

# Functional Programming WS 2010/11

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October 6, 2010

# Organization

#### Lecture

- LV-Nr. 703017
- VO 2
- http://cl-informatik.uibk.ac.at/teaching/ws10/fp/
- slides are also available online
- office hours: Tuesday 12:00 14:00 in 3N01
- online registration required before 23:59 on October 30
- grading: written exam (closed book)

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- PS 1
- three groups: group 1 Friday 8:15-9:00 HS 11 group 2 Friday 9:15-10:00 HS 11 group 3 Friday 9:15-10:00 SR 12
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# Schedule

week 1	October	6	week 8	November	24
week 2	October	13	week 9	December	1
week 3	October	20	week 10	December	15
week 4	October	27	week 11	January	12
week 5	November	3	week 12	January	19
week 6	November	10	week 13	January	26
week 7	November	17	week 14	February	2

# Schedule

week 1	October	6	November 26: 1st test
week 2	October	13	week 9 December 1
week 3	October	20	week 10 December 15
week 4	October	27	week 11 January 12
week 5	November	3	January 21: 2nd test
week 6	November	10	week 13 January 26
week 7	November	17	February 2: 1st exam

## Practical Topics

- lists
- strings
- trees
- sets
- combinator parsing
- lazy lists
- monads
- . . .

## Theoretical Topics

- $\lambda$ -calculus
- evaluation strategies
- induction
- reasoning about programs
- efficiency
- type checking/inference
- . . .

# Today's Topics

- Historical Overview
- Notions
- A Taste of Haskell
- First Steps

# History



1936 Alonzo Church:  $\lambda$ -calculus

1924 2010



1936 Alonzo Church:  $\lambda$ -calculus

1924 2010

1937 Alan Turing: turing machines





Alonzo Church:  $\lambda$ -calculus

Moses Schönfinkel: combinatory logic

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Haskell Curry: combinatory logic





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**Z3:** 1st programmable, fully automatic computing machine

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Peter Landin:

logic 1924

Moses

Schönfinkel:

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1950

2010

ZU1



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Alan Turing: 1937 turing machines



Haskell Curry: 1930

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Haskell Curry: combinatory logic 1977

John Backus: FΡ



John McCarthy: LISP

Robin Milner: LCF, Standard ML



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2010

Haskell2010

2010







# Notions

a state is the "content" of a certain "region in memory"

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# Example - Assignment

after x := 10, the region called "x" has state 10

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### Side Effects

a function or expression has side effects if it modifies some state

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# Side Effects

a function or expression has side effects if it modifies some state

# Example - $\sum_{i=0}^{n} i$

```
count := 0
total := 0
while count < n
  count := count + 1
  total := total + count</pre>
```

# Example $-\sum_{i=0}^{n} i$

the Haskell way of summing up the numbers from 0 to n is

```
sum [1..n]
```

- [1..4] generates the list [1,2,3,4]
- sum is a predefined function, summing up the elements of a list

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### Self-Made Definitions

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• [m..n] computes the range of numbers from m to n

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- [1..4] generates the list [1,2,3,4]
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#### Self-Made Definitions

• [m..n] computes the range of numbers from m to n

sum xs computes the sum of all elements in xs

```
mySum [] = 0
mySum (x:xs) = x + mySum xs
```

# Pure Functions

- a function is pure if it
  - evaluates the same result, given the same arguments; and
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#### Example - Random Numbers

