Functional Programming

WS 2010/11

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Today's Topics

- Module Basics
- Lists and Strings
- Recursive Functions
- Example - Printing a Calendar

Module Basics

Structuring Code

- split source code into several files
- separate namespaces for functions and types

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Splitting Source Code

- for each module `Module` create file `Module.hs`
- module names always start with uppercase letters
- start module by module header (with optional export list)

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module Module (⟨export list⟩) where
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```

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Example

```
module Stack (Stack, empty, push, pop) where
type Stack a = [a]
empty = []
push  = (:)
pop s = (head s, tail s)
```

Type Synonyms

- `type Stack a = [a]` is a type synonym
- just gives an alternative name for `[a]`
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Example

```
push :: a -> Stack a -> Stack a
push = (:)
```

- note the partial application of `(:)`
- this is equivalent to `push x s = x : s`

Lists and Strings

Strings are Lists

- the type `String` is just a type synonym for `[Char]`
- i.e., a string is just a list of characters
- all list functions are applicable to `String`s

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Some Implications

- `[]` is the same as `""` for strings
- `['h','e','l','l','o']` is the same as `"hello"` for strings

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Useful Functions on Strings

- `lines :: String -> [String]` - breaks string at newlines
- `unlines :: [String] -> String` - concatenates strings, inserting newlines
- `words :: String -> [String]` - breaks strings at white space
- `unwords :: [String] -> String` - concatenates strings, separated by spaces

Interlude - Function Composition

- in mathematics $f \circ g$ usually denotes applying f after g
- i.e., $(f \circ g)(x) = f(g(x))$
- only possible if output of g is compatible with input of f: $f\colon B \to C$ and $g\colon A \to B$
- in Haskell: `(.) :: (b -> c) -> (a -> b) -> (a -> c)`
- try "`:info (.)`" in GHCi

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Examples

- `map (f . g) xs` - on every element of `xs`, first apply `g` and then `f`
- equivalent to `map f (map g xs)`
- what's the result of `unwords . words`?

List Comprehensions - Generators

- in mathematics set comprehensions can be used to construct new sets from existing sets
- e.g., $\{x^2 \mid x \in \{1, \ldots, 5\}\}$ produces $\{1, 4, 9, 16, 25\}$
- in Haskell `[x^2 | x <- [1..5]]`
- here, `x <- [1..5]` is called a generator
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- order is important: rightmost generators are evaluated first

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Examples

- `concat xss = [x | xs <- xss, x <- xs]`
- `firsts ps = [x | (x,_) <- ps]`
- `length xs = sum [1 | _ <- xs]`

List Comprehensions - Guards

- filter values before generating result
- e.g., $\{x^2 \mid x \in \mathbb{N}, x > 5\}$
- in Haskell: `[x^2 | x <- xs, x > 5]`; square every number in `xs` that is greater than 5

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Examples

- `[x | x <- [1..10], even x]`
- `find k t = [v | (k', v) <- t, k == k']`
- ``factors n = [x | x <- [1..n], n `mod` x == 0]``
- `primes = [n | n <- [1..], factors n == [1,n]]`

Recursive Functions

Basic Concepts

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  2. enumerate the cases (e.g., `[]` and `x:xs`)
  3. define the simple cases (e.g., `product [] = 1`)
  4. define the other cases (e.g., `product (x:xs) = x * product xs`)
  5. generalize and simplify (e.g., `product :: Num a => [a] -> a` and `product = foldr (*) 1`)

Example - `drop`

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- generalize and simplify:

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drop :: Integer -> [a] -> [a]
drop n xs | n <= 0  =  xs
drop n []           =  []
drop n (_:xs)       =  drop (n-1) xs
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- generalize and simplify

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Example - Printing a Calendar

Printing a Calendar

- given a month and a year, print the corresponding calendar
- separate construction phase (computing of days, leap year, ...) from printing
- we concentrate on printing, assuming machinery for construction

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Example - October 2010

```
    October 2010
Su Mo Tu We Th Fr Sa
                1  2
 3  4  5  6  7  8  9
10 11 12 13 14 15 16
17 18 19 20 21 22 23
24 25 26 27 28 29 30
31
```

