Java Source and Bytecode Formalizations in Isabelle: Bali

Gerwin Klein Tobias Nipkow David von Oheimb Leonor Prensa Nieto
Norbert Schirmer Martin Strecker

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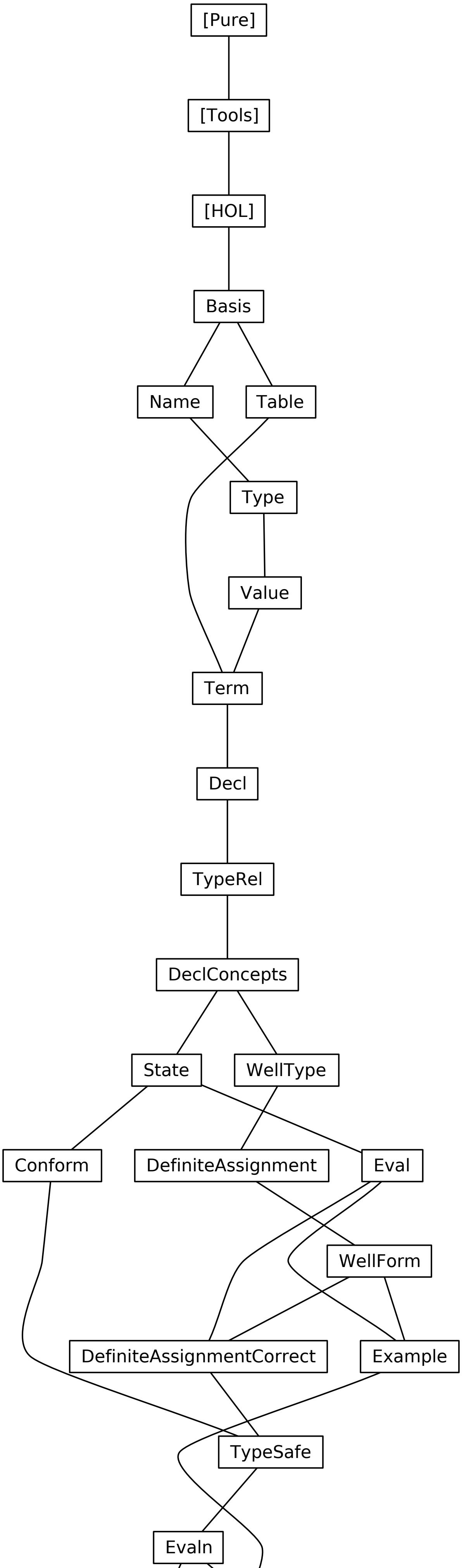
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Chapter 1

Overview

These theories, called Bali, model and analyse different aspects of the JavaCard **source language**. The basis is an abstract model of the JavaCard source language. On it, a type system, an operational semantics and an axiomatic semantics (Hoare logic) are built. The execution of a wellformed program (with respect to the type system) according to the operational semantics is proved to be typesafe. The axiomatic semantics is proved to be sound and relative complete with respect to the operational semantics.

We have modelled large parts of the original JavaCard source language. It models features such as:

- The basic "primitive types" of Java
- Classes and related concepts
- Class fields and methods
- Instance fields and methods
- Interfaces and related concepts
- Arrays
- Static initialisation
- Static overloading of fields and methods
- Inheritance, overriding and hiding of methods, dynamic binding
- All cases of abrupt termination
  - Exception throwing and handling
  - `break`, `continue` and `return`
- Packages
- Access Modifiers (`private`, `protected`, `public`)
- A "definite assignment" check

The following features are missing in Bali wrt. JavaCard:

- Some primitive types (`byte, short`)
- Syntactic variants of statements (`do`-loop, `for`-loop)
- Interface fields

- Inner Classes

In addition, features are missing that are not part of the JavaCard language, such as multithreading and garbage collection. No attempt has been made to model peculiarities of JavaCard such as the applet firewall or the transaction mechanism.
Overview of the theories:

Basis Some basic definitions and settings not specific to JavaCard but missing in HOL.

Table Definition and some properties of a lookup table to map various names (like class names or method names) to some content (like classes or methods).

Name Definition of various names (class names, variable names, package names,...)

Value JavaCard expression values (Boolean, Integer, Addresses,...)

Type JavaCard types. Primitive types (Boolean, Integer,...) and reference types (Classes, Interfaces, Arrays,...)

Term JavaCard terms. Variables, expressions and statements.

Decl Class, interface and program declarations. Recursion operators for the class and the interface hierarchy.

TypeRel Various relations on types like the subclass-, subinterface-, widening-, narrowing- and casting-relation.

DeclConcepts Advanced concepts on the class and interface hierarchy like inheritance, overriding, hiding, accessibility of types and members according to the access modifiers, method lookup.

WellType Typesystem on the JavaCard term level.

DefiniteAssignment The definite assignment analysis on the JavaCard term level.

WellForm Typesystem on the JavaCard class, interface and program level.

State The program state (like object store) for the execution of JavaCard. Abrupt completion (exceptions, break, continue, return) is modelled as flag inside the state.

Eval Operational (big step) semantics for JavaCard.

Example An concrete example of a JavaCard program to validate the typesystem and the operational semantics.

Conform Conformance predicate for states. When does an execution state conform to the static types of the program given by the typesystem.

DefiniteAssignmentCorrect Correctness of the definite assignment analysis. If the analysis regards a variable as definitely assigned at a certain program point, the variable will actually be assigned there during execution.

TypeSafe Typesafety proof of the execution of JavaCard. "Welltyped programs don't go wrong" or more technical: The execution of a welltyped JavaCard program preserves the conformance of execution states.

Evaln Copy of the operational semantics given in theory Eval expanded with an annotation for the maximal recursive depth. The semantics is not altered. The annotation is needed for the soundness proof of the axiomatic semantics.

Trans A smallstep operational semantics for JavaCard.

AxSem An axiomatic semantics (Hoare logic) for JavaCard.

AxSound The soundness proof of the axiomatic semantics with respect to the operational semantics.

AxCompl The proof of (relative) completeness of the axiomatic semantics with respect to the operational semantics.

AxExample An concrete example of the axiomatic semantics at work, applied to prove some properties of the JavaCard example given in theory Example.

Chapter 2

Basis

1 Definitions extending HOL as logical basis of Bali

theory *Basis*
imports *Main*
begin

misc

ML ‹*fun strip-tac ctxt i* = *REPEAT* (*resolve-tac ctxt* [*impI*, *allI*] *i*)›

declare *if-split-asm* [*split*] *option.split* [*split*] *option.split-asm* [*split*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt addloop* (*split-all-tac*, *split-all-tac*))›
declare *if-weak-cong* [*cong del*] *option.case-cong-weak* [*cong del*]
declare *length-Suc-conv* [*iff*]

lemma *Collect-split-eq*: $\{p.\ P\ (\mathit{case\text{-}prod}\ f\ p)\} = \{(a,b).\ P\ (f\ a\ b)\}$
  by *auto*

lemma *subset-insertD*: $A \subseteq \mathit{insert}\ x\ B \Longrightarrow A \subseteq B \wedge x \notin A \vee (\exists B'.\ A = \mathit{insert}\ x\ B' \wedge B' \subseteq B)$
  apply (*case-tac* $x \in A$)
   apply (*rule disjI2*)
   apply (*rule-tac* $x = A - \{x\}$ **in** *exI*)
   apply *fast*+
  done

abbreviation *nat3* :: *nat* (*3*) **where** $3 \equiv \mathit{Suc}\ 2$
abbreviation *nat4* :: *nat* (*4*) **where** $4 \equiv \mathit{Suc}\ 3$

lemma *irrefl-tranclI'*: $r^{-1} \cap r^{+} = \{\} \Longrightarrow \forall x.\ (x, x) \notin r^{+}$
  by (*blast elim*: *tranclE dest*: *trancl-into-rtrancl*)

lemma *trancl-rtrancl-trancl*: $[\![(x, y) \in r^{+}; (y, z) \in r^{*}]\!] \Longrightarrow (x, z) \in r^{+}$
  by (*auto dest*: *tranclD rtrancl-trans rtrancl-into-trancl2*)

lemma *rtrancl-into-trancl3*: $[\![(a, b) \in r^{*}; a \neq b]\!] \Longrightarrow (a, b) \in r^{+}$
  apply (*drule rtranclD*)
  apply *auto*

```
  done

lemma rtrancl-into-rtrancl2: ⟦(a, b) ∈ r; (b, c) ∈ r*⟧ ⟹ (a, c) ∈ r*
  by (auto intro: rtrancl-trans)

lemma triangle-lemma:
  assumes unique: ⋀a b c. ⟦(a,b)∈r; (a,c)∈r⟧ ⟹ b = c
    and ax: (a,x)∈r* and ay: (a,y)∈r*
  shows (x,y)∈r* ∨ (y,x)∈r*
  using ax ay
proof (induct rule: converse-rtrancl-induct)
  assume (x,y)∈r*
  then show ?thesis by blast
next
  fix a v
  assume a-v-r: (a, v) ∈ r
    and v-x-rt: (v, x) ∈ r*
    and a-y-rt: (a, y) ∈ r*
    and hyp: (v, y) ∈ r* ⟹ (x, y) ∈ r* ∨ (y, x) ∈ r*
  from a-y-rt show (x, y) ∈ r* ∨ (y, x) ∈ r*
  proof (cases rule: converse-rtranclE)
    assume a = y
    with a-v-r v-x-rt have (y,x) ∈ r*
      by (auto intro: rtrancl-trans)
    then show ?thesis by blast
  next
    fix w
    assume a-w-r: (a, w) ∈ r
      and w-y-rt: (w, y) ∈ r*
    from a-v-r a-w-r unique have v=w by auto
    with w-y-rt hyp show ?thesis by blast
  qed
qed

lemma rtrancl-cases:
  assumes (a,b)∈r*
  obtains (Refl) a = b
    | (Trancl) (a,b)∈r+
  apply (rule rtranclE [OF assms])
   apply (auto dest: rtrancl-into-trancl1)
  done

lemma Ball-weaken: ⟦Ball s P; ⋀ x. P x⟶Q x⟧⟹Ball s Q
  by auto

lemma finite-SetCompr2:
  finite {f y x |x y. P y} if finite (Collect P)
    ∀ y. P y ⟶ finite (range (f y))
proof –
  have {f y x |x y. P y} = (⋃y∈Collect P. range (f y))
    by auto
  with that show ?thesis by simp
qed
```

lemma *list-all2-trans*: $\forall a\ b\ c.\ P1\ a\ b \longrightarrow P2\ b\ c \longrightarrow P3\ a\ c \Longrightarrow$
$\forall xs2\ xs3.\ \textit{list-all2}\ P1\ xs1\ xs2 \longrightarrow \textit{list-all2}\ P2\ xs2\ xs3 \longrightarrow \textit{list-all2}\ P3\ xs1\ xs3$
apply (*induct-tac xs1*)
apply *simp*
apply (*rule allI*)
apply (*induct-tac xs2*)
apply *simp*
apply (*rule allI*)
apply (*induct-tac xs3*)
apply *auto*
done

pairs

lemma *surjective-pairing5*:
$p = (\textit{fst}\ p,\ \textit{fst}\ (\textit{snd}\ p),\ \textit{fst}\ (\textit{snd}\ (\textit{snd}\ p)),\ \textit{fst}\ (\textit{snd}\ (\textit{snd}\ (\textit{snd}\ p))),$
$\textit{snd}\ (\textit{snd}\ (\textit{snd}\ (\textit{snd}\ p))))$
by *auto*

lemma *fst-splitE* [*elim*!]:
assumes $\textit{fst}\ s' = x'$
obtains $x\ s$ **where** $s' = (x,s)$ **and** $x = x'$
using *assms* **by** (*cases* s') *auto*

lemma *fst-in-set-lemma*: $(x,\ y) \in \textit{set}\ l \Longrightarrow x \in \textit{fst}\ \text{`}\ \textit{set}\ l$
by (*induct l*) *auto*

quantifiers

lemma *All-Ex-refl-eq2* [*simp*]: $(\forall x.\ (\exists b.\ x = f\ b \wedge Q\ b) \longrightarrow P\ x) = (\forall b.\ Q\ b \longrightarrow P\ (f\ b))$
by *auto*

lemma *ex-ex-miniscope1* [*simp*]: $(\exists w\ v.\ P\ w\ v \wedge Q\ v) = (\exists v.\ (\exists w.\ P\ w\ v) \wedge Q\ v)$
by *auto*

lemma *ex-miniscope2* [*simp*]: $(\exists v.\ P\ v \wedge Q \wedge R\ v) = (Q \wedge (\exists v.\ P\ v \wedge R\ v))$
by *auto*

lemma *ex-reorder31*: $(\exists z\ x\ y.\ P\ x\ y\ z) = (\exists x\ y\ z.\ P\ x\ y\ z)$
by *auto*

lemma *All-Ex-refl-eq1* [*simp*]: $(\forall x.\ (\exists b.\ x = f\ b) \longrightarrow P\ x) = (\forall b.\ P\ (f\ b))$
by *auto*

sums

notation *case-sum* (**infixr** $'(+')$ *80*)

primrec *the-Inl* :: $'a + 'b \Rightarrow 'a$
where $\textit{the-Inl}\ (\textit{Inl}\ a) = a$

primrec $\textit{the-Inr} :: 'a + 'b \Rightarrow 'b$
  where $\textit{the-Inr}\ (\textit{Inr}\ b) = b$

datatype $('a, 'b, 'c)\ \textit{sum3} = \textit{In1}\ 'a \mid \textit{In2}\ 'b \mid \textit{In3}\ 'c$

primrec $\textit{the-In1} :: ('a, 'b, 'c)\ \textit{sum3} \Rightarrow 'a$
  where $\textit{the-In1}\ (\textit{In1}\ a) = a$

primrec $\textit{the-In2} :: ('a, 'b, 'c)\ \textit{sum3} \Rightarrow 'b$
  where $\textit{the-In2}\ (\textit{In2}\ b) = b$

primrec $\textit{the-In3} :: ('a, 'b, 'c)\ \textit{sum3} \Rightarrow 'c$
  where $\textit{the-In3}\ (\textit{In3}\ c) = c$

abbreviation $\textit{In1l} :: 'al \Rightarrow ('al + 'ar, 'b, 'c)\ \textit{sum3}$
  where $\textit{In1l}\ e \equiv \textit{In1}\ (\textit{Inl}\ e)$

abbreviation $\textit{In1r} :: 'ar \Rightarrow ('al + 'ar, 'b, 'c)\ \textit{sum3}$
  where $\textit{In1r}\ c \equiv \textit{In1}\ (\textit{Inr}\ c)$

abbreviation $\textit{the-In1l} :: ('al + 'ar, 'b, 'c)\ \textit{sum3} \Rightarrow 'al$
  where $\textit{the-In1l} \equiv \textit{the-Inl} \circ \textit{the-In1}$

abbreviation $\textit{the-In1r} :: ('al + 'ar, 'b, 'c)\ \textit{sum3} \Rightarrow 'ar$
  where $\textit{the-In1r} \equiv \textit{the-Inr} \circ \textit{the-In1}$

ML ‹
```
fun sum3-instantiate ctxt thm =
  map (fn s =>
    simplify (ctxt delsimps @{thms not-None-eq})
      (Rule-Insts.read-instantiate ctxt [((("t", 0), Position.none), "In" ^ s ^ " x")] ["x"] thm))
    ["1l","2","3","1r"]
```
›

quantifiers for option type

syntax
  $\textit{-Oall} :: [\textit{pttrn}, 'a\ \textit{option}, \textit{bool}] \Rightarrow \textit{bool}$ ((3! -:-:/ -) [0,0,10] 10)
  $\textit{-Oex} :: [\textit{pttrn}, 'a\ \textit{option}, \textit{bool}] \Rightarrow \textit{bool}$ ((3? -:-:/ -) [0,0,10] 10)

syntax (*symbols*)
  $\textit{-Oall} :: [\textit{pttrn}, 'a\ \textit{option}, \textit{bool}] \Rightarrow \textit{bool}$ ((3∀ -∈-:/ -) [0,0,10] 10)
  $\textit{-Oex} :: [\textit{pttrn}, 'a\ \textit{option}, \textit{bool}] \Rightarrow \textit{bool}$ ((3∃ -∈-:/ -) [0,0,10] 10)

translations
  $\forall x{\in}A\colon P \rightleftharpoons \forall x{\in}\textit{CONST set-option}\ A.\ P$
  $\exists x{\in}A\colon P \rightleftharpoons \exists x{\in}\textit{CONST set-option}\ A.\ P$

Special map update

Deemed too special for theory Map.

definition $\textit{chg-map} :: ('b \Rightarrow 'b) \Rightarrow 'a \Rightarrow ('a \rightharpoonup 'b) \Rightarrow ('a \rightharpoonup 'b)$
  where $\textit{chg-map}\ f\ a\ m = (\textit{case}\ m\ a\ \textit{of None} \Rightarrow m \mid \textit{Some}\ b \Rightarrow m(a \mapsto f\ b))$

lemma *chg-map-new*[*simp*]: $m\ a = \textit{None} \Longrightarrow \textit{chg-map}\ f\ a\ m = m$
  unfolding *chg-map-def* **by** *auto*

lemma *chg-map-upd*[*simp*]: $m\ a = Some\ b \Longrightarrow chg\text{-}map\ f\ a\ m = m(a \mapsto f\ b)$
unfolding *chg-map-def* **by** *auto*

lemma *chg-map-other* [*simp*]: $a \neq b \Longrightarrow chg\text{-}map\ f\ a\ m\ b = m\ b$
by (*auto simp*: *chg-map-def*)

unique association lists

definition *unique* :: $('a \times 'b)\ list \Rightarrow bool$
where $unique = distinct \circ map\ fst$

lemma *uniqueD*: $unique\ l \Longrightarrow (x,\ y) \in set\ l \Longrightarrow (x',\ y') \in set\ l \Longrightarrow x = x' \Longrightarrow y = y'$
unfolding *unique-def o-def*
by (*induct l*) (*auto dest*: *fst-in-set-lemma*)

lemma *unique-Nil* [*simp*]: *unique* []
by (*simp add*: *unique-def*)

lemma *unique-Cons* [*simp*]: $unique\ ((x,y)\#l) = (unique\ l \wedge (\forall\, y.\ (x,y) \notin set\ l))$
by (*auto simp*: *unique-def dest*: *fst-in-set-lemma*)

lemma *unique-ConsD*: $unique\ (x\#xs) \Longrightarrow unique\ xs$
by (*simp add*: *unique-def*)

lemma *unique-single* [*simp*]: $\bigwedge p.\ unique\ [p]$
by *simp*

lemma *unique-append* [*rule-format* (*no-asm*)]: $unique\ l' \Longrightarrow unique\ l \Longrightarrow$
$(\forall\,(x,y){\in}set\ l.\ \forall\,(x',y'){\in}set\ l'.\ x' \neq x) \longrightarrow unique\ (l\ @\ l')$
by (*induct l*) (*auto dest*: *fst-in-set-lemma*)

lemma *unique-map-inj*: $unique\ l \Longrightarrow inj\ f \Longrightarrow unique\ (map\ (\lambda(k,x).\ (f\ k,\ g\ k\ x))\ l)$
by (*induct l*) (*auto dest*: *fst-in-set-lemma simp add*: *inj-eq*)

lemma *map-of-SomeI*: $unique\ l \Longrightarrow (k,\ x) \in set\ l \Longrightarrow map\text{-}of\ l\ k = Some\ x$
by (*induct l*) *auto*

list patterns

definition *lsplit* :: $[['a,\ 'a\ list] \Rightarrow 'b,\ 'a\ list] \Rightarrow 'b$
where $lsplit = (\lambda f\ l.\ f\ (hd\ l)\ (tl\ l))$

list patterns – extends pre-defined type "pttrn" used in abstractions

syntax
-lpttrn :: $[pttrn,\ pttrn] \Rightarrow pttrn$ (*-#/-* [*901*,*900*] *900*)
translations
$\lambda y\ \#\ x\ \#\ xs.\ b \rightleftharpoons CONST\ lsplit\ (\lambda y\ x\ \#\ xs.\ b)$
$\lambda x\ \#\ xs.\ b \rightleftharpoons CONST\ lsplit\ (\lambda x\ xs.\ b)$

lemma *lsplit* [*simp*]: *lsplit c* (*x*#*xs*) = *c x xs*
by (*simp add*: *lsplit-def*)

lemma *lsplit2* [*simp*]: *lsplit P* (*x*#*xs*) *y z* = *P x xs y z*
by (*simp add*: *lsplit-def*)

end

Chapter 3

Table

1 Abstract tables and their implementation as lists

theory *Table* **imports** *Basis* **begin**

design issues:

- definition of table: infinite map vs. list vs. finite set list chosen, because:
  - + a priori finite
  - + lookup is more operational than for finite set
  - - not very abstract, but function table converts it to abstract mapping
- coding of lookup result: Some/None vs. value/arbitrary Some/None chosen, because:
  - ++ makes definedness check possible (applies also to finite set), which is important for the type standard, hiding/overriding, etc. (though it may perhaps be possible at least for the operational semantics to treat programs as infinite, i.e. where classes, fields, methods etc. of any name are considered to be defined)
  - - sometimes awkward case distinctions, alleviated by operator 'the'

type-synonym $('a, 'b)\ table$ — table with key type 'a and contents type 'b
  $= 'a \rightharpoonup 'b$
type-synonym $('a, 'b)\ tables$ — non-unique table with key 'a and contents 'b
  $= 'a \Rightarrow 'b\ set$

map of / table of

abbreviation
  $table\text{-}of :: ('a \times 'b)\ list \Rightarrow ('a, 'b)\ table$ — concrete table
  where $table\text{-}of \equiv map\text{-}of$

translations
  $(type)\ ('a, 'b)\ table <= (type)\ 'a \rightharpoonup 'b$

lemma *map-add-find-left*[*simp*]: $n\ k = None \Longrightarrow (m\ {+}{+}\ n)\ k = m\ k$
  by (*simp add*: *map-add-def*)

Conditional Override

definition $cond\text{-}override :: ('b \Rightarrow 'b \Rightarrow bool) \Rightarrow ('a, 'b)table \Rightarrow ('a, 'b)table \Rightarrow ('a, 'b)\ table$ **where**

— when merging tables old and new, only override an entry of table old when the condition cond holds

$$
\begin{array}{l}
\textit{cond-override cond old new} = \\
\ (\lambda k. \\
\ \ (\textit{case new k of} \\
\quad\ \ \textit{None} \qquad\quad \Rightarrow \textit{old k} \\
\quad \mid \textit{Some new-val} \Rightarrow (\textit{case old k of} \\
\qquad\qquad\qquad\qquad\quad\ \textit{None} \qquad\quad \Rightarrow \textit{Some new-val} \\
\qquad\qquad\qquad\qquad\quad \mid \textit{Some old-val} \Rightarrow (\textit{if cond new-val old-val} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\ \textit{then Some new-val} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\ \textit{else Some old-val}))))
\end{array}
$$

lemma *cond-override-empty1*[*simp*]: *cond-override c Map.empty t* = *t*
 by (*simp add*: *cond-override-def fun-eq-iff*)

lemma *cond-override-empty2*[*simp*]: *cond-override c t Map.empty* = *t*
 by (*simp add*: *cond-override-def fun-eq-iff*)

lemma *cond-override-None*[*simp*]:
 $\textit{old } k = \textit{None} \Longrightarrow (\textit{cond-override c old new})\ k = \textit{new } k$
 by (*simp add*: *cond-override-def*)

lemma *cond-override-override*:
 $[\![\textit{old } k = \textit{Some ov};\textit{new } k = \textit{Some nv};\ C\ \textit{nv ov}]\!]$
 $\Longrightarrow (\textit{cond-override } C \textit{ old new})\ k = \textit{Some nv}$
 by (*auto simp add*: *cond-override-def*)

lemma *cond-override-noOverride*:
 $[\![\textit{old } k = \textit{Some ov};\textit{new } k = \textit{Some nv};\ \neg\ (C\ \textit{nv ov})]\!]$
 $\Longrightarrow (\textit{cond-override } C \textit{ old new})\ k = \textit{Some ov}$
 by (*auto simp add*: *cond-override-def*)

lemma *dom-cond-override*: $\textit{dom}\ (\textit{cond-override } C\ s\ t) \subseteq \textit{dom } s \cup \textit{dom } t$
 by (*auto simp add*: *cond-override-def dom-def*)

lemma *finite-dom-cond-override*:
 $[\![\ \textit{finite}\ (\textit{dom } s);\ \textit{finite}\ (\textit{dom } t)\ ]\!] \Longrightarrow \textit{finite}\ (\textit{dom}\ (\textit{cond-override } C\ s\ t))$
apply (*rule-tac* $B{=}\textit{dom } s \cup \textit{dom } t$ **in** *finite-subset*)
apply (*rule dom-cond-override*)
by (*rule finite-UnI*)

Filter on Tables

definition $\textit{filter-tab} :: ('a \Rightarrow 'b \Rightarrow \textit{bool}) \Rightarrow ('a,\ 'b)\ \textit{table} \Rightarrow ('a,\ 'b)\ \textit{table}$
 where

$$
\begin{array}{l}
\textit{filter-tab c t} = (\lambda k.\ (\textit{case t k of} \\
\qquad\qquad\qquad\qquad\quad\ \textit{None} \quad \Rightarrow \textit{None} \\
\qquad\qquad\qquad\qquad\ \mid \textit{Some x} \Rightarrow \textit{if c k x then Some x else None}))
\end{array}
$$

lemma *filter-tab-empty*[*simp*]: *filter-tab c Map.empty* = *Map.empty*
by (*simp add*: *filter-tab-def empty-def*)

lemma *filter-tab-True*[*simp*]: *filter-tab* ($\lambda x\ y.\ True$) $t = t$
by (*simp add*: *fun-eq-iff filter-tab-def*)

lemma *filter-tab-False*[*simp*]: *filter-tab* ($\lambda x\ y.\ False$) $t = Map.empty$
by (*simp add*: *fun-eq-iff filter-tab-def empty-def*)

lemma *filter-tab-ran-subset*: *ran* (*filter-tab* $c\ t$) $\subseteq$ *ran* t
by (*auto simp add*: *filter-tab-def ran-def*)

lemma *filter-tab-range-subset*: *range* (*filter-tab* $c\ t$) $\subseteq$ *range* $t \cup \{None\}$
apply (*auto simp add*: *filter-tab-def*)
apply (*drule sym*, *blast*)
done

lemma *finite-range-filter-tab*:
finite (*range* t) $\Longrightarrow$ *finite* (*range* (*filter-tab* $c\ t$))
apply (*rule-tac* B=*range* $t \cup \{None\}$ **in** *finite-subset*)
apply (*rule filter-tab-range-subset*)
apply (*auto intro*: *finite-UnI*)
done

lemma *filter-tab-SomeD*[*dest*!]:
filter-tab $c\ t\ k = Some\ x \Longrightarrow (t\ k = Some\ x) \wedge c\ k\ x$
by (*auto simp add*: *filter-tab-def*)

lemma *filter-tab-SomeI*: $[\![t\ k = Some\ x;C\ k\ x]\!] \Longrightarrow$*filter-tab* $C\ t\ k = Some\ x$
by (*simp add*: *filter-tab-def*)

lemma *filter-tab-all-True*:
$\forall\ k\ y.\ t\ k = Some\ y \longrightarrow p\ k\ y \Longrightarrow$*filter-tab* $p\ t = t$
apply (*auto simp add*: *filter-tab-def fun-eq-iff*)
done

lemma *filter-tab-all-True-Some*:
$[\![\forall\ k\ y.\ t\ k = Some\ y \longrightarrow p\ k\ y;\ t\ k = Some\ v]\!] \Longrightarrow$ *filter-tab* $p\ t\ k = Some\ v$
by (*auto simp add*: *filter-tab-def fun-eq-iff*)

lemma *filter-tab-all-False*:
$\forall\ k\ y.\ t\ k = Some\ y \longrightarrow \neg\ p\ k\ y \Longrightarrow$*filter-tab* $p\ t = Map.empty$
by (*auto simp add*: *filter-tab-def fun-eq-iff*)

lemma *filter-tab-None*: $t\ k = None \Longrightarrow$ *filter-tab* $p\ t\ k = None$
apply (*simp add*: *filter-tab-def fun-eq-iff*)
done

lemma *filter-tab-dom-subset*: *dom* (*filter-tab* $C\ t$) $\subseteq$ *dom* t
by (*auto simp add*: *filter-tab-def dom-def*)

lemma *filter-tab-eq*: $[\![a{=}b]\!] \Longrightarrow$ *filter-tab C a* = *filter-tab C b*
by (*auto simp add*: *fun-eq-iff filter-tab-def*)

lemma *finite-dom-filter-tab*:
finite (*dom t*) $\Longrightarrow$ *finite* (*dom* (*filter-tab C t*))
apply (*rule-tac B=dom t* **in** *finite-subset*)
by (*rule filter-tab-dom-subset*)

lemma *filter-tab-weaken*:
$[\![\forall$ *a* $\in$ *t k*: $\exists$ *b* $\in$ *s k*: *P a b*;
$\bigwedge$ *k x y*. $[\![$*t k* = *Some x*;*s k* = *Some y*$]\!] \Longrightarrow$ *cond k x* $\longrightarrow$ *cond k y*
$]\!] \Longrightarrow \forall$ *a* $\in$ *filter-tab cond t k*: $\exists$ *b* $\in$ *filter-tab cond s k*: *P a b*
by (*force simp add*: *filter-tab-def*)

lemma *cond-override-filter*:
$[\![\bigwedge$ *k old new*. $[\![$*s k* = *Some new*; *t k* = *Some old*$]\!]$
$\Longrightarrow$ ($\neg$ *overC new old* $\longrightarrow$ $\neg$ *filterC k new*) $\wedge$
(*overC new old* $\longrightarrow$ *filterC k old* $\longrightarrow$ *filterC k new*)
$]\!] \Longrightarrow$
cond-override overC (*filter-tab filterC t*) (*filter-tab filterC s*)
= *filter-tab filterC* (*cond-override overC t s*)
by (*auto simp add*: *fun-eq-iff cond-override-def filter-tab-def*)

Misc

lemma *Ball-set-table*: ($\forall$ (*x*,*y*)$\in$ *set l*. *P x y*) $\Longrightarrow \forall$ *x*. $\forall$ *y*$\in$ *map-of l x*: *P x y*
apply (*erule rev-mp*)
apply (*induct l*)
apply *simp*
apply (*simp* (*no-asm*))
apply *auto*
done

lemma *Ball-set-tableD*:
$[\![$($\forall$ (*x*,*y*)$\in$ *set l*. *P x y*); *x* $\in$ *set-option* (*table-of l xa*)$]\!] \Longrightarrow$ *P xa x*
apply (*frule Ball-set-table*)
by *auto*

declare *map-of-SomeD* [*elim*]

lemma *table-of-Some-in-set*:
table-of l k = *Some x* $\Longrightarrow$ (*k*,*x*) $\in$ *set l*
by *auto*

lemma *set-get-eq*:
unique l $\Longrightarrow$ (*k*, *the* (*table-of l k*)) $\in$ *set l* = (*table-of l k* $\neq$ *None*)
by (*auto dest*!: *weak-map-of-SomeI*)

lemma *inj-Pair-const2*: *inj* (λ*k*. (*k*, *C*))

apply (*rule inj-onI*)
apply *auto*
done

lemma *table-of-mapconst-SomeI*:
$[\![$*table-of t k* = *Some y'*; *snd y*=*y'*; *fst y*=*c*$]\!] \Longrightarrow$
table-of (*map* (λ(*k*,*x*). (*k*,*c*,*x*)) *t*) *k* = *Some y*
by (*induct t*) *auto*

lemma *table-of-mapconst-NoneI*:
$[\![$*table-of t k* = *None*$]\!] \Longrightarrow$
table-of (*map* (λ(*k*,*x*). (*k*,*c*,*x*)) *t*) *k* = *None*
by (*induct t*) *auto*

lemmas *table-of-map2-SomeI* = *inj-Pair-const2* [*THEN map-of-mapk-SomeI*]

lemma *table-of-map-SomeI*: *table-of t k* = *Some x* $\Longrightarrow$
table-of (*map* (λ(*k*,*x*). (*k*, *f x*)) *t*) *k* = *Some* (*f x*)
by (*induct t*) *auto*

lemma *table-of-remap-SomeD*:
table-of (*map* (λ((*k*,*k'*),*x*). (*k*,(*k'*,*x*))) *t*) *k* = *Some* (*k'*,*x*) $\Longrightarrow$
table-of t (*k*, *k'*) = *Some x*
by (*induct t*) *auto*

lemma *table-of-mapf-Some*:
$\forall$ *x y*. *f x* = *f y* $\longrightarrow$ *x* = *y* $\Longrightarrow$
table-of (*map* (λ(*k*,*x*). (*k*,*f x*)) *t*) *k* = *Some* (*f x*) $\Longrightarrow$ *table-of t k* = *Some x*
by (*induct t*) *auto*

lemma *table-of-mapf-SomeD* [*dest*!]:
table-of (*map* (λ(*k*,*x*). (*k*, *f x*)) *t*) *k* = *Some z* $\Longrightarrow$ ($\exists$ *y*$\in$*table-of t k*: *z*=*f y*)
by (*induct t*) *auto*

lemma *table-of-mapf-NoneD* [*dest*!]:
table-of (*map* (λ(*k*,*x*). (*k*, *f x*)) *t*) *k* = *None* $\Longrightarrow$ (*table-of t k* = *None*)
by (*induct t*) *auto*

lemma *table-of-mapkey-SomeD* [*dest*!]:
table-of (*map* (λ(*k*,*x*). ((*k*,*C*),*x*)) *t*) (*k*,*D*) = *Some x* $\Longrightarrow$ *C* = *D* $\wedge$ *table-of t k* = *Some x*
by (*induct t*) *auto*

lemma *table-of-mapkey-SomeD2* [*dest*!]:
table-of (*map* (λ(*k*,*x*). ((*k*,*C*),*x*)) *t*) *ek* = *Some x* $\Longrightarrow$
C = *snd ek* $\wedge$ *table-of t* (*fst ek*) = *Some x*
by (*induct t*) *auto*

lemma *table-append-Some-iff*: *table-of* (*xs*@*ys*) *k* = *Some z* =
(*table-of xs k* = *Some z* $\vee$ (*table-of xs k* = *None* $\wedge$ *table-of ys k* = *Some z*))

apply (*simp*)
apply (*rule map-add-Some-iff*)
done

lemma *table-of-filter-unique-SomeD* [*rule-format* (*no-asm*)]:
table-of (*filter P xs*) k = *Some* $z \Longrightarrow$ *unique xs* $\longrightarrow$ *table-of xs* k = *Some* z
by (*induct xs*) (*auto del*: *map-of-SomeD intro*!: *map-of-SomeD*)

definition *Un-tables* :: ('*a*, '*b*) *tables set* $\Rightarrow$ ('*a*, '*b*) *tables*
where *Un-tables ts* = (λk. $\bigcup t{\in}ts.\ t\ k$)

definition *overrides-t* :: ('*a*, '*b*) *tables* $\Rightarrow$ ('*a*, '*b*) *tables* $\Rightarrow$ ('*a*, '*b*) *tables*
(**infixl** $\oplus\oplus$ *100*)
where $s \oplus\oplus t$ = (λk. *if* $t\ k = \{\}$ *then* $s\ k$ *else* $t\ k$)

definition
hidings-entails :: ('*a*, '*b*) *tables* $\Rightarrow$ ('*a*, '*c*) *tables* $\Rightarrow$ ('*b* $\Rightarrow$ '*c* $\Rightarrow$ *bool*) $\Rightarrow$ *bool*
(- *hidings* - *entails* - *20*)
where (t *hidings* s *entails* R) = ($\forall k.\ \forall x{\in}t\ k.\ \forall y{\in}s\ k.\ R\ x\ y$)

definition
— variant for unique table:
hiding-entails :: ('*a*, '*b*) *table* $\Rightarrow$ ('*a*, '*c*) *table* $\Rightarrow$ ('*b* $\Rightarrow$ '*c* $\Rightarrow$ *bool*) $\Rightarrow$ *bool*
(- *hiding* - *entails* - *20*)
where (t *hiding* s *entails* R) = ($\forall k.\ \forall x{\in}t\ k{:}\ \forall y{\in}s\ k{:}\ R\ x\ y$)

definition
— variant for a unique table and conditional overriding:
cond-hiding-entails :: ('*a*, '*b*) *table* $\Rightarrow$ ('*a*, '*c*) *table*
$\Rightarrow$ ('*b* $\Rightarrow$ '*c* $\Rightarrow$ *bool*) $\Rightarrow$ ('*b* $\Rightarrow$ '*c* $\Rightarrow$ *bool*) $\Rightarrow$ *bool*
(- *hiding* - *under* - *entails* - *20*)
where (t *hiding* s *under* C *entails* R) = ($\forall k.\ \forall x{\in}t\ k{:}\ \forall y{\in}s\ k{:}\ C\ x\ y \longrightarrow R\ x\ y$)

Untables

lemma *Un-tablesI* [*intro*]: $t \in ts \Longrightarrow x \in t\ k \Longrightarrow x \in$ *Un-tables ts k*
by (*auto simp add*: *Un-tables-def*)

lemma *Un-tablesD* [*dest*!]: $x \in$ *Un-tables ts k* $\Longrightarrow \exists t.\ t \in ts \wedge x \in t\ k$
by (*auto simp add*: *Un-tables-def*)

lemma *Un-tables-empty* [*simp*]: *Un-tables* {} = (λk. {})
by (*simp add*: *Un-tables-def*)

overrides

lemma *empty-overrides-t* [*simp*]: (λk. {}) $\oplus\oplus$ m = m
by (*simp add*: *overrides-t-def*)

lemma *overrides-empty-t* [*simp*]: m $\oplus\oplus$ (λk. {}) = m
by (*simp add*: *overrides-t-def*)

lemma *overrides-t-Some-iff*:

$(x \in (s \oplus\oplus\ t)\ k) = (x \in t\ k \vee t\ k = \{\} \wedge x \in s\ k)$
by (*simp add*: *overrides-t-def*)

lemmas *overrides-t-SomeD* = *overrides-t-Some-iff* [*THEN iffD1*, *dest*!]

lemma *overrides-t-right-empty* [*simp*]: $n\ k = \{\} \Longrightarrow (m \oplus\oplus\ n)\ k = m\ k$
by (*simp add*: *overrides-t-def*)

lemma *overrides-t-find-right* [*simp*]: $n\ k \neq \{\} \Longrightarrow (m \oplus\oplus\ n)\ k = n\ k$
by (*simp add*: *overrides-t-def*)

hiding entails

lemma *hiding-entailsD*:
t hiding s entails R $\Longrightarrow$ *t k = Some x* $\Longrightarrow$ *s k = Some y* $\Longrightarrow$ *R x y*
by (*simp add*: *hiding-entails-def*)

lemma *empty-hiding-entails* [*simp*]: *Map.empty hiding s entails R*
by (*simp add*: *hiding-entails-def*)

lemma *hiding-empty-entails* [*simp*]: *t hiding Map.empty entails R*
by (*simp add*: *hiding-entails-def*)

cond hiding entails

lemma *cond-hiding-entailsD*:
⟦*t hiding s under C entails R*; *t k = Some x*; *s k = Some y*; *C x y*⟧ $\Longrightarrow$ *R x y*
by (*simp add*: *cond-hiding-entails-def*)

lemma *empty-cond-hiding-entails*[*simp*]: *Map.empty hiding s under C entails R*
by (*simp add*: *cond-hiding-entails-def*)

lemma *cond-hiding-empty-entails*[*simp*]: *t hiding Map.empty under C entails R*
by (*simp add*: *cond-hiding-entails-def*)

lemma *hidings-entailsD*: ⟦*t hidings s entails R*; $x \in t\ k$; $y \in s\ k$⟧ $\Longrightarrow$ *R x y*
by (*simp add*: *hidings-entails-def*)

lemma *hidings-empty-entails* [*intro*!]: *t hidings* (λk. {}) *entails R*
apply (*unfold hidings-entails-def*)
apply (*simp* (*no-asm*))
done

lemma *empty-hidings-entails* [*intro*!]:
(λk. {}) *hidings s entails R***apply** (*unfold hidings-entails-def*)
by (*simp* (*no-asm*))

primrec *atleast-free* :: ($'a \rightharpoonup 'b$) => *nat* => *bool*

where
atleast-free m 0 = *True*
| *atleast-free-Suc*: *atleast-free m* (*Suc n*) = ($\exists$ *a*. *m a* = *None* $\wedge$ ($\forall$ *b*. *atleast-free* (*m*(*a*$\mapsto$*b*)) *n*))

lemma *atleast-free-weaken* [*rule-format* (*no-asm*)]:
$\forall$ *m*. *atleast-free m* (*Suc n*) $\longrightarrow$ *atleast-free m n*
apply (*induct-tac n*)
apply (*simp* (*no-asm*))
apply *clarify*
apply (*simp* (*no-asm*))
apply (*drule atleast-free-Suc* [*THEN iffD1*])
apply *fast*
done

lemma *atleast-free-SucI*:
[| *h a* = *None*; $\forall$ *obj*. *atleast-free* (*h*(*a*|−>*obj*)) *n* |] ==> *atleast-free h* (*Suc n*)
by *force*

declare *fun-upd-apply* [*simp del*]

lemma *atleast-free-SucD-lemma* [*rule-format* (*no-asm*)]:
$\forall$ *m a*. *m a* = *None* $\longrightarrow$ ($\forall$ *c*. *atleast-free* (*m*(*a*$\mapsto$*c*)) *n*) $\longrightarrow$
($\forall$ *b d*. *a* $\neq$ *b* $\longrightarrow$ *atleast-free* (*m*(*b*$\mapsto$*d*)) *n*)
apply (*induct-tac n*)
apply *auto*
apply (*rule-tac x* = *a* **in** *exI*)
apply (*rule conjI*)
apply (*force simp add*: *fun-upd-apply*)
apply (*erule-tac V* = *m a* = *None* **in** *thin-rl*)
apply *clarify*
apply (*subst fun-upd-twist*)
apply (*erule not-sym*)
apply (*rename-tac ba*)
apply (*drule-tac x* = *ba* **in** *spec*)
apply *clarify*
apply (*tactic smp-tac* ***context*** *2 1*)
apply (*erule* (*1*) *notE impE*)
apply (*case-tac aa* = *b*)
apply *fast+*
done
declare *fun-upd-apply* [*simp*]

lemma *atleast-free-SucD*: *atleast-free h* (*Suc n*) ==> *atleast-free* (*h*(*a*|−>*b*)) *n*
apply *auto*
apply (*case-tac aa* = *a*)
apply *auto*
apply (*erule atleast-free-SucD-lemma*)
apply *auto*
done

declare *atleast-free-Suc* [*simp del*]

end

Chapter 4

Name

1 Java names

theory *Name* **imports** *Basis* **begin**

typedecl *tnam* — ordinary type name, i.e. class or interface name
typedecl *pname* — package name
typedecl *mname* — method name
typedecl *vname* — variable or field name
typedecl *label* — label as destination of break or continue

datatype *ename* — expression name
  $= VNam\ vname$
  $|\ Res$ — special name to model the return value of methods

datatype *lname* — names for local variables and the This pointer
  $= EName\ ename$
  $|\ This$
abbreviation $VName :: vname \Rightarrow lname$
  where $VName\ n == EName\ (VNam\ n)$

abbreviation $Result :: lname$
  where $Result == EName\ Res$

datatype *xname* — names of standard exceptions
  $= Throwable$
  $|\ NullPointer\ |\ OutOfMemory\ |\ ClassCast$
  $|\ NegArrSize\ |\ IndOutBound\ |\ ArrStore$

lemma *xn-cases*:
$xn = Throwable \lor xn = NullPointer \lor$
$xn = OutOfMemory \lor xn = ClassCast \lor$
$xn = NegArrSize \lor xn = IndOutBound \lor xn = ArrStore$
apply (*induct xn*)
apply *auto*
done

datatype *tname* — type names for standard classes and other type names
  $= Object'$
  $|\ SXcpt'\ xname$
  $|\ TName\ tnam$

record *qtname* = — qualified tname cf. 6.5.3, 6.5.4
  pid :: *pname*
  tid :: *tname*

class *has-pname* =
  fixes *pname* :: *'a* ⇒ *pname*

instantiation *pname* :: *has-pname*
begin

definition
  pname-pname-def: *pname* (*p*::*pname*) ≡ *p*

instance ..

end

class *has-tname* =
  fixes *tname* :: *'a* ⇒ *tname*

instantiation *tname* :: *has-tname*
begin

definition
  tname-tname-def: *tname* (*t*::*tname*) = *t*

instance ..

end

definition
  qtname-qtname-def: *qtname* (*q*::*'a qtname-scheme*) = *q*

translations
  (*type*) *qtname* <= (*type*) (|*pid*::*pname*,*tid*::*tname*|)
  (*type*) *'a qtname-scheme* <= (*type*) (|*pid*::*pname*,*tid*::*tname*,...::*'a*|)

axiomatization *java-lang*::*pname* — package java.lang

definition
  Object :: *qtname*
  where *Object* = (|*pid* = *java-lang*, *tid* = *Object'*|)

definition *SXcpt* :: *xname* ⇒ *qtname*
  where *SXcpt* = (λ*x*. (|*pid* = *java-lang*, *tid* = *SXcpt' x*|))

lemma *Object-neq-SXcpt* [*simp*]: *Object* ≠ *SXcpt xn*
by (*simp add*: *Object-def SXcpt-def*)

lemma *SXcpt-inject* [*simp*]: (*SXcpt xn* = *SXcpt xm*) = (*xn* = *xm*)
by (*simp add*: *SXcpt-def*)

end

Chapter 5

Value

1 Java values

theory *Value* **imports** *Type* **begin**

typedecl *loc* — locations, i.e. abstract references on objects

datatype *val*
  = *Unit* — dummy result value of void methods
  | *Bool bool* — Boolean value
  | *Intg int* — integer value
  | *Null* — null reference
  | *Addr loc* — addresses, i.e. locations of objects

primrec *the-Bool* :: *val* ⇒ *bool*
  where *the-Bool* (*Bool b*) = *b*

primrec *the-Intg* :: *val* ⇒ *int*
  where *the-Intg* (*Intg i*) = *i*

primrec *the-Addr* :: *val* ⇒ *loc*
  where *the-Addr* (*Addr a*) = *a*

type-synonym *dyn-ty* = *loc* ⇒ *ty option*

primrec *typeof* :: *dyn-ty* ⇒ *val* ⇒ *ty option*
where
  typeof dt Unit = *Some* (*PrimT Void*)
| *typeof dt* (*Bool b*) = *Some* (*PrimT Boolean*)
| *typeof dt* (*Intg i*) = *Some* (*PrimT Integer*)
| *typeof dt Null* = *Some NT*
| *typeof dt* (*Addr a*) = *dt a*

primrec *defpval* :: *prim-ty* ⇒ *val* — default value for primitive types
where
  defpval Void = *Unit*
| *defpval Boolean* = *Bool False*
| *defpval Integer* = *Intg 0*

primrec *default-val* :: *ty* ⇒ *val* — default value for all types
where
  default-val (*PrimT pt*) = *defpval pt*
| *default-val* (*RefT r*) = *Null*

end

Chapter 6

Type

1 Java types

theory *Type* **imports** *Name* **begin**

simplifications:

- only the most important primitive types
- the null type is regarded as reference type

```
datatype prim-ty        — primitive type, cf. 4.2
        = Void          — result type of void methods
        | Boolean
        | Integer


datatype ref-ty         — reference type, cf. 4.3
        = NullT         — null type, cf. 4.1
        | IfaceT qtname — interface type
        | ClassT qtname — class type
        | ArrayT ty     — array type

and ty                  — any type, cf. 4.1
        = PrimT prim-ty — primitive type
        | RefT  ref-ty  — reference type

abbreviation NT == RefT NullT
abbreviation Iface I == RefT (IfaceT I)
abbreviation Class C == RefT (ClassT C)
abbreviation Array :: ty ⇒ ty  (-.[] [90] 90)
  where T.[] == RefT (ArrayT T)

definition
  the-Class :: ty ⇒ qtname
  where the-Class T = (SOME C. T = Class C)


lemma the-Class-eq [simp]: the-Class (Class C)= C
by (auto simp add: the-Class-def)

end
```

Chapter 7

Term

1 Java expressions and statements

theory *Term* **imports** *Value Table* **begin**

design issues:

- invocation frames for local variables could be reduced to special static objects (one per method). This would reduce redundancy, but yield a rather non-standard execution model more difficult to understand.
- method bodies separated from calls to handle assumptions in axiomat. semantics NB: Body is intended to be in the environment of the called method.
- class initialization is regarded as (auxiliary) statement (required for AxSem)
- result expression of method return is handled by a special result variable result variable is treated uniformly with local variables
  - + welltypedness and existence of the result/return expression is ensured without extra efford

simplifications:

- expression statement allowed for any expression
- This is modeled as a special non-assignable local variable
- Super is modeled as a general expression with the same value as This
- access to field x in current class via This.x
- NewA creates only one-dimensional arrays; initialization of further subarrays may be simulated with nested NewAs
- The 'Lit' constructor is allowed to contain a reference value. But this is assumed to be prohibited in the input language, which is enforced by the type-checking rules.
- a call of a static method via a type name may be simulated by a dummy variable
- no nested blocks with inner local variables
- no synchronized statements
- no secondary forms of if, while (e.g. no for) (may be easily simulated)
- no switch (may be simulated with if)