The function rand (producing random numbers) is not pure

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rand() = 0

rand() = 10

rand() = 42
```

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$$rand() = 0$$
  
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 $rand() = 42$ 

sometimes called referential transparent

data whose state does not change after the initial creation

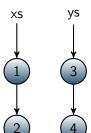
data whose state does not change after the initial creation

# Example - Linked Lists

consider 2 linked lists

$$xs = [1, 2]$$

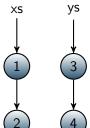
$$ys = [3, 4]$$



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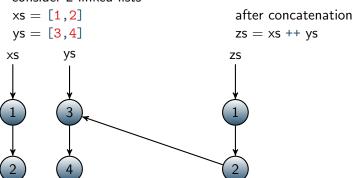


after concatenation zs = xs ++ ys

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# Example - Linked Lists

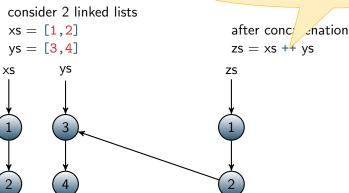
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#### Example - Linked Lists

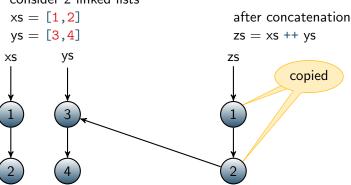
append elements of ys to xs



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# Example - Linked Lists

consider 2 linked lists



# Recursion

a function is recursive if it is used in its own definition

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#### Example - Factorial Numbers

```
factorial n =
  if n < 2
    then 1
    else n * factorial (n - 1)</pre>
```

- functions are defined by equations and pattern matching
- general idea: "replace equals by equals"

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#### Example - mySum

given the following two equations for mySum

$$mySum [] = 0 (1)$$

$$mySum (x:xs) = x + mySum xs$$
 (2)

we evaluate mySum [1, 2, 3] like

$$\begin{array}{lll} \mbox{mySum} \; [1,2,3] & = 1 + \mbox{mySum} \; [2,3] & \mbox{using} \; (2) \\ & = 1 + (2 + \mbox{mySum} \; [3]) & \mbox{using} \; (2) \\ & = 1 + (2 + (3 + \mbox{mySum} \; [])) & \mbox{using} \; (2) \\ & = 1 + (2 + (3 + 0)) & \mbox{using} \; (1) \\ & = 6 & \mbox{by def. of } + \end{array}$$

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empty list

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- general idea: "replace equals by equals"

#### Example - mySum

given the following two equations for mySum

list with "head" 
$$x$$
 and "tail"  $xs$ 

mySum [] = 0 (1)

mySum ( $x:xs$ ) =  $x$  + mySum  $xs$  (2)

we evaluate mySum [1, 2, 3] like

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# A Taste of Haskell

# Haskell

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#### Example - qsort

- sort list of elements smaller than or equal to x
- sort list of elements larger than x
- insert x in between

# First Steps

#### The Haskell Platform

- available from http://hackage.haskell.org/platform/
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- and its interpreter GHCi

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# Starting the Interpreter (GHCi)

```
$ ghci
GHCi, version 6.12.1: http://www.haskell.org/ghc/
:? for help
...
Prelude>
```

# The Standard Prelude

• on startup GHCi loads the file Prelude.hs, importing many standard functions

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# Examples

- arithmetic: +, -, \*, /, ^, mod, div
- lists

drop	drop the first $n$ elements from a list
head	extract the first element from a list
length	number of elements in a list
reverse	reverse the order of elements in a list
sum	sum up the elements of a list
tail	obtain the list without its first element
take	take the first $n$ elements from a list

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 note: in code examples Prelude functions are denoted like this and others like this

#### Function Application

- in mathematics: function application is denoted by enclosing the arguments in parenthesis, whereas multiplication of two arguments is often implicit (by juxtaposition)
- in Haskell: reflecting its primary status, function application is denoted silently (by juxtaposition), whereas multiplication is denoted explicitly by \*

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- in Haskell: reflecting its primary status, function application is denoted silently (by juxtaposition), whereas multiplication is denoted explicitly by \*

#### Examples

Mathematics	Haskell
f(x)	f x
f(x, y)	f x y
f(g(x))	f (g x)
f(x,g(y))	f x (g y)
f(x)g(y)	f x * g y
f(a,b)+cd	f a b + c*d

# Haskell Scripts

- define new functions inside scripts
- text file containing definitions
- common suffix .hs

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#### My First Script - test.hs

- set editor from inside GHCi :set editor vim
- start editor :edit test.hs and type