The Picture Analogon

pictures:

- atomic part: pixel
- height and width
- white pixel

strings:

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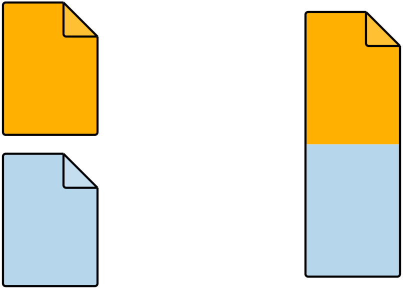
Auxiliary Types

```
type Height  = Int
type Width   = Int
type Picture = (Height, Width, [[Char]])
```

Stacking 2 Pictures Above Each Other



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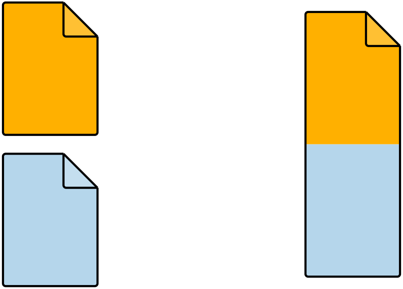

above

```
above :: Picture -> Picture -> Picture
(h,w,css) `above` (h',w',css')
  | w == w'    = (h+h',w,css ++ css')
  | otherwise = error "above: different widths"
```

Stacking Several Pictures Above Each Other

```
stack :: [Picture] -> Picture
stack = foldr1 above
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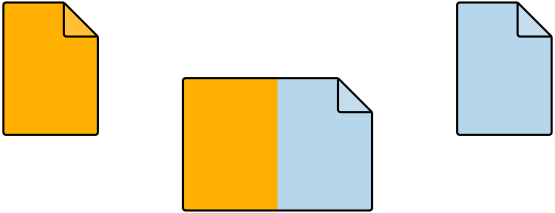
Notes

- `error :: String -> a`, indicates a runtime error, given as string
- `foldr1` - special version of `foldr`, without base value (this implies that it does not work on empty lists)

  ```
  foldr1 :: (a -> a -> a) -> [a] -> a
  foldr1 f [x]    = x
  foldr1 f (x:xs) = x `f` foldr1 f xs
  ```

Spreading 2 Pictures Beside Each Other



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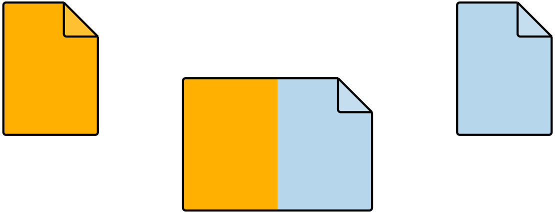

beside

```
beside :: Picture -> Picture -> Picture
(h,w,css) `beside` (h', w', css')
  | h == h'   = (h, w+w', zipWith (++) css css')
  | otherwise = error "beside: different heights"
```

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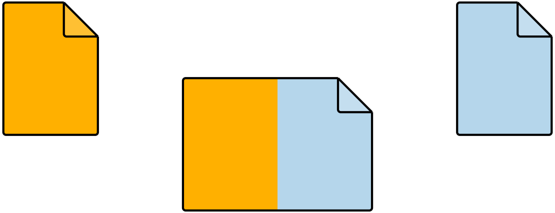

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```

Spreading Several Pictures Beside Each Other

```
spread :: [Picture] -> Picture
spread = foldr1 beside
```

Combining 2 Lists via a Function

- `zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]`
- `zipWith f` $[x_1, \ldots, x_m]$ $[y_1, \ldots, y_n] =$
  $[x_1$ `` `f` `` $y_1, \ldots, x_{\min\{m,n\}}$ `` `f` `` $y_{\min\{m,n\}}]$
- specialization `zip :: [a] -> [b] -> [(a,b)]`,

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Examples

- `zip [1,2,3] ['a','b']` = `[(1,'a'),(2,'b')]`
- `zipWith (*) [1,2] [3,4,5]` = `[1*3,2*4]` = `[3,8]`
- `zipWith drop [1,0] ["a","b"]` =
  `[drop 1 "a",drop 0 "b"]` = `["","b"]`

Creating Pictures

- single pixels