- the *try-catch-finally* statement is divided into the *try-catch* statement and a finally statement, which may be considered as try..finally with empty catch
- the *try-catch* statement has exactly one catch clause; multiple ones can be simulated with instanceof
- the compiler is supposed to add the annotations - during type-checking. This transformation is left out as its result is checked by the type rules anyway

type-synonym *locals* = (*lname*, *val*) *table* — local variables

datatype *jump*
 = *Break label* — break
 | *Cont label* — continue
 | *Ret* — return from method

datatype *xcpt* — exception
 = *Loc loc* — location of allocated execption object
 | *Std xname* — intermediate standard exception, see Eval.thy

datatype *error*
 = *AccessViolation* — Access to a member that isn't permitted
 | *CrossMethodJump* — Method exits with a break or continue

datatype *abrupt* — abrupt completion
 = *Xcpt xcpt* — exception
 | *Jump jump* — break, continue, return
 | *Error error* — runtime errors, we wan't to detect and proof absent in welltyped programms
type-synonym
abopt = *abrupt option*

Local variable store and exception. Anticipation of State.thy used by smallstep semantics. For a method call, we save the local variables of the caller in the term Callee to restore them after method return. Also an exception must be restored after the finally statement

translations
(*type*) *locals* <= (*type*) (*lname*, *val*) *table*

datatype *inv-mode* — invocation mode for method calls
 = *Static* — static
 | *SuperM* — super
 | *IntVir* — interface or virtual

record *sig* = — signature of a method, cf. 8.4.2
 name ::*mname* — acutally belongs to Decl.thy
 parTs::*ty list*

translations
(*type*) *sig* <= (*type*) (|*name*::*mname*,*parTs*::*ty list*|)
(*type*) *sig* <= (*type*) (|*name*::*mname*,*parTs*::*ty list*,...::'*a*|)

— function codes for unary operations
datatype *unop* = *UPlus* — + unary plus
 | *UMinus* — - unary minus
 | *UBitNot* — bitwise NOT
 | *UNot* — ! logical complement

— function codes for binary operations

datatype *binop* = *Mul* — `*` multiplication
| *Div* — `/` division
| *Mod* — `%` remainder
| *Plus* — `+` addition
| *Minus* — `-` subtraction
| *LShift* — `<<` left shift
| *RShift* — `>>` signed right shift
| *RShiftU* — `>>>` unsigned right shift
| *Less* — `<` less than
| *Le* — `<=` less than or equal
| *Greater* — `>` greater than
| *Ge* — `>=` greater than or equal
| *Eq* — `==` equal
| *Neq* — `!=` not equal
| *BitAnd* — `&` bitwise AND
| *And* — `&` boolean AND
| *BitXor* — `^` bitwise Xor
| *Xor* — `^` boolean Xor
| *BitOr* — `|` bitwise Or
| *Or* — `|` boolean Or
| *CondAnd* — `&&` conditional And
| *CondOr* — `||` conditional Or

The boolean operators `&` and `|` strictly evaluate both of their arguments. The conditional operators `&&` and `||` only evaluate the second argument if the value of the whole expression isn't allready determined by the first argument. e.g.: `false && e` e is not evaluated; `true || e` e is not evaluated;

datatype *var*
= *LVar lname* — local variable (incl. parameters)
| *FVar qtname qtname bool expr vname* ({-,-,-}-..-[*10*,*10*,*10*,*85*,*99*]*90*)
— class field
— {*accC*,*statDeclC*,*stat*}*e*..*fn*
— *accC*: accessing class (static class were
— the code is declared. Annotation only needed for
— evaluation to check accessibility)
— *statDeclC*: static declaration class of field
— *stat*: static or instance field?
— *e*: reference to object
— *fn*: field name
| *AVar expr expr* (-.[-][*90*,*10*]*90*)
— array component
— *e1*.[*e2*]: e1 array reference; e2 index
| *InsInitV stmt var*
— insertion of initialization before evaluation
— of var (technical term for smallstep semantics.)

and *expr*
= *NewC qtname* — class instance creation
| *NewA ty expr* (*New* -[-][*99*,*10*]*85*)
— array creation
| *Cast ty expr* — type cast
| *Inst expr ref-ty* (- *InstOf* -[*85*,*99*] *85*)
— instanceof
| *Lit val* — literal value, references not allowed
| *UnOp unop expr* — unary operation
| *BinOp binop expr expr* — binary operation

| *Super* — special Super keyword
| *Acc var* — variable access

| *Ass var expr* (-:=- [*90*,*85*]*85*)
— variable assign
| *Cond expr expr expr* (- *?* - : - [*85*,*85*,*80*]*80*) — conditional
| *Call qtname ref-ty inv-mode expr mname* (*ty list*) (*expr list*)
({-,-,-}-·-′({-}-′)[*10*,*10*,*10*,*85*,*99*,*10*,*10*]*85*)
— method call
— {*accC*,*statT*,*mode*}*e*·*mn*({*pTs*}*args*) "
— *accC*: accessing class (static class were
— the call code is declared. Annotation only needed for
— evaluation to check accessibility)
— *statT*: static declaration class/interface of
— method
— *mode*: invocation mode
— *e*: reference to object
— *mn*: field name
— *pTs*: types of parameters
— *args*: the actual parameters/arguments
| *Methd qtname sig* — (folded) method (see below)
| *Body qtname stmt* — (unfolded) method body
| *InsInitE stmt expr*
— insertion of initialization before
— evaluation of expr (technical term for smallstep sem.)
| *Callee locals expr* — save callers locals in callee-Frame
— (technical term for smallstep semantics)

and *stmt*
= *Skip* — empty statement
| *Expr expr* — expression statement
| *Lab jump stmt* (-· - [*99*,*66*]*66*)
— labeled statement; handles break
| *Comp stmt stmt* (-;; - [*66*,*65*]*65*)
| *If′ expr stmt stmt* (*If′*(-′) - *Else* - [*80*,*79*,*79*]*70*)
| *Loop label expr stmt* (-· *While′*(-′) - [*99*,*80*,*79*]*70*)
| *Jmp jump* — break, continue, return
| *Throw expr*
| *TryC stmt qtname vname stmt* (*Try* - *Catch′*(- -′) - [*79*,*99*,*80*,*79*]*70*)
— *Try c1 Catch*(*C vn*) *c2*
— *c1*: block were exception may be thrown
— *C*: execption class to catch
— *vn*: local name for exception used in *c2*
— *c2*: block to execute when exception is cateched
| *Fin stmt stmt* (- *Finally* - [*79*,*79*]*70*)
| *FinA abopt stmt* — Save abruption of first statement
— technical term for smallstep sem.)
| *Init qtname* — class initialization

datatype-compat *var expr stmt*

The expressions Methd and Body are artificial program constructs, in the sense that they are not used to define a concrete Bali program. In the operational semantic's they are "generated on the fly" to decompose the task to define the behaviour of the Call expression. They are crucial for the axiomatic semantics to give a syntactic hook to insert some assertions (cf. AxSem.thy, Eval.thy). The Init statement (to initialize a class on its first use) is inserted in various places by the semantics. Callee, InsInitV, InsInitE,FinA are only needed as intermediate steps in the smallstep (transition) semantics (cf. Trans.thy). Callee is used to save the local variables of the caller for method return. So ve avoid modelling a frame stack. The InsInitV/E terms are only used by the smallstep semantics to model the intermediate steps of class-initialisation.

type-synonym *term* = (*expr*+*stmt*,*var*,*expr list*) *sum3*
translations

(*type*) *sig* <= (*type*) *mname* × *ty list*
(*type*) *term* <= (*type*) (*expr+stmt,var,expr list*) *sum3*

abbreviation *this* :: *expr*
where *this* == *Acc* (*LVar This*)

abbreviation *LAcc* :: *vname* ⇒ *expr* (!!)
where !!*v* == *Acc* (*LVar* (*EName* (*VNam v*)))

abbreviation
LAss :: *vname* ⇒ *expr* ⇒*stmt* (-:==- [*90,85*] *85*)
where *v*:==*e* == *Expr* (*Ass* (*LVar* (*EName* (*VNam v*))) *e*)

abbreviation
Return :: *expr* ⇒ *stmt*
where *Return e* == *Expr* (*Ass* (*LVar* (*EName Res*)) *e*);; *Jmp Ret* — `Res := e;; Jmp Ret`

abbreviation
StatRef :: *ref-ty* ⇒ *expr*
where *StatRef rt* == *Cast* (*RefT rt*) (*Lit Null*)

definition
is-stmt :: *term* ⇒ *bool*
where *is-stmt t* = (∃ *c*. *t*=*In1r c*)

ML ‹*ML-Thms.bind-thms* (*is-stmt-rews*, *sum3-instantiate* ***context*** @{*thm is-stmt-def*})›

declare *is-stmt-rews* [*simp*]

Here is some syntactic stuff to handle the injections of statements, expressions, variables and expression lists into general terms.

abbreviation (*input*)
expr-inj-term :: *expr* ⇒ *term* ($\langle$-$\rangle_e$ *1000*)
where $\langle e\rangle_e$ == *In1l e*

abbreviation (*input*)
stmt-inj-term :: *stmt* ⇒ *term* ($\langle$-$\rangle_s$ *1000*)
where $\langle c\rangle_s$ == *In1r c*

abbreviation (*input*)
var-inj-term :: *var* ⇒ *term* ($\langle$-$\rangle_v$ *1000*)
where $\langle v\rangle_v$ == *In2 v*

abbreviation (*input*)
lst-inj-term :: *expr list* ⇒ *term* ($\langle$-$\rangle_l$ *1000*)
where $\langle es\rangle_l$ == *In3 es*

It seems to be more elegant to have an overloaded injection like the following.

class *inj-term* =
fixes *inj-term*:: *'a* ⇒ *term* (⟨-⟩ *1000*)

How this overloaded injections work can be seen in the theory *DefiniteAssignment*. Other big inductive relations on terms defined in theories *WellType*, *Eval*, *Evaln* and *AxSem* don't follow this convention right now, but introduce subtle syntactic sugar in the relations themselves to make a distinction on expressions, statements and so on. So unfortunately you will encounter a mixture of dealing with these injections. The abbreviations above are used as bridge between the different conventions.

instantiation *stmt* :: *inj-term*

begin

definition
  stmt-inj-term-def: $\langle c{::}stmt\rangle = In1r\ c$

instance ..

end

lemma *stmt-inj-term-simp*: $\langle c{::}stmt\rangle = In1r\ c$
by (*simp add*: *stmt-inj-term-def*)

lemma *stmt-inj-term* [*iff*]: $\langle x{::}stmt\rangle = \langle y\rangle \equiv x = y$
  by (*simp add*: *stmt-inj-term-simp*)

instantiation *expr* :: *inj-term*
begin

definition
  expr-inj-term-def: $\langle e{::}expr\rangle = In1l\ e$

instance ..

end

lemma *expr-inj-term-simp*: $\langle e{::}expr\rangle = In1l\ e$
by (*simp add*: *expr-inj-term-def*)

lemma *expr-inj-term* [*iff*]: $\langle x{::}expr\rangle = \langle y\rangle \equiv x = y$
  by (*simp add*: *expr-inj-term-simp*)

instantiation *var* :: *inj-term*
begin

definition
  var-inj-term-def: $\langle v{::}var\rangle = In2\ v$

instance ..

end

lemma *var-inj-term-simp*: $\langle v{::}var\rangle = In2\ v$
by (*simp add*: *var-inj-term-def*)

lemma *var-inj-term* [*iff*]: $\langle x{::}var\rangle = \langle y\rangle \equiv x = y$
  by (*simp add*: *var-inj-term-simp*)

class *expr-of* =
  fixes *expr-of* :: $'a \Rightarrow expr$

instantiation *expr* :: *expr-of*
begin

definition
  expr-of = $(\lambda(e::expr).\ e)$

instance ..

end

instantiation *list* :: (*expr-of*) *inj-term*
begin

definition
  $\langle es::'a\ list\rangle = In3\ (map\ expr\text{-}of\ es)$

instance ..

end

lemma *expr-list-inj-term-def*:
  $\langle es::expr\ list\rangle \equiv In3\ es$
  by (*simp add*: *inj-term-list-def expr-of-expr-def*)

lemma *expr-list-inj-term-simp*: $\langle es::expr\ list\rangle = In3\ es$
by (*simp add*: *expr-list-inj-term-def*)

lemma *expr-list-inj-term* [*iff*]: $\langle x::expr\ list\rangle = \langle y\rangle \equiv x = y$
  by (*simp add*: *expr-list-inj-term-simp*)

lemmas *inj-term-simps* = *stmt-inj-term-simp expr-inj-term-simp var-inj-term-simp*
  expr-list-inj-term-simp
lemmas *inj-term-sym-simps* = *stmt-inj-term-simp* [*THEN sym*]
  expr-inj-term-simp [*THEN sym*]
  var-inj-term-simp [*THEN sym*]
  expr-list-inj-term-simp [*THEN sym*]

lemma *stmt-expr-inj-term* [*iff*]: $\langle t::stmt\rangle \neq \langle w::expr\rangle$
  by (*simp add*: *inj-term-simps*)

lemma *expr-stmt-inj-term* [*iff*]: $\langle t::expr\rangle \neq \langle w::stmt\rangle$
  by (*simp add*: *inj-term-simps*)

lemma *stmt-var-inj-term* [*iff*]: $\langle t::stmt\rangle \neq \langle w::var\rangle$
  by (*simp add*: *inj-term-simps*)

lemma *var-stmt-inj-term* [*iff*]: $\langle t::var\rangle \neq \langle w::stmt\rangle$
  by (*simp add*: *inj-term-simps*)

lemma *stmt-elist-inj-term* [*iff*]: $\langle t::stmt\rangle \neq \langle w::expr\ list\rangle$
  by (*simp add*: *inj-term-simps*)

lemma *elist-stmt-inj-term* [*iff*]: $\langle t::expr\ list\rangle \neq \langle w::stmt\rangle$
  by (*simp add*: *inj-term-simps*)

lemma *expr-var-inj-term* [*iff*]: $\langle t::expr\rangle \neq \langle w::var\rangle$
  by (*simp add*: *inj-term-simps*)

lemma *var-expr-inj-term* [*iff*]: $\langle t::var\rangle \neq \langle w::expr\rangle$
by (*simp add*: *inj-term-simps*)

lemma *expr-elist-inj-term* [*iff*]: $\langle t::expr\rangle \neq \langle w::expr\ list\rangle$
by (*simp add*: *inj-term-simps*)

lemma *elist-expr-inj-term* [*iff*]: $\langle t::expr\ list\rangle \neq \langle w::expr\rangle$
by (*simp add*: *inj-term-simps*)

lemma *var-elist-inj-term* [*iff*]: $\langle t::var\rangle \neq \langle w::expr\ list\rangle$
by (*simp add*: *inj-term-simps*)

lemma *elist-var-inj-term* [*iff*]: $\langle t::expr\ list\rangle \neq \langle w::var\rangle$
by (*simp add*: *inj-term-simps*)

lemma *term-cases*:
$[\![\bigwedge\ v.\ P\ \langle v\rangle_v;\ \bigwedge\ e.\ P\ \langle e\rangle_e;\bigwedge\ c.\ P\ \langle c\rangle_s;\bigwedge\ l.\ P\ \langle l\rangle_l]\!]$
$\Longrightarrow P\ t$
apply (*cases t*)
apply (*rename-tac a*, *case-tac a*)
apply *auto*
done

Evaluation of unary operations

primrec *eval-unop* :: $unop \Rightarrow val \Rightarrow val$
where
eval-unop UPlus v = *Intg* (*the-Intg v*)
| *eval-unop UMinus v* = *Intg* (− (*the-Intg v*))
| *eval-unop UBitNot v* = *Intg 42* — FIXME: Not yet implemented
| *eval-unop UNot v* = *Bool* (¬ *the-Bool v*)

Evaluation of binary operations

primrec *eval-binop* :: $binop \Rightarrow val \Rightarrow val \Rightarrow val$
where
eval-binop Mul v1 v2 = *Intg* ((*the-Intg v1*) ∗ (*the-Intg v2*))
| *eval-binop Div v1 v2* = *Intg* ((*the-Intg v1*) *div* (*the-Intg v2*))
| *eval-binop Mod v1 v2* = *Intg* ((*the-Intg v1*) *mod* (*the-Intg v2*))
| *eval-binop Plus v1 v2* = *Intg* ((*the-Intg v1*) + (*the-Intg v2*))
| *eval-binop Minus v1 v2* = *Intg* ((*the-Intg v1*) − (*the-Intg v2*))

— Be aware of the explicit coercion of the shift distance to nat
| *eval-binop LShift v1 v2* = *Intg* ((*the-Intg v1*) ∗ (*2*^(*nat* (*the-Intg v2*))))
| *eval-binop RShift v1 v2* = *Intg* ((*the-Intg v1*) *div* (*2*^(*nat* (*the-Intg v2*))))
| *eval-binop RShiftU v1 v2* = *Intg 42* — FIXME: Not yet implemented

| *eval-binop Less v1 v2* = *Bool* ((*the-Intg v1*) < (*the-Intg v2*))
| *eval-binop Le v1 v2* = *Bool* ((*the-Intg v1*) ≤ (*the-Intg v2*))
| *eval-binop Greater v1 v2* = *Bool* ((*the-Intg v2*) < (*the-Intg v1*))
| *eval-binop Ge v1 v2* = *Bool* ((*the-Intg v2*) ≤ (*the-Intg v1*))

| *eval-binop Eq v1 v2* = *Bool* (*v1*=*v2*)
| *eval-binop Neq v1 v2* = *Bool* (*v1*≠*v2*)
| *eval-binop BitAnd v1 v2* = *Intg 42* — FIXME: Not yet implemented
| *eval-binop And v1 v2* = *Bool* ((*the-Bool v1*) ∧ (*the-Bool v2*))
| *eval-binop BitXor v1 v2* = *Intg 42* — FIXME: Not yet implemented
| *eval-binop Xor v1 v2* = *Bool* ((*the-Bool v1*) ≠ (*the-Bool v2*))

| *eval-binop BitOr v1 v2* = *Intg 42* — FIXME: Not yet implemented
| *eval-binop Or v1 v2* = *Bool* ((*the-Bool v1*) ∨ (*the-Bool v2*))
| *eval-binop CondAnd v1 v2* = *Bool* ((*the-Bool v1*) ∧ (*the-Bool v2*))
| *eval-binop CondOr v1 v2* = *Bool* ((*the-Bool v1*) ∨ (*the-Bool v2*))

definition
need-second-arg :: *binop* ⇒ *val* ⇒ *bool* **where**
need-second-arg binop v1 = (¬ ((*binop*=*CondAnd* ∧ ¬ *the-Bool v1*) ∨
(*binop*=*CondOr* ∧ *the-Bool v1*)))

CondAnd and *CondOr* only evalulate the second argument if the value isn't already determined by the first argument

lemma *need-second-arg-CondAnd* [*simp*]: *need-second-arg CondAnd* (*Bool b*) = *b*
by (*simp add*: *need-second-arg-def*)

lemma *need-second-arg-CondOr* [*simp*]: *need-second-arg CondOr* (*Bool b*) = (¬ *b*)
by (*simp add*: *need-second-arg-def*)

lemma *need-second-arg-strict*[*simp*]:
⟦*binop*≠*CondAnd*; *binop*≠*CondOr*⟧ ⟹ *need-second-arg binop b*
by (*cases binop*)
(*simp-all add*: *need-second-arg-def*)
end

Chapter 8

Decl

1 Field, method, interface, and class declarations, whole Java programs

theory *Decl*
imports *Term Table*

begin

improvements:

- clarification and correction of some aspects of the package/access concept (Also submitted as bug report to the Java Bug Database: Bug Id: 4485402 and Bug Id: 4493343 http://developer.java.sun.com/developer/bugParade/index.jshtml)

simplifications:

- the only field and method modifiers are static and the access modifiers
- no constructors, which may be simulated by new + suitable methods
- there is just one global initializer per class, which can simulate all others
- no throws clause
- a void method is replaced by one that returns Unit (of dummy type Void)
- no interface fields
- every class has an explicit superclass (unused for Object)
- the (standard) methods of Object and of standard exceptions are not specified
- no main method

2 Modifier

Access modifier

datatype *acc-modi*
    = *Private* | *Package* | *Protected* | *Public*

We can define a linear order for the access modifiers. With Private yielding the most restrictive access and public the most liberal access policy: Private < Package < Protected < Public

instantiation *acc-modi* :: *linorder*
begin

definition
less-acc-def: $a < b$
$\longleftrightarrow$ (*case a of*
Private $\Rightarrow$ (b=*Package* $\vee$ b=*Protected* $\vee$ b=*Public*)
| *Package* $\Rightarrow$ (b=*Protected* $\vee$ b=*Public*)
| *Protected* $\Rightarrow$ (b=*Public*)
| *Public* $\Rightarrow$ *False*)

definition
le-acc-def: (a :: *acc-modi*) $\leq b \longleftrightarrow a < b \vee a = b$

instance
proof
fix $x\ y\ z$::*acc-modi*
show $(x < y) = (x \leq y \wedge \neg\ y \leq x)$
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.split*)
show $x \leq x$ — reflexivity
by (*auto simp add*: *le-acc-def*)
{
assume $x \leq y\ y \leq z$ — transitivity
then show $x \leq z$
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.split*)
next
assume $x \leq y\ y \leq x$ — antisymmetry
moreover have $\forall\ x\ y.\ x <$ (y::*acc-modi*) $\wedge\ y < x \longrightarrow$ *False*
by (*auto simp add*: *less-acc-def split*: *acc-modi.split*)
ultimately show $x = y$ **by** (*unfold le-acc-def*) *iprover*
next
fix $x\ y$:: *acc-modi*
show $x \leq y \vee y \leq x$
by (*auto simp add*: *less-acc-def le-acc-def split*: *acc-modi.split*)
}
qed

end

lemma *acc-modi-top* [*simp*]: *Public* $\leq a \Longrightarrow a =$ *Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-top1* [*simp*, *intro*!]: $a \leq$ *Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-le-Public*:
$a \leq$ *Public* $\Longrightarrow a$=*Private* $\vee\ a =$ *Package* $\vee\ a$=*Protected* $\vee\ a$=*Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-bottom*: $a \leq$ *Private* $\Longrightarrow a =$ *Private*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-Private-le*:
Private $\leq a \Longrightarrow a$=*Private* $\vee\ a =$ *Package* $\vee\ a$=*Protected* $\vee\ a$=*Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-Package-le*:
  $Package \leq a \Longrightarrow a = Package \vee a{=}Protected \vee a{=}Public$
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.split*)

lemma *acc-modi-le-Package*:
  $a \leq Package \Longrightarrow a{=}Private \vee a = Package$
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-Protected-le*:
  $Protected \leq a \Longrightarrow a{=}Protected \vee a{=}Public$
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-le-Protected*:
  $a \leq Protected \Longrightarrow a{=}Private \vee a = Package \vee a = Protected$
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemmas *acc-modi-le-Dests* = *acc-modi-top* *acc-modi-le-Public*
acc-modi-Private-le *acc-modi-bottom*
acc-modi-Package-le *acc-modi-le-Package*
acc-modi-Protected-le *acc-modi-le-Protected*

lemma *acc-modi-Package-le-cases*:
  assumes $Package \leq m$
  obtains (*Package*) $m = Package$
  | (*Protected*) $m = Protected$
  | (*Public*) $m = Public$
using *assms* **by** (*auto dest*: *acc-modi-Package-le*)

Static Modifier

type-synonym *stat-modi* = *bool*

3 Declaration (base "class" for member,interface and class declarations

record *decl* =
    access :: *acc-modi*

translations
  (*type*) *decl* <= (*type*) (|*access*::*acc-modi*|)
  (*type*) *decl* <= (*type*) (|*access*::*acc-modi*,...::'a|)

4 Member (field or method)

record *member* = *decl* +
    static :: *stat-modi*

translations
  (*type*) *member* <= (*type*) (|*access*::*acc-modi*,*static*::*bool*|)
  (*type*) *member* <= (*type*) (|*access*::*acc-modi*,*static*::*bool*,...::'a|)

5 Field

record *field* = *member* +

type :: *ty*
translations
(*type*) *field* <= (*type*) (|*access*::*acc-modi*, *static*::*bool*, *type*::*ty*|)
(*type*) *field* <= (*type*) (|*access*::*acc-modi*, *static*::*bool*, *type*::*ty*,...::'*a*|)

type-synonym *fdecl*
= *vname* × *field*

translations
(*type*) *fdecl* <= (*type*) *vname* × *field*

6 Method

record *mhead* = *member* +
pars ::*vname list*
resT ::*ty*

record *mbody* =
lcls:: (*vname* × *ty*) *list*
stmt:: *stmt*

record *methd* = *mhead* +
mbody::*mbody*

type-synonym *mdecl* = *sig* × *methd*

translations
(*type*) *mhead* <= (*type*) (|*access*::*acc-modi*, *static*::*bool*,
pars::*vname list*, *resT*::*ty*|)
(*type*) *mhead* <= (*type*) (|*access*::*acc-modi*, *static*::*bool*,
pars::*vname list*, *resT*::*ty*,...::'*a*|)
(*type*) *mbody* <= (*type*) (|*lcls*::(*vname* × *ty*) *list*,*stmt*::*stmt*|)
(*type*) *mbody* <= (*type*) (|*lcls*::(*vname* × *ty*) *list*,*stmt*::*stmt*,...::'*a*|)
(*type*) *methd* <= (*type*) (|*access*::*acc-modi*, *static*::*bool*,
pars::*vname list*, *resT*::*ty*,*mbody*::*mbody*|)
(*type*) *methd* <= (*type*) (|*access*::*acc-modi*, *static*::*bool*,
pars::*vname list*, *resT*::*ty*,*mbody*::*mbody*,...::'*a*|)
(*type*) *mdecl* <= (*type*) *sig* × *methd*

definition
mhead :: *methd* ⇒ *mhead*
where *mhead m* = (|*access*=*access m*, *static*=*static m*, *pars*=*pars m*, *resT*=*resT m*|)

lemma *access-mhead* [*simp*]:*access* (*mhead m*) = *access m*
by (*simp add*: *mhead-def*)

lemma *static-mhead* [*simp*]:*static* (*mhead m*) = *static m*
by (*simp add*: *mhead-def*)

lemma *pars-mhead* [*simp*]:*pars* (*mhead m*) = *pars m*
by (*simp add*: *mhead-def*)

lemma *resT-mhead* [*simp*]:*resT* (*mhead m*) = *resT m*
by (*simp add*: *mhead-def*)

To be able to talk uniformaly about field and method declarations we introduce the notion of a member declaration (e.g. useful to define accessiblity)

datatype *memberdecl* = *fdecl fdecl* | *mdecl mdecl*

datatype *memberid* = *fid vname* | *mid sig*

class *has-memberid* =
  fixes *memberid* :: *'a* ⇒ *memberid*

instantiation *memberdecl* :: *has-memberid*
begin

definition
memberdecl-memberid-def:
  memberid m = (*case m of*
                    fdecl (*vn*,*f*) ⇒ *fid vn*
                  | *mdecl* (*sig*,*m*) ⇒ *mid sig*)

instance ..

end

lemma *memberid-fdecl-simp*[*simp*]: *memberid* (*fdecl* (*vn*,*f*)) = *fid vn*
by (*simp add*: *memberdecl-memberid-def*)

lemma *memberid-fdecl-simp1*: *memberid* (*fdecl f*) = *fid* (*fst f*)
by (*cases f*) (*simp add*: *memberdecl-memberid-def*)

lemma *memberid-mdecl-simp*[*simp*]: *memberid* (*mdecl* (*sig*,*m*)) = *mid sig*
by (*simp add*: *memberdecl-memberid-def*)

lemma *memberid-mdecl-simp1*: *memberid* (*mdecl m*) = *mid* (*fst m*)
by (*cases m*) (*simp add*: *memberdecl-memberid-def*)

instantiation *prod* :: (*type*, *has-memberid*) *has-memberid*
begin

definition
pair-memberid-def:
  memberid p = *memberid* (*snd p*)

instance ..

end

lemma *memberid-pair-simp*[*simp*]: *memberid* (*c*,*m*) = *memberid m*
by (*simp add*: *pair-memberid-def*)

lemma *memberid-pair-simp1*: *memberid p* = *memberid* (*snd p*)
by (*simp add*: *pair-memberid-def*)

definition
is-field :: *qtname* × *memberdecl* ⇒ *bool*
where *is-field m* = (∃ *declC f*. *m*=(*declC*,*fdecl f*))

lemma *is-fieldD*: *is-field m* ⟹ ∃ *declC f*. *m*=(*declC*,*fdecl f*)
by (*simp add*: *is-field-def*)

lemma *is-fieldI*: *is-field* (*C*,*fdecl f*)
by (*simp add*: *is-field-def*)

definition
is-method :: *qtname* × *memberdecl* ⇒ *bool*
where *is-method membr* = (∃ *declC m*. *membr*=(*declC*,*mdecl m*))

lemma *is-methodD*: *is-method membr* ⟹ ∃ *declC m*. *membr*=(*declC*,*mdecl m*)
by (*simp add*: *is-method-def*)

lemma *is-methodI*: *is-method* (*C*,*mdecl m*)
by (*simp add*: *is-method-def*)

7 Interface

record *ibody* = *decl* + — interface body
imethods :: (*sig* × *mhead*) *list* — method heads

record *iface* = *ibody* + — interface
isuperIfs:: *qtname list* — superinterface list
type-synonym
idecl — interface declaration, cf. 9.1
= *qtname* × *iface*

translations
(*type*) *ibody* <= (*type*) (|*access*::*acc-modi*,*imethods*::(*sig* × *mhead*) *list*|)
(*type*) *ibody* <= (*type*) (|*access*::*acc-modi*,*imethods*::(*sig* × *mhead*) *list*,...::'*a*|)
(*type*) *iface* <= (*type*) (|*access*::*acc-modi*,*imethods*::(*sig* × *mhead*) *list*,
isuperIfs::*qtname list*|)
(*type*) *iface* <= (*type*) (|*access*::*acc-modi*,*imethods*::(*sig* × *mhead*) *list*,
isuperIfs::*qtname list*,...::'*a*|)
(*type*) *idecl* <= (*type*) *qtname* × *iface*

definition
ibody :: *iface* ⇒ *ibody*
where *ibody i* = (|*access*=*access i*,*imethods*=*imethods i*|)

lemma *access-ibody* [*simp*]: (*access* (*ibody i*)) = *access i*
by (*simp add*: *ibody-def*)

lemma *imethods-ibody* [*simp*]: (*imethods* (*ibody i*)) = *imethods i*
by (*simp add*: *ibody-def*)

8 Class

record *cbody* = *decl* + — class body
  cfields:: *fdecl list*
  methods:: *mdecl list*
  init :: *stmt* — initializer

record *class* = *cbody* + — class
  super :: *qtname* — superclass
  superIfs:: *qtname list* — implemented interfaces
type-synonym
  cdecl — class declaration, cf. 8.1
  = *qtname* × *class*

translations
  (*type*) *cbody* <= (*type*) (|*access*::*acc-modi*,*cfields*::*fdecl list*,
    methods::*mdecl list*,*init*::*stmt*|)
  (*type*) *cbody* <= (*type*) (|*access*::*acc-modi*,*cfields*::*fdecl list*,
    methods::*mdecl list*,*init*::*stmt*,...::'*a*|)
  (*type*) *class* <= (*type*) (|*access*::*acc-modi*,*cfields*::*fdecl list*,
    methods::*mdecl list*,*init*::*stmt*,
    super::*qtname*,*superIfs*::*qtname list*|)
  (*type*) *class* <= (*type*) (|*access*::*acc-modi*,*cfields*::*fdecl list*,
    methods::*mdecl list*,*init*::*stmt*,
    super::*qtname*,*superIfs*::*qtname list*,...::'*a*|)
  (*type*) *cdecl* <= (*type*) *qtname* × *class*

definition
  cbody :: *class* ⇒ *cbody*
  where *cbody c* = (|*access*=*access c*, *cfields*=*cfields c*,*methods*=*methods c*,*init*=*init c*|)

lemma *access-cbody* [*simp*]:*access* (*cbody c*) = *access c*
by (*simp add*: *cbody-def*)

lemma *cfields-cbody* [*simp*]:*cfields* (*cbody c*) = *cfields c*
by (*simp add*: *cbody-def*)

lemma *methods-cbody* [*simp*]:*methods* (*cbody c*) = *methods c*
by (*simp add*: *cbody-def*)

lemma *init-cbody* [*simp*]:*init* (*cbody c*) = *init c*
by (*simp add*: *cbody-def*)

standard classes

consts
  Object-mdecls :: *mdecl list* — methods of Object
  SXcpt-mdecls :: *mdecl list* — methods of SXcpts

definition
  ObjectC :: *cdecl* — declaration of root class **where**
  ObjectC = (*Object*,(|*access*=*Public*,*cfields*=[],*methods*=*Object-mdecls*,
    init=*Skip*,*super*=*undefined*,*superIfs*=[]|))

definition

SXcptC ::*xname* ⇒ *cdecl* — declarations of throwable classes **where**
SXcptC xn = (*SXcpt xn*,(|*access*=*Public*,*cfields*=[],*methods*=*SXcpt-mdecls*,
init=*Skip*,
super=*if xn* = *Throwable then Object*
else SXcpt Throwable,
superIfs=[]|))

lemma *ObjectC-neq-SXcptC* [*simp*]: *ObjectC* ≠ *SXcptC xn*
by (*simp add*: *ObjectC-def SXcptC-def Object-def SXcpt-def*)

lemma *SXcptC-inject* [*simp*]: (*SXcptC xn* = *SXcptC xm*) = (*xn* = *xm*)
by (*simp add*: *SXcptC-def*)

definition
standard-classes :: *cdecl list* **where**
standard-classes = [*ObjectC*, *SXcptC Throwable*,
SXcptC NullPointer, *SXcptC OutOfMemory*, *SXcptC ClassCast*,
SXcptC NegArrSize , *SXcptC IndOutBound*, *SXcptC ArrStore*]

programs

record *prog* =
ifaces ::*idecl list*
classes::*cdecl list*

translations
(*type*) *prog* <= (*type*) (|*ifaces*::*idecl list*,*classes*::*cdecl list*|)
(*type*) *prog* <= (*type*) (|*ifaces*::*idecl list*,*classes*::*cdecl list*,. . .::′*a*|)

abbreviation
iface :: *prog* ⇒ (*qtname*, *iface*) *table*
where *iface G I* == *table-of* (*ifaces G*) *I*

abbreviation
class :: *prog* ⇒ (*qtname*, *class*) *table*
where *class G C* == *table-of* (*classes G*) *C*

abbreviation
is-iface :: *prog* ⇒ *qtname* ⇒ *bool*
where *is-iface G I* == *iface G I* ≠ *None*

abbreviation
is-class :: *prog* ⇒ *qtname* ⇒ *bool*
where *is-class G C* == *class G C* ≠ *None*

is type

primrec *is-type* :: *prog* ⇒ *ty* ⇒ *bool*
and *isrtype* :: *prog* ⇒ *ref-ty* ⇒ *bool*
where
is-type G (*PrimT pt*) = *True*
| *is-type G* (*RefT rt*) = *isrtype G rt*
| *isrtype G* (*NullT*) = *True*
| *isrtype G* (*IfaceT tn*) = *is-iface G tn*
| *isrtype G* (*ClassT tn*) = *is-class G tn*
| *isrtype G* (*ArrayT T*) = *is-type G T*

lemma *type-is-iface*: *is-type* G (*Iface* I) $\Longrightarrow$ *is-iface* G I
by *auto*

lemma *type-is-class*: *is-type* G (*Class* C) $\Longrightarrow$ *is-class* G C
by *auto*

subinterface and subclass relation, in anticipation of TypeRel.thy

definition
  subint1 :: *prog* $\Rightarrow$ (*qtname* $\times$ *qtname*) *set* — direct subinterface
  where *subint1* G = $\{(I,J).\ \exists i \in$ *iface* G I: $J \in$ *set* (*isuperIfs* i)$\}$

definition
  subcls1 :: *prog* $\Rightarrow$ (*qtname* $\times$ *qtname*) *set* — direct subclass
  where *subcls1* G = $\{(C,D).\ C \neq Object \wedge (\exists c \in$ *class* G C: *super* $c = D)\}$

abbreviation
  subcls1-syntax :: *prog* => [*qtname*, *qtname*] => *bool* (-$\vdash$-$\prec_C$1- [71,71,71] 70)
  where $G \vdash C \prec_C 1\ D$ == $(C,D) \in$ *subcls1* G

abbreviation
  subclseq-syntax :: *prog* => [*qtname*, *qtname*] => *bool* (-$\vdash$-$\preceq_C$ - [71,71,71] 70)
  where $G \vdash C \preceq_C D$ == $(C,D) \in$(*subcls1* G)*

abbreviation
  subcls-syntax :: *prog* => [*qtname*, *qtname*] => *bool* (-$\vdash$-$\prec_C$ - [71,71,71] 70)
  where $G \vdash C \prec_C D$ == $(C,D) \in$(*subcls1* G)$^+$

notation (*ASCII*)
  subcls1-syntax (-|−-<:C1- [71,71,71] 70) **and**
  subclseq-syntax (-|−-<=:C -[71,71,71] 70) **and**
  subcls-syntax (-|−-<:C -[71,71,71] 70)

lemma *subint1I*: ⟦*iface* G I = *Some* i; $J \in$ *set* (*isuperIfs* i)⟧
  $\Longrightarrow (I,J) \in$ *subint1* G
apply (*simp add*: *subint1-def*)
done

lemma *subcls1I*:⟦*class* G C = *Some* c; $C \neq$ *Object*⟧ $\Longrightarrow$ $(C,$(*super* c)$) \in$ *subcls1* G
apply (*simp add*: *subcls1-def*)
done

lemma *subint1D*: $(I,J) \in$ *subint1* $G \Longrightarrow \exists i \in$ *iface* G I: $J \in$ *set* (*isuperIfs* i)
by (*simp add*: *subint1-def*)

lemma *subcls1D*:
  $(C,D) \in$ *subcls1* $G \Longrightarrow C \neq$ *Object* $\wedge$ ($\exists c.$ *class* G C = *Some* c $\wedge$ (*super* $c = D$))
apply (*simp add*: *subcls1-def*)
apply *auto*
done

lemma *subint1-def2*:
  $subint1\ G = (SIGMA\ I{:}\ \{I.\ is\text{-}iface\ G\ I\}.\ set\ (isuperIfs\ (the\ (iface\ G\ I))))$
apply (*unfold subint1-def*)
apply *auto*
done

lemma *subcls1-def2*:
  $subcls1\ G =$
    $(SIGMA\ C{:}\ \{C.\ is\text{-}class\ G\ C\}.\ \{D.\ C{\neq}Object \wedge super\ (the(class\ G\ C)){=}D\})$
apply (*unfold subcls1-def*)
apply *auto*
done

lemma *subcls-is-class*:
$[\![G{\vdash}C \prec_C D]\!] \Longrightarrow \exists\ c.\ class\ G\ C = Some\ c$
by (*auto simp add*: *subcls1-def dest*: *tranclD*)

lemma *no-subcls1-Object*:$G{\vdash}Object{\prec_C}1\ D \Longrightarrow P$
by (*auto simp add*: *subcls1-def*)

lemma *no-subcls-Object*: $G{\vdash}Object{\prec_C}\ D \Longrightarrow P$
apply (*erule trancl-induct*)
apply (*auto intro*: *no-subcls1-Object*)
done

well-structured programs

definition
  $ws\text{-}idecl :: prog \Rightarrow qtname \Rightarrow qtname\ list \Rightarrow bool$
  where $ws\text{-}idecl\ G\ I\ si = (\forall\, J{\in}set\ si.\ \ is\text{-}iface\ G\ J\ \ \wedge (J,I){\notin}(subint1\ G)^+)$

definition
  $ws\text{-}cdecl :: prog \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$
  where $ws\text{-}cdecl\ G\ C\ sc = (C{\neq}Object \longrightarrow is\text{-}class\ G\ sc \wedge (sc,C){\notin}(subcls1\ G)^+)$

definition
  $ws\text{-}prog :: prog \Rightarrow bool$ **where**
  $ws\text{-}prog\ G = ((\forall\,(I,i){\in}set\ (ifaces\ \ G).\ ws\text{-}idecl\ G\ I\ (isuperIfs\ i)) \wedge$
      $(\forall\,(C,c){\in}set\ (classes\ G).\ ws\text{-}cdecl\ G\ C\ (super\ c)))$

lemma *ws-progI*:
$[\![\forall\,(I,i){\in}set\ (ifaces\ G).\ \forall\,J{\in}set\ (isuperIfs\ i).\ is\text{-}iface\ G\ J\ \wedge$
  $(J,I) \notin (subint1\ G)^+;$
 $\forall\,(C,c){\in}set\ (classes\ G).\ C{\neq}Object \longrightarrow is\text{-}class\ G\ (super\ c) \wedge$
  $((super\ c),C) \notin (subcls1\ G)^+$
 $]\!] \Longrightarrow ws\text{-}prog\ G$
apply (*unfold ws-prog-def ws-idecl-def ws-cdecl-def*)
apply (*erule-tac conjI*)
apply *blast*
done

lemma *ws-prog-ideclD*:

$[\![$*iface G I* = *Some i*; *J*∈*set* (*isuperIfs i*); *ws-prog G*$]\!]$ $\Longrightarrow$
is-iface G J ∧ (*J*,*I*)∉(*subint1 G*)$^+$
apply (*unfold ws-prog-def ws-idecl-def*)
apply *clarify*
apply (*drule-tac map-of-SomeD*)
apply *auto*
done

lemma *ws-prog-cdeclD*:
$[\![$*class G C* = *Some c*; *C*≠*Object*; *ws-prog G*$]\!]$ $\Longrightarrow$
is-class G (*super c*) ∧ (*super c*,*C*)∉(*subcls1 G*)$^+$
apply (*unfold ws-prog-def ws-cdecl-def*)
apply *clarify*
apply (*drule-tac map-of-SomeD*)
apply *auto*
done

well-foundedness

lemma *finite-is-iface*: *finite* {*I*. *is-iface G I*}
apply (*fold dom-def*)
apply (*rule-tac finite-dom-map-of*)
done

lemma *finite-is-class*: *finite* {*C*. *is-class G C*}
apply (*fold dom-def*)
apply (*rule-tac finite-dom-map-of*)
done

lemma *finite-subint1*: *finite* (*subint1 G*)
apply (*subst subint1-def2*)
apply (*rule finite-SigmaI*)
apply (*rule finite-is-iface*)
apply (*simp* (*no-asm*))
done

lemma *finite-subcls1*: *finite* (*subcls1 G*)
apply (*subst subcls1-def2*)
apply (*rule finite-SigmaI*)
apply (*rule finite-is-class*)
apply (*rule-tac B* = {*super* (*the* (*class G C*))} **in** *finite-subset*)
apply *auto*
done

lemma *subint1-irrefl-lemma1*:
ws-prog G $\Longrightarrow$ (*subint1 G*)$^{-1}$ ∩ (*subint1 G*)$^+$ = {}
apply (*force dest*: *subint1D ws-prog-ideclD conjunct2*)
done

lemma *subcls1-irrefl-lemma1*:
ws-prog G $\Longrightarrow$ (*subcls1 G*)$^{-1}$ ∩ (*subcls1 G*)$^+$ = {}
apply (*force dest*: *subcls1D ws-prog-cdeclD conjunct2*)
done

lemmas *subint1-irrefl-lemma2* = *subint1-irrefl-lemma1* [*THEN irrefl-tranclI'*]
lemmas *subcls1-irrefl-lemma2* = *subcls1-irrefl-lemma1* [*THEN irrefl-tranclI'*]

lemma *subint1-irrefl*: $[\![(x, y) \in \mathit{subint1}\ G;\ \mathit{ws\text{-}prog}\ G]\!] \Longrightarrow x \neq y$
apply (*rule irrefl-trancl-rD*)
apply (*rule subint1-irrefl-lemma2*)
apply *auto*
done

lemma *subcls1-irrefl*: $[\![(x, y) \in \mathit{subcls1}\ G;\ \mathit{ws\text{-}prog}\ G]\!] \Longrightarrow x \neq y$
apply (*rule irrefl-trancl-rD*)
apply (*rule subcls1-irrefl-lemma2*)
apply *auto*
done

lemmas *subint1-acyclic* = *subint1-irrefl-lemma2* [*THEN acyclicI*]
lemmas *subcls1-acyclic* = *subcls1-irrefl-lemma2* [*THEN acyclicI*]

lemma *wf-subint1*: $\mathit{ws\text{-}prog}\ G \Longrightarrow \mathit{wf}\ ((\mathit{subint1}\ G)^{-1})$
by (*auto intro*: *finite-acyclic-wf-converse finite-subint1 subint1-acyclic*)

lemma *wf-subcls1*: $\mathit{ws\text{-}prog}\ G \Longrightarrow \mathit{wf}\ ((\mathit{subcls1}\ G)^{-1})$
by (*auto intro*: *finite-acyclic-wf-converse finite-subcls1 subcls1-acyclic*)

lemma *subint1-induct*:
$[\![\mathit{ws\text{-}prog}\ G;\ \bigwedge x.\ \forall y.\ (x, y) \in \mathit{subint1}\ G \longrightarrow P\ y \Longrightarrow P\ x]\!] \Longrightarrow P\ a$
apply (*frule wf-subint1*)
apply (*erule wf-induct*)
apply (*simp* (*no-asm-use*) *only*: *converse-iff*)
apply *blast*
done

lemma *subcls1-induct* [*consumes 1*]:
$[\![\mathit{ws\text{-}prog}\ G;\ \bigwedge x.\ \forall y.\ (x, y) \in \mathit{subcls1}\ G \longrightarrow P\ y \Longrightarrow P\ x]\!] \Longrightarrow P\ a$
apply (*frule wf-subcls1*)
apply (*erule wf-induct*)
apply (*simp* (*no-asm-use*) *only*: *converse-iff*)
apply *blast*
done

lemma *ws-subint1-induct*:
$[\![\mathit{is\text{-}iface}\ G\ I;\ \mathit{ws\text{-}prog}\ G;\ \bigwedge I\ i.\ [\![\mathit{iface}\ G\ I = \mathit{Some}\ i\ \wedge$
$(\forall J \in \mathit{set}\ (\mathit{isuperIfs}\ i).\ (I,J) \in \mathit{subint1}\ G \wedge P\ J \wedge \mathit{is\text{-}iface}\ G\ J)]\!] \Longrightarrow P\ I$
$]\!] \Longrightarrow P\ I$
apply (*erule rev-mp*)
apply (*rule subint1-induct*)
apply *assumption*
apply (*simp* (*no-asm*))
apply *safe*

apply (*blast dest*: *subint1I ws-prog-ideclD*)
done

lemma *ws-subcls1-induct*: $[\![$*is-class G C*; *ws-prog G*;
$\bigwedge$*C c*. $[\![$*class G C* = *Some c*;
($C \neq$ *Object* $\longrightarrow$ (*C*,(*super c*))$\in$*subcls1 G* $\wedge$
P (*super c*) $\wedge$ *is-class G* (*super c*))$]\!] \Longrightarrow$ *P C*
$]\!] \Longrightarrow$ *P C*
apply (*erule rev-mp*)
apply (*rule subcls1-induct*)
apply *assumption*
apply (*simp* (*no-asm*))
apply *safe*
apply (*fast dest*: *subcls1I ws-prog-cdeclD*)
done

lemma *ws-class-induct* [*consumes 2*, *case-names Object Subcls*]:
$[\![$*class G C* = *Some c*; *ws-prog G*;
$\bigwedge$ *co*. *class G Object* = *Some co* $\Longrightarrow$ *P Object*;
$\bigwedge$ *C c*. $[\![$*class G C* = *Some c*; $C \neq$ *Object*; *P* (*super c*)$]\!] \Longrightarrow$ *P C*
$]\!] \Longrightarrow$ *P C*
proof −
assume *clsC*: *class G C* = *Some c*
and *init*: $\bigwedge$ *co*. *class G Object* = *Some co* $\Longrightarrow$ *P Object*
and *step*: $\bigwedge$ *C c*. $[\![$*class G C* = *Some c*; $C \neq$ *Object*; *P* (*super c*)$]\!] \Longrightarrow$ *P C*
assume *ws*: *ws-prog G*
then have *is-class G C* $\Longrightarrow$ *P C*
proof (*induct rule*: *subcls1-induct*)
fix *C*
assume *hyp*:$\forall$ *S*. *G*$\vdash$*C* $\prec_C$*1 S* $\longrightarrow$ *is-class G S* $\longrightarrow$ *P S*
and *iscls*:*is-class G C*
show *P C*
proof (*cases C*=*Object*)
case *True* **with** *iscls init* **show** *P C* **by** *auto*
next
case *False* **with** *ws step hyp iscls*
show *P C* **by** (*auto dest*: *subcls1I ws-prog-cdeclD*)
qed
qed
with *clsC* **show** *?thesis* **by** *simp*
qed

lemma *ws-class-induct′* [*consumes 2*, *case-names Object Subcls*]:
$[\![$*is-class G C*; *ws-prog G*;
$\bigwedge$ *co*. *class G Object* = *Some co* $\Longrightarrow$ *P Object*;
$\bigwedge$ *C c*. $[\![$*class G C* = *Some c*; $C \neq$ *Object*; *P* (*super c*)$]\!] \Longrightarrow$ *P C*
$]\!] \Longrightarrow$ *P C*
by (*auto intro*: *ws-class-induct*)

lemma *ws-class-induct′′* [*consumes 2*, *case-names Object Subcls*]:
$[\![$*class G C* = *Some c*; *ws-prog G*;
$\bigwedge$ *co*. *class G Object* = *Some co* $\Longrightarrow$ *P Object co*;
$\bigwedge$ *C c sc*. $[\![$*class G C* = *Some c*; *class G* (*super c*) = *Some sc*;
$C \neq$ *Object*; *P* (*super c*) *sc*$]\!] \Longrightarrow$ *P C c*