```
double x = x + x
quadruple x = double (double x)
```

load script

```
Prelude> :load test.hs
[1 of 1] Compiling Main ( test.hs, interpreted )
Ok, modules loaded: Main.
*Main>
```

# Interpreter Commands

Command	Meaning
:load $\langle name \rangle$ :reload :edit $\langle name \rangle$ :edit :type $\langle expr \rangle$ :set $\langle prop \rangle$	load script $\langle name \rangle$ reload current script edit script $\langle name \rangle$ edit current script show type of $\langle expr \rangle$ change various settings
:show $\langle info  angle$ :! $\langle cmd  angle$	show various information execute $\langle cmd \rangle$ in shell
:? :quit	show help text bye-bye!

# **Example Session**

- :load test.hs
- > quadruple 10
- 40
- > take (double 2) [1,2,3,4,5,6]
- [1,2,3,4]
- > :edit test.hs

:reload

3628800

factorial n = product [1..n]

> average [1,2,3,4,5]

- average ns = sum ns `div` length ns

  - factorial 10

#### **Example Session**

```
:load test.hs
> quadruple 10
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> take (double 2) [1,2,3,4,5,6]
[1,2,3,4]
> :edit test.hs
factorial n = product [1..n]
average ns = sum ns `div` length ns
 :reload
 factorial 10
3628800
> average [1,2,3,4,5]
```

enclosing a function in `...` turns it infix

names of functions and their arguments have to conform to the following  $\mbox{syntax}$ 

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```
\begin{array}{lll} \langle \textit{lower} \rangle & \stackrel{\textit{def}}{=} & \text{a} \mid \dots \mid \text{z} \mid \_ \\ \langle \textit{upper} \rangle & \stackrel{\textit{def}}{=} & \text{A} \mid \dots \mid \text{Z} \\ \langle \textit{digit} \rangle & \stackrel{\textit{def}}{=} & \text{0} \mid \dots \mid \text{9} \\ \langle \textit{name} \rangle & \stackrel{\textit{def}}{=} & \langle \textit{lower} \rangle (\langle \textit{lower} \rangle \mid \langle \textit{upper} \rangle \mid \langle \textit{digit} \rangle \mid ^{+})^{*} \end{array}
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#### Reserved Names

case class data default deriving do else foreign if import in infix infixl infixr instance let module newtype of then type where \_

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case class data default deriving do else foreign if import in infix infixl infixr instance let module newtype of then type where \_

# Examples |

```
myFun fun1 arg_2 x'
```

#### The Layout Rule

- items that start in the same column are grouped together
- by increasing indentation, items may span multiple lines
- groups end at EOF or when indentation decreases
- ignore layout: enclosing groups in braces ({,}) and separating items by semicolons (;)
- the content of a script is a group, nested groups are started by one of where, let, do, and of

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#### Examples '

```
main =
  let x = 1
    y = 1
  in
  putStrLn (take
    (x+y) (zs++us))
  where
  zs = []
  us = "abc"
```

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    (x+y) (zs++us))
  where
  zs = []
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```

without using layout

```
main =
  let {x = 1; y = 1} in
  putStrLn (take (x+y) (zs++us))
  where {zs = []; us = "abc"}
```

# Comments

there are two kinds of comments

- single-line comments: starting with -- and extending to EOL
- multi-line comments: enclosed in {- and -}

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#### **Examples**

```
-- Factorial of a positive integer:
factorial n = product [1..n]
-- Average of a list of integers:
average ns = sum ns `div` length ns
{- currently not used
double x = x + x
quadruple x = double (double x)
```

# Exercises (for October 15th)

- 1. read
   http://haskell.org/haskellwiki/Functional\_programming
   http://haskell.org/haskellwiki/Haskell\_in\_5\_steps
- 2. work through lessons 1 to 3 on http://tryhaskell.org/
- 3. explain and correct the 3 syntactic errors in the script:

```
N = a 'div' length xs
where
    a = 10
    xs = [1,2,3,4,5]
```

- 4. Show how the library function last (selecting the last element of a non-empty list) could be defined in terms of the Prelude functions used in this lecture. Can you think of another possible definition?
- 5. Show two possible definitions of the library function init (removing the last element from a list) in terms of the functions introduced so far.
- 6. Use recursion to define a function gcd, computing the greatest common divisor of two given integers.