and of


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## Examples

```
main =
    let x = 1
        y = 1
```

in
putStrLn (take ( $\mathrm{x}+\mathrm{y}$ ) ( $\mathrm{zs}++\mathrm{us}$ ))
where
zs = []
us = "abc"

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## Examples

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main =
    let x = 1
        y = 1
```

in
putStrLn (take (x+y) (zs++us))
where
without using layout

```
main =
    let {x = 1; y = 1} in
    putStrLn (take (x+y) (zs++us))
    where {zs = []; us = "abc"}
```

zs = []
us = "abc"

## Comments

there are two kinds of comments

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## Examples

```
-- Factorial of a positive integer:
factorial n = product [1..n]
-- Average of a list of integers:
average ns = sum ns `div` length ns
{- currently not used
double x = x + x
quadruple x = double (double x)
-}
```


## Exercises (for October 15th)

1. read
http://haskell.org/haskellwiki/Functional_programming
http://haskell.org/haskellwiki/Haskell_in_5_steps
2. work through lessons 1 to 3 on http://tryhaskell.org/
3. explain and correct the 3 syntactic errors in the script:
```
N = a 'div' length xs
```

where

$$
\begin{aligned}
\mathrm{a} & =10 \\
\mathrm{xs} & =[1,2,3,4,5]
\end{aligned}
$$

4. Show how the library function last (selecting the last element of a non-empty list) could be defined in terms of the Prelude functions used in this lecture. Can you think of another possible definition?
5. Show two possible definitions of the library function init (removing the last element from a list) in terms of the functions introduced so far.
6. Use recursion to define a function gcd, computing the greatest common divisor of two given integers.