$\rrbracket \Longrightarrow P\ C\ c$

proof −
  assume *clsC*: $class\ G\ C = Some\ c$
  and *init*: $\bigwedge co.\ class\ G\ Object = Some\ co \Longrightarrow P\ Object\ co$
  and *step*: $\bigwedge C\ c\ sc\ .\ \llbracket class\ G\ C = Some\ c;\ class\ G\ (super\ c) = Some\ sc;$
$C \neq Object;\ P\ (super\ c)\ sc\rrbracket \Longrightarrow P\ C\ c$
  assume *ws*: *ws-prog G*
  then have $\bigwedge c.\ class\ G\ C = Some\ c \Longrightarrow P\ C\ c$
  proof (*induct rule*: *subcls1-induct*)
    fix $C\ c$
    assume *hyp*: $\forall\ S.\ G\vdash C \prec_C 1\ S \longrightarrow (\forall\ s.\ class\ G\ S = Some\ s \longrightarrow P\ S\ s)$
      and *iscls*: $class\ G\ C = Some\ c$
    show $P\ C\ c$
    proof (*cases* C=*Object*)
      case *True* **with** *iscls init* **show** $P\ C\ c$ **by** *auto*
    next
      case *False*
      with *ws iscls* **obtain** *sc* **where**
        sc: $class\ G\ (super\ c) = Some\ sc$
        by (*auto dest*: *ws-prog-cdeclD*)
      from *iscls False* **have** $G\vdash C \prec_C 1\ (super\ c)$ **by** (*rule subcls1I*)
      with *False ws step hyp iscls sc*
      show $P\ C\ c$
        by (*auto*)
    qed
  qed
  with *clsC* **show** $P\ C\ c$ **by** *auto*
qed

lemma *ws-interface-induct* [*consumes 2*, *case-names Step*]:
  assumes *is-if-I*: *is-iface G I* **and**
    ws: *ws-prog G* **and**
    hyp-sub: $\bigwedge I\ i.\ \llbracket iface\ G\ I = Some\ i;$
$\forall\ J \in set\ (isuperIfs\ i).$
$(I,J){\in}subint1\ G \wedge P\ J \wedge is\text{-}iface\ G\ J\rrbracket \Longrightarrow P\ I$
  shows $P\ I$
proof −
  from *is-if-I ws*
  show $P\ I$
  proof (*rule ws-subint1-induct*)
    fix $I\ i$
    assume *hyp*: $iface\ G\ I = Some\ i \wedge$
$(\forall\ J{\in}set\ (isuperIfs\ i).\ (I,J) \in subint1\ G \wedge P\ J \wedge is\text{-}iface\ G\ J)$
    then have *if-I*: $iface\ G\ I = Some\ i$
      by *blast*
    show $P\ I$
    proof (*cases isuperIfs i*)
      case *Nil*
      with *if-I hyp-sub*
      show $P\ I$
        by *auto*
    next
      case (*Cons hd tl*)
      with *hyp if-I hyp-sub*
      show $P\ I$
        by *auto*
    qed
  qed

qed

general recursion operators for the interface and class hiearchies

function *iface-rec* :: *prog* ⇒ *qtname* ⇒ (*qtname* ⇒ *iface* ⇒ ′*a set* ⇒ ′*a*) ⇒ ′*a*
where
[*simp del*]: *iface-rec G I f* =
 (*case iface G I of*
   None ⇒ *undefined*
  | *Some i* ⇒ *if ws-prog G*
     then f I i
      ((λ*J*. *iface-rec G J f*)‘*set* (*isuperIfs i*))
     else undefined)
by *auto*
termination
by (*relation inv-image* (*same-fst ws-prog* (λ*G*. (*subint1 G*)$^{-1}$)) (%(*x*,*y*,*z*). (*x*,*y*)))
 (*auto simp*: *wf-subint1 subint1I wf-same-fst*)

lemma *iface-rec*:
⟦*iface G I* = *Some i*; *ws-prog G*⟧ ⟹
 iface-rec G I f = *f I i* ((λ*J*. *iface-rec G J f*)‘*set* (*isuperIfs i*))
apply (*subst iface-rec.simps*)
apply *simp*
done

function
 class-rec :: *prog* ⇒ *qtname* ⇒ ′*a* ⇒ (*qtname* ⇒ *class* ⇒ ′*a* ⇒ ′*a*) ⇒ ′*a*
where
[*simp del*]: *class-rec G C t f* =
 (*case class G C of*
   None ⇒ *undefined*
  | *Some c* ⇒ *if ws-prog G*
     then f C c
      (*if C* = *Object then t*
        else class-rec G (*super c*) *t f*)
     else undefined)

by *auto*
termination
by (*relation inv-image* (*same-fst ws-prog* (λ*G*. (*subcls1 G*)$^{-1}$)) (%(*x*,*y*,*z*,*w*). (*x*,*y*)))
 (*auto simp*: *wf-subcls1 subcls1I wf-same-fst*)

lemma *class-rec*: ⟦*class G C* = *Some c*; *ws-prog G*⟧ ⟹
 class-rec G C t f =
  f C c (*if C* = *Object then t else class-rec G* (*super c*) *t f*)
apply (*subst class-rec.simps*)
apply *simp*
done

definition
 imethds :: *prog* ⇒ *qtname* ⇒ (*sig*,*qtname* × *mhead*) *tables* **where**
 — methods of an interface, with overriding and inheritance, cf. 9.2
 imethds G I = *iface-rec G I*
  (λ*I i ts*. (*Un-tables ts*) ⊕⊕
   (*set-option* ∘ *table-of* (*map* (λ(*s*,*m*). (*s*,*I*,*m*)) (*imethods i*))))

end

Chapter 9

TypeRel

1 The relations between Java types

theory *TypeRel* **imports** *Decl* **begin**

simplifications:

- subinterface, subclass and widening relation includes identity

improvements over Java Specification 1.0:

- narrowing reference conversion also in cases where the return types of a pair of methods common to both types are in widening (rather identity) relation
- one could add similar constraints also for other cases

design issues:

- the type relations do not require *is-type* for their arguments
- the subint1 and subcls1 relations imply *is-iface*/*is-class* for their first arguments, which is required for their finiteness

definition
  $implmt1$ :: $prog \Rightarrow (qtname \times qtname)\ set$ — direct implementation
  — direct implementation, cf. 8.1.3
  where $implmt1\ G = \{(C,I).\ C{\neq}Object \wedge (\exists\, c{\in}class\ G\ C{:}\ I{\in}set\ (superIfs\ c))\}$

abbreviation
  subint1-syntax :: *prog* => [*qtname*, *qtname*] => *bool* (-⊢-≺I1- [71,71,71] 70)
  where $G{\vdash}I \prec I1\ J == (I,J) \in subint1\ G$

abbreviation
  subint-syntax :: *prog* => [*qtname*, *qtname*] => *bool* (-⊢-≼I - [71,71,71] 70)
  where $G{\vdash}I \preceq I\ J == (I,J) \in (subint1\ G)^*$ — cf. 9.1.3

abbreviation
  implmt1-syntax :: *prog* => [*qtname*, *qtname*] => *bool* (-⊢-↝1- [71,71,71] 70)
  where $G{\vdash}C \leadsto 1\ I == (C,I) \in implmt1\ G$

notation (*ASCII*)
  subint1-syntax (-|−-<:I1- [71,71,71] 70) **and**
  subint-syntax (-|−-<=:I -[71,71,71] 70) **and**
  implmt1-syntax (-|−-~>1- [71,71,71] 70)

subclass and subinterface relations

lemmas *subcls-direct* = *subcls1I* [*THEN r-into-rtrancl*]

lemma *subcls-direct1*:
$[\![$*class G C = Some c*; $C \neq$ *Object*;*D=super c*$]\!] \Longrightarrow G \vdash C \preceq_C D$
apply (*auto dest*: *subcls-direct*)
done

lemma *subcls1I1*:
$[\![$*class G C = Some c*; $C \neq$ *Object*;*D=super c*$]\!] \Longrightarrow G \vdash C \prec_C 1\ D$
apply (*auto dest*: *subcls1I*)
done

lemma *subcls-direct2*:
$[\![$*class G C = Some c*; $C \neq$ *Object*;*D=super c*$]\!] \Longrightarrow G \vdash C \prec_C D$
apply (*auto dest*: *subcls1I1*)
done

lemma *subclseq-trans*: $[\![G \vdash A \preceq_C B;\ G \vdash B \preceq_C C]\!] \Longrightarrow G \vdash A \preceq_C C$
by (*blast intro*: *rtrancl-trans*)

lemma *subcls-trans*: $[\![G \vdash A \prec_C B;\ G \vdash B \prec_C C]\!] \Longrightarrow G \vdash A \prec_C C$
by (*blast intro*: *trancl-trans*)

lemma *SXcpt-subcls-Throwable-lemma*:
$[\![$*class G* (*SXcpt xn*) = *Some xc*;
super xc = (*if xn* = *Throwable then Object else SXcpt Throwable*)$]\!]$
$\Longrightarrow G \vdash$ *SXcpt xn*$\preceq_C$ *SXcpt Throwable*
apply (*case-tac xn* = *Throwable*)
apply *simp-all*
apply (*drule subcls-direct*)
apply (*auto dest*: *sym*)
done

lemma *subcls-ObjectI*: $[\![$*is-class G C*; *ws-prog G*$]\!] \Longrightarrow G \vdash C \preceq_C$ *Object*
apply (*erule ws-subcls1-induct*)
apply *clarsimp*
apply (*case-tac C* = *Object*)
apply (*fast intro*: *r-into-rtrancl* [*THEN rtrancl-trans*])+
done

lemma *subclseq-ObjectD* [*dest*!]: $G \vdash$ *Object*$\preceq_C C \Longrightarrow C$ = *Object*
apply (*erule rtrancl-induct*)
apply (*auto dest*: *subcls1D*)
done

lemma *subcls-ObjectD* [*dest*!]: $G \vdash$ *Object*$\prec_C C \Longrightarrow$ *False*
apply (*erule trancl-induct*)
apply (*auto dest*: *subcls1D*)

done

lemma *subcls-ObjectI1* [*intro*!]:
$[\![C \neq Object;is\text{-}class\ G\ C;ws\text{-}prog\ G]\!] \Longrightarrow G\vdash C \prec_C Object$
apply (*drule* (*1*) *subcls-ObjectI*)
apply (*auto intro*: *rtrancl-into-trancl3*)
done

lemma *subcls-is-class*: $(C,D) \in (subcls1\ G)^+ \Longrightarrow is\text{-}class\ G\ C$
apply (*erule trancl-trans-induct*)
apply (*auto dest*!: *subcls1D*)
done

lemma *subcls-is-class2* [*rule-format* (*no-asm*)]:
$G\vdash C\preceq_C D \Longrightarrow is\text{-}class\ G\ D \longrightarrow is\text{-}class\ G\ C$
apply (*erule rtrancl-induct*)
apply (*drule-tac* [*2*] *subcls1D*)
apply *auto*
done

lemma *single-inheritance*:
$[\![G\vdash A \prec_C 1\ B;\ G\vdash A \prec_C 1\ C]\!] \Longrightarrow B = C$
by (*auto simp add*: *subcls1-def*)

lemma *subcls-compareable*:
$[\![G\vdash A \preceq_C X;\ G\vdash A \preceq_C Y$
$]\!] \Longrightarrow G\vdash X \preceq_C Y \lor G\vdash Y \preceq_C X$
by (*rule triangle-lemma*) (*auto intro*: *single-inheritance*)

lemma *subcls1-irrefl*: $[\![G\vdash C \prec_C 1\ D;\ ws\text{-}prog\ G\ ]\!]$
$\Longrightarrow C \neq D$
proof
  assume *ws*: *ws-prog G* **and**
    subcls1: $G\vdash C \prec_C 1\ D$ **and**
     eq-C-D: $C=D$
  from *subcls1* **obtain** *c*
    where
      neq-C-Object: $C\neq Object$ **and**
              clsC: *class G C = Some c* **and**
           super-c: *super c = D*
    by (*auto simp add*: *subcls1-def*)
  with *super-c subcls1 eq-C-D*
  have *subcls-super-c-C*: $G\vdash super\ c \prec_C C$
    by *auto*
  from *ws clsC neq-C-Object*
  have $\neg\ G\vdash super\ c \prec_C C$
    by (*auto dest*: *ws-prog-cdeclD*)
  from *this subcls-super-c-C*
  show *False*
    by (*rule notE*)
qed

lemma *no-subcls-Object*: $G \vdash C \prec_C D \Longrightarrow C \neq Object$
by (*erule converse-trancl-induct*) (*auto dest*: *subcls1D*)

lemma *subcls-acyclic*: $[\![G \vdash C \prec_C D;\ ws\text{-}prog\ G]\!] \Longrightarrow \neg\ G \vdash D \prec_C C$
proof −
  assume *ws*: *ws-prog G*
  assume *subcls-C-D*: $G \vdash C \prec_C D$
  then show *?thesis*
  proof (*induct rule*: *converse-trancl-induct*)
    fix *C*
    assume *subcls1-C-D*: $G \vdash C \prec_C 1\ D$
    then obtain *c* **where**
        $C \neq Object$ **and**
        class G C = Some c **and**
        super c = D
      by (*auto simp add*: *subcls1-def*)
    with *ws*
    show $\neg\ G \vdash D \prec_C C$
      by (*auto dest*: *ws-prog-cdeclD*)
  next
    fix *C Z*
    assume *subcls1-C-Z*: $G \vdash C \prec_C 1\ Z$ **and**
          subcls-Z-D: $G \vdash Z \prec_C D$ **and**
          nsubcls-D-Z: $\neg\ G \vdash D \prec_C Z$
    show $\neg\ G \vdash D \prec_C C$
    proof
      assume *subcls-D-C*: $G \vdash D \prec_C C$
      show *False*
      proof −
        from *subcls-D-C subcls1-C-Z*
        have $G \vdash D \prec_C Z$
          by (*auto dest*: *r-into-trancl trancl-trans*)
        with *nsubcls-D-Z*
        show *?thesis*
          by (*rule notE*)
      qed
    qed
  qed
qed

lemma *subclseq-cases*:
  assumes $G \vdash C \preceq_C D$
  obtains (*Eq*) $C = D$ | (*Subcls*) $G \vdash C \prec_C D$
using *assms* **by** (*blast intro*: *rtrancl-cases*)

lemma *subclseq-acyclic*:
$[\![G \vdash C \preceq_C D;\ G \vdash D \preceq_C C;\ ws\text{-}prog\ G]\!] \Longrightarrow C{=}D$
by (*auto elim*: *subclseq-cases dest*: *subcls-acyclic*)

lemma *subcls-irrefl*: $[\![G \vdash C \prec_C D;\ ws\text{-}prog\ G]\!]$
$\Longrightarrow C \neq D$
proof −
  assume *ws*: *ws-prog G*
  assume *subcls*: $G \vdash C \prec_C D$
  then show *?thesis*

```
  proof (induct rule: converse-trancl-induct)
    fix C
    assume G⊢C ≺C1 D
    with ws
    show C≠D
      by (blast dest: subcls1-irrefl)
  next
    fix C Z
    assume subcls1-C-Z: G⊢C ≺C1 Z and
            subcls-Z-D: G⊢Z ≺C D and
               neq-Z-D: Z ≠ D
    show C≠D
    proof
      assume eq-C-D: C=D
      show False
      proof −
        from subcls1-C-Z eq-C-D
        have G⊢D ≺C Z
          by (auto)
        also
        from subcls-Z-D ws
        have ¬ G⊢D ≺C Z
          by (rule subcls-acyclic)
        ultimately
        show ?thesis
          by − (rule notE)
      qed
    qed
  qed
qed


lemma invert-subclseq:
⟦G⊢C ≼C D; ws-prog G⟧
 ⟹ ¬ G⊢D ≺C C
proof −
  assume       ws: ws-prog G and
    subclseq-C-D: G⊢C ≼C D
  show ?thesis
  proof (cases D=C)
    case True
    with ws
    show ?thesis
      by (auto dest: subcls-irrefl)
  next
    case False
    with subclseq-C-D
    have G⊢C ≺C D
      by (blast intro: rtrancl-into-trancl3)
    with ws
    show ?thesis
      by (blast dest: subcls-acyclic)
  qed
qed


lemma invert-subcls:
⟦G⊢C ≺C D; ws-prog G⟧
 ⟹ ¬ G⊢D ≼C C
```

proof –
  assume *ws*: *ws-prog* G **and**
    subcls-C-D: $G\vdash C \prec_C D$
  then
  have *nsubcls-D-C*: $\neg\ G\vdash D \prec_C C$
    by (*blast dest*: *subcls-acyclic*)
  show *?thesis*
  proof
    assume $G\vdash D \preceq_C C$
    then show *False*
    proof (*cases rule*: *subclseq-cases*)
      case *Eq*
      with *ws subcls-C-D*
      show *?thesis*
        by (*auto dest*: *subcls-irrefl*)
    next
      case *Subcls*
      with *nsubcls-D-C*
      show *?thesis*
        by *blast*
    qed
  qed
qed

lemma *subcls-superD*:
$[\![G\vdash C \prec_C D;\ class\ G\ C = Some\ c]\!] \Longrightarrow G\vdash(super\ c) \preceq_C D$
proof –
  assume *clsC*: $class\ G\ C = Some\ c$
  assume *subcls-C-C*: $G\vdash C \prec_C D$
  then obtain S **where**
    $G\vdash C \prec_C 1\ S$ **and**
    subclseq-S-D: $G\vdash S \preceq_C D$
    by (*blast dest*: *tranclD*)
  with *clsC*
  have $S{=}super\ c$
    by (*auto dest*: *subcls1D*)
  with *subclseq-S-D* **show** *?thesis* **by** *simp*
qed

lemma *subclseq-superD*:
$[\![G\vdash C \preceq_C D;\ C{\neq}D;class\ G\ C = Some\ c]\!] \Longrightarrow G\vdash(super\ c) \preceq_C D$
proof –
  assume *neq-C-D*: $C{\neq}D$
  assume *clsC*: $class\ G\ C = Some\ c$
  assume *subclseq-C-D*: $G\vdash C \preceq_C D$
  then show *?thesis*
  proof (*cases rule*: *subclseq-cases*)
    case *Eq* **with** *neq-C-D* **show** *?thesis* **by** *contradiction*
  next
    case *Subcls*
    with *clsC* **show** *?thesis* **by** (*blast dest*: *subcls-superD*)
  qed
qed

implementation relation

lemma *implmt1D*: $G\vdash C\leadsto 1I \Longrightarrow C\neq Object \wedge (\exists\, c\in class\ G\ C\colon I\in set\ (superIfs\ c))$
apply (*unfold implmt1-def*)
apply *auto*
done

inductive — implementation, cf. 8.1.4
implmt :: *prog* ⇒ *qtname* ⇒ *qtname* ⇒ *bool* $(\text{-}\vdash\text{-}\leadsto\text{-}\ [71,71,71]\ 70)$
for *G* :: *prog*
where
direct: $G\vdash C\leadsto 1J \Longrightarrow G\vdash C\leadsto J$
| *subint*: $G\vdash C\leadsto 1I \Longrightarrow G\vdash I\preceq I\ J \Longrightarrow G\vdash C\leadsto J$
| *subcls1*: $G\vdash C\prec_C 1D \Longrightarrow G\vdash D\leadsto J \Longrightarrow G\vdash C\leadsto J$

lemma *implmtD*: $G\vdash C\leadsto J \Longrightarrow (\exists\, I.\ G\vdash C\leadsto 1I \wedge G\vdash I\preceq I\ J) \vee (\exists\, D.\ G\vdash C\prec_C 1D \wedge G\vdash D\leadsto J)$
apply (*erule implmt.induct*)
apply *fast+*
done

lemma *implmt-ObjectE* [*elim*!]: $G\vdash Object\leadsto I \Longrightarrow R$
by (*auto dest*!: *implmtD implmt1D subcls1D*)

lemma *subcls-implmt* [*rule-format* (*no-asm*)]: $G\vdash A\preceq_C B \Longrightarrow G\vdash B\leadsto K \longrightarrow G\vdash A\leadsto K$
apply (*erule rtrancl-induct*)
apply (*auto intro*: *implmt.subcls1*)
done

lemma *implmt-subint2*: $[\![\ G\vdash A\leadsto J;\ G\vdash J\preceq I\ K]\!] \Longrightarrow G\vdash A\leadsto K$
apply (*erule rev-mp*, *erule implmt.induct*)
apply (*auto dest*: *implmt.subint rtrancl-trans implmt.subcls1*)
done

lemma *implmt-is-class*: $G\vdash C\leadsto I \Longrightarrow is\text{-}class\ G\ C$
apply (*erule implmt.induct*)
apply (*auto dest*: *implmt1D subcls1D*)
done

widening relation

inductive
— widening, viz. method invocation conversion, cf. 5.3 i.e. kind of syntactic subtyping
widen :: *prog* ⇒ *ty* ⇒ *ty* ⇒ *bool* $(\text{-}\vdash\text{-}\preceq\text{-}\ [71,71,71]\ 70)$
for *G* :: *prog*
where
refl: $G\vdash T\preceq T$ — identity conversion, cf. 5.1.1
| *subint*: $G\vdash I\preceq I\ J \Longrightarrow G\vdash Iface\ I\preceq\ Iface\ J$ — wid.ref.conv.,cf. 5.1.4
| *int-obj*: $G\vdash Iface\ I\preceq\ Class\ Object$
| *subcls*: $G\vdash C\preceq_C D \Longrightarrow G\vdash Class\ C\preceq\ Class\ D$
| *implmt*: $G\vdash C\leadsto I \Longrightarrow G\vdash Class\ C\preceq\ Iface\ I$
| *null*: $G\vdash NT\preceq\ RefT\ R$
| *arr-obj*: $G\vdash T.[]\preceq\ Class\ Object$
| *array*: $G\vdash RefT\ S\preceq RefT\ T \Longrightarrow G\vdash RefT\ S.[]\preceq\ RefT\ T.[]$

declare *widen.refl* [*intro*!]
declare *widen.intros* [*simp*]

lemma *widen-PrimT*: $G \vdash PrimT\ x \preceq T \Longrightarrow (\exists y.\ T = PrimT\ y)$
apply (*ind-cases* $G \vdash PrimT\ x \preceq T$)
by *auto*

lemma *widen-PrimT2*: $G \vdash S \preceq PrimT\ x \Longrightarrow \exists y.\ S = PrimT\ y$
apply (*ind-cases* $G \vdash S \preceq PrimT\ x$)
by *auto*

These widening lemmata hold in Bali but are to strong for ordinary Java. They would not work for real Java Integral Types, like short, long, int. These lemmata are just for documentation and are not used.

lemma *widen-PrimT-strong*: $G \vdash PrimT\ x \preceq T \Longrightarrow T = PrimT\ x$
by (*ind-cases* $G \vdash PrimT\ x \preceq T$) *simp-all*

lemma *widen-PrimT2-strong*: $G \vdash S \preceq PrimT\ x \Longrightarrow S = PrimT\ x$
by (*ind-cases* $G \vdash S \preceq PrimT\ x$) *simp-all*

Specialized versions for booleans also would work for real Java

lemma *widen-Boolean*: $G \vdash PrimT\ Boolean \preceq T \Longrightarrow T = PrimT\ Boolean$
by (*ind-cases* $G \vdash PrimT\ Boolean \preceq T$) *simp-all*

lemma *widen-Boolean2*: $G \vdash S \preceq PrimT\ Boolean \Longrightarrow S = PrimT\ Boolean$
by (*ind-cases* $G \vdash S \preceq PrimT\ Boolean$) *simp-all*

lemma *widen-RefT*: $G \vdash RefT\ R \preceq T \Longrightarrow \exists t.\ T = RefT\ t$
apply (*ind-cases* $G \vdash RefT\ R \preceq T$)
by *auto*

lemma *widen-RefT2*: $G \vdash S \preceq RefT\ R \Longrightarrow \exists t.\ S = RefT\ t$
apply (*ind-cases* $G \vdash S \preceq RefT\ R$)
by *auto*

lemma *widen-Iface*: $G \vdash Iface\ I \preceq T \Longrightarrow T = Class\ Object \lor (\exists J.\ T = Iface\ J)$
apply (*ind-cases* $G \vdash Iface\ I \preceq T$)
by *auto*

lemma *widen-Iface2*: $G \vdash S \preceq\ Iface\ J \Longrightarrow S = NT \lor (\exists I.\ S = Iface\ I) \lor (\exists D.\ S = Class\ D)$
apply (*ind-cases* $G \vdash S \preceq\ Iface\ J$)
by *auto*

lemma *widen-Iface-Iface*: $G \vdash Iface\ I \preceq\ Iface\ J \Longrightarrow G \vdash I \preceq I\ J$
apply (*ind-cases* $G \vdash Iface\ I \preceq\ Iface\ J$)
by *auto*

lemma *widen-Iface-Iface-eq* [*simp*]: $G\vdash Iface\ I\preceq\ Iface\ J = G\vdash I\preceq I\ J$
apply (*rule iffI*)
apply (*erule widen-Iface-Iface*)
apply (*erule widen.subint*)
done

lemma *widen-Class*: $G\vdash Class\ C\preceq T \Longrightarrow (\exists D.\ T=Class\ D) \vee (\exists I.\ T=Iface\ I)$
apply (*ind-cases* $G\vdash Class\ C\preceq T$)
by *auto*

lemma *widen-Class2*: $G\vdash S\preceq\ Class\ C \Longrightarrow C = Object \vee S = NT \vee (\exists D.\ S = Class\ D)$
apply (*ind-cases* $G\vdash S\preceq\ Class\ C$)
by *auto*

lemma *widen-Class-Class*: $G\vdash Class\ C\preceq\ Class\ cm \Longrightarrow G\vdash C\preceq_C\ cm$
apply (*ind-cases* $G\vdash Class\ C\preceq\ Class\ cm$)
by *auto*

lemma *widen-Class-Class-eq* [*simp*]: $G\vdash Class\ C\preceq\ Class\ cm = G\vdash C\preceq_C\ cm$
apply (*rule iffI*)
apply (*erule widen-Class-Class*)
apply (*erule widen.subcls*)
done

lemma *widen-Class-Iface*: $G\vdash Class\ C\preceq\ Iface\ I \Longrightarrow G\vdash C\leadsto I$
apply (*ind-cases* $G\vdash Class\ C\preceq\ Iface\ I$)
by *auto*

lemma *widen-Class-Iface-eq* [*simp*]: $G\vdash Class\ C\preceq\ Iface\ I = G\vdash C\leadsto I$
apply (*rule iffI*)
apply (*erule widen-Class-Iface*)
apply (*erule widen.implmt*)
done

lemma *widen-Array*: $G\vdash S.[]\preceq T \Longrightarrow T=Class\ Object \vee (\exists T'.\ T=T'.[] \wedge G\vdash S\preceq T')$
apply (*ind-cases* $G\vdash S.[]\preceq T$)
by *auto*

lemma *widen-Array2*: $G\vdash S\preceq T.[] \Longrightarrow S = NT \vee (\exists S'.\ S=S'.[] \wedge G\vdash S'\preceq T)$
apply (*ind-cases* $G\vdash S\preceq T.[]$)
by *auto*

lemma *widen-ArrayPrimT*: $G\vdash PrimT\ t.[]\preceq T \Longrightarrow T=Class\ Object \vee T=PrimT\ t.[]$
apply (*ind-cases* $G\vdash PrimT\ t.[]\preceq T$)
by *auto*

lemma *widen-ArrayRefT*:

$G \vdash RefT\ t.[] \preceq T \Longrightarrow T = Class\ Object \lor (\exists\, s.\ T = RefT\ s.[] \land G \vdash RefT\ t \preceq RefT\ s)$
apply (*ind-cases* $G \vdash RefT\ t.[] \preceq T$)
by *auto*

lemma *widen-ArrayRefT-ArrayRefT-eq* [*simp*]:
$G \vdash RefT\ T.[] \preceq RefT\ T'.[] = G \vdash RefT\ T \preceq RefT\ T'$
apply (*rule iffI*)
apply (*drule widen-ArrayRefT*)
apply *simp*
apply (*erule widen.array*)
done

lemma *widen-Array-Array*: $G \vdash T.[] \preceq T'.[] \Longrightarrow G \vdash T \preceq T'$
apply (*drule widen-Array*)
apply *auto*
done

lemma *widen-Array-Class*: $G \vdash S.[] \preceq Class\ C \Longrightarrow C = Object$
by (*auto dest*: *widen-Array*)

lemma *widen-NT2*: $G \vdash S \preceq NT \Longrightarrow S = NT$
apply (*ind-cases* $G \vdash S \preceq NT$)
by *auto*

lemma *widen-Object*:
assumes *isrtype G T* **and** *ws-prog G*
shows $G \vdash RefT\ T \preceq Class\ Object$
proof (*cases T*)
case (*ClassT C*) **with** *assms* **have** $G \vdash C \preceq_C Object$ **by** (*auto intro*: *subcls-ObjectI*)
with *ClassT* **show** *?thesis* **by** *auto*
qed *simp-all*

lemma *widen-trans-lemma* [*rule-format* (*no-asm*)]:
$[\![G \vdash S \preceq U; \forall\, C.\ is\text{-}class\ G\ C \longrightarrow G \vdash C \preceq_C Object]\!] \Longrightarrow \forall\, T.\ G \vdash U \preceq T \longrightarrow G \vdash S \preceq T$
apply (*erule widen.induct*)
apply *safe*
prefer *5* **apply** (*drule widen-RefT*) **apply** *clarsimp*
apply (*frule-tac* [*1*] *widen-Iface*)
apply (*frule-tac* [*2*] *widen-Class*)
apply (*frule-tac* [*3*] *widen-Class*)
apply (*frule-tac* [*4*] *widen-Iface*)
apply (*frule-tac* [*5*] *widen-Class*)
apply (*frule-tac* [*6*] *widen-Array*)
apply *safe*
apply (*rule widen.int-obj*)
prefer *6* **apply** (*drule implmt-is-class*) **apply** *simp*
apply (*erule-tac* [!] *thin-rl*)
prefer *6* **apply** *simp*
apply (*rule-tac* [*9*] *widen.arr-obj*)
apply (*rotate-tac* [*9*] *−1*)
apply (*frule-tac* [*9*] *widen-RefT*)
apply (*auto elim*!: *rtrancl-trans subcls-implmt implmt-subint2*)

done

lemma *ws-widen-trans*: $[\![G{\vdash}S{\preceq}U;\ G{\vdash}U{\preceq}T;\ ws\text{-}prog\ G]\!] \Longrightarrow G{\vdash}S{\preceq}T$
by (*auto intro*: *widen-trans-lemma subcls-ObjectI*)

lemma *widen-antisym-lemma* [*rule-format* (*no-asm*)]: $[\![G{\vdash}S{\preceq}T;$
$\forall I\ J.\ G{\vdash}I{\preceq}I\ J \land G{\vdash}J{\preceq}I\ I \longrightarrow I = J;$
$\forall C\ D.\ G{\vdash}C{\preceq}_C\ D \land G{\vdash}D{\preceq}_C\ C \longrightarrow C = D;$
$\forall I\ .\ G{\vdash}Object{\leadsto}I \qquad \longrightarrow False]\!] \Longrightarrow G{\vdash}T{\preceq}S \longrightarrow S = T$
apply (*erule widen.induct*)
apply (*auto dest*: *widen-Iface widen-NT2 widen-Class*)
done

lemmas *subint-antisym* =
subint1-acyclic [*THEN acyclic-impl-antisym-rtrancl*]
lemmas *subcls-antisym* =
subcls1-acyclic [*THEN acyclic-impl-antisym-rtrancl*]

lemma *widen-antisym*: $[\![G{\vdash}S{\preceq}T;\ G{\vdash}T{\preceq}S;\ ws\text{-}prog\ G]\!] \Longrightarrow S{=}T$
by (*fast elim*: *widen-antisym-lemma subint-antisym* [*THEN antisymD*]
subcls-antisym [*THEN antisymD*])

lemma *widen-ObjectD* [*dest*!]: $G{\vdash}Class\ Object{\preceq}T \Longrightarrow T{=}Class\ Object$
apply (*frule widen-Class*)
apply (*fast dest*: *widen-Class-Class widen-Class-Iface*)
done

definition
widens :: *prog* ⇒ [*ty list*, *ty list*] ⇒ *bool* (-⊢-[⪯]- [*71*,*71*,*71*] *70*)
where $G{\vdash}Ts[\preceq]Ts' = list\text{-}all2\ (\lambda T\ T'.\ G{\vdash}T{\preceq}T')\ Ts\ Ts'$

lemma *widens-Nil* [*simp*]: $G{\vdash}[][\preceq][]$
apply (*unfold widens-def*)
apply *auto*
done

lemma *widens-Cons* [*simp*]: $G{\vdash}(S\#Ss)[\preceq](T\#Ts) = (G{\vdash}S{\preceq}T \land G{\vdash}Ss[\preceq]Ts)$
apply (*unfold widens-def*)
apply *auto*
done

narrowing relation

inductive — narrowing reference conversion, cf. 5.1.5
narrow :: *prog* ⇒ *ty* ⇒ *ty* ⇒ *bool* (-⊢-≻- [*71*,*71*,*71*] *70*)
for *G* :: *prog*
where
subcls: $G{\vdash}C{\preceq}_C\ D \Longrightarrow G{\vdash} \quad Class\ D{\succ}Class\ C$
| *implmt*: $\neg G{\vdash}C{\leadsto}I \Longrightarrow G{\vdash} \quad Class\ C{\succ}Iface\ I$
| *obj-arr*: $G{\vdash}Class\ Object{\succ}T.[]$
| *int-cls*: $G{\vdash} \quad Iface\ I{\succ}Class\ C$
| *subint*: *imethds G I hidings imethds G J entails*
$(\lambda(md,\ mh\quad)\ (md',mh').\ G{\vdash}mrt\ mh{\preceq}mrt\ mh') \Longrightarrow$

$\neg G \vdash I \preceq I\ J \quad \Longrightarrow G \vdash \quad Iface\ I \succ Iface\ J$
| *array*: $G \vdash RefT\ S \succ RefT\ T \Longrightarrow G \vdash \quad RefT\ S.[] \succ RefT\ T.[]$

lemma *narrow-RefT*: $G \vdash RefT\ R \succ T \Longrightarrow \exists\, t.\ T = RefT\ t$
apply (*ind-cases* $G \vdash RefT\ R \succ T$)
by *auto*

lemma *narrow-RefT2*: $G \vdash S \succ RefT\ R \Longrightarrow \exists\, t.\ S = RefT\ t$
apply (*ind-cases* $G \vdash S \succ RefT\ R$)
by *auto*

lemma *narrow-PrimT*: $G \vdash PrimT\ pt \succ T \Longrightarrow \exists\, t.\ T = PrimT\ t$
by (*ind-cases* $G \vdash PrimT\ pt \succ T$)

lemma *narrow-PrimT2*: $G \vdash S \succ PrimT\ pt \Longrightarrow$
$\exists\, t.\ S = PrimT\ t \wedge G \vdash PrimT\ t \preceq PrimT\ pt$
by (*ind-cases* $G \vdash S \succ PrimT\ pt$)

These narrowing lemmata hold in Bali but are to strong for ordinary Java. They would not work for real Java Integral Types, like short, long, int. These lemmata are just for documentation and are not used.

lemma *narrow-PrimT-strong*: $G \vdash PrimT\ pt \succ T \Longrightarrow T = PrimT\ pt$
by (*ind-cases* $G \vdash PrimT\ pt \succ T$)

lemma *narrow-PrimT2-strong*: $G \vdash S \succ PrimT\ pt \Longrightarrow S = PrimT\ pt$
by (*ind-cases* $G \vdash S \succ PrimT\ pt$)

Specialized versions for booleans also would work for real Java

lemma *narrow-Boolean*: $G \vdash PrimT\ Boolean \succ T \Longrightarrow T = PrimT\ Boolean$
by (*ind-cases* $G \vdash PrimT\ Boolean \succ T$)

lemma *narrow-Boolean2*: $G \vdash S \succ PrimT\ Boolean \Longrightarrow S = PrimT\ Boolean$
by (*ind-cases* $G \vdash S \succ PrimT\ Boolean$)

casting relation

inductive — casting conversion, cf. 5.5
cast :: *prog* ⇒ *ty* ⇒ *ty* ⇒ *bool* (-⊢-≼? - [*71*,*71*,*71*] *70*)
for *G* :: *prog*
where
widen: $G \vdash S \preceq T \Longrightarrow G \vdash S \preceq ?\ T$
| *narrow*: $G \vdash S \succ T \Longrightarrow G \vdash S \preceq ?\ T$

lemma *cast-RefT*: $G \vdash RefT\ R \preceq ?\ T \Longrightarrow \exists\, t.\ T = RefT\ t$
apply (*ind-cases* $G \vdash RefT\ R \preceq ?\ T$)
by (*auto dest*: *widen-RefT narrow-RefT*)

```
lemma cast-RefT2: G⊢S≼? RefT R ⟹ ∃ t. S=RefT t
apply (ind-cases G⊢S≼? RefT R)
by (auto dest: widen-RefT2 narrow-RefT2)
```

```
lemma cast-PrimT: G⊢PrimT pt≼? T ⟹ ∃ t. T=PrimT t
apply (ind-cases G⊢PrimT pt≼? T)
by (auto dest: widen-PrimT narrow-PrimT)
```

```
lemma cast-PrimT2: G⊢S≼? PrimT pt ⟹ ∃ t. S=PrimT t ∧ G⊢PrimT t≼PrimT pt
apply (ind-cases G⊢S≼? PrimT pt)
by (auto dest: widen-PrimT2 narrow-PrimT2)
```

```
lemma cast-Boolean:
  assumes bool-cast: G⊢PrimT Boolean≼? T
  shows T=PrimT Boolean
using bool-cast
proof (cases)
  case widen
  hence G⊢PrimT Boolean≼ T
    by simp
  thus ?thesis by  (rule widen-Boolean)
next
  case narrow
  hence G⊢PrimT Boolean≻T
    by simp
  thus ?thesis by (rule narrow-Boolean)
qed
```

```
lemma cast-Boolean2:
  assumes bool-cast: G⊢S≼? PrimT Boolean
  shows S = PrimT Boolean
using bool-cast
proof (cases)
  case widen
  hence G⊢S≼ PrimT Boolean
    by simp
  thus ?thesis by  (rule widen-Boolean2)
next
  case narrow
  hence G⊢S≻PrimT Boolean
    by simp
  thus ?thesis by (rule narrow-Boolean2)
qed

end
```

Chapter 10

DeclConcepts

1 Advanced concepts on Java declarations like overriding, inheritance, dynamic method lookup

theory *DeclConcepts* **imports** *TypeRel* **begin**

access control (cf. 6.6), overriding and hiding (cf. 8.4.6.1)

definition *is-public* :: *prog* ⇒ *qtname* ⇒ *bool* **where**
is-public G qn = (*case class G qn of*
    None ⇒ (*case iface G qn of*
        None ⇒ *False*
      | *Some i* ⇒ *access i* = *Public*)
  | *Some c* ⇒ *access c* = *Public*)

2 accessibility of types (cf. 6.6.1)

Primitive types are always accessible, interfaces and classes are accessible in their package or if they are defined public, an array type is accessible if its element type is accessible

primrec
  accessible-in :: *prog* ⇒ *ty* ⇒ *pname* ⇒ *bool* (- ⊢ - *accessible′-in* - [*61*,*61*,*61*] *60*) **and**
  rt-accessible-in :: *prog* ⇒ *ref-ty* ⇒ *pname* ⇒ *bool* (- ⊢ - *accessible′-in″* - [*61*,*61*,*61*] *60*)
where
  G⊢(*PrimT p*) *accessible-in pack* = *True*
| *accessible-in-RefT-simp*:
  G⊢(*RefT r*) *accessible-in pack* = *G*⊢*r accessible-in′ pack*
| *G*⊢(*NullT*) *accessible-in′ pack* = *True*
| *G*⊢(*IfaceT I*) *accessible-in′ pack* = ((*pid I* = *pack*) ∨ *is-public G I*)
| *G*⊢(*ClassT C*) *accessible-in′ pack* = ((*pid C* = *pack*) ∨ *is-public G C*)
| *G*⊢(*ArrayT ty*) *accessible-in′ pack* = *G*⊢*ty accessible-in pack*

declare *accessible-in-RefT-simp* [*simp del*]

definition
  is-acc-class :: *prog* ⇒ *pname* ⇒ *qtname* ⇒ *bool*
  where *is-acc-class G P C* = (*is-class G C* ∧ *G*⊢(*Class C*) *accessible-in P*)

definition
  is-acc-iface :: *prog* ⇒ *pname* ⇒ *qtname* ⇒ *bool*
  where *is-acc-iface G P I* = (*is-iface G I* ∧ *G*⊢(*Iface I*) *accessible-in P*)

definition
  is-acc-type :: *prog* ⇒ *pname* ⇒ *ty* ⇒ *bool*
  where *is-acc-type G P T* = (*is-type G T* ∧ *G*⊢*T accessible-in P*)

definition
$is\text{-}acc\text{-}reftype$ :: $prog \Rightarrow pname \Rightarrow ref\text{-}ty \Rightarrow bool$
where $is\text{-}acc\text{-}reftype\ G\ P\ T = (isrtype\ G\ T\ \wedge\ G\vdash T\ accessible\text{-}in'\ P)$

lemma $is\text{-}acc\text{-}classD$:
$is\text{-}acc\text{-}class\ G\ P\ C \Longrightarrow\ is\text{-}class\ G\ C\ \wedge\ G\vdash(Class\ C)\ accessible\text{-}in\ P$
by $(simp\ add{:}\ is\text{-}acc\text{-}class\text{-}def)$

lemma $is\text{-}acc\text{-}class\text{-}is\text{-}class$: $is\text{-}acc\text{-}class\ G\ P\ C \Longrightarrow is\text{-}class\ G\ C$
by $(auto\ simp\ add{:}\ is\text{-}acc\text{-}class\text{-}def)$

lemma $is\text{-}acc\text{-}ifaceD$:
$is\text{-}acc\text{-}iface\ G\ P\ I \Longrightarrow is\text{-}iface\ G\ I\ \wedge\ G\vdash(Iface\ I)\ accessible\text{-}in\ P$
by $(simp\ add{:}\ is\text{-}acc\text{-}iface\text{-}def)$

lemma $is\text{-}acc\text{-}typeD$:
$is\text{-}acc\text{-}type\ \ G\ P\ T \Longrightarrow is\text{-}type\ G\ T\ \ \wedge\ G\vdash T\ accessible\text{-}in\ P$
by $(simp\ add{:}\ is\text{-}acc\text{-}type\text{-}def)$

lemma $is\text{-}acc\text{-}reftypeD$:
$is\text{-}acc\text{-}reftype\ \ G\ P\ T \Longrightarrow isrtype\ G\ T\ \ \wedge\ G\vdash T\ accessible\text{-}in'\ P$
by $(simp\ add{:}\ is\text{-}acc\text{-}reftype\text{-}def)$

3 accessibility of members

The accessibility of members is more involved as the accessibility of types. We have to distinguish several cases to model the different effects of accessibility during inheritance, overriding and ordinary member access

Various technical conversion and selection functions

overloaded selector *accmodi* to select the access modifier out of various HOL types

class $has\text{-}accmodi$ =
fixes $accmodi{::}\ 'a \Rightarrow acc\text{-}modi$

instantiation $acc\text{-}modi$ :: $has\text{-}accmodi$
begin

definition
$acc\text{-}modi\text{-}accmodi\text{-}def$: $accmodi\ (a{::}acc\text{-}modi) = a$

instance ..

end

lemma $acc\text{-}modi\text{-}accmodi\text{-}simp[simp]$: $accmodi\ (a{::}acc\text{-}modi) = a$
by $(simp\ add{:}\ acc\text{-}modi\text{-}accmodi\text{-}def)$

instantiation $decl\text{-}ext$ :: $(type)\ has\text{-}accmodi$
begin

definition
decl-acc-modi-def: $accmodi\ (d::('a::\ type)\ decl\text{-}scheme) = access\ d$

instance ..

end

lemma *decl-acc-modi-simp*[*simp*]: $accmodi\ (d::('a::type)\ decl\text{-}scheme) = access\ d$
by (*simp add*: *decl-acc-modi-def*)

instantiation *prod* :: (*type*, *has-accmodi*) *has-accmodi*
begin

definition
pair-acc-modi-def: $accmodi\ p = accmodi\ (snd\ p)$

instance ..

end

lemma *pair-acc-modi-simp*[*simp*]: $accmodi\ (x,a) = (accmodi\ a)$
by (*simp add*: *pair-acc-modi-def*)

instantiation *memberdecl* :: *has-accmodi*
begin

definition
memberdecl-acc-modi-def: *accmodi m* = (*case m of*
fdecl f ⇒ *accmodi f*
| *mdecl m* ⇒ *accmodi m*)

instance ..

end

lemma *memberdecl-fdecl-acc-modi-simp*[*simp*]:
accmodi (*fdecl m*) = *accmodi m*
by (*simp add*: *memberdecl-acc-modi-def*)

lemma *memberdecl-mdecl-acc-modi-simp*[*simp*]:
accmodi (*mdecl m*) = *accmodi m*
by (*simp add*: *memberdecl-acc-modi-def*)

overloaded selector *declclass* to select the declaring class out of various HOL types

class *has-declclass* =
fixes *declclass*:: *'a* ⇒ *qtname*

instantiation *qtname-ext* :: (*type*) *has-declclass*
begin

definition
declclass q = (| *pid* = *pid q*, *tid* = *tid q* |)

instance ..

end

lemma *qtname-declclass-def*:
  declclass q $\equiv$ (*q*::*qtname*)
  by (*induct q*) (*simp add*: *declclass-qtname-ext-def*)

lemma *qtname-declclass-simp*[*simp*]: *declclass* (*q*::*qtname*) = *q*
by (*simp add*: *qtname-declclass-def*)

instantiation *prod* :: (*has-declclass*, *type*) *has-declclass*
begin

definition
  pair-declclass-def: *declclass p* = *declclass* (*fst p*)

instance ..

end

lemma *pair-declclass-simp*[*simp*]: *declclass* (*c*,*x*) = *declclass c*
by (*simp add*: *pair-declclass-def*)

overloaded selector *is-static* to select the static modifier out of various HOL types

class *has-static* =
  fixes *is-static* :: *'a* $\Rightarrow$ *bool*

instantiation *decl-ext* :: (*has-static*) *has-static*
begin

instance ..

end

instantiation *member-ext* :: (*type*) *has-static*
begin

instance ..

end

axiomatization where
  static-field-type-is-static-def: *is-static* (*m*::(*'a member-scheme*)) $\equiv$ *static m*

lemma *member-is-static-simp*: *is-static* (*m*::*'a member-scheme*) = *static m*
by (*simp add*: *static-field-type-is-static-def*)

instantiation *prod* :: (*type*, *has-static*) *has-static*
begin

definition
  pair-is-static-def: *is-static p* = *is-static* (*snd p*)

instance ..

end

lemma *pair-is-static-simp* [*simp*]: *is-static* (*x*,*s*) = *is-static s*
by (*simp add*: *pair-is-static-def*)

lemma *pair-is-static-simp1*: *is-static p* = *is-static* (*snd p*)
by (*simp add*: *pair-is-static-def*)

instantiation *memberdecl* :: *has-static*
begin

definition
memberdecl-is-static-def:
is-static m = (*case m of*
  fdecl f ⇒ *is-static f*
  | *mdecl m* ⇒ *is-static m*)

instance ..

end

lemma *memberdecl-is-static-fdecl-simp*[*simp*]:
is-static (*fdecl f*) = *is-static f*
by (*simp add*: *memberdecl-is-static-def*)

lemma *memberdecl-is-static-mdecl-simp*[*simp*]:
is-static (*mdecl m*) = *is-static m*
by (*simp add*: *memberdecl-is-static-def*)

lemma *mhead-static-simp* [*simp*]: *is-static* (*mhead m*) = *is-static m*
by (*cases m*) (*simp add*: *mhead-def member-is-static-simp*)

— some mnemotic selectors for various pairs

definition
decliface :: *qtname* × *'a decl-scheme* ⇒ *qtname* **where**
decliface = *fst* — get the interface component

definition
mbr :: *qtname* × *memberdecl* ⇒ *memberdecl* **where**
mbr = *snd* — get the memberdecl component

definition
mthd :: *'b* × *'a* ⇒ *'a* **where**
mthd = *snd* — get the method component
— also used for mdecl, mhead

definition
fld :: *'b* × *'a decl-scheme* ⇒ *'a decl-scheme* **where**
fld = *snd* — get the field component
— also used for ((*vname* × *qtname*)× *field*)

— some mnemotic selectors for (*vname* × *qtname*)

definition
fname:: *vname* × *'a* ⇒ *vname*
where *fname* = *fst*
— also used for fdecl

definition
declclassf:: (*vname* × *qtname*) ⇒ *qtname*
where *declclassf* = *snd*

lemma *decliface-simp*[*simp*]: *decliface* (*I*,*m*) = *I*
by (*simp add*: *decliface-def*)

lemma *mbr-simp*[*simp*]: *mbr* (*C*,*m*) = *m*
by (*simp add*: *mbr-def*)

lemma *access-mbr-simp* [*simp*]: (*accmodi* (*mbr m*)) = *accmodi m*
by (*cases m*) (*simp add*: *mbr-def*)

lemma *mthd-simp*[*simp*]: *mthd* (*C*,*m*) = *m*
by (*simp add*: *mthd-def*)

lemma *fld-simp*[*simp*]: *fld* (*C*,*f*) = *f*
by (*simp add*: *fld-def*)

lemma *accmodi-simp*[*simp*]: *accmodi* (*C*,*m*) = *access m*
by (*simp*)

lemma *access-mthd-simp* [*simp*]: (*access* (*mthd m*)) = *accmodi m*
by (*cases m*) (*simp add*: *mthd-def*)

lemma *access-fld-simp* [*simp*]: (*access* (*fld f*)) = *accmodi f*
by (*cases f*) (*simp add*: *fld-def*)

lemma *static-mthd-simp*[*simp*]: *static* (*mthd m*) = *is-static m*
by (*cases m*) (*simp add*: *mthd-def member-is-static-simp*)

lemma *mthd-is-static-simp* [*simp*]: *is-static* (*mthd m*) = *is-static m*
by (*cases m*) *simp*

lemma *static-fld-simp*[*simp*]: *static* (*fld f*) = *is-static f*
by (*cases f*) (*simp add*: *fld-def member-is-static-simp*)

lemma *ext-field-simp* [*simp*]: (*declclass f*,*fld f*) = *f*
by (*cases f*) (*simp add*: *fld-def*)

lemma *ext-method-simp* [*simp*]: (*declclass m*,*mthd m*) = *m*
by (*cases m*) (*simp add*: *mthd-def*)

lemma *ext-mbr-simp* [*simp*]: (*declclass m*,*mbr m*) = *m*
by (*cases m*) (*simp add*: *mbr-def*)

lemma *fname-simp*[*simp*]:*fname* (*n*,*c*) = *n*
by (*simp add*: *fname-def*)

lemma *declclassf-simp*[*simp*]:*declclassf* (*n*,*c*) = *c*
by (*simp add*: *declclassf-def*)