```
pixel :: Char -> Picture
pixel c = (1,1,[[c]])
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- blank

```
blank = (Int,Int) -> Picture
blank = stack . map row . blanks
  where blanks (h,w) =
    replicate h (replicate w ' ')
```

Constructing a Month

- assume function `monthInfo :: Int -> Int -> (Int,Int)`, returning the first weekday of the month together with the number of days for the month
- where days are 0 (Sunday), 1 (Monday), ...
- e.g., `monthInfo 10 2010` = `(5,31)`, meaning that the first weekday of October 2010 is a Friday and the month has 31 days

```
daysOfMonth :: (Month,Year) -> [Picture]
daysOfMonth (m,y) =
  map (row . rjustify 3 . pic) [1-d..42-d]
  where (d,t) = monthInfo m y
        pic n = if 1 <= n && n <= t then show n
                                    else ""

month :: (Month,Year) -> Picture
month = tile . group 7 . daysOfMonth
```

Missing Functions

- `rjustify` - right-justify given text inside box of given width

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rjustify :: Int -> String -> String
rjustify n xs =
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- `group` - split list into sublists of given length

```
group :: Int -> [a] -> [[a]]
group n xs = if null ys then []
                        else ys : group n zs
  where (ys,zs) = splitAt n xs
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- `tile` - tile a list of lists of pictures

```
tile :: [[Picture]] -> Picture
tile = stack . map spread
```

Printing a Month

- transform a `Picture` into a `String`

```
showPic :: Picture -> String
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- putting it all together

```
module Main where
import System
...
main = do
  args <- getArgs
  case args of
    [m,y] -> printMonth (read m,read y)
    _     -> error "expecting month and year"
```

Exercise Preparation - Caesar Cipher

- Caesar Cipher encodes text by replacing each letter by another one, some fixed positions (the key) down the alphabet
- e.g., encoding `hello` with a key of 2, yields `jgnnq`.
- in the following we restrict to lowercase letters
- approximate letter frequency list for English

```
tableEn = [8.2,1.5,2.8,4.3,12.7,2.2,2.0,6.1,7.0,
           0.2,0.8,4.0,2.4,6.7,7.5,1.9,0.1,6.0,
           6.3,9.1,2.8,1.0,2.4,0.2,2.0,0.1]
```

- chi-square statistic

$$\sum_{i=0}^{n-1} \frac{(os_i - es_i)^2}{es_i}$$

- where *os* is list of observed frequencies
- and *es* list of expected frequencies (e.g., `tableEn` for English)
- the lower chi-square, the better the match between *os* and *es*

Exercises (for October 29th)

1. read chapter 3 of Real World Haskell
2. Implement a function `rotate :: Int -> [a] -> [a]` that rotates the elements of a list to the left (wrapping around at the start of the list). E.g., `rotate 3 [1,2,3,4,5]` = `[4,5,1,2,3]`.
3. Implement a function `encode :: Int -> String -> String` that applies the Caesar cipher, e.g., `encode 2 "hello"` = `"jgnnq"`. (Note that decoding is just encoding with the negated key.)
4. Implement a function `freqs :: String -> [Float]` that produces a frequency list for the 26 lowercase letters. E.g., `freqs "aaab"` = `[75.0,25.0,0.0,...,0.0]`.
5. Implement the chi-square statistic by a function `chisqr :: [Float] -> [Float] -> Float`, taking two frequency lists.
6. Implement a function `crack :: String -> String` that is able to break the ciphertext `"rhn vktvdxw max vhwx"`. You may use all the previous functions and `tableEn`.

Hints

- a function `f` from module `M`, will be denoted by `M.f`
- in order to use `f` you need **`import`** `M` at start of file
- converting between integers and characters
  - `Data.Char.chr :: Int -> Char`
  - `Data.Char.ord :: Char -> Int`
- converting from integer to float `fromIntegral`