— some mnemotic selectors for (*vname* × *qtname*)

definition
fldname :: *vname* × *qtname* ⇒ *vname*
where *fldname* = *fst*

definition
fldclass :: *vname* × *qtname* ⇒ *qtname*
where *fldclass* = *snd*

lemma *fldname-simp*[*simp*]: *fldname* (*n*,*c*) = *n*
by (*simp add*: *fldname-def*)

lemma *fldclass-simp*[*simp*]: *fldclass* (*n*,*c*) = *c*
by (*simp add*: *fldclass-def*)

lemma *ext-fieldname-simp*[*simp*]: (*fldname f*,*fldclass f*) = *f*
by (*simp add*: *fldname-def fldclass-def*)

Convert a qualified method declaration (qualified with its declaring class) to a qualified member declaration: *methdMembr*

definition
methdMembr :: *qtname* × *mdecl* ⇒ *qtname* × *memberdecl*
where *methdMembr m* = (*fst m*, *mdecl* (*snd m*))

lemma *methdMembr-simp*[*simp*]: *methdMembr* (*c*,*m*) = (*c*,*mdecl m*)
by (*simp add*: *methdMembr-def*)

lemma *accmodi-methdMembr-simp*[*simp*]: *accmodi* (*methdMembr m*) = *accmodi m*
by (*cases m*) (*simp add*: *methdMembr-def*)

lemma *is-static-methdMembr-simp*[*simp*]: *is-static* (*methdMembr m*) = *is-static m*
by (*cases m*) (*simp add*: *methdMembr-def*)

lemma *declclass-methdMembr-simp*[*simp*]: *declclass* (*methdMembr m*) = *declclass m*
by (*cases m*) (*simp add*: *methdMembr-def*)

Convert a qualified method (qualified with its declaring class) to a qualified member declaration: *method*

definition
method :: *sig* ⇒ (*qtname* × *methd*) ⇒ (*qtname* × *memberdecl*)
where *method sig m* = (*declclass m*, *mdecl* (*sig*, *mthd m*))

lemma *method-simp*[*simp*]: *method sig* (*C*,*m*) = (*C*,*mdecl* (*sig*,*m*))
by (*simp add*: *method-def*)

lemma *accmodi-method-simp*[*simp*]: *accmodi* (*method sig m*) = *accmodi m*
by (*simp add*: *method-def*)

lemma *declclass-method-simp*[*simp*]: *declclass* (*method sig m*) = *declclass m*
by (*simp add*: *method-def*)

lemma *is-static-method-simp*[*simp*]: *is-static* (*method sig m*) = *is-static m*
by (*cases m*) (*simp add*: *method-def*)

lemma *mbr-method-simp*[*simp*]: *mbr* (*method sig m*) = *mdecl* (*sig*,*mthd m*)
by (*simp add*: *mbr-def method-def*)

lemma *memberid-method-simp*[*simp*]: *memberid* (*method sig m*) = *mid sig*
by (*simp add*: *method-def*)

definition
fieldm :: *vname* ⇒ (*qtname* × *field*) ⇒ (*qtname* × *memberdecl*)
where *fieldm n f* = (*declclass f*, *fdecl* (*n*, *fld f*))

lemma *fieldm-simp*[*simp*]: *fieldm n* (*C*,*f*) = (*C*,*fdecl* (*n*,*f*))
by (*simp add*: *fieldm-def*)

lemma *accmodi-fieldm-simp*[*simp*]: *accmodi* (*fieldm n f*) = *accmodi f*
by (*simp add*: *fieldm-def*)

lemma *declclass-fieldm-simp*[*simp*]: *declclass* (*fieldm n f*) = *declclass f*
by (*simp add*: *fieldm-def*)

lemma *is-static-fieldm-simp*[*simp*]: *is-static* (*fieldm n f*) = *is-static f*
by (*cases f*) (*simp add*: *fieldm-def*)

lemma *mbr-fieldm-simp*[*simp*]: *mbr* (*fieldm n f*) = *fdecl* (*n*,*fld f*)
by (*simp add*: *mbr-def fieldm-def*)

lemma *memberid-fieldm-simp*[*simp*]: *memberid* (*fieldm n f*) = *fid n*
by (*simp add*: *fieldm-def*)

Select the signature out of a qualified method declaration: *msig*

definition
msig :: (*qtname* × *mdecl*) ⇒ *sig*
where *msig m* = *fst* (*snd m*)

lemma *msig-simp*[*simp*]: *msig* (*c*,(*s*,*m*)) = *s*
by (*simp add*: *msig-def*)

Convert a qualified method (qualified with its declaring class) to a qualified method declaration: *qmdecl*

definition
qmdecl :: *sig* ⇒ (*qtname* × *methd*) ⇒ (*qtname* × *mdecl*)
where *qmdecl sig m* = (*declclass m*, (*sig*,*mthd m*))

lemma *qmdecl-simp*[*simp*]: *qmdecl sig* (*C*,*m*) = (*C*,(*sig*,*m*))
by (*simp add*: *qmdecl-def*)

lemma *declclass-qmdecl-simp*[*simp*]: *declclass* (*qmdecl sig m*) = *declclass m*
by (*simp add*: *qmdecl-def*)

lemma *accmodi-qmdecl-simp*[*simp*]: *accmodi* (*qmdecl sig m*) = *accmodi m*
by (*simp add*: *qmdecl-def*)

lemma *is-static-qmdecl-simp*[*simp*]: *is-static* (*qmdecl sig m*) = *is-static m*
by (*cases m*) (*simp add*: *qmdecl-def*)

lemma *msig-qmdecl-simp*[*simp*]: *msig* (*qmdecl sig m*) = *sig*
by (*simp add*: *qmdecl-def*)

lemma *mdecl-qmdecl-simp*[*simp*]:
mdecl (*mthd* (*qmdecl sig new*)) = *mdecl* (*sig*, *mthd new*)
by (*simp add*: *qmdecl-def*)

lemma *methdMembr-qmdecl-simp* [*simp*]:
methdMembr (*qmdecl sig old*) = *method sig old*
by (*simp add*: *methdMembr-def qmdecl-def method-def*)

overloaded selector *resTy* to select the result type out of various HOL types

class *has-resTy* =
fixes *resTy*:: *'a* ⇒ *ty*

instantiation *decl-ext* :: (*has-resTy*) *has-resTy*
begin

instance ..

end

instantiation *member-ext* :: (*has-resTy*) *has-resTy*
begin

instance ..

end

instantiation *mhead-ext* :: (*type*) *has-resTy*
begin

instance ..

end

axiomatization where
mhead-ext-type-resTy-def: *resTy* (*m*::(′*b mhead-scheme*)) ≡ *resT m*

lemma *mhead-resTy-simp*: *resTy* (*m*::′*a mhead-scheme*) = *resT m*
by (*simp add*: *mhead-ext-type-resTy-def*)

lemma *resTy-mhead* [*simp*]:*resTy* (*mhead m*) = *resTy m*
by (*simp add*: *mhead-def mhead-resTy-simp*)

instantiation *prod* :: (*type*, *has-resTy*) *has-resTy*
begin

definition
pair-resTy-def: *resTy p* = *resTy* (*snd p*)

instance ..

end

lemma *pair-resTy-simp*[*simp*]: *resTy* (*x*,*m*) = *resTy m*
by (*simp add*: *pair-resTy-def*)

lemma *qmdecl-resTy-simp* [*simp*]: *resTy* (*qmdecl sig m*) = *resTy m*
by (*cases m*) (*simp*)

lemma *resTy-mthd* [*simp*]:*resTy* (*mthd m*) = *resTy m*
by (*cases m*) (*simp add*: *mthd-def*)

inheritable-in

G⊢*m inheritable-in P*: m can be inherited by classes in package P if:

- the declaration class of m is accessible in P and
- the member m is declared with protected or public access or if it is declared with default (package) access, the package of the declaration class of m is also P. If the member m is declared with private access it is not accessible for inheritance at all.

definition
inheritable-in :: *prog* ⇒ (*qtname* × *memberdecl*) ⇒ *pname* ⇒ *bool* (- ⊢ - *inheritable*′*-in* - [*61*,*61*,*61*] *60*)
where
G⊢*membr inheritable-in pack* =
(*case* (*accmodi membr*) *of*

Private $\Rightarrow$ *False*
| *Package* $\Rightarrow$ (*pid* (*declclass membr*)) = *pack*
| *Protected* $\Rightarrow$ *True*
| *Public* $\Rightarrow$ *True*)

abbreviation
Method-inheritable-in-syntax::
prog $\Rightarrow$ (*qtname* $\times$ *mdecl*) $\Rightarrow$ *pname* $\Rightarrow$ *bool*
(- $\vdash$*Method* - *inheritable'-in* - [*61*,*61*,*61*] *60*)
where *G*$\vdash$*Method m inheritable-in p* == *G*$\vdash$*methdMembr m inheritable-in p*

abbreviation
Methd-inheritable-in::
prog $\Rightarrow$ *sig* $\Rightarrow$ (*qtname* $\times$ *methd*) $\Rightarrow$ *pname* $\Rightarrow$ *bool*
(- $\vdash$*Methd* - - *inheritable'-in* - [*61*,*61*,*61*,*61*] *60*)
where *G*$\vdash$*Methd s m inheritable-in p* == *G*$\vdash$(*method s m*) *inheritable-in p*

declared-in/undeclared-in

definition
cdeclaredmethd :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ (*sig*,*methd*) *table* **where**
cdeclaredmethd G C =
(*case class G C of*
None $\Rightarrow$ λ *sig*. *None*
| *Some c* $\Rightarrow$ *table-of* (*methods c*))

definition
cdeclaredfield :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ (*vname*,*field*) *table* **where**
cdeclaredfield G C =
(*case class G C of*
None $\Rightarrow$ λ *sig*. *None*
| *Some c* $\Rightarrow$ *table-of* (*cfields c*))

definition
declared-in :: *prog* $\Rightarrow$ *memberdecl* $\Rightarrow$ *qtname* $\Rightarrow$ *bool* (-$\vdash$ - *declared'-in* - [*61*,*61*,*61*] *60*)
where
G$\vdash$*m declared-in C* = (*case m of*
fdecl (*fn*,*f*) $\Rightarrow$ *cdeclaredfield G C fn* = *Some f*
| *mdecl* (*sig*,*m*) $\Rightarrow$ *cdeclaredmethd G C sig* = *Some m*)

abbreviation
method-declared-in:: *prog* $\Rightarrow$ (*qtname* $\times$ *mdecl*) $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
(-$\vdash$*Method* - *declared'-in* - [*61*,*61*,*61*] *60*)
where *G*$\vdash$*Method m declared-in C* == *G*$\vdash$*mdecl* (*mthd m*) *declared-in C*

abbreviation
methd-declared-in:: *prog* $\Rightarrow$ *sig* $\Rightarrow$(*qtname* $\times$ *methd*) $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
(-$\vdash$*Methd* - - *declared'-in* - [*61*,*61*,*61*,*61*] *60*)
where *G*$\vdash$*Methd s m declared-in C* == *G*$\vdash$*mdecl* (*s*,*mthd m*) *declared-in C*

lemma *declared-in-classD*:
G$\vdash$*m declared-in C* $\Longrightarrow$ *is-class G C*
by (*cases m*)
(*auto simp add*: *declared-in-def cdeclaredmethd-def cdeclaredfield-def*)

definition
undeclared-in :: *prog* $\Rightarrow$ *memberid* $\Rightarrow$ *qtname* $\Rightarrow$ *bool* (-$\vdash$ - *undeclared'-in* - [*61*,*61*,*61*] *60*)
where

$G \vdash m$ *undeclared-in* $C = ($*case* m *of*
fid fn $\Rightarrow$ *cdeclaredfield* G C *fn* $=$ *None*
$|$ *mid sig* $\Rightarrow$ *cdeclaredmethd* G C *sig* $=$ *None*$)$

members

inductive
members :: *prog* $\Rightarrow$ (*qtname* $\times$ *memberdecl*) $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
(- $\vdash$ - *member'-of* - [*61*,*61*,*61*] *60*)
for G :: *prog*
where

Immediate: $[\![G \vdash$*mbr m declared-in C*;*declclass m* $= C]\!] \Longrightarrow G \vdash m$ *member-of* C
| *Inherited*: $[\![G \vdash m$ *inheritable-in* (*pid* C); $G \vdash$*memberid m undeclared-in C*;
$G \vdash C \prec_C 1$ S; $G \vdash$(*Class* S) *accessible-in* (*pid* C);$G \vdash m$ *member-of* S
$]\!] \Longrightarrow G \vdash m$ *member-of* C

Note that in the case of an inherited member only the members of the direct superclass are concerned. If a member of a superclass of the direct superclass isn't inherited in the direct superclass (not member of the direct superclass) than it can't be a member of the class. E.g. If a member of a class A is defined with package access it isn't member of a subclass S if S isn't in the same package as A. Any further subclasses of S will not inherit the member, regardless if they are in the same package as A or not.

abbreviation
method-member-of:: *prog* $\Rightarrow$ (*qtname* $\times$ *mdecl*) $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
(- $\vdash$*Method* - *member'-of* - [*61*,*61*,*61*] *60*)
where $G \vdash$*Method m member-of C* $==$ $G \vdash$(*methdMembr m*) *member-of C*

abbreviation
methd-member-of:: *prog* $\Rightarrow$ *sig* $\Rightarrow$ (*qtname* $\times$ *methd*) $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
(- $\vdash$*Methd* - - *member'-of* - [*61*,*61*,*61*,*61*] *60*)
where $G \vdash$*Methd s m member-of C* $==$ $G \vdash$(*method s m*) *member-of C*

abbreviation
fieldm-member-of:: *prog* $\Rightarrow$ *vname* $\Rightarrow$ (*qtname* $\times$ *field*) $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
(- $\vdash$*Field* - - *member'-of* - [*61*,*61*,*61*] *60*)
where $G \vdash$*Field n f member-of C* $==$ $G \vdash$*fieldm n f member-of C*

definition
inherits :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ (*qtname* $\times$ *memberdecl*) $\Rightarrow$ *bool* (- $\vdash$ - *inherits* - [*61*,*61*,*61*] *60*)
where
$G \vdash C$ *inherits* m $=$
($G \vdash m$ *inheritable-in* (*pid* C) $\wedge$ $G \vdash$*memberid m undeclared-in C* $\wedge$
($\exists S.$ $G \vdash C \prec_C 1$ S $\wedge$ $G \vdash$(*Class* S) *accessible-in* (*pid* C) $\wedge$ $G \vdash m$ *member-of* S))

lemma *inherits-member*: $G \vdash C$ *inherits* $m \Longrightarrow G \vdash m$ *member-of* C
by (*auto simp add*: *inherits-def intro*: *members.Inherited*)

definition
member-in :: *prog* $\Rightarrow$ (*qtname* $\times$ *memberdecl*) $\Rightarrow$ *qtname* $\Rightarrow$ *bool* (- $\vdash$ - *member'-in* - [*61*,*61*,*61*] *60*)
where $G \vdash m$ *member-in* C $=$ ($\exists$ *provC*. $G \vdash$ $C \preceq_C$ *provC* $\wedge$ $G \vdash m$ *member-of provC*)

A member is in a class if it is member of the class or a superclass. If a member is in a class we can select this member. This additional notion is necessary since not all members are inherited to subclasses. So such members are not member-of the subclass but member-in the subclass.

abbreviation

method-member-in:: *prog* ⇒ (*qtname* × *mdecl*) ⇒ *qtname* ⇒ *bool*
(- ⊢*Method* - *member′-in* - [*61*,*61*,*61*] *60*)
where *G*⊢*Method m member-in C* == *G*⊢(*methdMembr m*) *member-in C*

abbreviation
methd-member-in:: *prog* ⇒ *sig* ⇒ (*qtname* × *methd*) ⇒ *qtname* ⇒ *bool*
(- ⊢*Methd* - - *member′-in* - [*61*,*61*,*61*,*61*] *60*)
where *G*⊢*Methd s m member-in C* == *G*⊢(*method s m*) *member-in C*

lemma *member-inD*: *G*⊢*m member-in C*
⟹ ∃ *provC*. *G*⊢ *C* $\preceq_C$ *provC* ∧ *G* ⊢ *m member-of provC*
by (*auto simp add*: *member-in-def*)

lemma *member-inI*: ⟦*G* ⊢ *m member-of provC*;*G*⊢ *C* $\preceq_C$ *provC*⟧ ⟹ *G*⊢*m member-in C*
by (*auto simp add*: *member-in-def*)

lemma *member-of-to-member-in*: *G* ⊢ *m member-of C* ⟹ *G* ⊢*m member-in C*
by (*auto intro*: *member-inI*)

overriding

Unfortunately the static notion of overriding (used during the typecheck of the compiler) and the dynamic notion of overriding (used during execution in the JVM) are not exactly the same.

Static overriding (used during the typecheck of the compiler)

inductive
stat-overridesR :: *prog* ⇒ (*qtname* × *mdecl*) ⇒ (*qtname* × *mdecl*) ⇒ *bool*
(- ⊢ - $overrides_S$ - [*61*,*61*,*61*] *60*)
for *G* :: *prog*
where

Direct: ⟦¬ *is-static new*; *msig new* = *msig old*;
G⊢*Method new declared-in* (*declclass new*);
G⊢*Method old declared-in* (*declclass old*);
G⊢*Method old inheritable-in pid* (*declclass new*);
G⊢(*declclass new*) $\prec_C 1$ *superNew*;
G ⊢*Method old member-of superNew*
⟧ ⟹ *G*⊢*new* $overrides_S$ *old*

| *Indirect*: ⟦*G*⊢*new* $overrides_S$ *intr*; *G*⊢*intr* $overrides_S$ *old*⟧
⟹ *G*⊢*new* $overrides_S$ *old*

Dynamic overriding (used during the typecheck of the compiler)

inductive
overridesR :: *prog* ⇒ (*qtname* × *mdecl*) ⇒ (*qtname* × *mdecl*) ⇒ *bool*
(- ⊢ - *overrides* - [*61*,*61*,*61*] *60*)
for *G* :: *prog*
where

Direct: ⟦¬ *is-static new*; ¬ *is-static old*; *accmodi new* ≠ *Private*;
msig new = *msig old*;
G⊢(*declclass new*) $\prec_C$ (*declclass old*);
G⊢*Method new declared-in* (*declclass new*);
G⊢*Method old declared-in* (*declclass old*);
G⊢*Method old inheritable-in pid* (*declclass new*);

$G \vdash resTy\ new \preceq resTy\ old$
$\rrbracket \Longrightarrow G \vdash new\ overrides\ old$

$|$ *Indirect*: $\llbracket G \vdash new\ overrides\ intr;\ G \vdash intr\ overrides\ old \rrbracket$
$\Longrightarrow G \vdash new\ overrides\ old$

abbreviation (*input*)
sig-stat-overrides::
$prog \Rightarrow sig \Rightarrow (qtname \times methd) \Rightarrow (qtname \times methd) \Rightarrow bool$
$(\text{-,-}\vdash\ \text{-}\ overrides_S\ \text{-}\ [61,61,61,61]\ 60)$
where $G,s \vdash new\ overrides_S\ old == G \vdash (qmdecl\ s\ new)\ overrides_S\ (qmdecl\ s\ old)$

abbreviation (*input*)
sig-overrides:: $prog \Rightarrow sig \Rightarrow (qtname \times methd) \Rightarrow (qtname \times methd) \Rightarrow bool$
$(\text{-,-}\vdash\ \text{-}\ overrides\ \text{-}\ [61,61,61,61]\ 60)$
where $G,s \vdash new\ overrides\ old == G \vdash (qmdecl\ s\ new)\ overrides\ (qmdecl\ s\ old)$

Hiding

definition
$hides :: prog \Rightarrow (qtname \times mdecl) \Rightarrow (qtname \times mdecl) \Rightarrow bool\ (\text{-}\vdash\ \text{-}\ hides\ \text{-}\ [61,61,61]\ 60)$
where
$G \vdash new\ hides\ old =$
$(is\text{-}static\ new \wedge msig\ new = msig\ old \wedge$
$G \vdash (declclass\ new) \prec_C (declclass\ old) \wedge$
$G \vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G \vdash Method\ old\ declared\text{-}in\ (declclass\ old) \wedge$
$G \vdash Method\ old\ inheritable\text{-}in\ pid\ (declclass\ new))$

abbreviation
sig-hides:: $prog \Rightarrow sig \Rightarrow (qtname \times methd) \Rightarrow (qtname \times methd) \Rightarrow bool$
$(\text{-,-}\vdash\ \text{-}\ hides\ \text{-}\ [61,61,61,61]\ 60)$
where $G,s \vdash new\ hides\ old == G \vdash (qmdecl\ s\ new)\ hides\ (qmdecl\ s\ old)$

lemma *hidesI*:
$\llbracket is\text{-}static\ new;\ msig\ new = msig\ old;$
$G \vdash (declclass\ new) \prec_C (declclass\ old);$
$G \vdash Method\ new\ declared\text{-}in\ (declclass\ new);$
$G \vdash Method\ old\ declared\text{-}in\ (declclass\ old);$
$G \vdash Method\ old\ inheritable\text{-}in\ pid\ (declclass\ new)$
$\rrbracket \Longrightarrow G \vdash new\ hides\ old$
by (*auto simp add*: *hides-def*)

lemma *hidesD*:
$\llbracket G \vdash new\ hides\ old \rrbracket \Longrightarrow$
$declclass\ new \neq Object \wedge is\text{-}static\ new \wedge msig\ new = msig\ old \wedge$
$G \vdash (declclass\ new) \prec_C (declclass\ old) \wedge$
$G \vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G \vdash Method\ old\ declared\text{-}in\ (declclass\ old)$
by (*auto simp add*: *hides-def*)

lemma *overrides-commonD*:
$\llbracket G \vdash new\ overrides\ old \rrbracket \Longrightarrow$
$declclass\ new \neq Object \wedge \neg\ is\text{-}static\ new \wedge \neg\ is\text{-}static\ old \wedge$
$accmodi\ new \neq Private \wedge$
$msig\ new = msig\ old\ \wedge$

$G \vdash (declclass\ new) \prec_C (declclass\ old) \wedge$
$G \vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G \vdash Method\ old\ declared\text{-}in\ (declclass\ old)$
by (*induct set*: *overridesR*) (*auto intro*: *trancl-trans*)

lemma *ws-overrides-commonD*:
$[\![G \vdash new\ overrides\ old; ws\text{-}prog\ G]\!] \Longrightarrow$
$declclass\ new \neq Object \wedge \neg\ is\text{-}static\ new \wedge \neg\ is\text{-}static\ old \wedge$
$accmodi\ new \neq Private \wedge G \vdash resTy\ new \preceq resTy\ old \wedge$
$msig\ new = msig\ old \wedge$
$G \vdash (declclass\ new) \prec_C (declclass\ old) \wedge$
$G \vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G \vdash Method\ old\ declared\text{-}in\ (declclass\ old)$
by (*induct set*: *overridesR*) (*auto intro*: *trancl-trans ws-widen-trans*)

lemma *overrides-eq-sigD*:
$[\![G \vdash new\ overrides\ old]\!] \Longrightarrow msig\ old = msig\ new$
by (*auto dest*: *overrides-commonD*)

lemma *hides-eq-sigD*:
$[\![G \vdash new\ hides\ old]\!] \Longrightarrow msig\ old = msig\ new$
by (*auto simp add*: *hides-def*)

permits access

definition
permits-acc :: $prog \Rightarrow (qtname \times memberdecl) \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$ (- ⊢ - *in* - *permits′-acc′-from* - [*61*,*61*,*61*,*61*] *60*)
where
$G \vdash membr\ in\ cls\ permits\text{-}acc\text{-}from\ accclass =$
$(case\ (accmodi\ membr)\ of$
$\quad Private \Rightarrow (declclass\ membr = accclass)$
$\mid Package \Rightarrow (pid\ (declclass\ membr) = pid\ accclass)$
$\mid Protected \Rightarrow (pid\ (declclass\ membr) = pid\ accclass)$
$\qquad \vee$
$\qquad (G \vdash accclass \prec_C declclass\ membr$
$\qquad \wedge (G \vdash cls \preceq_C accclass \vee is\text{-}static\ membr))$
$\mid Public \Rightarrow True)$

The subcondition of the *Protected* case: $G \vdash accclass \prec_C declclass\ membr$ could also be relaxed to: $G \vdash accclass \preceq_C declclass\ membr$ since in case both classes are the same the other condition $pid\ (declclass\ membr) = pid\ accclass$ holds anyway.

Like in case of overriding, the static and dynamic accessibility of members is not uniform.

- Statically the class/interface of the member must be accessible for the member to be accessible. During runtime this is not necessary. For Example, if a class is accessible and we are allowed to access a member of this class (statically) we expect that we can access this member in an arbitrary subclass (during runtime). It's not intended to restrict the access to accessible subclasses during runtime.
- Statically the member we want to access must be "member of" the class. Dynamically it must only be "member in" the class.

inductive
accessible-fromR :: $prog \Rightarrow qtname \Rightarrow (qtname \times memberdecl) \Rightarrow qtname \Rightarrow bool$

and *accessible-from* :: $prog \Rightarrow (qtname \times memberdecl) \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$
  (- ⊢ - *of* - *accessible'-from* - [*61*,*61*,*61*,*61*] *60*)
and *method-accessible-from* :: $prog \Rightarrow (qtname \times mdecl) \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$
  (- ⊢*Method* - *of* - *accessible'-from* - [*61*,*61*,*61*,*61*] *60*)
for *G* :: *prog* **and** *accclass* :: *qtname*
where
G⊢*membr of cls accessible-from accclass* ≡ *accessible-fromR G accclass membr cls*

| *G*⊢*Method m of cls accessible-from accclass* ≡ *accessible-fromR G accclass* (*methdMembr m*) *cls*

| *Immediate*: !!*membr class*.
  ⟦*G*⊢*membr member-of class*;
  G⊢(*Class class*) *accessible-in* (*pid accclass*);
  G⊢*membr in class permits-acc-from accclass*
  ⟧ ⟹ *G*⊢*membr of class accessible-from accclass*

| *Overriding*: !!*membr class C new old supr*.
  ⟦*G*⊢*membr member-of class*;
  G⊢(*Class class*) *accessible-in* (*pid accclass*);
  membr=(*C*,*mdecl new*);
  G⊢(*C*,*new*) $overrides_S$ *old*;
  G⊢*class* $\prec_C$ *supr*;
  G⊢*Method old of supr accessible-from accclass*
  ⟧⟹ *G*⊢*membr of class accessible-from accclass*

abbreviation
methd-accessible-from::
$prog \Rightarrow sig \Rightarrow (qtname \times methd) \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$
  (- ⊢*Methd* - - *of* - *accessible'-from* - [*61*,*61*,*61*,*61*,*61*] *60*)
where
G⊢*Methd s m of cls accessible-from accclass* ==
  G⊢(*method s m*) *of cls accessible-from accclass*

abbreviation
field-accessible-from::
$prog \Rightarrow vname \Rightarrow (qtname \times field) \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$
  (- ⊢*Field* - - *of* - *accessible'-from* - [*61*,*61*,*61*,*61*,*61*] *60*)
where
G⊢*Field fn f of C accessible-from accclass* ==
 G⊢(*fieldm fn f*) *of C accessible-from accclass*

inductive
dyn-accessible-fromR :: $prog \Rightarrow qtname \Rightarrow (qtname \times memberdecl) \Rightarrow qtname \Rightarrow bool$
and *dyn-accessible-from'* :: $prog \Rightarrow (qtname \times memberdecl) \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$
  (- ⊢ - *in* - *dyn'-accessible'-from* - [*61*,*61*,*61*,*61*] *60*)
and *method-dyn-accessible-from* :: $prog \Rightarrow (qtname \times mdecl) \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$
  (- ⊢*Method* - *in* - *dyn'-accessible'-from* - [*61*,*61*,*61*,*61*] *60*)
for *G* :: *prog* **and** *accclass* :: *qtname*
where
G⊢*membr in C dyn-accessible-from accC* ≡ *dyn-accessible-fromR G accC membr C*

| *G*⊢*Method m in C dyn-accessible-from accC* ≡ *dyn-accessible-fromR G accC* (*methdMembr m*) *C*

| *Immediate*: !!*class*. ⟦*G*⊢*membr member-in class*;
  G⊢*membr in class permits-acc-from accclass*
  ⟧ ⟹ *G*⊢*membr in class dyn-accessible-from accclass*

| *Overriding*: !!*class*. ⟦*G*⊢*membr member-in class*;
  membr=(*C*,*mdecl new*);

$G \vdash (C,new)$ *overrides old*;
$G \vdash$*class* $\prec_C$ *supr*;
$G \vdash$*Method old in supr dyn-accessible-from accclass*
$\rrbracket \Longrightarrow G \vdash$*membr in class dyn-accessible-from accclass*

abbreviation
methd-dyn-accessible-from::
prog $\Rightarrow$ *sig* $\Rightarrow$ (*qtname* $\times$ *methd*) $\Rightarrow$ *qtname* $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
(- $\vdash$*Methd - - in - dyn′-accessible′-from -* [*61,61,61,61,61*] *60*)
where
$G \vdash$*Methd s m in C dyn-accessible-from accC* ==
$G \vdash$(*method s m*) *in C dyn-accessible-from accC*

abbreviation
field-dyn-accessible-from::
prog $\Rightarrow$ *vname* $\Rightarrow$ (*qtname* $\times$ *field*) $\Rightarrow$ *qtname* $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
(- $\vdash$*Field - - in - dyn′-accessible′-from -* [*61,61,61,61,61*] *60*)
where
$G \vdash$*Field fn f in dynC dyn-accessible-from accC* ==
$G \vdash$(*fieldm fn f*) *in dynC dyn-accessible-from accC*

lemma *accessible-from-commonD*: $G \vdash$*m of C accessible-from S*
$\Longrightarrow G \vdash$*m member-of C* $\wedge$ $G \vdash$(*Class C*) *accessible-in* (*pid S*)
by (*auto elim*: *accessible-fromR.induct*)

lemma *unique-declaration*:
$\llbracket G \vdash$*m declared-in C*; $G \vdash$*n declared-in C*; *memberid m* = *memberid n* $\rrbracket$
$\Longrightarrow m = n$
apply (*cases m*)
apply (*cases n*,
auto simp add: *declared-in-def cdeclaredmethd-def cdeclaredfield-def*)+
done

lemma *declared-not-undeclared*:
$G \vdash$*m declared-in C* $\Longrightarrow \neg$ $G \vdash$ *memberid m undeclared-in C*
by (*cases m*) (*auto simp add*: *declared-in-def undeclared-in-def*)

lemma *undeclared-not-declared*:
$G \vdash$ *memberid m undeclared-in C* $\Longrightarrow \neg$ $G \vdash$ *m declared-in C*
by (*cases m*) (*auto simp add*: *declared-in-def undeclared-in-def*)

lemma *not-undeclared-declared*:
$\neg$ $G \vdash$ *membr-id undeclared-in C* $\Longrightarrow$ ($\exists$ *m*. $G \vdash$*m declared-in C* $\wedge$
membr-id = *memberid m*)
proof −
assume *not-undecl*:$\neg$ $G \vdash$ *membr-id undeclared-in C*
show *?thesis* (**is** *?P membr-id*)
proof (*cases membr-id*)
case (*fid vname*)
with *not-undecl*
obtain *fld* **where**
$G \vdash$*fdecl* (*vname,fld*) *declared-in C*
by (*auto simp add*: *undeclared-in-def declared-in-def*

cdeclaredfield-def)
with *fid* **show** *?thesis*
by *auto*
next
case (*mid sig*)
with *not-undecl*
obtain *mthd* **where**
$G\vdash$*mdecl* (*sig*,*mthd*) *declared-in C*
by (*auto simp add*: *undeclared-in-def declared-in-def*
cdeclaredmethd-def)
with *mid* **show** *?thesis*
by *auto*
qed
qed

lemma *unique-declared-in*:
$[\![G\vdash m$ *declared-in* C; $G\vdash n$ *declared-in* C; *memberid* m = *memberid* $n]\!]$
$\Longrightarrow m = n$
by (*auto simp add*: *declared-in-def cdeclaredmethd-def cdeclaredfield-def*
split: *memberdecl.splits*)

lemma *unique-member-of*:
assumes *n*: $G\vdash n$ *member-of* C **and**
m: $G\vdash m$ *member-of* C **and**
eqid: *memberid* n = *memberid* m
shows $n=m$
proof −
from *n m eqid*
show $n=m$
proof (*induct*)
case (*Immediate n C*)
assume *member-n*: $G\vdash$ *mbr n declared-in C declclass n* = C
assume *eqid*: *memberid* n = *memberid* m
assume $G \vdash m$ *member-of* C
then show $n=m$
proof (*cases*)
case *Immediate*
with *eqid member-n*
show *?thesis*
by (*cases n*, *cases m*)
(*auto simp add*: *declared-in-def*
cdeclaredmethd-def cdeclaredfield-def
split: *memberdecl.splits*)
next
case *Inherited*
with *eqid member-n*
show *?thesis*
by (*cases n*) (*auto dest*: *declared-not-undeclared*)
qed
next
case (*Inherited n C S*)
assume *undecl*: $G\vdash$ *memberid n undeclared-in C*
assume *super*: $G\vdash C\prec_C 1S$
assume *hyp*: $[\![G \vdash m$ *member-of* S; *memberid* n = *memberid* $m]\!] \Longrightarrow n = m$
assume *eqid*: *memberid* n = *memberid* m
assume $G \vdash m$ *member-of* C
then show $n=m$

```
    proof (cases)
      case Immediate
      then have G⊢ mbr m declared-in C by simp
      with eqid undecl
      show ?thesis
        by (cases m) (auto dest: declared-not-undeclared)
    next
      case Inherited
      with super have G ⊢ m member-of S
        by (auto dest!: subcls1D)
      with eqid hyp
      show ?thesis
        by blast
    qed
  qed
qed

lemma member-of-is-classD: G⊢m member-of C ⟹ is-class G C
proof (induct set: members)
  case (Immediate m C)
  assume G⊢ mbr m declared-in C
  then show is-class G C
    by (cases mbr m)
       (auto simp add: declared-in-def cdeclaredmethd-def cdeclaredfield-def)
next
  case (Inherited m C S)
  assume G⊢C≺C1S and is-class G S
  then show is-class G C
    by − (rule subcls-is-class2,auto)
qed

lemma member-of-declC:
 G⊢m member-of C
 ⟹ G⊢mbr m declared-in (declclass m)
by (induct set: members) auto

lemma member-of-member-of-declC:
 G⊢m member-of C
 ⟹ G⊢m member-of (declclass m)
by (auto dest: member-of-declC intro: members.Immediate)

lemma member-of-class-relation:
  G⊢m member-of C ⟹ G⊢C ≼C declclass m
proof (induct set: members)
  case (Immediate m C)
  then show G⊢C ≼C declclass m by simp
next
  case (Inherited m C S)
  then show G⊢C ≼C declclass m
    by (auto dest: r-into-rtrancl intro: rtrancl-trans)
qed

lemma member-in-class-relation:
  G⊢m member-in C ⟹ G⊢C ≼C declclass m
```

by (*auto dest*: *member-inD member-of-class-relation*
intro: *rtrancl-trans*)

lemma *stat-override-declclasses-relation*:
$[\![G\vdash(declclass\ new) \prec_C 1\ superNew;\ G \vdash Method\ old\ member\text{-}of\ superNew\ ]\!]$
$\Longrightarrow G\vdash(declclass\ new) \prec_C (declclass\ old)$
apply (*rule trancl-rtrancl-trancl*)
apply (*erule r-into-trancl*)
apply (*cases old*)
apply (*auto dest*: *member-of-class-relation*)
done

lemma *stat-overrides-commonD*:
$[\![G\vdash new\ overrides_S\ old]\!] \Longrightarrow$
$declclass\ new \neq Object \wedge \neg\ is\text{-}static\ new \wedge msig\ new = msig\ old \wedge$
$G\vdash(declclass\ new) \prec_C (declclass\ old) \wedge$
$G\vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G\vdash Method\ old\ declared\text{-}in\ (declclass\ old)$
apply (*induct set*: *stat-overridesR*)
apply (*frule* (*1*) *stat-override-declclasses-relation*)
apply (*auto intro*: *trancl-trans*)
done

lemma *member-of-Package*:
assumes $G\vdash m\ member\text{-}of\ C$
and $accmodi\ m = Package$
shows $pid\ (declclass\ m) = pid\ C$
using *assms*
proof *induct*
case *Immediate*
then show *?case* **by** *simp*
next
case *Inherited*
then show *?case* **by** (*auto simp add*: *inheritable-in-def*)
qed

lemma *member-in-declC*: $G\vdash m\ member\text{-}in\ C \Longrightarrow G\vdash m\ member\text{-}in\ (declclass\ m)$
proof −
assume *member-in-C*: $G\vdash m\ member\text{-}in\ C$
from *member-in-C*
obtain *provC* **where**
subclseq-C-provC: $G\vdash C \preceq_C provC$ **and**
member-of-provC: $G \vdash m\ member\text{-}of\ provC$
by (*auto simp add*: *member-in-def*)
from *member-of-provC*
have $G \vdash m\ member\text{-}of\ declclass\ m$
by (*rule member-of-member-of-declC*)
moreover
from *member-in-C*
have $G\vdash C \preceq_C declclass\ m$
by (*rule member-in-class-relation*)
ultimately
show *?thesis*
by (*auto simp add*: *member-in-def*)
qed

lemma *dyn-accessible-from-commonD*: $G \vdash m$ *in* C *dyn-accessible-from* S
$\Longrightarrow G \vdash m$ *member-in* C
by (*auto elim*: *dyn-accessible-fromR.induct*)

lemma *no-Private-stat-override*:
$[\![G \vdash new\ overrides_S\ old]\!] \Longrightarrow accmodi\ old \neq Private$
by (*induct set*: *stat-overridesR*) (*auto simp add*: *inheritable-in-def*)

lemma *no-Private-override*: $[\![G \vdash new\ overrides\ old]\!] \Longrightarrow accmodi\ old \neq Private$
by (*induct set*: *overridesR*) (*auto simp add*: *inheritable-in-def*)

lemma *permits-acc-inheritance*:
$[\![G \vdash m$ *in statC permits-acc-from accC*; $G \vdash dynC \preceq_C statC$
$]\!] \Longrightarrow G \vdash m$ *in dynC permits-acc-from accC*
by (*cases accmodi m*)
(*auto simp add*: *permits-acc-def*
intro: *subclseq-trans*)

lemma *permits-acc-static-declC*:
$[\![G \vdash m$ *in C permits-acc-from accC*; $G \vdash m$ *member-in C*; *is-static m*
$]\!] \Longrightarrow G \vdash m$ *in* (*declclass m*) *permits-acc-from accC*
by (*cases accmodi m*) (*auto simp add*: *permits-acc-def*)

lemma *dyn-accessible-from-static-declC*:
assumes *acc-C*: $G \vdash m$ *in C dyn-accessible-from accC* **and**
static: *is-static m*
shows $G \vdash m$ *in* (*declclass m*) *dyn-accessible-from accC*
proof −
from *acc-C static*
show $G \vdash m$ *in* (*declclass m*) *dyn-accessible-from accC*
proof (*induct*)
case (*Immediate m C*)
then show *?case*
by (*auto intro*!: *dyn-accessible-fromR.Immediate*
dest: *member-in-declC permits-acc-static-declC*)
next
case (*Overriding m C declCNew new old sup*)
then have ¬ *is-static m*
by (*auto dest*: *overrides-commonD*)
moreover
assume *is-static m*
ultimately show *?case*
by *contradiction*
qed
qed

lemma *field-accessible-fromD*:
$[\![G \vdash membr$ *of C accessible-from accC*;*is-field membr*$]\!]$
$\Longrightarrow G \vdash membr$ *member-of C* $\wedge$
$G \vdash$(*Class C*) *accessible-in* (*pid accC*) $\wedge$
$G \vdash membr$ *in C permits-acc-from accC*

by (*cases set*: *accessible-fromR*)
(*auto simp add*: *is-field-def split*: *memberdecl.splits*)

lemma *field-accessible-from-permits-acc-inheritance*:
$[\![G\vdash membr\ of\ statC\ accessible\text{-}from\ accC;\ is\text{-}field\ membr;\ G \vdash dynC \preceq_C statC]\!]$
$\Longrightarrow G\vdash membr\ in\ dynC\ permits\text{-}acc\text{-}from\ accC$
by (*auto dest*: *field-accessible-fromD intro*: *permits-acc-inheritance*)

lemma *accessible-fieldD*:
$[\![G\vdash membr\ of\ C\ accessible\text{-}from\ accC;\ is\text{-}field\ membr]\!]$
$\Longrightarrow G\vdash membr\ member\text{-}of\ C \wedge$
$G\vdash(Class\ C)\ accessible\text{-}in\ (pid\ accC) \wedge$
$G\vdash membr\ in\ C\ permits\text{-}acc\text{-}from\ accC$
by (*induct rule*: *accessible-fromR.induct*) (*auto dest*: *is-fieldD*)

lemma *member-of-Private*:
$[\![G\vdash m\ member\text{-}of\ C;\ accmodi\ m = Private]\!] \Longrightarrow declclass\ m = C$
by (*induct set*: *members*) (*auto simp add*: *inheritable-in-def*)

lemma *member-of-subclseq-declC*:
$G\vdash m\ member\text{-}of\ C \Longrightarrow G\vdash C \preceq_C declclass\ m$
by (*induct set*: *members*) (*auto dest*: *r-into-rtrancl intro*: *rtrancl-trans*)

lemma *member-of-inheritance*:
assumes *m*: $G\vdash m\ member\text{-}of\ D$ **and**
subclseq-D-C: $G\vdash D \preceq_C C$ **and**
subclseq-C-m: $G\vdash C \preceq_C declclass\ m$ **and**
ws: *ws-prog G*
shows $G\vdash m\ member\text{-}of\ C$
proof −
from *m subclseq-D-C subclseq-C-m*
show *?thesis*
proof (*induct*)
case (*Immediate m D*)
assume $declclass\ m = D$ **and**
$G\vdash D\preceq_C C$ **and** $G\vdash C\preceq_C declclass\ m$
with *ws* **have** $D=C$
by (*auto intro*: *subclseq-acyclic*)
with *Immediate*
show $G\vdash m\ member\text{-}of\ C$
by (*auto intro*: *members.Immediate*)
next
case (*Inherited m D S*)
assume *member-of-D-props*:
$G \vdash m\ inheritable\text{-}in\ pid\ D$
$G\vdash memberid\ m\ undeclared\text{-}in\ D$
$G \vdash Class\ S\ accessible\text{-}in\ pid\ D$
$G \vdash m\ member\text{-}of\ S$
assume *super*: $G\vdash D\prec_C 1 S$

assume *hyp*: $[\![G\vdash S\preceq_C C;\ G\vdash C\preceq_C \textit{declclass } m]\!] \Longrightarrow G \vdash m \textit{ member-of } C$
assume *subclseq-C-m*: $G\vdash C\preceq_C \textit{declclass } m$
assume $G\vdash D\preceq_C C$
then show $G\vdash m \textit{ member-of } C$
proof (*cases rule*: *subclseq-cases*)
case *Eq*
assume $D=C$
with *super member-of-D-props*
show *?thesis*
by (*auto intro*: *members.Inherited*)
next
case *Subcls*
assume $G\vdash D\prec_C C$
with *super*
have $G\vdash S\preceq_C C$
by (*auto dest*: *subcls1D subcls-superD*)
with *subclseq-C-m hyp* **show** *?thesis*
by *blast*
qed
qed
qed

lemma *member-of-subcls*:
assumes *old*: $G\vdash \textit{old member-of } C$ **and**
new: $G\vdash \textit{new member-of } D$ **and**
eqid: *memberid new* = *memberid old* **and**
subclseq-D-C: $G\vdash D \preceq_C C$ **and**
subcls-new-old: $G\vdash \textit{declclass new} \prec_C \textit{declclass old}$ **and**
ws: *ws-prog G*
shows $G\vdash D \prec_C C$
proof −
from *old*
have *subclseq-C-old*: $G\vdash C \preceq_C \textit{declclass old}$
by (*auto dest*: *member-of-subclseq-declC*)
from *new*
have *subclseq-D-new*: $G\vdash D \preceq_C \textit{declclass new}$
by (*auto dest*: *member-of-subclseq-declC*)
from *subcls-new-old ws*
have *neq-new-old*: $\textit{new}\neq\textit{old}$
by (*cases new*,*cases old*) (*auto dest*: *subcls-irrefl*)
from *subclseq-D-new subclseq-D-C*
have $G\vdash(\textit{declclass new}) \preceq_C C \vee G\vdash C \preceq_C (\textit{declclass new})$
by (*rule subcls-compareable*)
then have $G\vdash(\textit{declclass new}) \preceq_C C$
proof
assume $G\vdash \textit{declclass new}\preceq_C C$ **then show** *?thesis* **.**
next
assume $G\vdash C \preceq_C (\textit{declclass new})$
with *new subclseq-D-C ws*
have $G\vdash \textit{new member-of } C$
by (*blast intro*: *member-of-inheritance*)
with *eqid old*
have *new*=*old*
by (*blast intro*: *unique-member-of*)
with *neq-new-old*
show *?thesis*
by *contradiction*
qed

then show *?thesis*
proof (*cases rule*: *subclseq-cases*)
case *Eq*
assume *declclass new* = *C*
with *new* **have** $G\vdash$*new member-of C*
by (*auto dest*: *member-of-member-of-declC*)
with *eqid old*
have *new=old*
by (*blast intro*: *unique-member-of*)
with *neq-new-old*
show *?thesis*
by *contradiction*
next
case *Subcls*
assume $G\vdash$*declclass new*$\prec_C$ *C*
with *subclseq-D-new*
show $G\vdash D\prec_C C$
by (*rule rtrancl-trancl-trancl*)
qed
qed

corollary *member-of-overrides-subcls*:
$[\![G\vdash$*Methd sig old member-of C*; $G\vdash$*Methd sig new member-of D*;$G\vdash D \preceq_C C$;
G,*sig*$\vdash$*new overrides old*; *ws-prog G*$]\!]$
$\Longrightarrow G\vdash D \prec_C C$
by (*drule overrides-commonD*) (*auto intro*: *member-of-subcls*)

corollary *member-of-stat-overrides-subcls*:
$[\![G\vdash$*Methd sig old member-of C*; $G\vdash$*Methd sig new member-of D*;$G\vdash D \preceq_C C$;
G,*sig*$\vdash$*new overrides*$_S$ *old*; *ws-prog G*$]\!]$
$\Longrightarrow G\vdash D \prec_C C$
by (*drule stat-overrides-commonD*) (*auto intro*: *member-of-subcls*)

lemma *inherited-field-access*:
assumes *stat-acc*: $G\vdash$*membr of statC accessible-from accC* **and**
is-field: *is-field membr* **and**
subclseq: $G \vdash$ *dynC* $\preceq_C$ *statC*
shows $G\vdash$*membr in dynC dyn-accessible-from accC*
proof −
from *stat-acc is-field subclseq*
show *?thesis*
by (*auto dest*: *accessible-fieldD*
intro: *dyn-accessible-fromR.Immediate*
member-inI
permits-acc-inheritance)
qed

lemma *accessible-inheritance*:
assumes *stat-acc*: $G\vdash$*m of statC accessible-from accC* **and**
subclseq: $G\vdash$*dynC* $\preceq_C$ *statC* **and**
member-dynC: $G\vdash$*m member-of dynC* **and**
dynC-acc: $G\vdash$(*Class dynC*) *accessible-in* (*pid accC*)
shows $G\vdash$*m of dynC accessible-from accC*
proof −
from *stat-acc*

have *member-statC*: *G*⊢*m member-of statC*
  by (*auto dest*: *accessible-from-commonD*)
from *stat-acc*
show *?thesis*
proof (*cases*)
  case *Immediate*
  with *member-dynC member-statC subclseq dynC-acc*
  show *?thesis*
    by (*auto intro*: *accessible-fromR.Immediate permits-acc-inheritance*)
next
  case *Overriding*
  with *member-dynC subclseq dynC-acc*
  show *?thesis*
    by (*auto intro*: *accessible-fromR.Overriding rtrancl-trancl-trancl*)
qed
qed

fields and methods

type-synonym
  fspec = *vname* × *qtname*

translations
  (*type*) *fspec* <= (*type*) *vname* × *qtname*

definition
  imethds :: *prog* ⇒ *qtname* ⇒ (*sig*,*qtname* × *mhead*) *tables* **where**
  imethds G I =
    iface-rec G I (λ*I i ts*. (*Un-tables ts*) ⊕⊕
        (*set-option* ∘ *table-of* (*map* (λ(*s*,*m*). (*s*,*I*,*m*)) (*imethods i*))))

methods of an interface, with overriding and inheritance, cf. 9.2

definition
  accimethds :: *prog* ⇒ *pname* ⇒ *qtname* ⇒ (*sig*,*qtname* × *mhead*) *tables* **where**
  accimethds G pack I =
    (*if G*⊢*Iface I accessible-in pack*
     then imethds G I
     else (λ *k*. {}))

only returns imethds if the interface is accessible

definition
  methd :: *prog* ⇒ *qtname* ⇒ (*sig*,*qtname* × *methd*) *table* **where**
  methd G C =
    class-rec G C Map.empty
        (λ*C c subcls-mthds*.
          filter-tab (λ*sig m*. *G*⊢*C inherits method sig m*)
              subcls-mthds
          ++
          table-of (*map* (λ(*s*,*m*). (*s*,*C*,*m*)) (*methods c*)))

methd G C: methods of a class C (statically visible from C), with inheritance and hiding cf. 8.4.6; Overriding is captured by *dynmethd*. Every new method with the same signature coalesces the method of a superclass.

definition
  accmethd :: *prog* ⇒ *qtname* ⇒ *qtname* ⇒ (*sig*,*qtname* × *methd*) *table* **where**
  accmethd G S C =
    filter-tab (λ*sig m*. *G*⊢*method sig m of C accessible-from S*) (*methd G C*)

accmethd G S C: only those methods of *methd G C*, accessible from S

Note the class component in the accessibility filter. The class where method m is declared (*declC*) isn't necessarily accessible from the current scope S. The method can be made accessible through inheritance, too. So we must test accessibility of method m of class C (not *declclass* m)

definition
```
  dynmethd :: prog  ⇒ qtname ⇒ qtname ⇒ (sig,qtname × methd) table where
  dynmethd G statC dynC =
    (λsig.
       (if G⊢dynC ≼C statC
          then (case methd G statC sig of
                  None ⇒ None
                | Some statM
                    ⇒ (class-rec G dynC Map.empty
                         (λC c subcls-mthds.
                            subcls-mthds
                            ++
                            (filter-tab
                              (λ - dynM. G,sig⊢dynM overrides statM ∨ dynM=statM)
                              (methd G C) ))
                        ) sig
                )
          else None))
```

dynmethd G statC dynC: dynamic method lookup of a reference with dynamic class *dynC* and static class *statC*

Note some kind of duality between *methd* and *dynmethd* in the *class-rec* arguments. Whereas *methd* filters the subclass methods (to get only the inherited ones), *dynmethd* filters the new methods (to get only those methods which actually override the methods of the static class)

definition
```
  dynimethd :: prog ⇒ qtname ⇒ qtname ⇒ (sig,qtname × methd) table where
  dynimethd G I dynC =
    (λsig. if imethds G I sig ≠ {}
           then methd G dynC sig
           else dynmethd G Object dynC sig)
```

dynimethd G I dynC: dynamic method lookup of a reference with dynamic class dynC and static interface type I

When calling an interface method, we must distinguish if the method signature was defined in the interface or if it must be an Object method in the other case. If it was an interface method we search the class hierarchy starting at the dynamic class of the object up to Object to find the first matching method (*methd*). Since all interface methods have public access the method can't be coalesced due to some odd visibility effects like in case of dynmethd. The method will be inherited or overridden in all classes from the first class implementing the interface down to the actual dynamic class.

definition
```
  dynlookup :: prog  ⇒ ref-ty ⇒ qtname ⇒ (sig,qtname × methd) table where
  dynlookup G statT dynC =
    (case statT of
      NullT         ⇒ Map.empty
    | IfaceT I      ⇒ dynimethd G I       dynC
    | ClassT statC ⇒ dynmethd  G statC  dynC
    | ArrayT ty     ⇒ dynmethd  G Object dynC)
```

dynlookup G statT dynC: dynamic lookup of a method within the static reference type statT and the dynamic class dynC. In a wellformd context statT will not be NullT and in case statT is an array type, dynC=Object

definition

fields :: *prog* ⇒ *qtname* ⇒ ((*vname* × *qtname*) × *field*) *list* **where**
fields G C =
class-rec G C [] (λ*C c ts. map* (λ(*n*,*t*). ((*n*,*C*),*t*)) (*cfields c*) @ *ts*)

DeclConcepts.fields G C list of fields of a class, including all the fields of the superclasses (private, inherited and hidden ones) not only the accessible ones (an instance of a object allocates all these fields

definition
accfield :: *prog* ⇒ *qtname* ⇒ *qtname* ⇒ (*vname*, *qtname* × *field*) *table* **where**
accfield G S C =
(*let field-tab* = *table-of*((*map* (λ((*n*,*d*),*f*).(*n*,(*d*,*f*)))) (*fields G C*))
in filter-tab (λ*n* (*declC*,*f*). *G*⊢ (*declC*,*fdecl* (*n*,*f*)) *of C accessible-from S*)
field-tab)

accfield G C S: fields of a class *C* which are accessible from scope of class *S* with inheritance and hiding, cf. 8.3

note the class component in the accessibility filter (see also *methd*). The class declaring field *f* (*declC*) isn't necessarily accessible from scope *S*. The field can be made visible through inheritance, too. So we must test accessibility of field *f* of class *C* (not *declclass f*)

definition
is-methd :: *prog* ⇒ *qtname* ⇒ *sig* ⇒ *bool*
where *is-methd G* = (λ*C sig. is-class G C* ∧ *methd G C sig* ≠ *None*)

definition
efname :: ((*vname* × *qtname*) × *field*) ⇒ (*vname* × *qtname*)
where *efname* = *fst*

lemma *efname-simp*[*simp*]:*efname* (*n*,*f*) = *n*
by (*simp add*: *efname-def*)

4 imethds

lemma *imethds-rec*: ⟦*iface G I* = *Some i*; *ws-prog G*⟧ ⟹
imethds G I = *Un-tables* ((λ*J. imethds G J*)‘*set* (*isuperIfs i*)) ⊕⊕
(*set-option* ∘ *table-of* (*map* (λ(*s*,*mh*). (*s*,*I*,*mh*)) (*imethods i*)))
apply (*unfold imethds-def*)
apply (*rule iface-rec* [*THEN trans*])
apply *auto*
done

lemma *imethds-norec*:
⟦*iface G md* = *Some i*; *ws-prog G*; *table-of* (*imethods i*) *sig* = *Some mh*⟧ ⟹
(*md*, *mh*) ∈ *imethds G md sig*
apply (*subst imethds-rec*)
apply *assumption*+
apply (*rule iffD2*)
apply (*rule overrides-t-Some-iff*)
apply (*rule disjI1*)
apply (*auto elim*: *table-of-map-SomeI*)
done

lemma *imethds-declI*: ⟦*m* ∈ *imethds G I sig*; *ws-prog G*; *is-iface G I*⟧ ⟹

($\exists$ *i*. *iface G* (*decliface m*) = *Some i* $\wedge$
table-of (*imethods i*) *sig* = *Some* (*mthd m*)) $\wedge$
(*I*,*decliface m*) $\in$ (*subint1 G*)* $\wedge$ *m* $\in$ *imethds G* (*decliface m*) *sig*
apply (*erule rev-mp*)
apply (*rule ws-subint1-induct*, *assumption*, *assumption*)
apply (*subst imethds-rec*, *erule conjunct1*, *assumption*)
apply (*force elim*: *imethds-norec intro*: *rtrancl-into-rtrancl2*)
done

lemma *imethds-cases*:
assumes *im*: *im* $\in$ *imethds G I sig*
and *ifI*: *iface G I* = *Some i*
and *ws*: *ws-prog G*
obtains (*NewMethod*) *table-of* (*map* (λ(*s*, *mh*). (*s*, *I*, *mh*)) (*imethods i*)) *sig* = *Some im*
| (*InheritedMethod*) *J* **where** *J* $\in$ *set* (*isuperIfs i*) **and** *im* $\in$ *imethds G J sig*
using *assms* **by** (*auto simp add*: *imethds-rec*)

5 accimethd

lemma *accimethds-simp* [*simp*]:
G$\vdash$*Iface I accessible-in pack* $\Longrightarrow$ *accimethds G pack I* = *imethds G I*
by (*simp add*: *accimethds-def*)

lemma *accimethdsD*:
im $\in$ *accimethds G pack I sig*
$\Longrightarrow$ *im* $\in$ *imethds G I sig* $\wedge$ *G*$\vdash$*Iface I accessible-in pack*
by (*auto simp add*: *accimethds-def*)

lemma *accimethdsI*:
$[\![$*im* $\in$ *imethds G I sig*;*G*$\vdash$*Iface I accessible-in pack*$]\!]$
$\Longrightarrow$ *im* $\in$ *accimethds G pack I sig*
by *simp*

6 methd

lemma *methd-rec*: $[\![$*class G C* = *Some c*; *ws-prog G*$]\!]$ $\Longrightarrow$
methd G C
= (*if C* = *Object*
then Map.empty
else filter-tab (λ*sig m*. *G*$\vdash$*C inherits method sig m*)
(*methd G* (*super c*)))
++ *table-of* (*map* (λ(*s*,*m*). (*s*,*C*,*m*)) (*methods c*))
apply (*unfold methd-def*)
apply (*erule class-rec* [*THEN trans*], *assumption*)
apply (*simp*)
done

lemma *methd-norec*:
$[\![$*class G declC* = *Some c*; *ws-prog G*;*table-of* (*methods c*) *sig* = *Some m*$]\!]$
$\Longrightarrow$ *methd G declC sig* = *Some* (*declC*, *m*)
apply (*simp only*: *methd-rec*)
apply (*rule disjI1* [*THEN map-add-Some-iff* [*THEN iffD2*]])
apply (*auto elim*: *table-of-map-SomeI*)
done

lemma *methd-declC*:
$[\![$*methd G C sig = Some m*; *ws-prog G*;*is-class G C*$]\!] \Longrightarrow$
($\exists$ *d. class G* (*declclass m*)=*Some d* $\wedge$ *table-of* (*methods d*) *sig*=*Some* (*mthd m*)) $\wedge$
G$\vdash$*C* $\preceq_C$ (*declclass m*) $\wedge$ *methd G* (*declclass m*) *sig* = *Some m*
apply (*erule rev-mp*)
apply (*rule ws-subcls1-induct*, *assumption*, *assumption*)
apply (*subst methd-rec*, *assumption*)
apply (*case-tac Ca*=*Object*)
apply (*force elim*: *methd-norec*)

apply *simp*
apply (*case-tac table-of* (*map* (λ(*s*, *m*). (*s*, *Ca*, *m*)) (*methods c*)) *sig*)
apply (*force intro*: *rtrancl-into-rtrancl2*)

apply (*auto intro*: *methd-norec*)
done

lemma *methd-inheritedD*:
$[\![$*class G C = Some c*; *ws-prog G*;*methd G C sig = Some m*$]\!]$
$\Longrightarrow$ (*declclass m* $\neq$ *C* $\longrightarrow$ *G* $\vdash$*C inherits method sig m*)
by (*auto simp add*: *methd-rec*)

lemma *methd-diff-cls*:
$[\![$*ws-prog G*; *is-class G C*; *is-class G D*;
methd G C sig = m; *methd G D sig = n*; *m*$\neq$*n*
$]\!] \Longrightarrow$ *C*$\neq$*D*
by (*auto simp add*: *methd-rec*)

lemma *method-declared-inI*:
$[\![$*table-of* (*methods c*) *sig = Some m*; *class G C = Some c*$]\!]$
$\Longrightarrow$ *G*$\vdash$*mdecl* (*sig*,*m*) *declared-in C*
by (*auto simp add*: *cdeclaredmethd-def declared-in-def*)

lemma *methd-declared-in-declclass*:
$[\![$*methd G C sig = Some m*; *ws-prog G*;*is-class G C*$]\!]$
$\Longrightarrow$ *G*$\vdash$*Methd sig m declared-in* (*declclass m*)
by (*auto dest*: *methd-declC method-declared-inI*)

lemma *member-methd*:
assumes *member-of*: *G*$\vdash$*Methd sig m member-of C* **and**
ws: *ws-prog G*
shows *methd G C sig = Some m*
proof −
from *member-of*
have *iscls-C*: *is-class G C*
by (*rule member-of-is-classD*)
from *iscls-C ws member-of*
show *?thesis* (**is** *?Methd C*)
proof (*induct rule*: *ws-class-induct'*)
case (*Object co*)
assume *G* $\vdash$*Methd sig m member-of Object*

```
    then have G⊢Methd sig m declared-in Object ∧ declclass m = Object
      by (cases set: members) (cases m, auto dest: subcls1D)
    with ws Object
    show ?Methd Object
      by (cases m)
         (auto simp add: declared-in-def cdeclaredmethd-def methd-rec
                intro: table-of-mapconst-SomeI)
  next
    case (Subcls C c)
    assume clsC: class G C = Some c and
      neq-C-Obj: C ≠ Object and
           hyp: G ⊢Methd sig m member-of super c ⟹ ?Methd (super c) and
      member-of: G ⊢Methd sig m member-of C
    from member-of
    show ?Methd C
    proof (cases)
      case Immediate
      with clsC
      have table-of (map (λ(s, m). (s, C, m)) (methods c)) sig = Some m
        by (cases m)
           (auto simp add: declared-in-def cdeclaredmethd-def
                  intro: table-of-mapconst-SomeI)
      with clsC neq-C-Obj ws
      show ?thesis
        by (simp add: methd-rec)
    next
      case (Inherited S)
      with clsC
      have  undecl: G⊢mid sig undeclared-in C and
            super: G ⊢Methd sig m member-of (super c)
        by (auto dest: subcls1D)
      from clsC undecl
      have table-of (map (λ(s, m). (s, C, m)) (methods c)) sig = None
        by (auto simp add: undeclared-in-def cdeclaredmethd-def
                  intro: table-of-mapconst-NoneI)
      moreover
      from Inherited have G ⊢ C inherits (method sig m)
        by (auto simp add: inherits-def)
      moreover
      note clsC neq-C-Obj ws super hyp
      ultimately
      show ?thesis
        by (auto simp add: methd-rec intro: filter-tab-SomeI)
    qed
  qed
qed
```

```
lemma finite-methd:ws-prog G ⟹ finite {methd G C sig |sig C. is-class G C}
apply (rule finite-is-class [THEN finite-SetCompr2])
apply (intro strip)
apply (erule-tac ws-subcls1-induct, assumption)
apply (subst methd-rec)
apply (assumption)
apply (auto intro!: finite-range-map-of finite-range-filter-tab finite-range-map-of-map-add)
done
```

lemma *finite-dom-methd*:
⟦*ws-prog G*; *is-class G C*⟧ ⟹ *finite* (*dom* (*methd G C*))
apply (*erule-tac ws-subcls1-induct*)
apply *assumption*
apply (*subst methd-rec*)
apply (*assumption*)
apply (*auto intro*!: *finite-dom-map-of finite-dom-filter-tab*)
done

7 accmethd

lemma *accmethd-SomeD*:
accmethd G S C sig = *Some m*
⟹ *methd G C sig* = *Some m* ∧ *G*⊢*method sig m of C accessible-from S*
by (*auto simp add*: *accmethd-def*)

lemma *accmethd-SomeI*:
⟦*methd G C sig* = *Some m*; *G*⊢*method sig m of C accessible-from S*⟧
⟹ *accmethd G S C sig* = *Some m*
by (*auto simp add*: *accmethd-def intro*: *filter-tab-SomeI*)

lemma *accmethd-declC*:
⟦*accmethd G S C sig* = *Some m*; *ws-prog G*; *is-class G C*⟧ ⟹
(∃ *d*. *class G* (*declclass m*)=*Some d* ∧
table-of (*methods d*) *sig*=*Some* (*mthd m*)) ∧
G⊢*C* $\preceq_C$ (*declclass m*) ∧ *methd G* (*declclass m*) *sig* = *Some m* ∧
G⊢*method sig m of C accessible-from S*
by (*auto dest*: *accmethd-SomeD methd-declC accmethd-SomeI*)

lemma *finite-dom-accmethd*:
⟦*ws-prog G*; *is-class G C*⟧ ⟹ *finite* (*dom* (*accmethd G S C*))
by (*auto simp add*: *accmethd-def intro*: *finite-dom-filter-tab finite-dom-methd*)

8 dynmethd

lemma *dynmethd-rec*:
⟦*class G dynC* = *Some c*; *ws-prog G*⟧ ⟹
dynmethd G statC dynC sig
= (*if G*⊢*dynC* $\preceq_C$ *statC*
then (*case methd G statC sig of*
None ⇒ *None*
| *Some statM*
⇒ (*case methd G dynC sig of*
None ⇒ *dynmethd G statC* (*super c*) *sig*
| *Some dynM* ⇒
(*if G*,*sig*⊢ *dynM overrides statM* ∨ *dynM* = *statM*
then Some dynM
else (*dynmethd G statC* (*super c*) *sig*)
)))
else None)
(**is** - ⟹ - ⟹ *?Dynmethd-def dynC sig* = *?Dynmethd-rec dynC c sig*)
proof −
assume *clsDynC*: *class G dynC* = *Some c* **and**
ws: *ws-prog G*
then show *?Dynmethd-def dynC sig* = *?Dynmethd-rec dynC c sig*

```
proof (induct rule: ws-class-induct'')
  case (Object co)
  show ?Dynmethd-def Object sig = ?Dynmethd-rec Object co sig
  proof (cases G⊢Object ≼C statC)
    case False
    then show ?thesis by (simp add: dynmethd-def)
  next
    case True
    then have eq-statC-Obj: statC = Object ..
    show ?thesis
    proof (cases methd G statC sig)
      case None then show ?thesis by (simp add: dynmethd-def)
    next
      case Some
      with True Object ws eq-statC-Obj
      show ?thesis
        by (auto simp add: dynmethd-def class-rec
                 intro: filter-tab-SomeI)
    qed
  qed
next
  case (Subcls dynC c sc)
  show ?Dynmethd-def dynC sig = ?Dynmethd-rec dynC c sig
  proof (cases G⊢dynC ≼C statC)
    case False
    then show ?thesis by (simp add: dynmethd-def)
  next
    case True
    note subclseq-dynC-statC = True
    show ?thesis
    proof (cases methd G statC sig)
      case None then show ?thesis by (simp add: dynmethd-def)
    next
      case (Some statM)
      note statM = Some
      let ?filter =
            λC. filter-tab
              (λ- dynM. G,sig ⊢ dynM overrides statM ∨ dynM = statM)
              (methd G C)
      let ?class-rec =
            λC. class-rec G C Map.empty
                        (λC c subcls-mthds. subcls-mthds ++ (?filter C))
      from statM Subcls ws subclseq-dynC-statC
      have dynmethd-dynC-def:
           ?Dynmethd-def dynC sig =
             ((?class-rec (super c))
              ++
             (?filter dynC)) sig
       by (simp (no-asm-simp) only: dynmethd-def class-rec)
          auto
       show ?thesis
       proof (cases dynC = statC)
         case True
         with subclseq-dynC-statC statM dynmethd-dynC-def
         have ?Dynmethd-def dynC sig = Some statM
           by (auto intro: map-add-find-right filter-tab-SomeI)
         with subclseq-dynC-statC True Some
         show ?thesis
           by auto
```

```
        next
          case False
          with subclseq-dynC-statC Subcls
          have subclseq-super-statC: G⊢(super c) ≼C statC
            by (blast dest: subclseq-superD)
          show ?thesis
          proof (cases methd G dynC sig)
            case None
            then have ?filter dynC sig = None
              by (rule filter-tab-None)
            then have ?Dynmethd-def dynC sig=?class-rec (super c) sig
              by (simp add: dynmethd-dynC-def)
            with  subclseq-super-statC statM None
            have ?Dynmethd-def dynC sig = ?Dynmethd-def (super c) sig
              by (auto simp add: empty-def dynmethd-def)
            with None subclseq-dynC-statC statM
            show ?thesis
              by simp
          next
            case (Some dynM)
            note dynM = Some
            let ?Termination = G ⊢ qmdecl sig dynM overrides qmdecl sig statM ∨
                                dynM = statM
            show ?thesis
            proof (cases ?filter dynC sig)
              case None
              with dynM
              have no-termination: ¬ ?Termination
                by (simp add: filter-tab-def)
              from None
              have ?Dynmethd-def dynC sig=?class-rec (super c) sig
                by (simp add: dynmethd-dynC-def)
              with subclseq-super-statC statM dynM no-termination
              show ?thesis
                by (auto simp add: empty-def dynmethd-def)
            next
              case Some
              with dynM
              have termination: ?Termination
                by (auto)
              with Some dynM
              have ?Dynmethd-def dynC sig=Some dynM
               by (auto simp add: dynmethd-dynC-def)
              with subclseq-super-statC statM dynM termination
              show ?thesis
                by (auto simp add: dynmethd-def)
            qed
          qed
        qed
      qed
    qed
  qed
qed
```

lemma *dynmethd-C-C*:⟦*is-class G C*; *ws-prog G*⟧
⟹ *dynmethd G C C sig = methd G C sig*
apply (*auto simp add*: *dynmethd-rec*)
done

lemma *dynmethdSomeD*:
$\llbracket$*dynmethd G statC dynC sig* = *Some dynM*; *is-class G dynC*; *ws-prog G*$\rrbracket$
$\Longrightarrow$ *G*$\vdash$*dynC* $\preceq_C$ *statC* $\wedge$ ($\exists$ *statM*. *methd G statC sig* = *Some statM*)
by (*auto simp add*: *dynmethd-rec*)

lemma *dynmethd-Some-cases*:
assumes *dynM*: *dynmethd G statC dynC sig* = *Some dynM*
and *is-cls-dynC*: *is-class G dynC*
and *ws*: *ws-prog G*
obtains (*Static*) *methd G statC sig* = *Some dynM*
| (*Overrides*) *statM*
where *methd G statC sig* = *Some statM*
and *dynM* $\neq$ *statM*
and *G*,*sig*$\vdash$*dynM overrides statM*
proof −
from *is-cls-dynC* **obtain** *dc* **where** *clsDynC*: *class G dynC* = *Some dc* **by** *blast*
from *clsDynC ws dynM Static Overrides*
show *?thesis*
proof (*induct rule*: *ws-class-induct*)
case (*Object co*)
with *ws* **have** *statC* = *Object*
by (*auto simp add*: *dynmethd-rec*)
with *ws Object* **show** *?thesis* **by** (*auto simp add*: *dynmethd-C-C*)
next
case (*Subcls C c*)
with *ws* **show** *?thesis*
by (*auto simp add*: *dynmethd-rec*)
qed
qed

lemma *no-override-in-Object*:
assumes *dynM*: *dynmethd G statC dynC sig* = *Some dynM* **and**
is-cls-dynC: *is-class G dynC* **and**
ws: *ws-prog G* **and**
statM: *methd G statC sig* = *Some statM* **and**
neq-dynM-statM: *dynM*$\neq$*statM*
shows *dynC* $\neq$ *Object*
proof −
from *is-cls-dynC* **obtain** *dc* **where** *clsDynC*: *class G dynC* = *Some dc* **by** *blast*
from *clsDynC ws dynM statM neq-dynM-statM*
show *?thesis* (**is** *?P dynC*)
proof (*induct rule*: *ws-class-induct*)
case (*Object co*)
with *ws* **have** *statC* = *Object*
by (*auto simp add*: *dynmethd-rec*)
with *ws Object* **show** *?P Object* **by** (*auto simp add*: *dynmethd-C-C*)
next
case (*Subcls dynC c*)
with *ws* **show** *?P dynC*
by (*auto simp add*: *dynmethd-rec*)
qed
qed

lemma *dynmethd-Some-rec-cases*:
  assumes *dynM*: *dynmethd G statC dynC sig = Some dynM*
    and *clsDynC*: *class G dynC = Some c*
    and *ws*: *ws-prog G*
  obtains (*Static*) *methd G statC sig = Some dynM*
    | (*Override*) *statM* **where** *methd G statC sig = Some statM*
        and *methd G dynC sig = Some dynM* **and** *statM* $\neq$ *dynM*
        and *G*,*sig*$\vdash$ *dynM overrides statM*
    | (*Recursion*) *dynC* $\neq$ *Object* **and** *dynmethd G statC* (*super c*) *sig = Some dynM*
proof −
  from *clsDynC* **have** ∗: *is-class G dynC* **by** *simp*
  from *ws clsDynC dynM Static Override Recursion*
  show *?thesis*
    by (*auto simp add*: *dynmethd-rec dest*: *no-override-in-Object* [*OF dynM* ∗ *ws*])
qed

lemma *dynmethd-declC*:
[[*dynmethd G statC dynC sig = Some m*;
  is-class G statC;*ws-prog G*
 ]] $\Longrightarrow$
  ($\exists$ *d*. *class G* (*declclass m*)=*Some d* $\wedge$ *table-of* (*methods d*) *sig*=*Some* (*mthd m*)) $\wedge$
  G$\vdash$*dynC* $\preceq_C$ (*declclass m*) $\wedge$ *methd G* (*declclass m*) *sig = Some m*
proof −
  assume *is-cls-statC*: *is-class G statC*
  assume *ws*: *ws-prog G*
  assume *m*: *dynmethd G statC dynC sig = Some m*
  from *m*
  have *G*$\vdash$*dynC* $\preceq_C$ *statC* **by** (*auto simp add*: *dynmethd-def*)
  from *this is-cls-statC*
  have *is-cls-dynC*: *is-class G dynC* **by** (*rule subcls-is-class2*)
  from *is-cls-dynC ws m*
  show *?thesis* (**is** *?P dynC*)
  proof (*induct rule*: *ws-class-induct*$'$)
    case (*Object co*)
    with *ws* **have** *statC*=*Object* **by** (*auto simp add*: *dynmethd-rec*)
    with *ws Object*
    show *?P Object*
      by (*auto simp add*: *dynmethd-C-C dest*: *methd-declC*)
  next
    case (*Subcls dynC c*)
    assume *hyp*: *dynmethd G statC* (*super c*) *sig = Some m* $\Longrightarrow$ *?P* (*super c*) **and**
          clsDynC: *class G dynC = Some c* **and**
              m$'$: *dynmethd G statC dynC sig = Some m* **and**
    neq-dynC-Obj: *dynC* $\neq$ *Object*
    from *ws this* **obtain** *statM* **where**
      subclseq-dynC-statC: *G*$\vdash$*dynC* $\preceq_C$ *statC* **and**
                    statM: *methd G statC sig = Some statM*
      by (*blast dest*: *dynmethdSomeD*)
    from *clsDynC neq-dynC-Obj*
    have *subclseq-dynC-super*: *G*$\vdash$*dynC* $\preceq_C$ (*super c*)
      by (*auto intro*: *subcls1I*)
    from *m*$'$ *clsDynC ws*
    show *?P dynC*
    proof (*cases rule*: *dynmethd-Some-rec-cases*)
      case *Static*
      with *is-cls-statC ws subclseq-dynC-statC*
      show *?thesis*
        by (*auto intro*: *rtrancl-trans dest*: *methd-declC*)

next
case *Override*
with *clsDynC ws*
show *?thesis*
by (*auto dest*: *methd-declC*)
next
case *Recursion*
with *hyp subclseq-dynC-super*
show *?thesis*
by (*auto intro*: *rtrancl-trans*)
qed
qed
qed

lemma *methd-Some-dynmethd-Some*:
assumes *statM*: *methd G statC sig* = *Some statM* **and**
subclseq: $G\vdash dynC \preceq_C statC$ **and**
is-cls-statC: *is-class G statC* **and**
ws: *ws-prog G*
shows ∃ *dynM*. *dynmethd G statC dynC sig* = *Some dynM*
(**is** *?P dynC*)
proof −
from *subclseq is-cls-statC*
have *is-cls-dynC*: *is-class G dynC* **by** (*rule subcls-is-class2*)
then obtain *dc* **where**
clsDynC: *class G dynC* = *Some dc* **by** *blast*
from *clsDynC ws subclseq*
show *?thesis*
proof (*induct rule*: *ws-class-induct*)
case (*Object co*)
with *ws* **have** *statC* = *Object*
by (*auto*)
with *ws Object statM*
show *?P Object*
by (*auto simp add*: *dynmethd-C-C*)
next
case (*Subcls dynC dc*)
assume *clsDynC′*: *class G dynC* = *Some dc*
assume *neq-dynC-Obj*: *dynC* ≠ *Object*
assume *hyp*: $G\vdash super\ dc \preceq_C statC \Longrightarrow ?P\ (super\ dc)$
assume *subclseq′*: $G\vdash dynC \preceq_C statC$
then
show *?P dynC*
proof (*cases rule*: *subclseq-cases*)
case *Eq*
with *ws statM clsDynC′*
show *?thesis*
by (*auto simp add*: *dynmethd-rec*)
next
case *Subcls*
assume $G\vdash dynC \prec_C statC$
from *this clsDynC′*
have $G\vdash super\ dc \preceq_C statC$ **by** (*rule subcls-superD*)
with *hyp ws clsDynC′ subclseq′ statM*
show *?thesis*
by (*auto simp add*: *dynmethd-rec*)
qed
qed

qed

lemma *dynmethd-cases*:
assumes *statM*: *methd G statC sig* = *Some statM*
and *subclseq*: $G\vdash dynC \preceq_C statC$
and *is-cls-statC*: *is-class G statC*
and *ws*: *ws-prog G*
obtains (*Static*) *dynmethd G statC dynC sig* = *Some statM*
| (*Overrides*) *dynM* **where** *dynmethd G statC dynC sig* = *Some dynM*
and *dynM* ≠ *statM* **and** *G*,*sig*⊢*dynM overrides statM*
proof −
note *hyp-static* = *Static* **and** *hyp-override* = *Overrides*
from *subclseq is-cls-statC*
have *is-cls-dynC*: *is-class G dynC* **by** (*rule subcls-is-class2*)
then obtain *dc* **where**
clsDynC: *class G dynC* = *Some dc* **by** *blast*
from *statM subclseq is-cls-statC ws*
obtain *dynM* **where** *dynM*: *dynmethd G statC dynC sig* = *Some dynM*
by (*blast dest*: *methd-Some-dynmethd-Some*)
from *dynM is-cls-dynC ws*
show *?thesis*
proof (*cases rule*: *dynmethd-Some-cases*)
case *Static*
with *hyp-static dynM statM* **show** *?thesis* **by** *simp*
next
case *Overrides*
with *hyp-override dynM statM* **show** *?thesis* **by** *simp*
qed
qed

lemma *ws-dynmethd*:
assumes *statM*: *methd G statC sig* = *Some statM* **and**
subclseq: $G\vdash dynC \preceq_C statC$ **and**
is-cls-statC: *is-class G statC* **and**
ws: *ws-prog G*
shows
∃ *dynM*. *dynmethd G statC dynC sig* = *Some dynM* ∧
is-static dynM = *is-static statM* ∧ *G*⊢*resTy dynM*≼*resTy statM*
proof −
from *statM subclseq is-cls-statC ws*
show *?thesis*
proof (*cases rule*: *dynmethd-cases*)
case *Static*
with *statM*
show *?thesis*
by *simp*
next
case *Overrides*
with *ws*
show *?thesis*
by (*auto dest*: *ws-overrides-commonD*)
qed
qed

9 dynlookup

lemma *dynlookup-cases*:

assumes *dynlookup G statT dynC sig* = *x*
obtains (*NullT*) *statT* = *NullT* **and** *Map.empty sig* = *x*
| (*IfaceT*) *I* **where** *statT* = *IfaceT I* **and** *dynimethd G I dynC sig* = *x*
| (*ClassT*) *statC* **where** *statT* = *ClassT statC* **and** *dynmethd G statC dynC sig* = *x*
| (*ArrayT*) *ty* **where** *statT* = *ArrayT ty* **and** *dynmethd G Object dynC sig* = *x*
using *assms* **by** (*cases statT*) (*auto simp add*: *dynlookup-def*)

10 fields

lemma *fields-rec*: ⟦*class G C* = *Some c*; *ws-prog G*⟧ ⟹
fields G C = *map* (λ(*fn*,*ft*). ((*fn*,*C*),*ft*)) (*cfields c*) @
(*if C* = *Object then* [] *else fields G* (*super c*))
apply (*simp only*: *fields-def*)
apply (*erule class-rec* [*THEN trans*])
apply *assumption*
apply *clarsimp*
done

lemma *fields-norec*:
⟦*class G fd* = *Some c*; *ws-prog G*; *table-of* (*cfields c*) *fn* = *Some f*⟧
⟹ *table-of* (*fields G fd*) (*fn*,*fd*) = *Some f*
apply (*subst fields-rec*)
apply *assumption*+
apply (*subst map-of-append*)
apply (*rule disjI1* [*THEN map-add-Some-iff* [*THEN iffD2*]])
apply (*auto elim*: *table-of-map2-SomeI*)
done

lemma *table-of-fieldsD*:
table-of (*map* (λ(*fn*,*ft*). ((*fn*,*C*),*ft*)) (*cfields c*)) *efn* = *Some f*
⟹ (*declclassf efn*) = *C* ∧ *table-of* (*cfields c*) (*fname efn*) = *Some f*
apply (*case-tac efn*)
by *auto*

lemma *fields-declC*:
⟦*table-of* (*fields G C*) *efn* = *Some f*; *ws-prog G*; *is-class G C*⟧ ⟹
(∃ *d*. *class G* (*declclassf efn*) = *Some d* ∧
table-of (*cfields d*) (*fname efn*)=*Some f*) ∧
G⊢*C* $\preceq_C$ (*declclassf efn*) ∧ *table-of* (*fields G* (*declclassf efn*)) *efn* = *Some f*
apply (*erule rev-mp*)
apply (*rule ws-subcls1-induct*, *assumption*, *assumption*)
apply (*subst fields-rec*, *assumption*)
apply *clarify*
apply (*simp only*: *map-of-append*)
apply (*case-tac table-of* (*map* (*case-prod* (λ*fn*. *Pair* (*fn*, *Ca*))) (*cfields c*)) *efn*)
apply (*force intro*:*rtrancl-into-rtrancl2 simp add*: *map-add-def*)

apply (*frule-tac fd*=*Ca* **in** *fields-norec*)
apply *assumption*
apply *blast*
apply (*frule table-of-fieldsD*)
apply (*frule-tac n*=*table-of* (*map* (*case-prod* (λ*fn*. *Pair* (*fn*, *Ca*))) (*cfields c*))
and *m*=*table-of* (*if Ca* = *Object then* [] *else fields G* (*super c*))
in *map-add-find-right*)

apply (*case-tac efn*)
apply (*simp*)
done

lemma *fields-emptyI*: $\bigwedge y$. $[\![$*ws-prog G*; *class G C* = *Some c*;*cfields c* = []; $C \neq$ *Object* $\longrightarrow$ *class G* (*super c*) = *Some y* $\wedge$ *fields G* (*super c*) = []$]\!] \Longrightarrow$ *fields G C* = []
apply (*subst fields-rec*)
apply *assumption*
apply *auto*
done

lemma *fields-mono-lemma*:
$[\![x \in$ *set* (*fields G C*); $G \vdash D \preceq_C C$; *ws-prog G*$]\!]$
$\Longrightarrow x \in$ *set* (*fields G D*)
apply (*erule rev-mp*)
apply (*erule converse-rtrancl-induct*)
apply *fast*
apply (*drule subcls1D*)
apply *clarsimp*
apply (*subst fields-rec*)
apply *auto*
done

lemma *ws-unique-fields-lemma*:
$[\![$(*efn*,*fd*) $\in$ *set* (*fields G* (*super c*)); *fc* $\in$ *set* (*cfields c*); *ws-prog G*;
fname efn = *fname fc*; *declclassf efn* = *C*;
class G C = *Some c*; $C \neq$ *Object*; *class G* (*super c*) = *Some d*$]\!] \Longrightarrow R$
apply (*frule-tac ws-prog-cdeclD* [*THEN conjunct2*], *assumption*, *assumption*)
apply (*drule-tac weak-map-of-SomeI*)
apply (*frule-tac subcls1I* [*THEN subcls1-irrefl*], *assumption*, *assumption*)
apply (*auto dest*: *fields-declC* [*THEN conjunct2* [*THEN conjunct1*[*THEN rtranclD*]]])
done

lemma *ws-unique-fields*: $[\![$*is-class G C*; *ws-prog G*;
$\bigwedge C\ c$. $[\![$*class G C* = *Some c*$]\!] \Longrightarrow$ *unique* (*cfields c*) $]\!] \Longrightarrow$
unique (*fields G C*)
apply (*rule ws-subcls1-induct*, *assumption*, *assumption*)
apply (*subst fields-rec*, *assumption*)
apply (*auto intro*!: *unique-map-inj inj-onI*
elim!: *unique-append ws-unique-fields-lemma fields-norec*)
done

11 accfield

lemma *accfield-fields*:
accfield G S C fn = *Some f*
$\Longrightarrow$ *table-of* (*fields G C*) (*fn*, *declclass f*) = *Some* (*fld f*)
apply (*simp only*: *accfield-def Let-def*)
apply (*rule table-of-remap-SomeD*)
apply *auto*
done

lemma *accfield-declC-is-class*:
$[\![$*is-class G C*; *accfield G S C en* = *Some* (*fd*, *f*); *ws-prog G*$]\!]$ $\Longrightarrow$
is-class G fd
apply (*drule accfield-fields*)
apply (*drule fields-declC* [*THEN conjunct1*], *assumption*)
apply *auto*
done

lemma *accfield-accessibleD*:
accfield G S C fn = *Some f* $\Longrightarrow$ *G*$\vdash$*Field fn f of C accessible-from S*
by (*auto simp add*: *accfield-def Let-def*)

12 is methd

lemma *is-methdI*:
$[\![$*class G C* = *Some y*; *methd G C sig* = *Some b*$]\!]$ $\Longrightarrow$ *is-methd G C sig*
apply (*unfold is-methd-def*)
apply *auto*
done

lemma *is-methdD*:
is-methd G C sig $\Longrightarrow$ *class G C* $\neq$ *None* $\wedge$ *methd G C sig* $\neq$ *None*
apply (*unfold is-methd-def*)
apply *auto*
done

lemma *finite-is-methd*:
ws-prog G $\Longrightarrow$ *finite* (*Collect* (*case-prod* (*is-methd G*)))
apply (*unfold is-methd-def*)
apply (*subst Collect-case-prod-Sigma*)
apply (*rule finite-is-class* [*THEN finite-SigmaI*])
apply (*simp only*: *mem-Collect-eq*)
apply (*fold dom-def*)
apply (*erule finite-dom-methd*)
apply *assumption*
done

calculation of the superclasses of a class

definition
superclasses :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *qtname set* **where**
superclasses G C = *class-rec G C* {}
 (λ *C c supercls*. (*if C*=*Object*
 then {}
 else insert (*super c*) *supercls*))

lemma *superclasses-rec*: $[\![$*class G C* = *Some c*; *ws-prog G*$]\!]$ $\Longrightarrow$
superclasses G C
= (*if* (*C*=*Object*)
 then {}
 else insert (*super c*) (*superclasses G* (*super c*)))
apply (*unfold superclasses-def*)
apply (*erule class-rec* [*THEN trans*], *assumption*)

```
apply (simp)
done


lemma superclasses-mono:
  assumes clsrel: G⊢C≺C D
  and ws: ws-prog G
  and cls-C: class G C = Some c
  and wf: ⋀C c. ⟦class G C = Some c; C ≠ Object⟧
    ⟹ ∃ sc. class G (super c) = Some sc
  and x: x∈superclasses G D
  shows x∈superclasses G C using clsrel cls-C x
proof (induct arbitrary: c rule: converse-trancl-induct)
  case (base C)
  with wf ws show ?case
    by (auto    intro: no-subcls1-Object
             simp add: superclasses-rec subcls1-def)
next
  case (step C S)
  moreover note wf ws
  moreover from calculation
  have x∈superclasses G S
    by (force intro: no-subcls1-Object simp add: subcls1-def)
  moreover from calculation
  have super c = S
    by (auto intro: no-subcls1-Object simp add: subcls1-def)
  ultimately show ?case
    by (auto intro: no-subcls1-Object simp add: superclasses-rec)
qed


lemma subclsEval:
  assumes clsrel: G⊢C≺C D
  and ws: ws-prog G
  and cls-C: class G C = Some c
  and wf: ⋀C c. ⟦class G C = Some c; C ≠ Object⟧
    ⟹ ∃ sc. class G (super c) = Some sc
  shows D∈superclasses G C using clsrel cls-C
proof (induct arbitrary: c rule: converse-trancl-induct)
  case (base C)
  with ws wf show ?case
    by (auto intro: no-subcls1-Object simp add: superclasses-rec subcls1-def)
next
  case (step C S)
  with ws wf show ?case
    by − (rule superclasses-mono,
          auto dest: no-subcls1-Object simp add: subcls1-def )
qed

end
```

Chapter 11

WellType

1 Well-typedness of Java programs

theory *WellType*
imports *DeclConcepts*
begin

improvements over Java Specification 1.0:

- methods of Object can be called upon references of interface or array type

simplifications:

- the type rules include all static checks on statements and expressions, e.g. definedness of names (of parameters, locals, fields, methods)

design issues:

- unified type judgment for statements, variables, expressions, expression lists
- statements are typed like expressions with dummy type Void
- the typing rules take an extra argument that is capable of determining the dynamic type of objects. Therefore, they can be used for both checking static types and determining runtime types in transition semantics.

type-synonym *lenv*
  = (*lname*, *ty*) *table* — local variables, including This and Result

record *env* =
  prg:: *prog* — program
  cls:: *qtname* — current package and class name
  lcl:: *lenv* — local environment

translations
(*type*) *lenv* <= (*type*) (*lname*, *ty*) *table*
(*type*) *lenv* <= (*type*) *lname* ⇒ *ty option*
(*type*) *env* <= (*type*) (|*prg*::*prog*,*cls*::*qtname*,*lcl*::*lenv*|)
(*type*) *env* <= (*type*) (|*prg*::*prog*,*cls*::*qtname*,*lcl*::*lenv*,...::'*a*|)

abbreviation
pkg :: *env* ⇒ *pname* — select the current package from an environment
where *pkg e* == *pid* (*cls e*)

Static overloading: maximally specific methods

type-synonym
  emhead = *ref-ty* × *mhead*

— Some mnemotic selectors for emhead
definition
  declrefT :: *emhead* ⇒ *ref-ty*
  where *declrefT* = *fst*

definition
  mhd :: *emhead* ⇒ *mhead*
  where *mhd* ≡ *snd*

lemma *declrefT-simp*[*simp*]:*declrefT* (*r*,*m*) = *r*
by (*simp add*: *declrefT-def*)

lemma *mhd-simp*[*simp*]:*mhd* (*r*,*m*) = *m*
by (*simp add*: *mhd-def*)

lemma *static-mhd-simp*[*simp*]: *static* (*mhd m*) = *is-static m*
by (*cases m*) (*simp add*: *member-is-static-simp mhd-def*)

lemma *mhd-resTy-simp* [*simp*]: *resTy* (*mhd m*) = *resTy m*
by (*cases m*) *simp*

lemma *mhd-is-static-simp* [*simp*]: *is-static* (*mhd m*) = *is-static m*
by (*cases m*) *simp*

lemma *mhd-accmodi-simp* [*simp*]: *accmodi* (*mhd m*) = *accmodi m*
by (*cases m*) *simp*

definition
  cmheads :: *prog* ⇒ *qtname* ⇒ *qtname* ⇒ *sig* ⇒ *emhead set*
  where *cmheads G S C* = (λ*sig*. (λ(*Cls*,*mthd*). (*ClassT Cls*,(*mhead mthd*))) ‘ *set-option* (*accmethd G S C sig*))

definition
  Objectmheads :: *prog* ⇒ *qtname* ⇒ *sig* ⇒ *emhead set* **where**
  Objectmheads G S =
    (λ*sig*. (λ(*Cls*,*mthd*). (*ClassT Cls*,(*mhead mthd*)))
      ‘ *set-option* (*filter-tab* (λ*sig m*. *accmodi m* ≠ *Private*) (*accmethd G S Object*) *sig*))

definition
  accObjectmheads :: *prog* ⇒ *qtname* ⇒ *ref-ty* ⇒ *sig* ⇒ *emhead set*
where
  accObjectmheads G S T =
    (*if G*⊢*RefT T accessible-in* (*pid S*)
     then Objectmheads G S
     else (λ*sig*. {}))

primrec *mheads* :: *prog* ⇒ *qtname* ⇒ *ref-ty* ⇒ *sig* ⇒ *emhead set*
where

$\quad mheads\ G\ S\ \ NullT \quad\ \ = (\lambda sig.\ \{\})$
$|\ mheads\ G\ S\ (IfaceT\ I) = (\lambda sig.\ (\lambda(I,h).(IfaceT\ I,h))$
$\qquad\qquad\qquad\qquad\quad ` \ accimethds\ G\ (pid\ S)\ I\ sig \cup$
$\qquad\qquad\qquad\qquad\quad accObjectmheads\ G\ S\ (IfaceT\ I)\ sig)$
$|\ mheads\ G\ S\ (ClassT\ C) = cmheads\ G\ S\ C$
$|\ mheads\ G\ S\ (ArrayT\ T) = accObjectmheads\ G\ S\ (ArrayT\ T)$

definition
— applicable methods, cf. 15.11.2.1
$appl\text{-}methds :: prog \Rightarrow qtname \Rightarrow\ ref\text{-}ty \Rightarrow sig \Rightarrow (emhead \times ty\ list)\ set$ **where**
$appl\text{-}methds\ G\ S\ rt = (\lambda\ sig.$
$\quad \{(mh,pTs')\ |mh\ pTs'.\ mh \in mheads\ G\ S\ rt\ (|name=name\ sig,parTs=pTs'|) \wedge$
$\qquad\qquad\qquad G\vdash(parTs\ sig)[\preceq]pTs'\})$

definition
— more specific methods, cf. 15.11.2.2
$more\text{-}spec :: prog \Rightarrow emhead \times ty\ list \Rightarrow emhead \times ty\ list \Rightarrow bool$ **where**
$more\text{-}spec\ G = (\lambda(mh,pTs).\ \lambda(mh',pTs').\ G\vdash pTs[\preceq]pTs')$

definition
— maximally specific methods, cf. 15.11.2.2
$max\text{-}spec :: prog \Rightarrow qtname \Rightarrow ref\text{-}ty \Rightarrow sig \Rightarrow (emhead \times ty\ list)\ set$ **where**
$max\text{-}spec\ G\ S\ rt\ sig = \{m.\ m \in appl\text{-}methds\ G\ S\ rt\ sig \wedge$
$\qquad\qquad\qquad (\forall\ m'\in appl\text{-}methds\ G\ S\ rt\ sig.\ more\text{-}spec\ G\ m'\ m \longrightarrow m'=m)\}$

lemma *max-spec2appl-meths*:
$x \in max\text{-}spec\ G\ S\ T\ sig \Longrightarrow x \in appl\text{-}methds\ G\ S\ T\ sig$
by (*auto simp*: *max-spec-def*)

lemma *appl-methsD*: $(mh,pTs')\in appl\text{-}methds\ G\ S\ T\ (|name=mn,parTs=pTs|) \Longrightarrow$
$\quad mh \in mheads\ G\ S\ T\ (|name=mn,parTs=pTs'|) \wedge G\vdash pTs[\preceq]pTs'$
by (*auto simp*: *appl-methds-def*)

lemma *max-spec2mheads*:
$max\text{-}spec\ G\ S\ rt\ (|name=mn,parTs=pTs|) = insert\ (mh,\ pTs')\ A$
$\Longrightarrow mh \in mheads\ G\ S\ rt\ (|name=mn,parTs=pTs'|) \wedge G\vdash pTs[\preceq]pTs'$
apply (*auto dest*: *equalityD2 subsetD max-spec2appl-meths appl-methsD*)
done

definition
$empty\text{-}dt :: dyn\text{-}ty$
where $empty\text{-}dt = (\lambda a.\ None)$

definition
$invmode :: ('a::type)member\text{-}scheme \Rightarrow expr \Rightarrow inv\text{-}mode$ **where**
$invmode\ m\ e = (if\ is\text{-}static\ m$
$\qquad\qquad then\ Static$
$\qquad\qquad else\ if\ e=Super\ then\ SuperM\ else\ IntVir)$

lemma *invmode-nonstatic* [*simp*]:
$invmode\ (|access=a,static=False,\ldots=x|)\ (Acc\ (LVar\ e)) = IntVir$
apply (*unfold invmode-def*)

apply (*simp* (*no-asm*) *add*: *member-is-static-simp*)
done

lemma *invmode-Static-eq* [*simp*]: (*invmode m e = Static*) = *is-static m*
apply (*unfold invmode-def*)
apply (*simp* (*no-asm*))
done

lemma *invmode-IntVir-eq*: (*invmode m e = IntVir*) = (¬(*is-static m*) ∧ *e*≠*Super*)
apply (*unfold invmode-def*)
apply (*simp* (*no-asm*))
done

lemma *Null-staticD*:
a'=*Null* ⟶ (*is-static m*) ⟹ *invmode m e = IntVir* ⟶ $a' \neq$ *Null*
apply (*clarsimp simp add*: *invmode-IntVir-eq*)
done

Typing for unary operations

primrec *unop-type* :: *unop* ⇒ *prim-ty*
where
unop-type UPlus = Integer
| *unop-type UMinus = Integer*
| *unop-type UBitNot = Integer*
| *unop-type UNot = Boolean*

primrec *wt-unop* :: *unop* ⇒ *ty* ⇒ *bool*
where
wt-unop UPlus t = (*t = PrimT Integer*)
| *wt-unop UMinus t* = (*t = PrimT Integer*)
| *wt-unop UBitNot t* = (*t = PrimT Integer*)
| *wt-unop UNot t* = (*t = PrimT Boolean*)

Typing for binary operations

primrec *binop-type* :: *binop* ⇒ *prim-ty*
where
binop-type Mul = Integer
| *binop-type Div = Integer*
| *binop-type Mod = Integer*
| *binop-type Plus = Integer*
| *binop-type Minus = Integer*
| *binop-type LShift = Integer*
| *binop-type RShift = Integer*
| *binop-type RShiftU = Integer*
| *binop-type Less = Boolean*
| *binop-type Le = Boolean*
| *binop-type Greater = Boolean*
| *binop-type Ge = Boolean*
| *binop-type Eq = Boolean*
| *binop-type Neq = Boolean*
| *binop-type BitAnd = Integer*
| *binop-type And = Boolean*

| *binop-type BitXor* = *Integer*
| *binop-type Xor* = *Boolean*
| *binop-type BitOr* = *Integer*
| *binop-type Or* = *Boolean*
| *binop-type CondAnd* = *Boolean*
| *binop-type CondOr* = *Boolean*

primrec *wt-binop* :: *prog* ⇒ *binop* ⇒ *ty* ⇒ *ty* ⇒ *bool*
where
  wt-binop G Mul t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Div t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Mod t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Plus t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Minus t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G LShift t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G RShift t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G RShiftU t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Less t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Le t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Greater t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Ge t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Eq t1 t2* = (*G*⊢*t1*≼*t2* ∨ *G*⊢*t2*≼*t1*)
| *wt-binop G Neq t1 t2* = (*G*⊢*t1*≼*t2* ∨ *G*⊢*t2*≼*t1*)
| *wt-binop G BitAnd t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G And t1 t2* = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))
| *wt-binop G BitXor t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Xor t1 t2* = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))
| *wt-binop G BitOr t1 t2* = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
| *wt-binop G Or t1 t2* = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))
| *wt-binop G CondAnd t1 t2* = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))
| *wt-binop G CondOr t1 t2* = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))

Typing for terms

type-synonym *tys* = *ty* + *ty list*
translations
  (*type*) *tys* <= (*type*) *ty* + *ty list*

inductive *wt* :: *env* ⇒ *dyn-ty* ⇒ [*term*,*tys*] ⇒ *bool* (-,-⊨-::- [*51*,*51*,*51*,*51*] *50*)
  and *wt-stmt* :: *env* ⇒ *dyn-ty* ⇒ *stmt* ⇒ *bool* (-,-⊨-::√ [*51*,*51*,*51*] *50*)
  and *ty-expr* :: *env* ⇒ *dyn-ty* ⇒ [*expr* ,*ty*] ⇒ *bool* (-,-⊨-::−- [*51*,*51*,*51*,*51*] *50*)
  and *ty-var* :: *env* ⇒ *dyn-ty* ⇒ [*var* ,*ty*] ⇒ *bool* (-,-⊨-::=- [*51*,*51*,*51*,*51*] *50*)
  and *ty-exprs* :: *env* ⇒ *dyn-ty* ⇒ [*expr list*, *ty list*] ⇒ *bool*
    (-,-⊨-::≐- [*51*,*51*,*51*,*51*] *50*)
where

  E,*dt*⊨*s*::√ ≡ *E*,*dt*⊨*In1r s*::*Inl* (*PrimT Void*)
| *E*,*dt*⊨*e*::−*T* ≡ *E*,*dt*⊨*In1l e*::*Inl T*
| *E*,*dt*⊨*e*::=*T* ≡ *E*,*dt*⊨*In2 e*::*Inl T*
| *E*,*dt*⊨*e*::≐*T* ≡ *E*,*dt*⊨*In3 e*::*Inr T*

— well-typed statements

| *Skip*: *E*,*dt*⊨*Skip*::√

| *Expr*: ⟦*E*,*dt*⊨*e*::−*T*⟧ ⟹
  E,*dt*⊨*Expr e*::√
  — cf. 14.6

| *Lab*: $E,dt \models c::\surd \Longrightarrow$

$$E,dt \models l \bullet\ c::\surd$$

| *Comp*: $[\![E,dt \models c1::\surd;$
$E,dt \models c2::\surd]\!] \Longrightarrow$

$$E,dt \models c1;;\ c2::\surd$$

— cf. 14.8
| *If*: $[\![E,dt \models e::-PrimT\ Boolean;$
$E,dt \models c1::\surd;$
$E,dt \models c2::\surd]\!] \Longrightarrow$

$$E,dt \models If(e)\ c1\ Else\ c2::\surd$$

— cf. 14.10
| *Loop*: $[\![E,dt \models e::-PrimT\ Boolean;$
$E,dt \models c::\surd]\!] \Longrightarrow$

$$E,dt \models l \bullet\ While(e)\ c::\surd$$

— cf. 14.13, 14.15, 14.16
| *Jmp*: $E,dt \models Jmp\ jump::\surd$

— cf. 14.16
| *Throw*: $[\![E,dt \models e::-Class\ tn;$
$prg\ E \vdash tn \preceq_C\ SXcpt\ Throwable]\!] \Longrightarrow$

$$E,dt \models Throw\ e::\surd$$

— cf. 14.18
| *Try*: $[\![E,dt \models c1::\surd;\ prg\ E \vdash tn \preceq_C\ SXcpt\ Throwable;$
$lcl\ E\ (VName\ vn) = None;\ E\ (\!|lcl := lcl\ E(VName\ vn \mapsto Class\ tn)|\!),dt \models c2::\surd]\!]$
$\Longrightarrow$

$$E,dt \models Try\ c1\ Catch(tn\ vn)\ c2::\surd$$

— cf. 14.18
| *Fin*: $[\![E,dt \models c1::\surd;\ E,dt \models c2::\surd]\!] \Longrightarrow$

$$E,dt \models c1\ Finally\ c2::\surd$$

| *Init*: $[\![is\text{-}class\ (prg\ E)\ C]\!] \Longrightarrow$

$$E,dt \models Init\ C::\surd$$

— *Init* is created on the fly during evaluation (see Eval.thy). The class isn't necessarily accessible from the points *Init* is called. Therefor we only demand *is-class* and not *is-acc-class* here.

— well-typed expressions

— cf. 15.8
| *NewC*: $[\![is\text{-}acc\text{-}class\ (prg\ E)\ (pkg\ E)\ C]\!] \Longrightarrow$

$$E,dt \models NewC\ C::-Class\ C$$

— cf. 15.9
| *NewA*: $[\![is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ T;$
$E,dt \models i::-PrimT\ Integer]\!] \Longrightarrow$

$$E,dt \models New\ T[i]::-T.[]$$

— cf. 15.15
| *Cast*: $[\![E,dt \models e::-T;\ is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ T';$
$prg\ E \vdash T \preceq ?\ T']\!] \Longrightarrow$

$$E,dt \models Cast\ T'\ e::-T'$$

— cf. 15.19.2
| *Inst*: $[\![E,dt \models e::-RefT\ T;\ is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ (RefT\ T');$
$prg\ E \vdash RefT\ T \preceq ?\ RefT\ T']\!] \Longrightarrow$

$$E,dt \models e\ InstOf\ T'::-PrimT\ Boolean$$

— cf. 15.7.1

| *Lit*: $[\![typeof\ dt\ x = Some\ T]\!] \Longrightarrow$

$E,dt \models Lit\ x::-T$

| *UnOp*: $[\![E,dt \models e::-Te;\ wt\text{-}unop\ unop\ Te;\ T=PrimT\ (unop\text{-}type\ unop)]\!]$

$\Longrightarrow$

$E,dt \models UnOp\ unop\ e::-T$

| *BinOp*: $[\![E,dt \models e1::-T1;\ E,dt \models e2::-T2;\ wt\text{-}binop\ (prg\ E)\ binop\ T1\ T2;$

$T=PrimT\ (binop\text{-}type\ binop)]\!]$

$\Longrightarrow$

$E,dt \models BinOp\ binop\ e1\ e2::-T$

— cf. 15.10.2, 15.11.1

| *Super*: $[\![lcl\ E\ This = Some\ (Class\ C);\ C \neq Object;$

$class\ (prg\ E)\ C = Some\ c]\!] \Longrightarrow$

$E,dt \models Super::-Class\ (super\ c)$

— cf. 15.13.1, 15.10.1, 15.12

| *Acc*: $[\![E,dt \models va::=T]\!] \Longrightarrow$

$E,dt \models Acc\ va::-T$

— cf. 15.25, 15.25.1

| *Ass*: $[\![E,dt \models va::=T;\ va \neq LVar\ This;$

$E,dt \models v\ ::-T';$

$prg\ E \vdash T' \preceq T]\!] \Longrightarrow$

$E,dt \models va:=v::-T'$

— cf. 15.24

| *Cond*: $[\![E,dt \models e0::-PrimT\ Boolean;$

$E,dt \models e1::-T1;\ E,dt \models e2::-T2;$

$prg\ E \vdash T1 \preceq T2 \wedge T = T2 \ \vee\ prg\ E \vdash T2 \preceq T1 \wedge T = T1]\!] \Longrightarrow$

$E,dt \models e0\ ?\ e1 : e2::-T$

— cf. 15.11.1, 15.11.2, 15.11.3

| *Call*: $[\![E,dt \models e::-RefT\ statT;$

$E,dt \models ps::\dot{=}pTs;$

$max\text{-}spec\ (prg\ E)\ (cls\ E)\ statT\ (\!|name=mn,parTs=pTs|\!)$

$= \{((statDeclT,m),pTs')\}$

$]\!] \Longrightarrow$

$E,dt \models \{cls\ E,statT,invmode\ m\ e\}e\cdot mn(\{pTs'\}ps)::-(resTy\ m)$

| *Methd*: $[\![is\text{-}class\ (prg\ E)\ C;$

$methd\ (prg\ E)\ C\ sig = Some\ m;$

$E,dt \models Body\ (declclass\ m)\ (stmt\ (mbody\ (mthd\ m)))::-T]\!] \Longrightarrow$

$E,dt \models Methd\ C\ sig::-T$

— The class C is the dynamic class of the method call (cf. Eval.thy). It hasn't got to be directly accessible from the current package *pkg E*. Only the static class must be accessible (enshured indirectly by *Call*). Note that l is just a dummy value. It is only used in the smallstep semantics. To proof typesafety directly for the smallstep semantics we would have to assume conformance of l here!

| *Body*: $[\![is\text{-}class\ (prg\ E)\ D;$

$E,dt \models blk::\surd;$

$(lcl\ E)\ Result = Some\ T;$

$is\text{-}type\ (prg\ E)\ T]\!] \Longrightarrow$

$E,dt \models Body\ D\ blk::-T$

— The class D implementing the method must not directly be accessible from the current package *pkg E*, but can also be indirectly accessible due to inheritance (enshured in *Call*) The result type hasn't got to be accessible in Java! (If it is not accessible you can only assign it to Object). For dummy value l see rule *Methd*.

— well-typed variables

— cf. 15.13.1
| *LVar*: ⟦*lcl E vn* = *Some T*; *is-acc-type* (*prg E*) (*pkg E*) *T*⟧ ⟹
E,*dt*⊨*LVar vn*::=*T*
— cf. 15.10.1
| *FVar*: ⟦*E*,*dt*⊨*e*::−*Class C*;
accfield (*prg E*) (*cls E*) *C fn* = *Some* (*statDeclC*,*f*)⟧ ⟹
E,*dt*⊨{*cls E*,*statDeclC*,*is-static f*}*e*..*fn*::=(*type f*)
— cf. 15.12
| *AVar*: ⟦*E*,*dt*⊨*e*::−*T*.[];
E,*dt*⊨*i*::−*PrimT Integer*⟧ ⟹
E,*dt*⊨*e*.[*i*]::=*T*

— well-typed expression lists

— cf. 15.11.???
| *Nil*: *E*,*dt*⊨[]::≐[]

— cf. 15.11.???
| *Cons*: ⟦*E*,*dt*⊨*e* ::−*T*;
E,*dt*⊨*es*::≐*Ts*⟧ ⟹
E,*dt*⊨*e*#*es*::≐*T*#*Ts*

abbreviation
wt-syntax :: *env* ⇒ [*term*,*tys*] ⇒ *bool* (-⊢-::- [*51*,*51*,*51*] *50*)
where *E*⊢*t*::*T* == *E*,*empty-dt*⊨*t*:: *T*

abbreviation
wt-stmt-syntax :: *env* ⇒ *stmt* ⇒ *bool* (-⊢-::√ [*51*,*51*] *50*)
where *E*⊢*s*::√ == *E*⊢*In1r s* :: *Inl* (*PrimT Void*)

abbreviation
ty-expr-syntax :: *env* ⇒ [*expr*, *ty*] ⇒ *bool* (-⊢-::−- [*51*,*51*,*51*] *50*)
where *E*⊢*e*::−*T* == *E*⊢*In1l e* :: *Inl T*

abbreviation
ty-var-syntax :: *env* ⇒ [*var*, *ty*] ⇒ *bool* (-⊢-::=- [*51*,*51*,*51*] *50*)
where *E*⊢*e*::=*T* == *E*⊢*In2 e* :: *Inl T*

abbreviation
ty-exprs-syntax :: *env* ⇒ [*expr list*, *ty list*] ⇒ *bool* (-⊢-::≐- [*51*,*51*,*51*] *50*)
where *E*⊢*e*::≐*T* == *E*⊢*In3 e* :: *Inr T*

notation (*ASCII*)
wt-syntax (-|−-::- [*51*,*51*,*51*] *50*) **and**
wt-stmt-syntax (-|−-:<> [*51*,*51*] *50*) **and**
ty-expr-syntax (-|−-:−- [*51*,*51*,*51*] *50*) **and**
ty-var-syntax (-|−-:=- [*51*,*51*,*51*] *50*) **and**
ty-exprs-syntax (-|−-:#- [*51*,*51*,*51*] *50*)

declare *not-None-eq* [*simp del*]
declare *if-split* [*split del*] *if-split-asm* [*split del*]
declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt delloop split-all-tac*)›

inductive-cases *wt-elim-cases* [*cases set*]:
$E,dt \models In2\ \ (LVar\ vn)\qquad :: T$
$E,dt \models In2\ \ (\{accC,statDeclC,s\}e..fn) :: T$
$E,dt \models In2\ \ (e.[i])\qquad :: T$
$E,dt \models In1l\ (NewC\ C)\qquad :: T$
$E,dt \models In1l\ (New\ T'[i])\qquad :: T$
$E,dt \models In1l\ (Cast\ T'\ e)\qquad :: T$
$E,dt \models In1l\ (e\ InstOf\ T')\qquad :: T$
$E,dt \models In1l\ (Lit\ x)\qquad :: T$
$E,dt \models In1l\ (UnOp\ unop\ e)\qquad :: T$
$E,dt \models In1l\ (BinOp\ binop\ e1\ e2)\qquad :: T$
$E,dt \models In1l\ (Super)\qquad :: T$
$E,dt \models In1l\ (Acc\ va)\qquad :: T$
$E,dt \models In1l\ (Ass\ va\ v)\qquad :: T$
$E,dt \models In1l\ (e0\ ?\ e1\ :\ e2)\qquad :: T$
$E,dt \models In1l\ (\{accC,statT,mode\}e\cdot mn(\{pT'\}p)) :: T$
$E,dt \models In1l\ (Methd\ C\ sig)\qquad :: T$
$E,dt \models In1l\ (Body\ D\ blk)\qquad :: T$
$E,dt \models In3\ \ ([])\qquad :: Ts$
$E,dt \models In3\ \ (e\#es)\qquad :: Ts$
$E,dt \models In1r\ \ Skip\qquad :: x$
$E,dt \models In1r\ (Expr\ e)\qquad :: x$
$E,dt \models In1r\ (c1;;\ c2)\qquad :: x$
$E,dt \models In1r\ (l\bullet\ c)\qquad :: x$
$E,dt \models In1r\ (If(e)\ c1\ Else\ c2)\qquad :: x$
$E,dt \models In1r\ (l\bullet\ While(e)\ c)\qquad :: x$
$E,dt \models In1r\ (Jmp\ jump)\qquad :: x$
$E,dt \models In1r\ (Throw\ e)\qquad :: x$
$E,dt \models In1r\ (Try\ c1\ Catch(tn\ vn)\ c2) :: x$
$E,dt \models In1r\ (c1\ Finally\ c2)\qquad :: x$
$E,dt \models In1r\ (Init\ C)\qquad :: x$

declare *not-None-eq* [*simp*]
declare *if-split* [*split*] *if-split-asm* [*split*]
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt addloop* (*split-all-tac*, *split-all-tac*))›

lemma *is-acc-class-is-accessible*:
is-acc-class G P C $\Longrightarrow$ *G*⊢(*Class C*) *accessible-in P*
by (*auto simp add*: *is-acc-class-def*)

lemma *is-acc-iface-is-iface*: *is-acc-iface G P I* $\Longrightarrow$ *is-iface G I*
by (*auto simp add*: *is-acc-iface-def*)

lemma *is-acc-iface-Iface-is-accessible*:
is-acc-iface G P I $\Longrightarrow$ *G*⊢(*Iface I*) *accessible-in P*
by (*auto simp add*: *is-acc-iface-def*)

lemma *is-acc-type-is-type*: *is-acc-type G P T* $\Longrightarrow$ *is-type G T*
by (*auto simp add*: *is-acc-type-def*)

lemma *is-acc-iface-is-accessible*:
is-acc-type G P T $\Longrightarrow$ *G*⊢*T accessible-in P*
by (*auto simp add*: *is-acc-type-def*)

lemma *wt-Methd-is-methd*:
  $E\vdash In1l\ (Methd\ C\ sig)::T \Longrightarrow is\text{-}methd\ (prg\ E)\ C\ sig$
apply (*erule-tac wt-elim-cases*)
apply *clarsimp*
apply (*erule is-methdI*, *assumption*)
done

Special versions of some typing rules, better suited to pattern match the conclusion (no selectors in the conclusion)

lemma *wt-Call*:
$[\![E,dt\models e::-RefT\ statT;\ E,dt\models ps::\doteq pTs;$
  $max\text{-}spec\ (prg\ E)\ (cls\ E)\ statT\ (\![name=mn,parTs=pTs]\!)$
    $= \{((statDeclC,m),pTs')\};rT=(resTy\ m);accC=cls\ E;$
 $mode = invmode\ m\ e]\!] \Longrightarrow E,dt\models\{accC,statT,mode\}e\cdot mn(\{pTs'\}ps)::-rT$
by (*auto elim*: *wt.Call*)

lemma *invocationTypeExpr-noClassD*:
$[\![\ E\vdash e::-RefT\ statT]\!]$
 $\Longrightarrow (\forall\ statC.\ statT \neq ClassT\ statC) \longrightarrow invmode\ m\ e \neq SuperM$
proof −
  assume *wt*: $E\vdash e::-RefT\ statT$
  show *?thesis*
  proof (*cases e=Super*)
    case *True*
    with *wt* **obtain** *C* **where** $statT = ClassT\ C$ **by** (*blast elim*: *wt-elim-cases*)
    then show *?thesis* **by** *blast*
  next
    case *False* **then show** *?thesis*
      by (*auto simp add*: *invmode-def*)
  qed
qed

lemma *wt-Super*:
$[\![lcl\ E\ This = Some\ (Class\ C);\ C \neq Object;\ class\ (prg\ E)\ C = Some\ c;\ D=super\ c]\!]$
$\Longrightarrow E,dt\models Super::-Class\ D$
by (*auto elim*: *wt.Super*)

lemma *wt-FVar*:
$[\![E,dt\models e::-Class\ C;\ accfield\ (prg\ E)\ (cls\ E)\ C\ fn = Some\ (statDeclC,f);$
  $sf=is\text{-}static\ f;\ fT=(type\ f);\ accC=cls\ E]\!]$
$\Longrightarrow E,dt\models\{accC,statDeclC,sf\}e..fn::=fT$
by (*auto dest*: *wt.FVar*)

lemma *wt-init* [*iff*]: $E,dt\models Init\ C::\surd = is\text{-}class\ (prg\ E)\ C$
by (*auto elim*: *wt-elim-cases intro*: *wt.Init*)

declare *wt.Skip* [*iff*]

lemma *wt-StatRef*:
  $is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ (RefT\ rt) \Longrightarrow E\vdash StatRef\ rt::-RefT\ rt$

apply (*rule wt.Cast*)
apply (*rule wt.Lit*)
apply (*simp* (*no-asm*))
apply (*simp* (*no-asm-simp*))
apply (*rule cast.widen*)
apply (*simp* (*no-asm*))
done

lemma *wt-Inj-elim*:
$\bigwedge E.\ E,dt \models t::U \Longrightarrow case\ t\ of$
$\quad In1\ ec \Rightarrow (case\ ec\ of$
$\qquad Inl\ e \Rightarrow \exists\, T.\ U=Inl\ T$
$\qquad |\ Inr\ s \Rightarrow U=Inl\ (PrimT\ Void))$
$\quad |\ In2\ e \Rightarrow (\exists\, T.\ U=Inl\ T)$
$\quad |\ In3\ e \Rightarrow (\exists\, T.\ U=Inr\ T)$
apply (*erule wt.induct*)
apply *auto*
done

— In the special syntax to distinguish the typing judgements for expressions, statements, variables and expression lists the kind of term corresponds to the kind of type in the end e.g. An statement (injection *In3* into terms, always has type void (injection *Inl* into the generalised types. The following simplification procedures establish these kinds of correlation.

lemma *wt-expr-eq*: $E,dt \models In1l\ t::U = (\exists\, T.\ U=Inl\ T \wedge E,dt \models t::-T)$
by (*auto*, *frule wt-Inj-elim*, *auto*)

lemma *wt-var-eq*: $E,dt \models In2\ t::U = (\exists\, T.\ U=Inl\ T \wedge E,dt \models t::=T)$
by (*auto*, *frule wt-Inj-elim*, *auto*)

lemma *wt-exprs-eq*: $E,dt \models In3\ t::U = (\exists\, Ts.\ U=Inr\ Ts \wedge E,dt \models t::\doteq Ts)$
by (*auto*, *frule wt-Inj-elim*, *auto*)

lemma *wt-stmt-eq*: $E,dt \models In1r\ t::U = (U=Inl(PrimT\ Void) \wedge E,dt \models t::\surd)$
by (*auto*, *frule wt-Inj-elim*, *auto*, *frule wt-Inj-elim*, *auto*)

```
simproc-setup wt-expr (E,dt|=In1l t::U) = ‹
  fn - => fn - => fn ct =>
    (case Thm.term-of ct of
      (- $ - $ - $ - $ (Const - $ -)) => NONE
    | - => SOME (mk-meta-eq @{thm wt-expr-eq}))›

simproc-setup wt-var (E,dt|=In2 t::U) = ‹
  fn - => fn - => fn ct =>
    (case Thm.term-of ct of
      (- $ - $ - $ - $ (Const - $ -)) => NONE
    | - => SOME (mk-meta-eq @{thm wt-var-eq}))›

simproc-setup wt-exprs (E,dt|=In3 t::U) = ‹
  fn - => fn - => fn ct =>
    (case Thm.term-of ct of
      (- $ - $ - $ - $ (Const - $ -)) => NONE
    | - => SOME (mk-meta-eq @{thm wt-exprs-eq}))›
```

simproc-setup *wt-stmt* (*E*,*dt*⊨*In1r t*::*U*) = ‹
  fn - => fn - => fn ct =>
    (*case Thm.term-of ct of*
      (- $ - $ - $ - $ (*Const* - $ -)) => *NONE*
    | - => *SOME* (*mk-meta-eq* @{*thm wt-stmt-eq*}))›

lemma *wt-elim-BinOp*:
  ⟦*E*,*dt*⊨*In1l* (*BinOp binop e1 e2*)::*T*;
    ⋀*T1 T2 T3*.
      ⟦*E*,*dt*⊨*e1*::−*T1*; *E*,*dt*⊨*e2*::−*T2*; *wt-binop* (*prg E*) *binop T1 T2*;
         E,*dt*⊨(*if b then In1l e2 else In1r Skip*)::*T3*;
         T = *Inl* (*PrimT* (*binop-type binop*))⟧
      ⟹ *P*⟧
⟹ *P*
apply (*erule wt-elim-cases*)
apply (*cases b*)
apply *auto*
done

lemma *Inj-eq-lemma* [*simp*]:
  (∀ *T*. (∃ *T*′. *T* = *Inj T*′ ∧ *P T*′) ⟶ *Q T*) = (∀ *T*′. *P T*′ ⟶ *Q* (*Inj T*′))
by *auto*

lemma *single-valued-tys-lemma* [*rule-format* (*no-asm*)]:
  ∀ *S T*. *G*⊢*S*≼*T* ⟶ *G*⊢*T*≼*S* ⟶ *S* = *T* ⟹ *E*,*dt*⊨*t*::*T* ⟹
     G = *prg E* ⟶ (∀ *T*′. *E*,*dt*⊨*t*::*T*′ ⟶ *T* = *T*′)
apply (*cases E*, *erule wt.induct*)
apply (*safe del*: *disjE*)
apply (*simp-all* (*no-asm-use*) *split del*: *if-split-asm*)
apply (*safe del*: *disjE*)

apply (*tactic* ‹*ALLGOALS* (*fn i* =>
  if i = *11 then EVERY*′
   [*Rule-Insts.thin-tac* ***context*** *E*,*dt*⊨*e0*::−*PrimT Boolean* [(***binding***‹*E*›, *NONE*, *NoSyn*)],
      Rule-Insts.thin-tac ***context*** *E*,*dt*⊨*e1*::−*T1* [(***binding***‹*E*›, *NONE*, *NoSyn*), (***binding***‹*T1*›, *NONE*, *NoSyn*)],
      Rule-Insts.thin-tac ***context*** *E*,*dt*⊨*e2*::−*T2* [(***binding***‹*E*›, *NONE*, *NoSyn*), (***binding***‹*T2*›, *NONE*, *NoSyn*)]] *i*
  else Rule-Insts.thin-tac ***context*** *All P* [(***binding***‹*P*›, *NONE*, *NoSyn*)] *i*)›)

apply (*tactic* ‹*ALLGOALS* (*eresolve-tac* ***context*** @{*thms wt-elim-cases*})›)
apply (*simp-all* (*no-asm-use*) *split del*: *if-split-asm*)
apply (*erule-tac* [*12*] *V* = *All P* **for** *P* **in** *thin-rl*)
apply (*blast del*: *equalityCE dest*: *sym* [*THEN trans*])+
done

lemma *single-valued-tys*:
ws-prog (*prg E*) ⟹ *single-valued* {(*t*,*T*). *E*,*dt*⊨*t*::*T*}
apply (*unfold single-valued-def*)
apply *clarsimp*
apply (*rule single-valued-tys-lemma*)
apply (*auto intro*!: *widen-antisym*)
done

lemma *typeof-empty-is-type*: $typeof\ (\lambda a.\ None)\ v = Some\ T \Longrightarrow is\text{-}type\ G\ T$
by $(induct\ v)\ auto$

lemma *typeof-is-type*: $(\forall\, a.\ v \neq Addr\ a) \Longrightarrow \exists\, T.\ typeof\ dt\ v = Some\ T \wedge is\text{-}type\ G\ T$
by $(induct\ v)\ auto$

end

Chapter 12

DefiniteAssignment

1 Definite Assignment

theory *DefiniteAssignment* **imports** *WellType* **begin**

Definite Assignment Analysis (cf. 16)

The definite assignment analysis approximates the sets of local variables that will be assigned at a certain point of evaluation, and ensures that we will only read variables which previously were assigned. It should conform to the following idea: If the evaluation of a term completes normally (no abruption (exception, break, continue, return) appeared) , the set of local variables calculated by the analysis is a subset of the variables that were actually assigned during evaluation.

To get more precise information about the sets of assigned variables the analysis includes the following optimisations:

- Inside of a while loop we also take care of the variables assigned before break statements, since the break causes the while loop to continue normally.
- For conditional statements we take care of constant conditions to statically determine the path of evaluation.
- Inside a distinct path of a conditional statements we know to which boolean value the condition has evaluated to, and so can retrieve more information about the variables assigned during evaluation of the boolean condition.

Since in our model of Java the return values of methods are stored in a local variable we also ensure that every path of (normal) evaluation will assign the result variable, or in the sense of real Java every path ends up in and return instruction.

Not covered yet:

- analysis of definite unassigned
- special treatment of final fields

Correct nesting of jump statements

For definite assignment it becomes crucial, that jumps (break, continue, return) are nested correctly i.e. a continue jump is nested in a matching while statement, a break jump is nested in a proper label statement, a class initialiser does not terminate abruptly with a return. With this we can for example ensure that evaluation of an expression will never end up with a jump, since no breaks, continues or returns are allowed in an expression.

primrec *jumpNestingOkS* :: *jump set* $\Rightarrow$ *stmt* $\Rightarrow$ *bool*

where
jumpNestingOkS jmps (*Skip*) = *True*
| *jumpNestingOkS jmps* (*Expr e*) = *True*
| *jumpNestingOkS jmps* (*j*• *s*) = *jumpNestingOkS* ({*j*} ∪ *jmps*) *s*
| *jumpNestingOkS jmps* (*c1*;;*c2*) = (*jumpNestingOkS jmps c1* ∧
jumpNestingOkS jmps c2)
| *jumpNestingOkS jmps* (*If*(*e*) *c1 Else c2*) = (*jumpNestingOkS jmps c1* ∧
jumpNestingOkS jmps c2)
| *jumpNestingOkS jmps* (*l*• *While*(*e*) *c*) = *jumpNestingOkS* ({*Cont l*} ∪ *jmps*) *c*
— The label of the while loop only handles continue jumps. Breaks are only handled by *Lab*
| *jumpNestingOkS jmps* (*Jmp j*) = (*j* ∈ *jmps*)
| *jumpNestingOkS jmps* (*Throw e*) = *True*
| *jumpNestingOkS jmps* (*Try c1 Catch*(*C vn*) *c2*) = (*jumpNestingOkS jmps c1* ∧
jumpNestingOkS jmps c2)
| *jumpNestingOkS jmps* (*c1 Finally c2*) = (*jumpNestingOkS jmps c1* ∧
jumpNestingOkS jmps c2)
| *jumpNestingOkS jmps* (*Init C*) = *True*
— wellformedness of the program must enshure that for all initializers jumpNestingOkS holds
— Dummy analysis for intermediate smallstep term *FinA*
| *jumpNestingOkS jmps* (*FinA a c*) = *False*

definition *jumpNestingOk* :: *jump set* ⇒ *term* ⇒ *bool* **where**
jumpNestingOk jmps t = (*case t of*
In1 se ⇒ (*case se of*
Inl e ⇒ *True*
| *Inr s* ⇒ *jumpNestingOkS jmps s*)
| *In2 v* ⇒ *True*
| *In3 es* ⇒ *True*)

lemma *jumpNestingOk-expr-simp* [*simp*]: *jumpNestingOk jmps* (*In1l e*) = *True*
by (*simp add*: *jumpNestingOk-def*)

lemma *jumpNestingOk-expr-simp1* [*simp*]: *jumpNestingOk jmps* ⟨*e*::*expr*⟩ = *True*
by (*simp add*: *inj-term-simps*)

lemma *jumpNestingOk-stmt-simp* [*simp*]:
jumpNestingOk jmps (*In1r s*) = *jumpNestingOkS jmps s*
by (*simp add*: *jumpNestingOk-def*)

lemma *jumpNestingOk-stmt-simp1* [*simp*]:
jumpNestingOk jmps ⟨*s*::*stmt*⟩ = *jumpNestingOkS jmps s*
by (*simp add*: *inj-term-simps*)

lemma *jumpNestingOk-var-simp* [*simp*]: *jumpNestingOk jmps* (*In2 v*) = *True*
by (*simp add*: *jumpNestingOk-def*)

lemma *jumpNestingOk-var-simp1* [*simp*]: *jumpNestingOk jmps* ⟨*v*::*var*⟩ = *True*
by (*simp add*: *inj-term-simps*)

lemma *jumpNestingOk-expr-list-simp* [*simp*]: *jumpNestingOk jmps* (*In3 es*) = *True*
by (*simp add*: *jumpNestingOk-def*)

lemma *jumpNestingOk-expr-list-simp1* [*simp*]:
jumpNestingOk jmps $\langle es{::}expr\ list\rangle = True$
by (*simp add*: *inj-term-simps*)

Calculation of assigned variables for boolean expressions

2 Very restricted calculation fallback calculation

primrec *the-LVar-name* $::$ *var* $\Rightarrow$ *lname*
  where *the-LVar-name* (*LVar n*) = *n*

primrec *assignsE* $::$ *expr* $\Rightarrow$ *lname set*
  and *assignsV* $::$ *var* $\Rightarrow$ *lname set*
  and *assignsEs*$::$ *expr list* $\Rightarrow$ *lname set*
where
  assignsE (*NewC c*) $= \{\}$
| *assignsE* (*NewA t e*) = *assignsE e*
| *assignsE* (*Cast t e*) = *assignsE e*
| *assignsE* (*e InstOf r*) = *assignsE e*
| *assignsE* (*Lit val*) $= \{\}$
| *assignsE* (*UnOp unop e*) = *assignsE e*
| *assignsE* (*BinOp binop e1 e2*) = (*if binop*=*CondAnd* $\vee$ *binop*=*CondOr*
    then (*assignsE e1*)
    else (*assignsE e1*) $\cup$ (*assignsE e2*))
| *assignsE* (*Super*) $= \{\}$
| *assignsE* (*Acc v*) = *assignsV v*
| *assignsE* (*v*:=*e*) = (*assignsV v*) $\cup$ (*assignsE e*) $\cup$
    (*if* $\exists$ *n*. *v*=(*LVar n*) *then* $\{$*the-LVar-name v*$\}$
      else $\{\}$)
| *assignsE* (*b*? *e1* : *e2*) = (*assignsE b*) $\cup$ ((*assignsE e1*) $\cap$ (*assignsE e2*))
| *assignsE* ($\{$*accC*,*statT*,*mode*$\}$*objRef*$\cdot$*mn*($\{$*pTs*$\}$*args*))
    = (*assignsE objRef*) $\cup$ (*assignsEs args*)
— Only dummy analysis for intermediate expressions *Methd*, *Body*, *InsInitE* and *Callee*
| *assignsE* (*Methd C sig*) $= \{\}$
| *assignsE* (*Body C s*) $= \{\}$
| *assignsE* (*InsInitE s e*) $= \{\}$
| *assignsE* (*Callee l e*) $= \{\}$

| *assignsV* (*LVar n*) $= \{\}$
| *assignsV* ($\{$*accC*,*statDeclC*,*stat*$\}$*objRef*..*fn*) = *assignsE objRef*
| *assignsV* (*e1*.[*e2*]) = *assignsE e1* $\cup$ *assignsE e2*

| *assignsEs* [] $= \{\}$
| *assignsEs* (*e*#*es*) = *assignsE e* $\cup$ *assignsEs es*

definition *assigns* $::$ *term* $\Rightarrow$ *lname set* **where**
assigns t = (*case t of*
    In1 se $\Rightarrow$ (*case se of*
      Inl e $\Rightarrow$ *assignsE e*
      | *Inr s* $\Rightarrow$ $\{\}$)
  | *In2 v* $\Rightarrow$ *assignsV v*
  | *In3 es* $\Rightarrow$ *assignsEs es*)

lemma *assigns-expr-simp* [*simp*]: *assigns* (*In1l e*) = *assignsE e*
by (*simp add*: *assigns-def*)

lemma *assigns-expr-simp1* [*simp*]: *assigns* (⟨*e*⟩) = *assignsE e*
by (*simp add*: *inj-term-simps*)

lemma *assigns-stmt-simp* [*simp*]: *assigns* (*In1r s*) = {}
by (*simp add*: *assigns-def*)

lemma *assigns-stmt-simp1* [*simp*]: *assigns* (⟨*s*::*stmt*⟩) = {}
by (*simp add*: *inj-term-simps*)

lemma *assigns-var-simp* [*simp*]: *assigns* (*In2 v*) = *assignsV v*
by (*simp add*: *assigns-def*)

lemma *assigns-var-simp1* [*simp*]: *assigns* (⟨*v*⟩) = *assignsV v*
by (*simp add*: *inj-term-simps*)

lemma *assigns-expr-list-simp* [*simp*]: *assigns* (*In3 es*) = *assignsEs es*
by (*simp add*: *assigns-def*)

lemma *assigns-expr-list-simp1* [*simp*]: *assigns* (⟨*es*⟩) = *assignsEs es*
by (*simp add*: *inj-term-simps*)

3 Analysis of constant expressions

```
primrec constVal :: expr ⇒ val option
where
  constVal (NewC c)      = None
| constVal (NewA t e)    = None
| constVal (Cast t e)    = None
| constVal (Inst e r)    = None
| constVal (Lit val)     = Some val
| constVal (UnOp unop e) = (case (constVal e) of
                              None   ⇒ None
                            | Some v ⇒ Some (eval-unop unop v))
| constVal (BinOp binop e1 e2) = (case (constVal e1) of
                                    None    ⇒ None
                                  | Some v1 ⇒ (case (constVal e2) of
                                                 None    ⇒ None
                                               | Some v2 ⇒ Some (eval-binop
                                                                   binop v1 v2)))
| constVal (Super)       = None
| constVal (Acc v)       = None
| constVal (Ass v e)     = None
| constVal (Cond b e1 e2)  = (case (constVal b) of
                                None   ⇒ None
                              | Some bv⇒ (case the-Bool bv of
                                            True ⇒ (case (constVal e2) of
                                                       None   ⇒ None
                                                     | Some v ⇒ constVal e1)
                                          | False⇒ (case (constVal e1) of
                                                       None   ⇒ None
                                                     | Some v ⇒ constVal e2)))
```

— Note that *constVal* (*Cond b e1 e2*) is stricter as it could be. It requires that all tree expressions are

constant even if we can decide which branch to choose, provided the constant value of b
| *constVal* (*Call accC statT mode objRef mn pTs args*) = *None*
| *constVal* (*Methd C sig*) = *None*
| *constVal* (*Body C s*) = *None*
| *constVal* (*InsInitE s e*) = *None*
| *constVal* (*Callee l e*) = *None*

lemma *constVal-Some-induct* [*consumes 1*, *case-names Lit UnOp BinOp CondL CondR*]:
assumes *const*: *constVal e = Some v* **and**
hyp-Lit: $\bigwedge$ *v*. *P* (*Lit v*) **and**
hyp-UnOp: $\bigwedge$ *unop e'*. *P e'* $\Longrightarrow$ *P* (*UnOp unop e'*) **and**
hyp-BinOp: $\bigwedge$ *binop e1 e2*. ⟦*P e1*; *P e2*⟧ $\Longrightarrow$ *P* (*BinOp binop e1 e2*) **and**
hyp-CondL: $\bigwedge$ *b bv e1 e2*. ⟦*constVal b = Some bv*; *the-Bool bv*; *P b*; *P e1*⟧
$\Longrightarrow$ *P* (*b? e1 : e2*) **and**
hyp-CondR: $\bigwedge$ *b bv e1 e2*. ⟦*constVal b = Some bv*; ¬*the-Bool bv*; *P b*; *P e2*⟧
$\Longrightarrow$ *P* (*b? e1 : e2*)
shows *P e*
proof −
have $\bigwedge$ *v*. *constVal e = Some v* $\Longrightarrow$ *P e*
proof (*induct e*)
case *Lit*
show *?case* **by** (*rule hyp-Lit*)
next
case *UnOp*
thus *?case*
by (*auto intro*: *hyp-UnOp*)
next
case *BinOp*
thus *?case*
by (*auto intro*: *hyp-BinOp*)
next
case (*Cond b e1 e2*)
then obtain *v* **where** *v*: *constVal* (*b ? e1 : e2*) = *Some v*
by *blast*
then obtain *bv* **where** *bv*: *constVal b = Some bv*
by *simp*
show *?case*
proof (*cases the-Bool bv*)
case *True*
with *Cond* **show** *?thesis* **using** *v bv*
by (*auto intro*: *hyp-CondL*)
next
case *False*
with *Cond* **show** *?thesis* **using** *v bv*
by (*auto intro*: *hyp-CondR*)
qed
qed (*simp-all add*: *hyp-Lit*)
with *const*
show *?thesis*
by *blast*
qed

lemma *assignsE-const-simp*: *constVal e = Some v* $\Longrightarrow$ *assignsE e* = {}
by (*induct rule*: *constVal-Some-induct*) *simp-all*

4 Main analysis for boolean expressions

Assigned local variables after evaluating the expression if it evaluates to a specific boolean value. If the expression cannot evaluate to a *Boolean* value UNIV is returned. If we expect true/false the opposite constant false/true will also lead to UNIV.

```
primrec assigns-if :: bool ⇒ expr ⇒ lname set
where
  assigns-if b (NewC c)          = UNIV — can never evaluate to Boolean
| assigns-if b (NewA t e)        = UNIV — can never evaluate to Boolean
| assigns-if b (Cast t e)        = assigns-if b e
| assigns-if b (Inst e r)        = assignsE e — Inst has type Boolean but e is a reference type
| assigns-if b (Lit val)         = (if val=Bool b then {} else UNIV)
| assigns-if b (UnOp unop e)     = (case constVal (UnOp unop e) of
                                    None   ⇒ (if unop = UNot
                                               then assigns-if (¬b) e
                                               else UNIV)
                                  | Some v ⇒ (if v=Bool b
                                               then {}
                                               else UNIV))
| assigns-if b (BinOp binop e1 e2)
    = (case constVal (BinOp binop e1 e2) of
         None ⇒ (if binop=CondAnd then
                   (case b of
                       True  ⇒ assigns-if True  e1 ∪ assigns-if True e2
                     | False ⇒ assigns-if False e1 ∩
                                (assigns-if True e1 ∪ assigns-if False e2))
                 else
                (if binop=CondOr then
                   (case b of
                       True  ⇒ assigns-if True e1 ∩
                                (assigns-if False e1 ∪ assigns-if True e2)
                     | False ⇒ assigns-if False  e1 ∪ assigns-if False e2)
                 else assignsE e1 ∪ assignsE e2))
       | Some v ⇒ (if v=Bool b then {} else UNIV))

| assigns-if b (Super)      = UNIV — can never evaluate to Boolean
| assigns-if b (Acc v)      = (assignsV v)
| assigns-if b (v := e)     = (assignsE (Ass v e))
| assigns-if b (c? e1 : e2) = (assignsE c) ∪
                              (case (constVal c) of
                                 None    ⇒ (assigns-if b e1) ∩
                                            (assigns-if b e2)
                               | Some bv ⇒ (case the-Bool bv of
                                              True  ⇒ assigns-if b e1
                                            | False ⇒ assigns-if b e2))
| assigns-if b ({accC,statT,mode}objRef·mn({pTs}args))
           = assignsE ({accC,statT,mode}objRef·mn({pTs}args))
— Only dummy analysis for intermediate expressions Methd, Body, InsInitE and Callee
| assigns-if b (Methd C sig)   = {}
| assigns-if b (Body  C s)     = {}
| assigns-if b (InsInitE s e)  = {}
| assigns-if b (Callee l e)    = {}


lemma assigns-if-const-b-simp:
  assumes boolConst: constVal e = Some (Bool b) (is ?Const b e)
  shows   assigns-if b e = {} (is ?Ass b e)
proof −
```

have $\bigwedge b.\ ?Const\ b\ e \Longrightarrow ?Ass\ b\ e$
proof (*induct e*)
case *Lit*
thus *?case* **by** *simp*
next
case *UnOp*
thus *?case* **by** *simp*
next
case (*BinOp binop*)
thus *?case*
by (*cases binop*) (*simp-all*)
next
case (*Cond c e1 e2 b*)
note *hyp-c* = ‹$\bigwedge b.\ ?Const\ b\ c \Longrightarrow ?Ass\ b\ c$›
note *hyp-e1* = ‹$\bigwedge b.\ ?Const\ b\ e1 \Longrightarrow ?Ass\ b\ e1$›
note *hyp-e2* = ‹$\bigwedge b.\ ?Const\ b\ e2 \Longrightarrow ?Ass\ b\ e2$›
note *const* = ‹*constVal* (*c ? e1 : e2*) = *Some* (*Bool b*)›
then obtain *bv* **where** *bv*: *constVal c* = *Some bv*
by *simp*
hence *emptyC*: *assignsE c* = {} **by** (*rule assignsE-const-simp*)
show *?case*
proof (*cases the-Bool bv*)
case *True*
with *const bv*
have *?Const b e1* **by** *simp*
hence *?Ass b e1* **by** (*rule hyp-e1*)
with *emptyC bv True*
show *?thesis*
by *simp*
next
case *False*
with *const bv*
have *?Const b e2* **by** *simp*
hence *?Ass b e2* **by** (*rule hyp-e2*)
with *emptyC bv False*
show *?thesis*
by *simp*
qed
qed (*simp-all*)
with *boolConst*
show *?thesis*
by *blast*
qed

lemma *assigns-if-const-not-b-simp*:
assumes *boolConst*: *constVal e* = *Some* (*Bool b*) (**is** *?Const b e*)
shows *assigns-if* ($\neg b$) *e* = *UNIV* (**is** *?Ass b e*)
proof −
have $\bigwedge b.\ ?Const\ b\ e \Longrightarrow ?Ass\ b\ e$
proof (*induct e*)
case *Lit*
thus *?case* **by** *simp*
next
case *UnOp*
thus *?case* **by** *simp*
next
case (*BinOp binop*)
thus *?case*

by (*cases binop*) (*simp-all*)
next
case (*Cond c e1 e2 b*)
note *hyp-c* = ‹⋀ *b*. *?Const b c* ⟹ *?Ass b c*›
note *hyp-e1* = ‹⋀ *b*. *?Const b e1* ⟹ *?Ass b e1*›
note *hyp-e2* = ‹⋀ *b*. *?Const b e2* ⟹ *?Ass b e2*›
note *const* = ‹*constVal* (*c ? e1 : e2*) = *Some* (*Bool b*)›
then obtain *bv* **where** *bv*: *constVal c* = *Some bv*
by *simp*
show *?case*
proof (*cases the-Bool bv*)
case *True*
with *const bv*
have *?Const b e1* **by** *simp*
hence *?Ass b e1* **by** (*rule hyp-e1*)
with *bv True*
show *?thesis*
by *simp*
next
case *False*
with *const bv*
have *?Const b e2* **by** *simp*
hence *?Ass b e2* **by** (*rule hyp-e2*)
with *bv False*
show *?thesis*
by *simp*
qed
qed (*simp-all*)
with *boolConst*
show *?thesis*
by *blast*
qed

5 Lifting set operations to range of tables (map to a set)

definition
union-ts :: (′*a*,′*b*) *tables* ⇒ (′*a*,′*b*) *tables* ⇒ (′*a*,′*b*) *tables* (- ⇒∪ - [*67*,*67*] *65*)
where *A* ⇒∪ *B* = (λ *k*. *A k* ∪ *B k*)

definition
intersect-ts :: (′*a*,′*b*) *tables* ⇒ (′*a*,′*b*) *tables* ⇒ (′*a*,′*b*) *tables* (- ⇒∩ - [*72*,*72*] *71*)
where *A* ⇒∩ *B* = (λ*k*. *A k* ∩ *B k*)

definition
all-union-ts :: (′*a*,′*b*) *tables* ⇒ ′*b set* ⇒ (′*a*,′*b*) *tables* (**infixl** ⇒$\cup_\forall$ *40*)
where (*A* ⇒$\cup_\forall$ *B*) = (λ *k*. *A k* ∪ *B*)

Binary union of tables

lemma *union-ts-iff* [*simp*]: (*c* ∈ (*A* ⇒∪ *B*) *k*) = (*c* ∈ *A k* ∨ *c* ∈ *B k*)
by (*unfold union-ts-def*) *blast*

lemma *union-tsI1* [*elim?*]: *c* ∈ *A k* ⟹ *c* ∈ (*A* ⇒∪ *B*) *k*
by *simp*

lemma *union-tsI2* [*elim?*]: *c* ∈ *B k* ⟹ *c* ∈ (*A* ⇒∪ *B*) *k*
by *simp*

lemma *union-tsCI* [*intro*!]: $(c \notin B\ k \Longrightarrow c \in A\ k) \Longrightarrow c \in (A \Rightarrow\cup\ B)\ k$
by *auto*

lemma *union-tsE* [*elim*!]:
$[\![c \in (A \Rightarrow\cup\ B)\ k;\ (c \in A\ k \Longrightarrow P);\ (c \in B\ k \Longrightarrow P)]\!] \Longrightarrow P$
by (*unfold union-ts-def*) *blast*

Binary intersection of tables

lemma *intersect-ts-iff* [*simp*]: $c \in (A \Rightarrow\cap\ B)\ k = (c \in A\ k \wedge c \in B\ k)$
by (*unfold intersect-ts-def*) *blast*

lemma *intersect-tsI* [*intro*!]: $[\![c \in A\ k;\ c \in B\ k]\!] \Longrightarrow c \in (A \Rightarrow\cap\ B)\ k$
by *simp*

lemma *intersect-tsD1*: $c \in (A \Rightarrow\cap\ B)\ k \Longrightarrow c \in A\ k$
by *simp*

lemma *intersect-tsD2*: $c \in (A \Rightarrow\cap\ B)\ k \Longrightarrow c \in B\ k$
by *simp*

lemma *intersect-tsE* [*elim*!]:
$[\![c \in (A \Rightarrow\cap\ B)\ k;\ [\![c \in A\ k;\ c \in B\ k]\!] \Longrightarrow P]\!] \Longrightarrow P$
by *simp*

All-Union of tables and set

lemma *all-union-ts-iff* [*simp*]: $(c \in (A \Rightarrow\cup_\forall\ B)\ k) = (c \in A\ k \vee c \in B)$
by (*unfold all-union-ts-def*) *blast*

lemma *all-union-tsI1* [*elim?*]: $c \in A\ k \Longrightarrow c \in (A \Rightarrow\cup_\forall\ B)\ k$
by *simp*

lemma *all-union-tsI2* [*elim?*]: $c \in B \Longrightarrow c \in (A \Rightarrow\cup_\forall\ B)\ k$
by *simp*

lemma *all-union-tsCI* [*intro*!]: $(c \notin B \Longrightarrow c \in A\ k) \Longrightarrow c \in (A \Rightarrow\cup_\forall\ B)\ k$
by *auto*

lemma *all-union-tsE* [*elim*!]:
$[\![c \in (A \Rightarrow\cup_\forall\ B)\ k;\ (c \in A\ k \Longrightarrow P);\ (c \in B \Longrightarrow P)]\!] \Longrightarrow P$
by (*unfold all-union-ts-def*) *blast*

The rules of definite assignment

type-synonym *breakass* = (*label*, *lname*) *tables*
— Mapping from a break label, to the set of variables that will be assigned if the evaluation terminates with this break

record *assigned* =
  nrm :: *lname set* — Definetly assigned variables for normal completion
  brk :: *breakass* — Definetly assigned variables for abrupt completion with a break

definition
 $rmlab :: 'a \Rightarrow ('a,'b)\ tables \Rightarrow ('a,'b)\ tables$
 where $rmlab\ k\ A = (\lambda x.\ if\ x{=}k\ then\ UNIV\ else\ A\ x)$

definition
 $range\text{-}inter\text{-}ts :: ('a,'b)\ tables \Rightarrow 'b\ set\ (\Rightarrow\bigcap\text{-}\ 80)$
 where $\Rightarrow\bigcap A = \{x\ |x.\ \forall\ \ k.\ x \in A\ k\}$

In $E\vdash\ B\ »t»\ A$, B denotes the "assigned" variables before evaluating term t, whereas A denotes the "assigned" variables after evaluating term t. The environment E is only needed for the conditional *- ? - : -*. The definite assignment rules refer to the typing rules here to distinguish boolean and other expressions.

inductive
 $da :: env \Rightarrow lname\ set \Rightarrow term \Rightarrow assigned \Rightarrow bool\ (\text{-}\vdash\ \text{-}\ »\text{-}»\ \text{-}\ [65,65,65,65]\ 71)$
where
 Skip: $Env\vdash\ B\ »\langle Skip\rangle»\ (\!|nrm{=}B,brk{=}\lambda\ l.\ UNIV|\!)$

| *Expr*: $Env\vdash\ B\ »\langle e\rangle»\ A$
  $\Longrightarrow$
  $Env\vdash\ B\ »\langle Expr\ e\rangle»\ A$
| *Lab*: $[\![Env\vdash\ B\ »\langle c\rangle»\ C;\ nrm\ A = nrm\ C \cap (brk\ C)\ l;\ brk\ A = rmlab\ l\ (brk\ C)]\!]$
  $\Longrightarrow$
  $Env\vdash\ B\ »\langle Break\ l{\cdot}\ c\rangle»\ A$

| *Comp*: $[\![Env\vdash\ B\ »\langle c1\rangle»\ C1;\ Env\vdash\ nrm\ C1\ »\langle c2\rangle»\ C2;$
  $nrm\ A = nrm\ C2;\ brk\ A = (brk\ C1) \Rightarrow\cap (brk\ C2)]\!]$
  $\Longrightarrow$
  $Env\vdash\ B\ »\langle c1;;\ c2\rangle»\ A$

| *If*: $[\![Env\vdash\ B\ »\langle e\rangle»\ E;$
  $Env\vdash\ (B \cup assigns\text{-}if\ True\ \ e)\ »\langle c1\rangle»\ C1;$
  $Env\vdash\ (B \cup assigns\text{-}if\ False\ e)\ »\langle c2\rangle»\ C2;$
  $nrm\ A = nrm\ C1 \cap nrm\ C2;$
  $brk\ A = brk\ C1 \Rightarrow\cap brk\ C2\ ]\!]$
  $\Longrightarrow$
  $Env\vdash\ B\ »\langle If(e)\ c1\ Else\ c2\rangle»\ A$

— Note that E is not further used, because we take the specialized sets that also consider if the expression evaluates to true or false. Inside of e there is no `break` or `finally`, so the break map of E will be the trivial one. So $Env\vdash\ B\ »\langle e\rangle»\ E$ is just used to ensure the definite assignment in expression e. Notice the implicit analysis of a constant boolean expression e in this rule. For example, if e is constantly *True* then $assigns\text{-}if\ False\ e = UNIV$ and therefor $nrm\ C2 = UNIV$. So finally $nrm\ A = nrm\ C1$. For the break maps this trick workd too, because the trivial break map will map all labels to *UNIV*. In the example, if no break occurs in *c2* the break maps will trivially map to *UNIV* and if a break occurs it will map to *UNIV* too, because $assigns\text{-}if\ False\ e = UNIV$. So in the intersection of the break maps the path *c2* will have no contribution.

| *Loop*: $[\![Env\vdash\ B\ »\langle e\rangle»\ E;$
  $Env\vdash\ (B \cup assigns\text{-}if\ True\ e)\ »\langle c\rangle»\ C;$
  $nrm\ A = nrm\ C \cap (B \cup assigns\text{-}if\ False\ e);$
  $brk\ A = brk\ C]\!]$
  $\Longrightarrow$

$Env \vdash B \gg\langle l \bullet\ While(e)\ c\rangle\gg A$

— The *Loop* rule resembles some of the ideas of the *If* rule. For the *nrm A* the set $B \cup$ *assigns-if False e* will be *UNIV* if the condition is constantly true. To normally exit the while loop, we must consider the body *c* to be completed normally (*nrm C*) or with a break. But in this model, the label *l* of the loop only handles continue labels, not break labels. The break label will be handled by an enclosing *Lab* statement. So we don't have to handle the breaks specially.

$$
\begin{array}{l}
\mid Jmp: [\![jump{=}Ret \longrightarrow Result \in B;\\
\quad nrm\ A = UNIV;\\
\quad brk\ A = (case\ jump\ of\\
\qquad\quad Break\ l \Rightarrow \lambda\ k.\ if\ k{=}l\ then\ B\ else\ UNIV\\
\qquad \mid Cont\ l \Rightarrow \lambda\ k.\ UNIV\\
\qquad \mid Ret \quad \Rightarrow \lambda\ k.\ UNIV)]\!]\\
\quad \Longrightarrow\\
\quad Env \vdash B \gg\langle Jmp\ jump\rangle\gg A
\end{array}
$$

— In case of a break to label *l* the corresponding break set is all variables assigned before the break. The assigned variables for normal completion of the *Jmp* is *UNIV*, because the statement will never complete normally. For continue and return the break map is the trivial one. In case of a return we enshure that the result value is assigned.

$$
\begin{array}{l}
\mid Throw: [\![Env \vdash B \gg\langle e\rangle\gg E;\ nrm\ A = UNIV;\ brk\ A = (\lambda\ l.\ UNIV)]\!]\\
\quad \Longrightarrow Env \vdash B \gg\langle Throw\ e\rangle\gg A
\end{array}
$$

$$
\begin{array}{l}
\mid Try: [\![Env \vdash B \gg\langle c1\rangle\gg C1;\\
\quad Env(\!|lcl := lcl\ Env(VName\ vn \mapsto Class\ C)|\!) \vdash (B \cup \{VName\ vn\}) \gg\langle c2\rangle\gg C2;\\
\quad nrm\ A = nrm\ C1 \cap nrm\ C2;\\
\quad brk\ A = brk\ C1 \Rightarrow\cap\ brk\ C2]\!]\\
\quad \Longrightarrow Env \vdash B \gg\langle Try\ c1\ Catch(C\ vn)\ c2\rangle\gg A
\end{array}
$$

$$
\begin{array}{l}
\mid Fin: [\![Env \vdash B \gg\langle c1\rangle\gg C1;\\
\quad Env \vdash B \gg\langle c2\rangle\gg C2;\\
\quad nrm\ A = nrm\ C1 \cup nrm\ C2;\\
\quad brk\ A = ((brk\ C1) \Rightarrow\cup_\forall\ (nrm\ C2)) \Rightarrow\cap\ (brk\ C2)]\!]\\
\quad \Longrightarrow\\
\quad Env \vdash B \gg\langle c1\ Finally\ c2\rangle\gg A
\end{array}
$$

— The set of assigned variables before execution *c2* are the same as before execution *c1*, because *c1* could throw an exception and so we can't guarantee that any variable will be assigned in *c1*. The *Finally* statement completes normally if both *c1* and *c2* complete normally. If *c1* completes abruptly with a break, then *c2* also will be executed and may terminate normally or with a break. The overall break map then is the intersection of the maps of both paths. If *c2* terminates normally we have to extend all break sets in *brk C1* with *nrm C2* ($\Rightarrow\cup_\forall$). If *c2* exits with a break this break will appear in the overall result state. We don't know if *c1* completed normally or abruptly (maybe with an exception not only a break) so *c1* has no contribution to the break map following this path.

— Evaluation of expressions and the break sets of definite assignment: Thinking of a Java expression we assume that we can never have a break statement inside of a expression. So for all expressions the break sets could be set to the trivial one: $\lambda l.\ UNIV$. But we can't trivially proof, that evaluating an expression will never result in a break, allthough Java expressions allready syntactically don't allow nested stetements in them. The reason are the nested class initialzation statements which are inserted by the evaluation rules. So to proof the absence of a break we need to ensure, that the initialization statements will never end up in a break. In a wellfromed initialization statement, of course, were breaks are nested correctly inside of *Lab* or *Loop* statements evaluation of the whole initialization statement will never result in a break, because this break will be handled inside of the statement. But for simplicity we haven't added the analysis of the correct nesting of breaks in the typing judgments right now. So we have decided to adjust the rules of definite assignment to fit to these circumstances. If an initialization is involved during evaluation of the expression (evaluation rules *FVar*, *NewC* and *NewA*

$\mid Init: Env \vdash B \gg\langle Init\ C\rangle\gg (\!|nrm{=}B,brk{=}\lambda\ l.\ UNIV|\!)$

— Wellformedness of a program will ensure, that every static initialiser is definetly assigned and the jumps are nested correctly. The case here for *Init* is just for convenience, to get a proper precondition for the induction hypothesis in various proofs, so that we don't have to expand the initialisation on every point where it is triggerred by the evaluation rules.

| *NewC*: $Env\vdash\ B\ »\langle NewC\ C\rangle»\ (\!|nrm{=}B,brk{=}\lambda\ l.\ UNIV|\!)$

| *NewA*: $Env\vdash\ B\ »\langle e\rangle»\ A$
$\Longrightarrow$
$Env\vdash\ B\ »\langle New\ T[e]\rangle»\ A$

| *Cast*: $Env\vdash\ B\ »\langle e\rangle»\ A$
$\Longrightarrow$
$Env\vdash\ B\ »\langle Cast\ T\ e\rangle»\ A$

| *Inst*: $Env\vdash\ B\ »\langle e\rangle»\ A$
$\Longrightarrow$
$Env\vdash\ B\ »\langle e\ InstOf\ T\rangle»\ A$

| *Lit*: $Env\vdash\ B\ »\langle Lit\ v\rangle»\ (\!|nrm{=}B,brk{=}\lambda\ l.\ UNIV|\!)$

| *UnOp*: $Env\vdash\ B\ »\langle e\rangle»\ A$
$\Longrightarrow$
$Env\vdash\ B\ »\langle UnOp\ unop\ e\rangle»\ A$

| *CondAnd*: $[\![Env\vdash\ B\ »\langle e1\rangle»\ E1;\ Env\vdash\ (B \cup assigns\text{-}if\ True\ e1)\ »\langle e2\rangle»\ E2;$
$nrm\ A = B \cup (assigns\text{-}if\ True\ (BinOp\ CondAnd\ e1\ e2) \cap$
$assigns\text{-}if\ False\ (BinOp\ CondAnd\ e1\ e2));$
$brk\ A = (\lambda\ l.\ UNIV)\ ]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle BinOp\ CondAnd\ e1\ e2\rangle»\ A$

| *CondOr*: $[\![Env\vdash\ B\ »\langle e1\rangle»\ E1;\ Env\vdash\ (B \cup assigns\text{-}if\ False\ e1)\ »\langle e2\rangle»\ E2;$
$nrm\ A = B \cup (assigns\text{-}if\ True\ (BinOp\ CondOr\ e1\ e2) \cap$
$assigns\text{-}if\ False\ (BinOp\ CondOr\ e1\ e2));$
$brk\ A = (\lambda\ l.\ UNIV)\ ]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle BinOp\ CondOr\ e1\ e2\rangle»\ A$

| *BinOp*: $[\![Env\vdash\ B\ »\langle e1\rangle»\ E1;\ Env\vdash\ nrm\ E1\ »\langle e2\rangle»\ A;$
$binop \neq CondAnd;\ binop \neq CondOr]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle BinOp\ binop\ e1\ e2\rangle»\ A$

| *Super*: $This \in B$
$\Longrightarrow$
$Env\vdash\ B\ »\langle Super\rangle»\ (\!|nrm{=}B,brk{=}\lambda\ l.\ UNIV|\!)$

| *AccLVar*: $[\![vn \in B;$
$nrm\ A = B;\ brk\ A = (\lambda\ k.\ UNIV)]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle Acc\ (LVar\ vn)\rangle»\ A$

— To properly access a local variable we have to test the definite assignment here. The variable must occur in the set B

| *Acc*: $[\![\forall\ vn.\ v \neq LVar\ vn;$
$Env\vdash\ B\ »\langle v\rangle»\ A]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle Acc\ v\rangle»\ A$

| *AssLVar*: $[\![Env\vdash\ B\ »\langle e\rangle»\ E;\ nrm\ A = nrm\ E \cup \{vn\};\ brk\ A = brk\ E]\!]$

$\Longrightarrow$
$Env\vdash B \gg\langle(LVar\ vn) := e\rangle\gg A$

| *Ass*: $[\![\forall\ vn.\ v \neq LVar\ vn;\ Env\vdash B \gg\langle v\rangle\gg V;\ Env\vdash nrm\ V \gg\langle e\rangle\gg A]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle v := e\rangle\gg A$

| *CondBool*: $[\![Env\vdash(c\ ?\ e1 : e2)::-(PrimT\ Boolean);$
$Env\vdash B \gg\langle c\rangle\gg C;$
$Env\vdash (B \cup assigns\text{-}if\ True\ \ c) \gg\langle e1\rangle\gg E1;$
$Env\vdash (B \cup assigns\text{-}if\ False\ c) \gg\langle e2\rangle\gg E2;$
$nrm\ A = B \cup (assigns\text{-}if\ True\ \ (c\ ?\ e1 : e2) \cap$
$assigns\text{-}if\ False\ (c\ ?\ e1 : e2));$
$brk\ A = (\lambda\ l.\ UNIV)]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle c\ ?\ e1 : e2\rangle\gg A$

| *Cond*: $[\![\neg\ Env\vdash(c\ ?\ e1 : e2)::-(PrimT\ Boolean);$
$Env\vdash B \gg\langle c\rangle\gg C;$
$Env\vdash (B \cup assigns\text{-}if\ True\ \ c) \gg\langle e1\rangle\gg E1;$
$Env\vdash (B \cup assigns\text{-}if\ False\ c) \gg\langle e2\rangle\gg E2;$
$nrm\ A = nrm\ E1 \cap nrm\ E2;\ brk\ A = (\lambda\ l.\ UNIV)]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle c\ ?\ e1 : e2\rangle\gg A$

| *Call*: $[\![Env\vdash B \gg\langle e\rangle\gg E;\ Env\vdash nrm\ E \gg\langle args\rangle\gg A]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle\{accC,statT,mode\}e\cdot mn(\{pTs\}args)\rangle\gg A$

— The interplay of *Call*, *Methd* and *Body*: Why rules for *Methd* and *Body* at all? Note that a Java source program will not include bare *Methd* or *Body* terms. These terms are just introduced during evaluation. So definite assignment of *Call* does not consider *Methd* or *Body* at all. So for definite assignment alone we could omit the rules for *Methd* and *Body*. But since evaluation of the method invocation is split up into three rules we must ensure that we have enough information about the call even in the *Body* term to make sure that we can proof type safety. Also we must be able transport this information from *Call* to *Methd* and then further to *Body* during evaluation to establish the definite assignment of *Methd* during evaluation of *Call*, and of *Body* during evaluation of *Methd*. This is necessary since definite assignment will be a precondition for each induction hypothesis coming out of the evaluation rules, and therefor we have to establish the definite assignment of the sub-evaluation during the type-safety proof. Note that well-typedness is also a precondition for type-safety and so we can omit some assertion that are already ensured by well-typedness.
| *Methd*: $[\![methd\ (prg\ Env)\ D\ sig = Some\ m;$
$Env\vdash B \gg\langle Body\ (declclass\ m)\ (stmt\ (mbody\ (mthd\ m)))\rangle\gg A$
$]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle Methd\ D\ sig\rangle\gg A$

| *Body*: $[\![Env\vdash B \gg\langle c\rangle\gg C;\ jumpNestingOkS\ \{Ret\}\ c;\ Result \in nrm\ C;$
$nrm\ A = B;\ brk\ A = (\lambda\ l.\ UNIV)]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle Body\ D\ c\rangle\gg A$
— Note that *A* is not correlated to *C*. If the body statement returns abruptly with return, evaluation of *Body* will absorb this return and complete normally. So we cannot trivially get the assigned variables of the body statement since it has not completed normally or with a break. If the body completes normally we guarantee that the result variable is set with this rule. But if the body completes abruptly with a return we can't guarantee that the result variable is set here, since definite assignment only talks about normal completion and breaks. So for a return the *Jump* rule ensures that the result variable is set and then this information must be carried over to the *Body* rule by the conformance predicate of the state.
| *LVar*: $Env\vdash B \gg\langle LVar\ vn\rangle\gg (\!|nrm{=}B,\ brk{=}\lambda\ l.\ UNIV|\!)$

| *FVar*: *Env*⊢ *B* »⟨*e*⟩» *A*
⟹
Env⊢ *B* »⟨{*accC*,*statDeclC*,*stat*}*e*..*fn*⟩» *A*

| *AVar*: ⟦*Env*⊢ *B* »⟨*e1*⟩» *E1*; *Env*⊢ *nrm E1* »⟨*e2*⟩» *A*⟧
⟹
Env⊢ *B* »⟨*e1*.[*e2*]⟩» *A*

| *Nil*: *Env*⊢ *B* »⟨[]::*expr list*⟩» ⦇*nrm*=*B*, *brk*=λ *l*. *UNIV*⦈

| *Cons*: ⟦*Env*⊢ *B* »⟨*e*::*expr*⟩» *E*; *Env*⊢ *nrm E* »⟨*es*⟩» *A*⟧
⟹
Env⊢ *B* »⟨*e*#*es*⟩» *A*

declare *inj-term-sym-simps* [*simp*]
declare *assigns-if.simps* [*simp del*]
declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt delloop split-all-tac*)›

inductive-cases *da-elim-cases* [*cases set*]:
Env⊢ *B* »⟨*Skip*⟩» *A*
Env⊢ *B* »*In1r Skip*» *A*
Env⊢ *B* »⟨*Expr e*⟩» *A*
Env⊢ *B* »*In1r* (*Expr e*)» *A*
Env⊢ *B* »⟨*l*• *c*⟩» *A*
Env⊢ *B* »*In1r* (*l*• *c*)» *A*
Env⊢ *B* »⟨*c1*;; *c2*⟩» *A*
Env⊢ *B* »*In1r* (*c1*;; *c2*)» *A*
Env⊢ *B* »⟨*If*(*e*) *c1 Else c2*⟩» *A*
Env⊢ *B* »*In1r* (*If*(*e*) *c1 Else c2*)» *A*
Env⊢ *B* »⟨*l*• *While*(*e*) *c*⟩» *A*
Env⊢ *B* »*In1r* (*l*• *While*(*e*) *c*)» *A*
Env⊢ *B* »⟨*Jmp jump*⟩» *A*
Env⊢ *B* »*In1r* (*Jmp jump*)» *A*
Env⊢ *B* »⟨*Throw e*⟩» *A*
Env⊢ *B* »*In1r* (*Throw e*)» *A*
Env⊢ *B* »⟨*Try c1 Catch*(*C vn*) *c2*⟩» *A*
Env⊢ *B* »*In1r* (*Try c1 Catch*(*C vn*) *c2*)» *A*
Env⊢ *B* »⟨*c1 Finally c2*⟩» *A*
Env⊢ *B* »*In1r* (*c1 Finally c2*)» *A*
Env⊢ *B* »⟨*Init C*⟩» *A*
Env⊢ *B* »*In1r* (*Init C*)» *A*
Env⊢ *B* »⟨*NewC C*⟩» *A*
Env⊢ *B* »*In1l* (*NewC C*)» *A*
Env⊢ *B* »⟨*New T*[*e*]⟩» *A*
Env⊢ *B* »*In1l* (*New T*[*e*])» *A*
Env⊢ *B* »⟨*Cast T e*⟩» *A*
Env⊢ *B* »*In1l* (*Cast T e*)» *A*
Env⊢ *B* »⟨*e InstOf T*⟩» *A*
Env⊢ *B* »*In1l* (*e InstOf T*)» *A*
Env⊢ *B* »⟨*Lit v*⟩» *A*
Env⊢ *B* »*In1l* (*Lit v*)» *A*
Env⊢ *B* »⟨*UnOp unop e*⟩» *A*
Env⊢ *B* »*In1l* (*UnOp unop e*)» *A*
Env⊢ *B* »⟨*BinOp binop e1 e2*⟩» *A*
Env⊢ *B* »*In1l* (*BinOp binop e1 e2*)» *A*
Env⊢ *B* »⟨*Super*⟩» *A*
Env⊢ *B* »*In1l* (*Super*)» *A*

Env⊢ *B* »⟨*Acc v*⟩» *A*
Env⊢ *B* »*In1l* (*Acc v*)» *A*
Env⊢ *B* »⟨*v* := *e*⟩» *A*
Env⊢ *B* »*In1l* (*v* := *e*)» *A*
Env⊢ *B* »⟨*c* ? *e1* : *e2*⟩» *A*
Env⊢ *B* »*In1l* (*c* ? *e1* : *e2*)» *A*
Env⊢ *B* »⟨{*accC*,*statT*,*mode*}*e*·*mn*({*pTs*}*args*)⟩» *A*
Env⊢ *B* »*In1l* ({*accC*,*statT*,*mode*}*e*·*mn*({*pTs*}*args*))» *A*
Env⊢ *B* »⟨*Methd C sig*⟩» *A*
Env⊢ *B* »*In1l* (*Methd C sig*)» *A*
Env⊢ *B* »⟨*Body D c*⟩» *A*
Env⊢ *B* »*In1l* (*Body D c*)» *A*
Env⊢ *B* »⟨*LVar vn*⟩» *A*
Env⊢ *B* »*In2* (*LVar vn*)» *A*
Env⊢ *B* »⟨{*accC*,*statDeclC*,*stat*}*e*..*fn*⟩» *A*
Env⊢ *B* »*In2* ({*accC*,*statDeclC*,*stat*}*e*..*fn*)» *A*
Env⊢ *B* »⟨*e1*.[*e2*]⟩» *A*
Env⊢ *B* »*In2* (*e1*.[*e2*])» *A*
Env⊢ *B* »⟨[]::*expr list*⟩» *A*
Env⊢ *B* »*In3* ([]::*expr list*)» *A*
Env⊢ *B* »⟨*e*#*es*⟩» *A*
Env⊢ *B* »*In3* (*e*#*es*)» *A*
declare *inj-term-sym-simps* [*simp del*]
declare *assigns-if.simps* [*simp*]
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt addloop* (*split-all-tac*, *split-all-tac*))›

lemma *da-Skip*: *A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|) ⟹ *Env*⊢ *B* »⟨*Skip*⟩» *A*
by (*auto intro*: *da.Skip*)

lemma *da-NewC*: *A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|) ⟹ *Env*⊢ *B* »⟨*NewC C*⟩» *A*
by (*auto intro*: *da.NewC*)

lemma *da-Lit*: *A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|) ⟹ *Env*⊢ *B* »⟨*Lit v*⟩» *A*
by (*auto intro*: *da.Lit*)

lemma *da-Super*: ⟦*This* ∈ *B*;*A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|)⟧ ⟹ *Env*⊢ *B* »⟨*Super*⟩» *A*
by (*auto intro*: *da.Super*)

lemma *da-Init*: *A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|) ⟹ *Env*⊢ *B* »⟨*Init C*⟩» *A*
by (*auto intro*: *da.Init*)

lemma *assignsE-subseteq-assigns-ifs*:
assumes *boolEx*: *E*⊢*e*::−*PrimT Boolean* (**is** *?Boolean e*)
shows *assignsE e* ⊆ *assigns-if True e* ∩ *assigns-if False e* (**is** *?Incl e*)
proof −
obtain *vv* **where** *ex-lit*: *E*⊢*Lit vv*::− *PrimT Boolean*
using *typeof.simps*(*2*) *wt.Lit* **by** *blast*

```
have ?Boolean e ⟹ ?Incl e
proof (induct e)
  case (Cast T e)
  have E⊢e::− (PrimT Boolean)
  proof −
    from ‹E⊢(Cast T e)::− (PrimT Boolean)›
    obtain Te where E⊢e::−Te
                        prg E⊢Te≼? PrimT Boolean
      by cases simp
    thus ?thesis
      by − (drule cast-Boolean2,simp)
  qed
  with Cast.hyps
  show ?case
    by simp
next
  case (Lit val)
  thus ?case
    by − (erule wt-elim-cases, cases val, auto simp add: empty-dt-def)
next
  case (UnOp unop e)
  thus ?case
    by − (erule wt-elim-cases,cases unop,
          (fastforce simp add: assignsE-const-simp)+)
next
  case (BinOp binop e1 e2)
  from BinOp.prems obtain e1T e2T
    where E⊢e1::−e1T and E⊢e2::−e2T and wt-binop (prg E) binop e1T e2T
          and (binop-type binop) = Boolean
    by (elim wt-elim-cases) simp
  with BinOp.hyps
  show ?case
    by − (cases binop, auto simp add: assignsE-const-simp)
next
  case (Cond c e1 e2)
  note hyp-c = ‹?Boolean c ⟹ ?Incl c›
  note hyp-e1 = ‹?Boolean e1 ⟹ ?Incl e1›
  note hyp-e2 = ‹?Boolean e2 ⟹ ?Incl e2›
  note wt = ‹E⊢(c ? e1 : e2)::−PrimT Boolean›
  then obtain
    boolean-c: E⊢c::−PrimT Boolean and
    boolean-e1: E⊢e1::−PrimT Boolean and
    boolean-e2: E⊢e2::−PrimT Boolean
    by (elim wt-elim-cases) (auto dest: widen-Boolean2)
  show ?case
  proof (cases constVal c)
    case None
    with boolean-e1 boolean-e2
    show ?thesis
      using hyp-e1 hyp-e2
      by (auto)
  next
    case (Some bv)
    show ?thesis
    proof (cases the-Bool bv)
      case True
      with Some show ?thesis using hyp-e1 boolean-e1 by auto
    next
      case False
```

```
        with Some show ?thesis using hyp-e2 boolean-e2 by auto
      qed
    qed
  next
    show ⋀x. E⊢Lit vv::−PrimT Boolean
      by (rule ex-lit)
  qed (simp-all add: ex-lit)
  with boolEx
  show ?thesis
    by blast
qed
```

```
lemma rmlab-same-label [simp]: (rmlab l A) l = UNIV
  by (simp add: rmlab-def)
```

```
lemma rmlab-same-label1 [simp]: l=l′ ⟹ (rmlab l A) l′ = UNIV
  by (simp add: rmlab-def)
```

```
lemma rmlab-other-label [simp]: l≠l′⟹ (rmlab l A) l′ = A l′
  by (auto simp add: rmlab-def)
```

```
lemma range-inter-ts-subseteq [intro]: ∀ k. A k ⊆ B k ⟹ ⇒⋂A ⊆ ⇒⋂B
  by (auto simp add: range-inter-ts-def)
```

```
lemma range-inter-ts-subseteq′: ∀ k. A k ⊆ B k ⟹ x ∈ ⇒⋂A ⟹ x ∈ ⇒⋂B
  by (auto simp add: range-inter-ts-def)
```

```
lemma da-monotone:
  assumes da: Env⊢ B »t» A and
    B ⊆ B′ and
    da′: Env⊢ B′ »t» A′
  shows (nrm A ⊆ nrm A′) ∧ (∀ l. (brk A l ⊆ brk A′ l))
proof −
  from da
  have ⋀ B′ A′. ⟦Env⊢ B′ »t» A′; B ⊆ B′⟧
                ⟹ (nrm A ⊆ nrm A′) ∧ (∀ l. (brk A l ⊆ brk A′ l))
       (is PROP ?Hyp Env B t A)
  proof (induct)
    case Skip
    then show ?case by cases simp
  next
    case Expr
    from Expr.prems Expr.hyps
    show ?case by cases simp
  next
    case (Lab Env B c C A l B′ A′)
    note A = ‹nrm A = nrm C ∩ brk C l› ‹brk A = rmlab l (brk C)›
    note ‹PROP ?Hyp Env B ⟨c⟩ C›
    moreover
    note ‹B ⊆ B′›
```

```
    moreover
    obtain C′
      where Env⊢ B′ »⟨c⟩» C′
        and A′: nrm A′ = nrm C′ ∩ brk C′ l brk A′ = rmlab l (brk C′)
      using Lab.prems
      by cases simp
    ultimately
    have nrm C ⊆ nrm C′ and hyp-brk: (∀ l. brk C l ⊆ brk C′ l) by auto
    then
    have nrm C ∩ brk C l ⊆ nrm C′ ∩ brk C′ l by auto
    moreover
    {
      fix l′
      from hyp-brk
      have rmlab l (brk C) l′ ⊆ rmlab l (brk C′) l′
        by (cases l=l′) simp-all
    }
    moreover note A A′
    ultimately show ?case
      by simp
  next
    case (Comp Env B c1 C1 c2 C2 A B′ A′)
    note A = ‹nrm A = nrm C2› ‹brk A = brk C1 ⇒∩ brk C2›
    from ‹Env⊢ B′ »⟨c1;; c2⟩» A′›
    obtain C1′ C2′
      where da-c1: Env⊢ B′ »⟨c1⟩» C1′ and
            da-c2: Env⊢ nrm C1′ »⟨c2⟩» C2′ and
            A′: nrm A′ = nrm C2′ brk A′ = brk C1′ ⇒∩ brk C2′
      by cases auto
    note ‹PROP ?Hyp Env B ⟨c1⟩ C1›
    moreover note ‹B ⊆ B′›
    moreover note da-c1
    ultimately have C1′: nrm C1 ⊆ nrm C1′ (∀ l. brk C1 l ⊆ brk C1′ l)
      by auto
    note ‹PROP ?Hyp Env (nrm C1) ⟨c2⟩ C2›
    with da-c2 C1′
    have C2′: nrm C2 ⊆ nrm C2′ (∀ l. brk C2 l ⊆ brk C2′ l)
      by auto
    with A A′ C1′
    show ?case
      by auto
  next
    case (If Env B e E c1 C1 c2 C2 A B′ A′)
    note A = ‹nrm A = nrm C1 ∩ nrm C2› ‹brk A = brk C1 ⇒∩ brk C2›
    from ‹Env⊢ B′ »⟨If(e) c1 Else c2⟩» A′›
    obtain C1′ C2′
      where da-c1: Env⊢ B′ ∪ assigns-if True e »⟨c1⟩» C1′ and
            da-c2: Env⊢ B′ ∪ assigns-if False e »⟨c2⟩» C2′ and
              A′: nrm A′ = nrm C1′ ∩ nrm C2′ brk A′ = brk C1′ ⇒∩ brk C2′
      by cases auto
    note ‹PROP ?Hyp Env (B ∪ assigns-if True e) ⟨c1⟩ C1›
    moreover note B′ = ‹B ⊆ B′›
    moreover note da-c1
    ultimately obtain C1′: nrm C1 ⊆ nrm C1′ (∀ l. brk C1 l ⊆ brk C1′ l)
      by blast
    note ‹PROP ?Hyp Env (B ∪ assigns-if False e) ⟨c2⟩ C2›
    with da-c2 B′
    obtain C2′: nrm C2 ⊆ nrm C2′ (∀ l. brk C2 l ⊆ brk C2′ l)
      by blast
```

```
    with A A' C1'
    show ?case
      by auto
  next
    case (Loop Env B e E c C A l B' A')
    note A = ‹nrm A = nrm C ∩ (B ∪ assigns-if False e)› ‹brk A = brk C›
    from ‹Env⊢ B' »⟨l· While(e) c⟩» A'›
    obtain C'
      where
       da-c': Env⊢ B' ∪ assigns-if True e »⟨c⟩» C' and
          A': nrm A' = nrm C' ∩ (B' ∪ assigns-if False e)
              brk A' = brk C'
      by cases auto
    note ‹PROP ?Hyp Env (B ∪ assigns-if True e) ⟨c⟩ C›
    moreover note B' = ‹B ⊆ B'›
    moreover note da-c'
    ultimately obtain C': nrm C ⊆ nrm C' (∀ l. brk C l ⊆ brk C' l)
      by blast
    with A A' B'
    have nrm A ⊆ nrm A'
      by blast
    moreover
    { fix l'
      have  brk A l' ⊆ brk A' l'
      proof (cases constVal e)
        case None
        with A A' C'
        show ?thesis
           by (cases l=l') auto
      next
        case (Some bv)
        with A A' C'
        show ?thesis
          by (cases the-Bool bv, cases l=l') auto
      qed
    }
    ultimately show ?case
      by auto
  next
    case (Jmp jump B A Env B' A')
    thus ?case by (elim da-elim-cases) (auto split: jump.splits)
  next
    case Throw thus ?case by (elim da-elim-cases) auto
  next
    case (Try Env B c1 C1 vn C c2 C2 A B' A')
    note A = ‹nrm A = nrm C1 ∩ nrm C2› ‹brk A = brk C1 ⇒∩  brk C2›
    from ‹Env⊢ B' »⟨Try c1 Catch(C vn) c2⟩» A'›
    obtain C1' C2'
      where da-c1': Env⊢ B' »⟨c1⟩» C1' and
            da-c2': Env(|lcl := lcl Env(VName vn↦Class C)|)⊢ B' ∪ {VName vn}
                      »⟨c2⟩» C2' and
            A': nrm A' = nrm C1' ∩ nrm C2'
                brk A' = brk C1' ⇒∩  brk C2'
      by cases auto
    note ‹PROP ?Hyp Env B ⟨c1⟩ C1›
    moreover note B' = ‹B ⊆ B'›
    moreover note da-c1'
    ultimately obtain C1': nrm C1 ⊆ nrm C1' (∀ l. brk C1 l ⊆ brk C1' l)
      by blast
```

```
  note ‹PROP ?Hyp (Env⦇lcl := lcl Env(VName vn↦Class C)⦈)
                 (B ∪ {VName vn}) ⟨c2⟩ C2›
  with B' da-c2'
  obtain nrm C2 ⊆ nrm C2' (∀ l. brk C2 l ⊆ brk C2' l)
    by blast
  with C1' A A'
  show ?case
    by auto
next
  case (Fin Env B c1 C1 c2 C2 A B' A')
  note A = ‹nrm A = nrm C1 ∪ nrm C2›
    ‹brk A = (brk C1 ⇒∪∀ nrm C2) ⇒∩ (brk C2)›
  from ‹Env⊢ B' »⟨c1 Finally c2⟩» A'›
  obtain C1' C2'
    where  da-c1': Env⊢ B' »⟨c1⟩» C1' and
           da-c2': Env⊢ B' »⟨c2⟩» C2' and
           A':  nrm A' = nrm C1' ∪ nrm C2'
                brk A' = (brk C1' ⇒∪∀ nrm C2') ⇒∩ (brk C2')
    by cases auto
  note ‹PROP ?Hyp Env B ⟨c1⟩ C1›
  moreover note B' = ‹B ⊆ B'›
  moreover note da-c1'
  ultimately obtain C1': nrm C1 ⊆ nrm C1' (∀ l. brk C1 l ⊆ brk C1' l)
    by blast
  note hyp-c2 = ‹PROP ?Hyp Env B ⟨c2⟩ C2›
  from da-c2' B'
   obtain nrm C2 ⊆ nrm C2' (∀ l. brk C2 l ⊆ brk C2' l)
     by − (drule hyp-c2,auto)
   with A A' C1'
   show ?case
     by auto
 next
   case Init thus ?case by (elim da-elim-cases) auto
 next
   case NewC thus ?case by (elim da-elim-cases) auto
 next
   case NewA thus ?case by (elim da-elim-cases) auto
 next
   case Cast thus ?case by (elim da-elim-cases) auto
 next
   case Inst thus ?case by (elim da-elim-cases) auto
 next
   case Lit thus ?case by (elim da-elim-cases) auto
 next
   case UnOp thus ?case by (elim da-elim-cases) auto
 next
   case (CondAnd Env B e1 E1 e2 E2 A B' A')
   note A = ‹nrm A = B ∪
                   assigns-if True (BinOp CondAnd e1 e2) ∩
                   assigns-if False (BinOp CondAnd e1 e2)›
          ‹brk A = (λl. UNIV)›
   from ‹Env⊢ B' »⟨BinOp CondAnd e1 e2⟩» A'›
   obtain  A': nrm A' = B' ∪
                           assigns-if True (BinOp CondAnd e1 e2) ∩
                           assigns-if False (BinOp CondAnd e1 e2)
                   brk A' = (λl. UNIV)
     by cases auto
   note B' = ‹B ⊆ B'›
   with A A' show ?case
```

```
    by auto
  next
    case CondOr thus ?case by (elim da-elim-cases) auto
  next
    case BinOp thus ?case by (elim da-elim-cases) auto
  next
    case Super thus ?case by (elim da-elim-cases) auto
  next
    case AccLVar thus ?case by (elim da-elim-cases) auto
  next
    case Acc thus ?case by (elim da-elim-cases) auto
  next
    case AssLVar thus ?case by (elim da-elim-cases) auto
  next
    case Ass thus ?case by (elim da-elim-cases) auto
  next
    case (CondBool Env c e1 e2 B C E1 E2 A B' A')
    note A = ‹nrm A = B ∪
                  assigns-if True (c ? e1 : e2) ∩
                  assigns-if False (c ? e1 : e2)›
         ‹brk A = (λl. UNIV)›
    note ‹Env⊢ (c ? e1 : e2)::− (PrimT Boolean)›
    moreover
    note ‹Env⊢ B' »⟨c ? e1 : e2⟩» A'›
    ultimately
    obtain A': nrm A' = B' ∪
                          assigns-if True (c ? e1 : e2) ∩
                          assigns-if False (c ? e1 : e2)
               brk A' = (λl. UNIV)
      by (elim da-elim-cases) (auto simp add: inj-term-simps)

    note B' = ‹B ⊆ B'›
    with A A' show ?case
      by auto
  next
    case (Cond Env c e1 e2 B C E1 E2 A B' A')
    note A = ‹nrm A = nrm E1 ∩ nrm E2› ‹brk A = (λl. UNIV)›
    note not-bool = ‹¬ Env⊢ (c ? e1 : e2)::− (PrimT Boolean)›
    from ‹Env⊢ B' »⟨c ? e1 : e2⟩» A'›
    obtain E1' E2'
      where da-e1': Env⊢ B' ∪ assigns-if True c »⟨e1⟩» E1' and
            da-e2': Env⊢ B' ∪ assigns-if False c »⟨e2⟩» E2' and
              A': nrm A' = nrm E1' ∩ nrm E2'
                  brk A' = (λl. UNIV)
      using not-bool
      by (elim da-elim-cases) (auto simp add: inj-term-simps)

    note ‹PROP ?Hyp Env (B ∪ assigns-if True c) ⟨e1⟩ E1›
    moreover note B' = ‹B ⊆ B'›
    moreover note da-e1'
    ultimately obtain E1': nrm E1 ⊆ nrm E1' (∀ l. brk E1 l ⊆ brk E1' l)
      by blast
    note ‹PROP ?Hyp Env (B ∪ assigns-if False c) ⟨e2⟩ E2›
    with B' da-e2'
    obtain nrm E2 ⊆ nrm E2' (∀ l. brk E2 l ⊆ brk E2' l)
      by blast
   with E1' A A'
   show ?case
     by auto
```

next
  case *Call*
  from *Call.prems* **and** *Call.hyps*
  show *?case* **by** *cases auto*
next
  case *Methd* **thus** *?case* **by** (*elim da-elim-cases*) *auto*
next
  case *Body* **thus** *?case* **by** (*elim da-elim-cases*) *auto*
next
  case *LVar* **thus** *?case* **by** (*elim da-elim-cases*) *auto*
next
  case *FVar* **thus** *?case* **by** (*elim da-elim-cases*) *auto*
next
  case *AVar* **thus** *?case* **by** (*elim da-elim-cases*) *auto*
next
  case *Nil* **thus** *?case* **by** (*elim da-elim-cases*) *auto*
next
  case *Cons* **thus** *?case* **by** (*elim da-elim-cases*) *auto*
qed
from *this* [*OF da′* ‹$B \subseteq B'$›] **show** *?thesis* **.**
qed

lemma *da-weaken*:
  assumes *da*: $Env \vdash B \gg t \gg A$ **and** $B \subseteq B'$
  shows $\exists\ A'.\ Env \vdash B' \gg t \gg A'$
proof −
  note *assigned.select-convs* [*Pure.intro*]
  from *da*
  have $\bigwedge B'.\ B \subseteq B' \Longrightarrow \exists\ A'.\ Env \vdash B' \gg t \gg A'$ (**is** *PROP ?Hyp Env B t*)
  proof (*induct*)
    case *Skip* **thus** *?case* **by** (*iprover intro*: *da.Skip*)
  next
    case *Expr* **thus** *?case* **by** (*iprover intro*: *da.Expr*)
  next
    case (*Lab Env B c C A l B′*)
    note ‹*PROP ?Hyp Env B* ⟨*c*⟩›
    moreover
    note $B' =$ ‹$B \subseteq B'$›
    ultimately obtain C' **where** $Env \vdash B' \gg\langle c\rangle\gg C'$
      by *iprover*
    then obtain A' **where** $Env \vdash B' \gg\langle Break\ l \cdot\ c\rangle\gg A'$
      by (*iprover intro*: *da.Lab*)
    thus *?case* **..**
  next
    case (*Comp Env B c1 C1 c2 C2 A B′*)
    note *da-c1* = ‹$Env \vdash B \gg\langle c1\rangle\gg C1$›
    note ‹*PROP ?Hyp Env B* ⟨*c1*⟩›
    moreover
    note $B' =$ ‹$B \subseteq B'$›
    ultimately obtain $C1'$ **where** *da-c1′*: $Env \vdash B' \gg\langle c1\rangle\gg C1'$
      by *iprover*
    with *da-c1 B′*
    have
      $nrm\ C1 \subseteq nrm\ C1'$
      by (*rule da-monotone* [*elim-format*]) *simp*
    moreover
    note ‹*PROP ?Hyp Env* (*nrm C1*) ⟨*c2*⟩›
    ultimately obtain $C2'$ **where** $Env \vdash nrm\ C1' \gg\langle c2\rangle\gg C2'$

```
      by iprover
    with da-c1' obtain A' where Env⊢ B' »⟨c1;; c2⟩» A'
      by (iprover intro: da.Comp)
    thus ?case ..
  next
    case (If Env B e E c1 C1 c2 C2 A B')
    note B' = ‹B ⊆ B'›
    obtain  E' where Env⊢ B' »⟨e⟩» E'
    proof −
      have PROP ?Hyp Env B ⟨e⟩ by (rule If.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
    obtain C1' where Env⊢ (B' ∪ assigns-if True e) »⟨c1⟩» C1'
    proof −
      from B'
      have (B ∪ assigns-if True e) ⊆ (B' ∪ assigns-if True e)
        by blast
      moreover
      have PROP ?Hyp Env (B ∪ assigns-if True e) ⟨c1⟩ by (rule If.hyps)
      ultimately
      show ?thesis using that by iprover
    qed
    moreover
    obtain C2' where Env⊢ (B' ∪ assigns-if False e) »⟨c2⟩» C2'
    proof −
      from B' have (B ∪ assigns-if False e) ⊆ (B' ∪ assigns-if False e)
        by blast
      moreover
      have PROP ?Hyp Env (B ∪ assigns-if False e) ⟨c2⟩ by (rule If.hyps)
      ultimately
      show ?thesis using that by iprover
    qed
    ultimately
    obtain A' where Env⊢ B' »⟨If(e) c1 Else c2⟩» A'
      by (iprover intro: da.If)
    thus ?case ..
  next
    case (Loop Env B e E c C A l B')
    note B' = ‹B ⊆ B'›
    obtain E' where Env⊢ B' »⟨e⟩» E'
    proof −
      have PROP ?Hyp Env B ⟨e⟩ by (rule Loop.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
    obtain C' where Env⊢ (B' ∪ assigns-if True e) »⟨c⟩» C'
    proof −
      from B'
      have (B ∪ assigns-if True e) ⊆ (B' ∪ assigns-if True e)
        by blast
      moreover
      have PROP ?Hyp Env (B ∪ assigns-if True e) ⟨c⟩ by (rule Loop.hyps)
      ultimately
      show ?thesis using that by iprover
    qed
    ultimately
```

```
  obtain A' where Env⊢ B' »⟨l· While(e) c⟩» A'
    by (iprover intro: da.Loop )
  thus ?case ..
next
  case (Jmp jump B A Env B')
  note B' = ‹B ⊆ B'›
  with Jmp.hyps have jump = Ret ⟶ Result ∈ B'
    by auto
  moreover
  obtain A'::assigned
          where  nrm A' = UNIV
                 brk A' = (case jump of
                              Break l ⇒ λk. if k = l then B' else UNIV
                            | Cont l ⇒ λk. UNIV
                            | Ret ⇒ λk. UNIV)

    by  iprover
  ultimately have Env⊢ B' »⟨Jmp jump⟩» A'
    by (rule da.Jmp)
  thus ?case ..
next
  case Throw thus ?case by (iprover intro: da.Throw )
next
  case (Try Env B c1 C1 vn C c2 C2 A B')
  note B' = ‹B ⊆ B'›
  obtain C1' where Env⊢ B' »⟨c1⟩» C1'
  proof -
    have PROP ?Hyp Env B ⟨c1⟩ by (rule Try.hyps)
    with B'
    show ?thesis using that by iprover
  qed
  moreover
  obtain C2' where
    Env⦇lcl := lcl Env(VName vn↦Class C)⦈⊢ B' ∪ {VName vn} »⟨c2⟩» C2'
  proof -
    from B' have B ∪ {VName vn} ⊆ B' ∪ {VName vn} by blast
    moreover
    have PROP ?Hyp (Env⦇lcl := lcl Env(VName vn↦Class C)⦈)
                   (B ∪ {VName vn}) ⟨c2⟩
      by (rule Try.hyps)
    ultimately
    show ?thesis using that by iprover
  qed
  ultimately
  obtain A' where Env⊢ B' »⟨Try c1 Catch(C vn) c2⟩» A'
    by (iprover intro: da.Try )
  thus ?case ..
next
  case (Fin Env B c1 C1 c2 C2 A B')
  note B' = ‹B ⊆ B'›
  obtain C1' where C1': Env⊢ B' »⟨c1⟩» C1'
  proof -
    have PROP ?Hyp Env B ⟨c1⟩ by (rule Fin.hyps)
    with B'
    show ?thesis using that by iprover
  qed
  moreover
  obtain C2' where Env⊢ B' »⟨c2⟩» C2'
  proof -
```

have *PROP ?Hyp Env B* ⟨*c2*⟩ **by** (*rule Fin.hyps*)
with *B′*
show *?thesis* **using** *that* **by** *iprover*
qed
ultimately
obtain *A′* **where** *Env*⊢ *B′* »⟨*c1 Finally c2*⟩» *A′*
by (*iprover intro*: *da.Fin*)
thus *?case* **..**
next
case *Init* **thus** *?case* **by** (*iprover intro*: *da.Init*)
next
case *NewC* **thus** *?case* **by** (*iprover intro*: *da.NewC*)
next
case *NewA* **thus** *?case* **by** (*iprover intro*: *da.NewA*)
next
case *Cast* **thus** *?case* **by** (*iprover intro*: *da.Cast*)
next
case *Inst* **thus** *?case* **by** (*iprover intro*: *da.Inst*)
next
case *Lit* **thus** *?case* **by** (*iprover intro*: *da.Lit*)
next
case *UnOp* **thus** *?case* **by** (*iprover intro*: *da.UnOp*)
next
case (*CondAnd Env B e1 E1 e2 E2 A B′*)
note *B′* = ‹*B* ⊆ *B′*›
obtain *E1′* **where** *Env*⊢ *B′* »⟨*e1*⟩» *E1′*
proof −
have *PROP ?Hyp Env B* ⟨*e1*⟩ **by** (*rule CondAnd.hyps*)
with *B′*
show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain *E2′* **where** *Env*⊢ *B′* ∪ *assigns-if True e1* »⟨*e2*⟩» *E2′*
proof −
from *B′* **have** *B* ∪ *assigns-if True e1* ⊆ *B′* ∪ *assigns-if True e1*
by *blast*
moreover
have *PROP ?Hyp Env* (*B* ∪ *assigns-if True e1*) ⟨*e2*⟩ **by** (*rule CondAnd.hyps*)
ultimately show *?thesis* **using** *that* **by** *iprover*
qed
ultimately
obtain *A′* **where** *Env*⊢ *B′* »⟨*BinOp CondAnd e1 e2*⟩» *A′*
by (*iprover intro*: *da.CondAnd*)
thus *?case* **..**
next
case (*CondOr Env B e1 E1 e2 E2 A B′*)
note *B′* = ‹*B* ⊆ *B′*›
obtain *E1′* **where** *Env*⊢ *B′* »⟨*e1*⟩» *E1′*
proof −
have *PROP ?Hyp Env B* ⟨*e1*⟩ **by** (*rule CondOr.hyps*)
with *B′*
show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain *E2′* **where** *Env*⊢ *B′* ∪ *assigns-if False e1* »⟨*e2*⟩» *E2′*
proof −
from *B′* **have** *B* ∪ *assigns-if False e1* ⊆ *B′* ∪ *assigns-if False e1*
by *blast*
moreover

```
      have PROP ?Hyp Env (B ∪ assigns-if False e1) ⟨e2⟩ by (rule CondOr.hyps)
      ultimately show ?thesis using that by iprover
    qed
    ultimately
    obtain A′ where Env⊢ B′ »⟨BinOp CondOr e1 e2⟩» A′
      by (iprover intro: da.CondOr)
    thus ?case ..
  next
    case (BinOp Env B e1 E1 e2 A binop B′)
    note B′ = ‹B ⊆ B′›
    obtain E1′ where E1′: Env⊢ B′ »⟨e1⟩» E1′
    proof −
      have PROP ?Hyp Env B ⟨e1⟩ by (rule BinOp.hyps)
      with B′
      show ?thesis using that by iprover
    qed
    moreover
    obtain A′ where Env⊢ nrm E1′ »⟨e2⟩» A′
    proof −
      have Env⊢ B »⟨e1⟩» E1 by (rule BinOp.hyps)
      from this B′ E1′
      have nrm E1 ⊆ nrm E1′
        by (rule da-monotone [THEN conjE])
      moreover
      have PROP ?Hyp Env (nrm E1) ⟨e2⟩ by (rule BinOp.hyps)
      ultimately show ?thesis using that by iprover
    qed
    ultimately
    have Env⊢ B′ »⟨BinOp binop e1 e2⟩» A′
      using BinOp.hyps by (iprover intro: da.BinOp)
    thus ?case ..
  next
    case (Super B Env B′)
    note B′ = ‹B ⊆ B′›
    with Super.hyps have This ∈ B′
      by auto
    thus ?case by (iprover intro: da.Super)
  next
    case (AccLVar vn B A Env B′)
    note ‹vn ∈ B›
    moreover
    note ‹B ⊆ B′›
    ultimately have vn ∈ B′ by auto
    thus ?case by (iprover intro: da.AccLVar)
  next
    case Acc thus ?case by (iprover intro: da.Acc)
  next
    case (AssLVar Env B e E A vn B′)
    note B′ = ‹B ⊆ B′›
    then obtain E′ where Env⊢ B′ »⟨e⟩» E′
      by (rule AssLVar.hyps [elim-format]) iprover
    then obtain A′ where
      Env⊢ B′ »⟨LVar vn:=e⟩» A′
      by (iprover intro: da.AssLVar)
    thus ?case ..
  next
    case (Ass v Env B V e A B′)
    note B′ = ‹B ⊆ B′›
    note ‹∀ vn. v ≠ LVar vn›
```

```
  moreover
  obtain V' where V': Env⊢ B' »⟨v⟩» V'
  proof -
    have PROP ?Hyp Env B ⟨v⟩ by (rule Ass.hyps)
    with B'
    show ?thesis using that by iprover
  qed
  moreover
  obtain A' where Env⊢ nrm V' »⟨e⟩» A'
  proof -
    have Env⊢ B »⟨v⟩» V by (rule Ass.hyps)
    from this B' V'
    have nrm V ⊆ nrm V'
      by (rule da-monotone [THEN conjE])
    moreover
    have PROP ?Hyp Env (nrm V) ⟨e⟩ by (rule Ass.hyps)
    ultimately show ?thesis using that by iprover
  qed
  ultimately
  have Env⊢ B' »⟨v := e⟩» A'
    by (iprover intro: da.Ass)
  thus ?case ..
next
  case (CondBool Env c e1 e2 B C E1 E2 A B')
  note B' = ‹B ⊆ B'›
  note ‹Env⊢(c ? e1 : e2)::-(PrimT Boolean)›
  moreover obtain C' where C': Env⊢ B' »⟨c⟩» C'
  proof -
    have PROP ?Hyp Env B ⟨c⟩ by (rule CondBool.hyps)
    with B'
    show ?thesis using that by iprover
  qed
  moreover
  obtain E1' where Env⊢ B' ∪ assigns-if True c »⟨e1⟩» E1'
  proof -
    from B'
    have (B ∪ assigns-if True c) ⊆ (B' ∪ assigns-if True c)
      by blast
    moreover
    have PROP ?Hyp Env (B ∪ assigns-if True c) ⟨e1⟩ by (rule CondBool.hyps)
    ultimately
    show ?thesis using that by iprover
  qed
  moreover
  obtain E2' where Env⊢ B' ∪ assigns-if False c »⟨e2⟩» E2'
  proof -
    from B'
    have (B ∪ assigns-if False c) ⊆ (B' ∪ assigns-if False c)
      by blast
    moreover
    have PROP ?Hyp Env (B ∪ assigns-if False c) ⟨e2⟩ by(rule CondBool.hyps)
    ultimately
    show ?thesis using that by iprover
  qed
  ultimately
  obtain A' where Env⊢ B' »⟨c ? e1 : e2⟩» A'
    by (iprover intro: da.CondBool)
  thus ?case ..
next
```

```
  case (Cond Env c e1 e2 B C E1 E2 A B′)
  note B′ = ‹B ⊆ B′›
  note ‹¬ Env⊢(c ? e1 : e2)::−(PrimT Boolean)›
  moreover obtain C′ where C′: Env⊢ B′ »⟨c⟩» C′
  proof −
    have PROP ?Hyp Env B ⟨c⟩ by (rule Cond.hyps)
    with B′
    show ?thesis using that by iprover
  qed
  moreover
  obtain E1′ where Env⊢ B′ ∪ assigns-if True c »⟨e1⟩» E1′
  proof −
    from B′
    have (B ∪ assigns-if True c) ⊆ (B′ ∪ assigns-if True c)
      by blast
    moreover
    have PROP ?Hyp Env (B ∪ assigns-if True c) ⟨e1⟩ by (rule Cond.hyps)
    ultimately
    show ?thesis using that by iprover
  qed
  moreover
  obtain E2′ where Env⊢ B′ ∪ assigns-if False c »⟨e2⟩» E2′
  proof −
    from B′
    have (B ∪ assigns-if False c) ⊆ (B′ ∪ assigns-if False c)
      by blast
    moreover
    have PROP ?Hyp Env (B ∪ assigns-if False c) ⟨e2⟩ by (rule Cond.hyps)
    ultimately
    show ?thesis using that by iprover
  qed
  ultimately
  obtain A′ where Env⊢ B′ »⟨c ? e1 : e2⟩» A′
    by (iprover intro: da.Cond)
  thus ?case ..
next
  case (Call Env B e E args A accC statT mode mn pTs B′)
  note B′ = ‹B ⊆ B′›
  obtain E′ where E′: Env⊢ B′ »⟨e⟩» E′
  proof −
    have PROP ?Hyp Env B ⟨e⟩ by (rule Call.hyps)
    with B′
    show ?thesis using that by iprover
  qed
  moreover
  obtain A′ where Env⊢ nrm E′ »⟨args⟩» A′
  proof −
    have Env⊢ B »⟨e⟩» E by (rule Call.hyps)
    from this B′ E′
    have nrm E ⊆ nrm E′
      by (rule da-monotone [THEN conjE])
    moreover
    have PROP ?Hyp Env (nrm E) ⟨args⟩ by (rule Call.hyps)
    ultimately show ?thesis using that by iprover
  qed
  ultimately
  have Env⊢ B′ »⟨{accC,statT,mode}e·mn( {pTs}args)⟩» A′
    by (iprover intro: da.Call)
  thus ?case ..
```

```
next
  case Methd thus ?case by (iprover intro: da.Methd)
next
  case (Body Env B c C A D B′)
  note B′ = ‹B ⊆ B′›
  obtain C′ where C′: Env⊢ B′ »⟨c⟩» C′ and nrm-C′: nrm C ⊆ nrm C′
  proof −
    have Env⊢ B »⟨c⟩» C by (rule Body.hyps)
    moreover note B′
    moreover
    from B′ obtain C′ where da-c: Env⊢ B′ »⟨c⟩» C′
      by (rule Body.hyps [elim-format]) blast
    ultimately
    have nrm C ⊆ nrm C′
      by (rule da-monotone [THEN conjE])
    with da-c that show ?thesis by iprover
  qed
  moreover
  note ‹Result ∈ nrm C›
  with nrm-C′ have Result ∈ nrm C′
    by blast
  moreover note ‹jumpNestingOkS {Ret} c›
  ultimately obtain A′ where
    Env⊢ B′ »⟨Body D c⟩» A′
    by (iprover intro: da.Body)
  thus ?case ..
next
  case LVar thus ?case by (iprover intro: da.LVar)
next
  case FVar thus ?case by (iprover intro: da.FVar)
next
  case (AVar Env B e1 E1 e2 A B′)
  note B′ = ‹B ⊆ B′›
  obtain E1′ where E1′: Env⊢ B′ »⟨e1⟩» E1′
  proof −
    have PROP ?Hyp Env B ⟨e1⟩ by (rule AVar.hyps)
    with B′
    show ?thesis using that by iprover
  qed
  moreover
  obtain A′ where Env⊢ nrm E1′ »⟨e2⟩» A′
  proof −
    have Env⊢ B »⟨e1⟩» E1 by (rule AVar.hyps)
    from this B′ E1′
    have nrm E1 ⊆ nrm E1′
      by (rule da-monotone [THEN conjE])
    moreover
    have PROP ?Hyp Env (nrm E1) ⟨e2⟩ by (rule AVar.hyps)
    ultimately show ?thesis using that by iprover
  qed
  ultimately
  have Env⊢ B′ »⟨e1.[e2]⟩» A′
    by (iprover intro: da.AVar)
  thus ?case ..
next
  case Nil thus ?case by (iprover intro: da.Nil)
next
  case (Cons Env B e E es A B′)
  note B′ = ‹B ⊆ B′›
```

obtain E' **where** E': *Env*$\vdash$ B' »$\langle e\rangle$» E'
proof $-$
have *PROP ?Hyp Env B* $\langle e\rangle$ **by** (*rule Cons.hyps*)
with B'
show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain A' **where** *Env*$\vdash$ *nrm* E' »$\langle es\rangle$» A'
proof $-$
have *Env*$\vdash$ B »$\langle e\rangle$» E **by** (*rule Cons.hyps*)
from *this* B' E'
have *nrm* $E \subseteq$ *nrm* E'
by (*rule da-monotone* [*THEN conjE*])
moreover
have *PROP ?Hyp Env* (*nrm E*) $\langle es\rangle$ **by** (*rule Cons.hyps*)
ultimately show *?thesis* **using** *that* **by** *iprover*
qed
ultimately
have *Env*$\vdash$ B' »$\langle e \# es\rangle$» A'
by (*iprover intro*: *da.Cons*)
thus *?case* **..**
qed
from *this* [*OF* ‹$B \subseteq B'$›] **show** *?thesis* **.**
qed

corollary *da-weakenE* [*consumes 2*]:
assumes *da*: *Env*$\vdash$ B »t» A **and**
B': $B \subseteq B'$ **and**
ex-mono: $\bigwedge$ A'. $[\![$*Env*$\vdash$ B' »t» A'; *nrm* $A \subseteq$ *nrm* A';
$\bigwedge$ l. *brk* A $l \subseteq$ *brk* A' $l]\!] \Longrightarrow P$
shows P
proof $-$
from *da* B'
obtain A' **where** A': *Env*$\vdash$ B' »t» A'
by (*rule da-weaken* [*elim-format*]) *iprover*
with *da* B'
have *nrm* $A \subseteq$ *nrm* $A' \wedge (\forall\ l.$ *brk* A $l \subseteq$ *brk* A' $l)$
by (*rule da-monotone*)
with A' *ex-mono*
show *?thesis*
by *iprover*
qed

end

Chapter 13

WellForm

1 Well-formedness of Java programs

theory *WellForm* **imports** *DefiniteAssignment* **begin**

For static checks on expressions and statements, see WellType.thy
improvements over Java Specification 1.0 (cf. 8.4.6.3, 8.4.6.4, 9.4.1):

- a method implementing or overwriting another method may have a result type that widens to the result type of the other method (instead of identical type)
- if a method hides another method (both methods have to be static!) there are no restrictions to the result type since the methods have to be static and there is no dynamic binding of static methods
- if an interface inherits more than one method with the same signature, the methods need not have identical return types

simplifications:

- Object and standard exceptions are assumed to be declared like normal classes

well-formed field declarations

well-formed field declaration (common part for classes and interfaces), cf. 8.3 and (9.3)

definition
  wf-fdecl :: *prog* ⇒ *pname* ⇒ *fdecl* ⇒ *bool*
  where *wf-fdecl G P* = (λ(*fn*,*f*). *is-acc-type G P* (*type f*))

lemma *wf-fdecl-def2*: ⋀*fd*. *wf-fdecl G P fd* = *is-acc-type G P* (*type* (*snd fd*))
apply (*unfold wf-fdecl-def*)
apply *simp*
done

well-formed method declarations

A method head is wellformed if:

- the signature and the method head agree in the number of parameters
- all types of the parameters are visible

- the result type is visible
- the parameter names are unique

definition
wf-mhead :: *prog* ⇒ *pname* ⇒ *sig* ⇒ *mhead* ⇒ *bool* **where**
$wf\text{-}mhead\ G\ P = (\lambda\ sig\ mh.\ length\ (parTs\ sig) = length\ (pars\ mh) \wedge$
$(\forall\ T{\in}set\ (parTs\ sig).\ is\text{-}acc\text{-}type\ G\ P\ T) \wedge$
$is\text{-}acc\text{-}type\ G\ P\ (resTy\ mh) \wedge$
$distinct\ (pars\ mh))$

A method declaration is wellformed if:

- the method head is wellformed
- the names of the local variables are unique
- the types of the local variables must be accessible
- the local variables don't shadow the parameters
- the class of the method is defined
- the body statement is welltyped with respect to the modified environment of local names, were the local variables, the parameters the special result variable (Res) and This are assoziated with there types.

definition
callee-lcl :: *qtname* ⇒ *sig* ⇒ *methd* ⇒ *lenv* **where**
$callee\text{-}lcl\ C\ sig\ m =$
$(\lambda k.\ (case\ k\ of$
$EName\ e$
$\Rightarrow (case\ e\ of$
$VNam\ v$
$\Rightarrow (table\text{-}of\ (lcls\ (mbody\ m))((pars\ m)[\mapsto](parTs\ sig)))\ v$
$\mid Res \Rightarrow Some\ (resTy\ m))$
$\mid This \Rightarrow if\ is\text{-}static\ m\ then\ None\ else\ Some\ (Class\ C)))$

definition
parameters :: *methd* ⇒ *lname set* **where**
$parameters\ m = set\ (map\ (EName \circ VNam)\ (pars\ m)) \cup (if\ (static\ m)\ then\ \{\}\ else\ \{This\})$

definition
wf-mdecl :: *prog* ⇒ *qtname* ⇒ *mdecl* ⇒ *bool* **where**
$wf\text{-}mdecl\ G\ C =$
$(\lambda(sig,m).$
$wf\text{-}mhead\ G\ (pid\ C)\ sig\ (mhead\ m) \wedge$
$unique\ (lcls\ (mbody\ m)) \wedge$
$(\forall (vn,T){\in}set\ (lcls\ (mbody\ m)).\ is\text{-}acc\text{-}type\ G\ (pid\ C)\ T) \wedge$
$(\forall pn{\in}set\ (pars\ m).\ table\text{-}of\ (lcls\ (mbody\ m))\ pn = None) \wedge$
$jumpNestingOkS\ \{Ret\}\ (stmt\ (mbody\ m)) \wedge$
$is\text{-}class\ G\ C \wedge$
$(\!|prg{=}G,cls{=}C,lcl{=}callee\text{-}lcl\ C\ sig\ m|\!)\vdash(stmt\ (mbody\ m))::\surd \wedge$
$(\exists\ A.\ (\!|prg{=}G,cls{=}C,lcl{=}callee\text{-}lcl\ C\ sig\ m|\!)$
$\vdash parameters\ m\ \gg\langle stmt\ (mbody\ m)\rangle\gg\ A$
$\wedge\ Result \in nrm\ A))$

lemma *callee-lcl-VNam-simp* [*simp*]:

callee-lcl C sig m (*EName* (*VNam v*))
  = (*table-of* (*lcls* (*mbody m*))((*pars m*)[↦](*parTs sig*))) *v*
by (*simp add*: *callee-lcl-def*)

lemma *callee-lcl-Res-simp* [*simp*]:
callee-lcl C sig m (*EName Res*) = *Some* (*resTy m*)
by (*simp add*: *callee-lcl-def*)

lemma *callee-lcl-This-simp* [*simp*]:
callee-lcl C sig m (*This*) = (*if is-static m then None else Some* (*Class C*))
by (*simp add*: *callee-lcl-def*)

lemma *callee-lcl-This-static-simp*:
is-static m ⟹ *callee-lcl C sig m* (*This*) = *None*
by *simp*

lemma *callee-lcl-This-not-static-simp*:
¬ *is-static m* ⟹ *callee-lcl C sig m* (*This*) = *Some* (*Class C*)
by *simp*

lemma *wf-mheadI*:
⟦*length* (*parTs sig*) = *length* (*pars m*); ∀ *T*∈*set* (*parTs sig*). *is-acc-type G P T*;
  is-acc-type G P (*resTy m*); *distinct* (*pars m*)⟧ ⟹
  wf-mhead G P sig m
apply (*unfold wf-mhead-def*)
apply (*simp* (*no-asm-simp*))
done

lemma *wf-mdeclI*: ⟦
  wf-mhead G (*pid C*) *sig* (*mhead m*); *unique* (*lcls* (*mbody m*));
  (∀ *pn*∈*set* (*pars m*). *table-of* (*lcls* (*mbody m*)) *pn* = *None*);
  ∀ (*vn*,*T*)∈*set* (*lcls* (*mbody m*)). *is-acc-type G* (*pid C*) *T*;
  jumpNestingOkS {*Ret*} (*stmt* (*mbody m*));
  is-class G C;
  ⦇*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*⦈⊢(*stmt* (*mbody m*))::√;
  (∃ *A*. ⦇*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*⦈ ⊢ *parameters m* »⟨*stmt* (*mbody m*)⟩» *A*
        ∧ *Result* ∈ *nrm A*)
  ⟧ ⟹
  wf-mdecl G C (*sig*,*m*)
apply (*unfold wf-mdecl-def*)
apply *simp*
done

lemma *wf-mdeclE* [*consumes 1*]:
  ⟦*wf-mdecl G C* (*sig*,*m*);
    ⟦*wf-mhead G* (*pid C*) *sig* (*mhead m*); *unique* (*lcls* (*mbody m*));
     ∀ *pn*∈*set* (*pars m*). *table-of* (*lcls* (*mbody m*)) *pn* = *None*;
     ∀ (*vn*,*T*)∈*set* (*lcls* (*mbody m*)). *is-acc-type G* (*pid C*) *T*;
     jumpNestingOkS {*Ret*} (*stmt* (*mbody m*));
     is-class G C;
     ⦇*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*⦈⊢(*stmt* (*mbody m*))::√;
   (∃ *A*. ⦇*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*⦈⊢ *parameters m* »⟨*stmt* (*mbody m*)⟩» *A*

$\wedge$ *Result* $\in$ *nrm A*)
$]\!] \Longrightarrow P$
$]\!] \Longrightarrow P$
by (*unfold wf-mdecl-def*) *simp*

lemma *wf-mdeclD1*:
wf-mdecl G C (*sig*,*m*) $\Longrightarrow$
wf-mhead G (*pid C*) *sig* (*mhead m*) $\wedge$ *unique* (*lcls* (*mbody m*)) $\wedge$
($\forall$ *pn*$\in$*set* (*pars m*). *table-of* (*lcls* (*mbody m*)) *pn* = *None*) $\wedge$
($\forall$ (*vn*,*T*)$\in$*set* (*lcls* (*mbody m*)). *is-acc-type G* (*pid C*) *T*)
apply (*unfold wf-mdecl-def*)
apply *simp*
done

lemma *wf-mdecl-bodyD*:
wf-mdecl G C (*sig*,*m*) $\Longrightarrow$
($\exists$ *T*. (|*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*|)$\vdash$*Body C* (*stmt* (*mbody m*))::$-$*T* $\wedge$
G$\vdash$*T*$\preceq$(*resTy m*))
apply (*unfold wf-mdecl-def*)
apply *clarify*
apply (*rule-tac x*=(*resTy m*) **in** *exI*)
apply (*unfold wf-mhead-def*)
apply (*auto simp add*: *wf-mhead-def is-acc-type-def intro*: *wt.Body*)
done

lemma *rT-is-acc-type*:
wf-mhead G P sig m $\Longrightarrow$ *is-acc-type G P* (*resTy m*)
apply (*unfold wf-mhead-def*)
apply *auto*
done

well-formed interface declarations

A interface declaration is wellformed if:

- the interface hierarchy is wellstructured
- there is no class with the same name
- the method heads are wellformed and not static and have Public access
- the methods are uniquely named
- all superinterfaces are accessible
- the result type of a method overriding a method of Object widens to the result type of the overridden method. Shadowing static methods is forbidden.
- the result type of a method overriding a set of methods defined in the superinterfaces widens to each of the corresponding result types

definition

$wf\text{-}idecl :: prog \Rightarrow idecl \Rightarrow bool$ **where**
$wf\text{-}idecl\ G =$
$(\lambda(I,i).$
$ws\text{-}idecl\ G\ I\ (isuperIfs\ i) \wedge$
$\neg is\text{-}class\ G\ I \wedge$
$(\forall (sig,mh)\in set\ (imethods\ i).\ wf\text{-}mhead\ G\ (pid\ I)\ sig\ mh \wedge$
$\neg is\text{-}static\ mh \wedge$
$accmodi\ mh = Public) \wedge$
$unique\ (imethods\ i) \wedge$
$(\forall\ J\in set\ (isuperIfs\ i).\ is\text{-}acc\text{-}iface\ G\ (pid\ I)\ J) \wedge$
$(table\text{-}of\ (imethods\ i)$
$hiding\ (methd\ G\ Object)$
$under\ (\lambda\ new\ old.\ accmodi\ old \neq Private)$
$entails\ (\lambda new\ old.\ G\vdash resTy\ new\preceq resTy\ old \wedge$
$is\text{-}static\ new = is\text{-}static\ old)) \wedge$
$(set\text{-}option \circ table\text{-}of\ (imethods\ i)$
$hidings\ Un\text{-}tables((\lambda J.(imethds\ G\ J))`set\ (isuperIfs\ i))$
$entails\ (\lambda new\ old.\ G\vdash resTy\ new\preceq resTy\ old)))$

lemma $wf\text{-}idecl\text{-}mhead$: $[\![wf\text{-}idecl\ G\ (I,i);\ (sig,mh)\in set\ (imethods\ i)]\!] \Longrightarrow$
$wf\text{-}mhead\ G\ (pid\ I)\ sig\ mh \wedge \neg is\text{-}static\ mh \wedge accmodi\ mh = Public$
apply (*unfold wf-idecl-def*)
apply *auto*
done

lemma $wf\text{-}idecl\text{-}hidings$:
$wf\text{-}idecl\ G\ (I,\ i) \Longrightarrow$
$(\lambda s.\ set\text{-}option\ (table\text{-}of\ (imethods\ i)\ s))$
$hidings\ Un\text{-}tables\ ((\lambda J.\ imethds\ G\ J)\ `\ set\ (isuperIfs\ i))$
$entails\ \lambda new\ old.\ G\vdash resTy\ new\preceq resTy\ old$
apply (*unfold wf-idecl-def o-def*)
apply *simp*
done

lemma $wf\text{-}idecl\text{-}hiding$:
$wf\text{-}idecl\ G\ (I,\ i) \Longrightarrow$
$(table\text{-}of\ (imethods\ i)$
$hiding\ (methd\ G\ Object)$
$under\ (\lambda\ new\ old.\ accmodi\ old \neq Private)$
$entails\ (\lambda new\ old.\ G\vdash resTy\ new\preceq resTy\ old \wedge$
$is\text{-}static\ new = is\text{-}static\ old))$
apply (*unfold wf-idecl-def*)
apply *simp*
done

lemma $wf\text{-}idecl\text{-}supD$:
$[\![wf\text{-}idecl\ G\ (I,i);\ J \in set\ (isuperIfs\ i)]\!]$
$\Longrightarrow is\text{-}acc\text{-}iface\ G\ (pid\ I)\ J \wedge (J,\ I) \notin (subint1\ G)^+$
apply (*unfold wf-idecl-def ws-idecl-def*)
apply *auto*
done

well-formed class declarations

A class declaration is wellformed if:

- there is no interface with the same name
- all superinterfaces are accessible and for all methods implementing an interface method the result type widens to the result type of the interface method, the method is not static and offers at least as much access (this actually means that the method has Public access, since all interface methods have public access)
- all field declarations are wellformed and the field names are unique
- all method declarations are wellformed and the method names are unique
- the initialization statement is welltyped
- the classhierarchy is wellstructured
- Unless the class is Object:
  - the superclass is accessible
  - for all methods overriding another method (of a superclass)the result type widens to the result type of the overridden method, the access modifier of the new method provides at least as much access as the overwritten one.
  - for all methods hiding a method (of a superclass) the hidden method must be static and offer at least as much access rights. Remark: In contrast to the Java Language Specification we don't restrict the result types of the method (as in case of overriding), because there seems to be no reason, since there is no dynamic binding of static methods. (cf. 8.4.6.3 vs. 15.12.1). Stricly speaking the restrictions on the access rights aren't necessary to, since the static type and the access rights together determine which method is to be called statically. But if a class gains more then one static method with the same signature due to inheritance, it is confusing when the method selection depends on the access rights only: e.g. Class C declares static public method foo(). Class D is subclass of C and declares static method foo() with default package access. D.foo() ? if this call is in the same package as D then foo of class D is called, otherwise foo of class C.

definition
  entails :: $('a,'b)$ *table* $\Rightarrow$ $('b \Rightarrow$ *bool*$)$ $\Rightarrow$ *bool* (- *entails* - *20*)
  where $(t\ \textit{entails}\ P) = (\forall k.\ \forall\ x \in t\ k{:}\ P\ x)$

lemma *entailsD*:
$[\![t\ \textit{entails}\ P;\ t\ k = \textit{Some}\ x]\!] \Longrightarrow P\ x$
by (*simp add*: *entails-def*)

lemma *empty-entails*[*simp*]: *Map.empty entails P*
by (*simp add*: *entails-def*)

definition
  wf-cdecl :: *prog* $\Rightarrow$ *cdecl* $\Rightarrow$ *bool* **where**
  wf-cdecl G =
    $(\lambda(C,c).$
    $\neg$*is-iface G C* $\wedge$
    $(\forall I{\in}set\ (superIfs\ c).\ is\text{-}acc\text{-}iface\ G\ (pid\ C)\ I\ \wedge$
      $(\forall s.\ \forall\ im \in imethds\ G\ I\ s.$
        $(\exists\ cm \in methd\ \ G\ C\ s{:}\ G{\vdash}resTy\ cm{\preceq}resTy\ im\ \wedge$
                $\neg\ is\text{-}static\ cm\ \wedge$
                $accmodi\ im \le accmodi\ cm)))\ \wedge$
    $(\forall f{\in}set\ (cfields\ c).\ wf\text{-}fdecl\ G\ (pid\ C)\ f)\ \wedge\ unique\ (cfields\ c)\ \wedge$

(∀ m∈set (methods c). wf-mdecl G C m) ∧ unique (methods c) ∧
jumpNestingOkS {} (init c) ∧
(∃ A. (|prg=G,cls=C,lcl=Map.empty|)⊢ {} »⟨init c⟩» A) ∧
(|prg=G,cls=C,lcl=Map.empty|)⊢(init c)::√ ∧ ws-cdecl G C (super c) ∧
(C ≠ Object ⟶
(is-acc-class G (pid C) (super c) ∧
(table-of (map (λ (s,m). (s,C,m)) (methods c))
entails (λ new. ∀ old sig.
(G,sig⊢new overrides$_S$ old
⟶ (G⊢resTy new⪯resTy old ∧
accmodi old ≤ accmodi new ∧
¬is-static old)) ∧
(G,sig⊢new hides old
⟶ (accmodi old ≤ accmodi new ∧
is-static old))))
)))

lemma *wf-cdeclE* [*consumes 1*]:
⟦wf-cdecl G (C,c);
⟦¬is-iface G C;
(∀ I∈set (superIfs c). is-acc-iface G (pid C) I ∧
(∀ s. ∀ im ∈ imethds G I s.
(∃ cm ∈ methd G C s: G⊢resTy cm⪯resTy im ∧
¬ is-static cm ∧
accmodi im ≤ accmodi cm)));
∀ f∈set (cfields c). wf-fdecl G (pid C) f; unique (cfields c);
∀ m∈set (methods c). wf-mdecl G C m; unique (methods c);
jumpNestingOkS {} (init c);
∃ A. (|prg=G,cls=C,lcl=Map.empty|)⊢ {} »⟨init c⟩» A;
(|prg=G,cls=C,lcl=Map.empty|)⊢(init c)::√;
ws-cdecl G C (super c);
(C ≠ Object ⟶
(is-acc-class G (pid C) (super c) ∧
(table-of (map (λ (s,m). (s,C,m)) (methods c))
entails (λ new. ∀ old sig.
(G,sig⊢new overrides$_S$ old
⟶ (G⊢resTy new⪯resTy old ∧
accmodi old ≤ accmodi new ∧
¬is-static old)) ∧
(G,sig⊢new hides old
⟶ (accmodi old ≤ accmodi new ∧
is-static old))))
))⟧ ⟹ P
⟧ ⟹ P
by (*unfold wf-cdecl-def*) *simp*

lemma *wf-cdecl-unique*:
wf-cdecl G (C,c) ⟹ unique (cfields c) ∧ unique (methods c)
apply (*unfold wf-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-fdecl*:
⟦wf-cdecl G (C,c); f∈set (cfields c)⟧ ⟹ wf-fdecl G (pid C) f

apply (*unfold wf-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-mdecl*:
⟦*wf-cdecl G* (*C*,*c*); *m*∈*set* (*methods c*)⟧ ⟹ *wf-mdecl G C m*
apply (*unfold wf-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-impD*:
⟦*wf-cdecl G* (*C*,*c*); *I*∈*set* (*superIfs c*)⟧
⟹ *is-acc-iface G* (*pid C*) *I* ∧
(∀ *s*. ∀ *im* ∈ *imethds G I s*.
(∃ *cm* ∈ *methd G C s*: *G*⊢*resTy cm*≼*resTy im* ∧ ¬*is-static cm* ∧
accmodi im ≤ *accmodi cm*))
apply (*unfold wf-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-supD*:
⟦*wf-cdecl G* (*C*,*c*); *C* ≠ *Object*⟧ ⟹
is-acc-class G (*pid C*) (*super c*) ∧ (*super c*,*C*) ∉ $(subcls1\ G)^+$ ∧
(*table-of* (*map* (λ (*s*,*m*). (*s*,*C*,*m*)) (*methods c*))
entails (λ *new*. ∀ *old sig*.
(*G*,*sig*⊢*new* $overrides_S$ *old*
⟶ (*G*⊢*resTy new*≼*resTy old* ∧
accmodi old ≤ *accmodi new* ∧
¬*is-static old*)) ∧
(*G*,*sig*⊢*new hides old*
⟶ (*accmodi old* ≤ *accmodi new* ∧
is-static old))))
apply (*unfold wf-cdecl-def ws-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-overrides-SomeD*:
⟦*wf-cdecl G* (*C*,*c*); *C* ≠ *Object*; *table-of* (*methods c*) *sig* = *Some newM*;
G,*sig*⊢(*C*,*newM*) $overrides_S$ *old*
⟧ ⟹ *G*⊢*resTy newM*≼*resTy old* ∧
accmodi old ≤ *accmodi newM* ∧
¬ *is-static old*
apply (*drule* (*1*) *wf-cdecl-supD*)
apply (*clarify*)
apply (*drule entailsD*)
apply (*blast intro*: *table-of-map-SomeI*)
apply (*drule-tac x*=*old* **in** *spec*)
apply (*auto dest*: *overrides-eq-sigD simp add*: *msig-def*)
done

lemma *wf-cdecl-hides-SomeD*:
⟦*wf-cdecl G* (*C*,*c*); *C* ≠ *Object*; *table-of* (*methods c*) *sig* = *Some newM*;
G,*sig*⊢(*C*,*newM*) *hides old*

$\rrbracket \Longrightarrow$ *accmodi old* $\leq$ *access newM* $\wedge$
is-static old
apply (*drule* (*1*) *wf-cdecl-supD*)
apply (*clarify*)
apply (*drule entailsD*)
apply (*blast intro*: *table-of-map-SomeI*)
apply (*drule-tac x=old* **in** *spec*)
apply (*auto dest*: *hides-eq-sigD simp add*: *msig-def*)
done

lemma *wf-cdecl-wt-init*:
wf-cdecl G (*C*, *c*) $\Longrightarrow$ (|*prg=G*,*cls=C*,*lcl=Map.empty*|)$\vdash$*init c*::$\surd$
apply (*unfold wf-cdecl-def*)
apply *auto*
done

well-formed programs

A program declaration is wellformed if:

- the class ObjectC of Object is defined
- every method of Object has an access modifier distinct from Package. This is necessary since every interface automatically inherits from Object. We must know, that every time a Object method is "overriden" by an interface method this is also overriden by the class implementing the the interface (see *implement-dynmethd and class-mheadsD*)
- all standard Exceptions are defined
- all defined interfaces are wellformed
- all defined classes are wellformed

definition
wf-prog :: *prog* $\Rightarrow$ *bool* **where**
wf-prog G = (*let is* = *ifaces G*; *cs* = *classes G in*
ObjectC $\in$ *set cs* $\wedge$
($\forall$ *m*$\in$*set Object-mdecls*. *accmodi m* $\neq$ *Package*) $\wedge$
($\forall$ *xn*. *SXcptC xn* $\in$ *set cs*) $\wedge$
($\forall$ *i*$\in$*set is*. *wf-idecl G i*) $\wedge$ *unique is* $\wedge$
($\forall$ *c*$\in$*set cs*. *wf-cdecl G c*) $\wedge$ *unique cs*)

lemma *wf-prog-idecl*: $\llbracket$*iface G I* = *Some i*; *wf-prog G*$\rrbracket$ $\Longrightarrow$ *wf-idecl G* (*I*,*i*)
apply (*unfold wf-prog-def Let-def*)
apply *simp*
apply (*fast dest*: *map-of-SomeD*)
done

lemma *wf-prog-cdecl*: $\llbracket$*class G C* = *Some c*; *wf-prog G*$\rrbracket$ $\Longrightarrow$ *wf-cdecl G* (*C*,*c*)
apply (*unfold wf-prog-def Let-def*)
apply *simp*
apply (*fast dest*: *map-of-SomeD*)
done

lemma *wf-prog-Object-mdecls*:

wf-prog G $\Longrightarrow$ ($\forall$ *m*$\in$*set Object-mdecls*. *accmodi m* $\neq$ *Package*)
apply (*unfold wf-prog-def Let-def*)
apply *simp*
done

lemma *wf-prog-acc-superD*:
$[\![$*wf-prog G*; *class G C* = *Some c*; *C* $\neq$ *Object* $]\!]$
$\Longrightarrow$ *is-acc-class G* (*pid C*) (*super c*)
by (*auto dest*: *wf-prog-cdecl wf-cdecl-supD*)

lemma *wf-ws-prog* [*elim*!,*simp*]: *wf-prog G* $\Longrightarrow$ *ws-prog G*
apply (*unfold wf-prog-def Let-def*)
apply (*rule ws-progI*)
apply (*simp-all* (*no-asm*))
apply (*auto simp add*: *is-acc-class-def is-acc-iface-def*
dest!: *wf-idecl-supD wf-cdecl-supD*)+
done

lemma *class-Object* [*simp*]:
wf-prog G $\Longrightarrow$
class G Object = *Some* (|*access*=*Public*,*cfields*=[],*methods*=*Object-mdecls*,
init=*Skip*,*super*=*undefined*,*superIfs*=[]|)
apply (*unfold wf-prog-def Let-def ObjectC-def*)
apply (*fast dest*!: *map-of-SomeI*)
done

lemma *methd-Object*[*simp*]: *wf-prog G* $\Longrightarrow$ *methd G Object* =
table-of (*map* (λ(*s*,*m*). (*s*, *Object*, *m*)) *Object-mdecls*)
apply (*subst methd-rec*)
apply (*auto simp add*: *Let-def*)
done

lemma *wf-prog-Object-methd*:
$[\![$*wf-prog G*; *methd G Object sig* = *Some m*$]\!]$ $\Longrightarrow$ *accmodi m* $\neq$ *Package*
by (*auto dest*!: *wf-prog-Object-mdecls*) (*auto dest*!: *map-of-SomeD*)

lemma *wf-prog-Object-is-public*[*intro*]:
wf-prog G $\Longrightarrow$ *is-public G Object*
by (*auto simp add*: *is-public-def dest*: *class-Object*)

lemma *class-SXcpt* [*simp*]:
wf-prog G $\Longrightarrow$
class G (*SXcpt xn*) = *Some* (|*access*=*Public*,*cfields*=[],*methods*=*SXcpt-mdecls*,
init=*Skip*,
super=*if xn* = *Throwable then Object*
else SXcpt Throwable,
superIfs=[]|)
apply (*unfold wf-prog-def Let-def SXcptC-def*)
apply (*fast dest*!: *map-of-SomeI*)
done

lemma *wf-ObjectC* [*simp*]:
wf-cdecl G ObjectC = ($\neg$*is-iface G Object* $\wedge$ *Ball* (*set Object-mdecls*)
(*wf-mdecl G Object*) $\wedge$ *unique Object-mdecls*)
apply (*unfold wf-cdecl-def ws-cdecl-def ObjectC-def*)
apply (*auto intro*: *da.Skip*)
done

lemma *Object-is-class* [*simp*,*elim*!]: *wf-prog G* $\Longrightarrow$ *is-class G Object*
apply (*simp* (*no-asm-simp*))
done

lemma *Object-is-acc-class* [*simp*,*elim*!]: *wf-prog G* $\Longrightarrow$ *is-acc-class G S Object*
apply (*simp* (*no-asm-simp*) *add*: *is-acc-class-def is-public-def*
accessible-in-RefT-simp)
done

lemma *SXcpt-is-class* [*simp*,*elim*!]: *wf-prog G* $\Longrightarrow$ *is-class G* (*SXcpt xn*)
apply (*simp* (*no-asm-simp*))
done

lemma *SXcpt-is-acc-class* [*simp*,*elim*!]:
wf-prog G $\Longrightarrow$ *is-acc-class G S* (*SXcpt xn*)
apply (*simp* (*no-asm-simp*) *add*: *is-acc-class-def is-public-def*
accessible-in-RefT-simp)
done

lemma *fields-Object* [*simp*]: *wf-prog G* $\Longrightarrow$ *DeclConcepts.fields G Object* = []
by (*force intro*: *fields-emptyI*)

lemma *accfield-Object* [*simp*]:
wf-prog G $\Longrightarrow$ *accfield G S Object* = *Map.empty*
apply (*unfold accfield-def*)
apply (*simp* (*no-asm-simp*) *add*: *Let-def*)
done

lemma *fields-Throwable* [*simp*]:
wf-prog G $\Longrightarrow$ *DeclConcepts.fields G* (*SXcpt Throwable*) = []
by (*force intro*: *fields-emptyI*)

lemma *fields-SXcpt* [*simp*]: *wf-prog G* $\Longrightarrow$ *DeclConcepts.fields G* (*SXcpt xn*) = []
apply (*case-tac xn* = *Throwable*)
apply (*simp* (*no-asm-simp*))
by (*force intro*: *fields-emptyI*)

lemmas *widen-trans* = *ws-widen-trans* [*OF - - wf-ws-prog*, *elim*]

lemma *widen-trans2* [*elim*]: $[\![G\vdash U\preceq T;\ G\vdash S\preceq U;\ wf\text{-}prog\ G]\!] \Longrightarrow G\vdash S\preceq T$
apply (*erule* (*2*) *widen-trans*)
done

lemma *Xcpt-subcls-Throwable* [*simp*]:
wf-prog G $\Longrightarrow$ *G*$\vdash$*SXcpt xn*$\preceq_C$ *SXcpt Throwable*
apply (*rule SXcpt-subcls-Throwable-lemma*)
apply *auto*
done

lemma *unique-fields*:
$[\![$*is-class G C*; *wf-prog G*$]\!]$ $\Longrightarrow$ *unique* (*DeclConcepts.fields G C*)
apply (*erule ws-unique-fields*)
apply (*erule wf-ws-prog*)
apply (*erule* (*1*) *wf-prog-cdecl* [*THEN wf-cdecl-unique* [*THEN conjunct1*]])
done

lemma *fields-mono*:
$[\![$*table-of* (*DeclConcepts.fields G C*) *fn* = *Some f*; *G*$\vdash$*D*$\preceq_C$ *C*;
is-class G D; *wf-prog G*$]\!]$
$\Longrightarrow$ *table-of* (*DeclConcepts.fields G D*) *fn* = *Some f*
apply (*rule map-of-SomeI*)
apply (*erule* (*1*) *unique-fields*)
apply (*erule* (*1*) *map-of-SomeD* [*THEN fields-mono-lemma*])
apply (*erule wf-ws-prog*)
done

lemma *fields-is-type* [*elim*]:
$[\![$*table-of* (*DeclConcepts.fields G C*) *m* = *Some f*; *wf-prog G*; *is-class G C*$]\!]$ $\Longrightarrow$
is-type G (*type f*)
apply (*frule wf-ws-prog*)
apply (*force dest*: *fields-declC* [*THEN conjunct1*]
wf-prog-cdecl [*THEN wf-cdecl-fdecl*]
simp add: *wf-fdecl-def2 is-acc-type-def*)
done

lemma *imethds-wf-mhead* [*rule-format* (*no-asm*)]:
$[\![$*m* $\in$ *imethds G I sig*; *wf-prog G*; *is-iface G I*$]\!]$ $\Longrightarrow$
wf-mhead G (*pid* (*decliface m*)) *sig* (*mthd m*) $\wedge$
$\neg$ *is-static m* $\wedge$ *accmodi m* = *Public*
apply (*frule wf-ws-prog*)
apply (*drule* (*2*) *imethds-declI* [*THEN conjunct1*])
apply *clarify*
apply (*frule-tac I*=(*decliface m*) **in** *wf-prog-idecl*,*assumption*)
apply (*drule wf-idecl-mhead*)
apply (*erule map-of-SomeD*)
apply (*cases m*, *simp*)
done

lemma *methd-wf-mdecl*:
$[\![$*methd G C sig* = *Some m*; *wf-prog G*; *class G C* = *Some y*$]\!]$ $\Longrightarrow$
G$\vdash$*C*$\preceq_C$ (*declclass m*) $\wedge$ *is-class G* (*declclass m*) $\wedge$
wf-mdecl G (*declclass m*) (*sig*,(*mthd m*))
apply (*frule wf-ws-prog*)
apply (*drule* (*1*) *methd-declC*)
apply *fast*
apply *clarsimp*

apply (*frule* (*1*) *wf-prog-cdecl*, *erule wf-cdecl-mdecl*, *erule map-of-SomeD*)
done

lemma *methd-rT-is-type*:
$[\![$*wf-prog G*;*methd G C sig* = *Some m*;
  class G C = *Some y*$]\!]$
$\Longrightarrow$ *is-type G* (*resTy m*)
apply (*drule* (*2*) *methd-wf-mdecl*)
apply *clarify*
apply (*drule wf-mdeclD1*)
apply *clarify*
apply (*drule rT-is-acc-type*)
apply (*cases m*, *simp add*: *is-acc-type-def*)
done

lemma *accmethd-rT-is-type*:
$[\![$*wf-prog G*;*accmethd G S C sig* = *Some m*;
  class G C = *Some y*$]\!]$
$\Longrightarrow$ *is-type G* (*resTy m*)
by (*auto simp add*: *accmethd-def*
    intro: *methd-rT-is-type*)

lemma *methd-Object-SomeD*:
$[\![$*wf-prog G*;*methd G Object sig* = *Some m*$]\!]$
$\Longrightarrow$ *declclass m* = *Object*
by (*auto dest*: *class-Object simp add*: *methd-rec*)

lemmas *iface-rec-induct'* = *iface-rec.induct* [*of %x y z. P x y*] **for** *P*

lemma *wf-imethdsD*:
$[\![$*im* $\in$ *imethds G I sig*;*wf-prog G*; *is-iface G I*$]\!]$
$\Longrightarrow$ $\neg$*is-static im* $\wedge$ *accmodi im* = *Public*
proof −
 assume *asm*: *wf-prog G is-iface G I im* $\in$ *imethds G I sig*

 have *wf-prog G* $\longrightarrow$
   ($\forall$ *i im. iface G I* = *Some i* $\longrightarrow$ *im* $\in$ *imethds G I sig*
     $\longrightarrow$ $\neg$*is-static im* $\wedge$ *accmodi im* = *Public*) (**is** *?P G I*)
 proof (*induct G I rule*: *iface-rec-induct'*, *intro allI impI*)
  fix *G I i im*
  assume *hyp*: $\bigwedge$ *i J. iface G I* = *Some i* $\Longrightarrow$ *ws-prog G* $\Longrightarrow$ *J* $\in$ *set* (*isuperIfs i*)
     $\Longrightarrow$ *?P G J*
  assume *wf*: *wf-prog G* **and** *if-I*: *iface G I* = *Some i* **and**
    im: *im* $\in$ *imethds G I sig*
  show $\neg$*is-static im* $\wedge$ *accmodi im* = *Public*
  proof −
   let *?inherited* = *Un-tables* (*imethds G ‘ set* (*isuperIfs i*))
   let *?new* = (*set-option* $\circ$ *table-of* (*map* (λ(*s*, *mh*). (*s*, *I*, *mh*)) (*imethods i*)))
   from *if-I wf im* **have** *imethds*:*im* $\in$ (*?inherited* $\oplus\oplus$ *?new*) *sig*
    by (*simp add*: *imethds-rec*)
   from *wf if-I* **have**
    wf-supI: $\forall$ *J. J* $\in$ *set* (*isuperIfs i*) $\longrightarrow$ ($\exists$ *j. iface G J* = *Some j*)

        by (*blast dest*: *wf-prog-idecl wf-idecl-supD is-acc-ifaceD*)
      from *wf if-I* **have**
        $\forall$ *im* $\in$ *set* (*imethods i*). $\neg$ *is-static im* $\wedge$ *accmodi im* = *Public*
        by (*auto dest*!: *wf-prog-idecl wf-idecl-mhead*)
      then have *new-ok*: $\forall$ *im*. *table-of* (*imethods i*) *sig* = *Some im*
                $\longrightarrow$ $\neg$ *is-static im* $\wedge$ *accmodi im* = *Public*
        by (*auto dest*!: *table-of-Some-in-set*)
      show *?thesis*
        proof (*cases ?new sig* = {})
          case *True*
          from *True wf wf-supI if-I imethds hyp*
          show *?thesis* **by** (*auto simp del*: *split-paired-All*)
        next
          case *False*
          from *False wf wf-supI if-I imethds new-ok hyp*
          show *?thesis* **by** (*auto dest*: *wf-idecl-hidings hidings-entailsD*)
        qed
      qed
    qed
  with *asm* **show** *?thesis* **by** (*auto simp del*: *split-paired-All*)
qed

lemma *wf-prog-hidesD*:
  assumes *hides*: *G* $\vdash$*new hides old* **and** *wf*: *wf-prog G*
  shows
   accmodi old $\leq$ *accmodi new* $\wedge$
    is-static old
proof −
  from *hides*
  obtain *c* **where**
    clsNew: *class G* (*declclass new*) = *Some c* **and**
    neqObj: *declclass new* $\neq$ *Object*
    by (*auto dest*: *hidesD declared-in-classD*)
  with *hides* **obtain** *newM oldM* **where**
    newM: *table-of* (*methods c*) (*msig new*) = *Some newM* **and**
     new: *new* = (*declclass new*,(*msig new*),*newM*) **and**
     old: *old* = (*declclass old*,(*msig old*),*oldM*) **and**
          msig new = *msig old*
    by (*cases new*,*cases old*)
       (*auto dest*: *hidesD*
        simp add: *cdeclaredmethd-def declared-in-def*)
  with *hides*
  have *hides′*:
        G,(*msig new*)$\vdash$(*declclass new*,*newM*) *hides* (*declclass old*,*oldM*)
    by *auto*
  from *clsNew wf*
  have *wf-cdecl G* (*declclass new*,*c*) **by** (*blast intro*: *wf-prog-cdecl*)
  note *wf-cdecl-hides-SomeD* [*OF this neqObj newM hides′*]
  with *new old*
  show *?thesis*
    by (*cases new*, *cases old*) *auto*
qed

Compare this lemma about static overriding $G \vdash$ *new overrides*$_S$ *old* with the definition of dynamic overriding $G \vdash$ *new overrides old.* Conforming result types and restrictions on the access modifiers of the old and the new method are not part of the predicate for static overriding. But they are enshured in a wellfromed program. Dynamic overriding has no restrictions on the access modifiers but enforces confrom result types as precondition. But with some efford we can guarantee the access modifier restriction for dynamic overriding, too. See lemma *wf-prog-dyn-override-prop*.

lemma *wf-prog-stat-overridesD*:
  assumes *stat-override*: $G \vdash new\ overrides_S\ old$ **and** *wf*: *wf-prog G*
  shows
   $G\vdash resTy\ new \preceq resTy\ old\ \wedge$
   $accmodi\ old \leq accmodi\ new\ \wedge$
   $\neg$ *is-static old*
proof −
  from *stat-override*
  obtain *c* **where**
    clsNew: *class G* (*declclass new*) = *Some c* **and**
    neqObj: *declclass new* $\neq$ *Object*
    by (*auto dest*: *stat-overrides-commonD declared-in-classD*)
  with *stat-override* **obtain** *newM oldM* **where**
    newM: *table-of* (*methods c*) (*msig new*) = *Some newM* **and**
     new: *new* = (*declclass new*,(*msig new*),*newM*) **and**
     old: *old* = (*declclass old*,(*msig old*),*oldM*) **and**
        msig new = *msig old*
    by (*cases new*,*cases old*)
      (*auto dest*: *stat-overrides-commonD*
       simp add: *cdeclaredmethd-def declared-in-def*)
  with *stat-override*
  have *stat-override'*:
      $G,(msig\ new)\vdash(declclass\ new,newM)\ overrides_S\ (declclass\ old,oldM)$
    by *auto*
  from *clsNew wf*
  have *wf-cdecl G* (*declclass new*,*c*) **by** (*blast intro*: *wf-prog-cdecl*)
  note *wf-cdecl-overrides-SomeD* [*OF this neqObj newM stat-override'*]
  with *new old*
  show *?thesis*
    by (*cases new*, *cases old*) *auto*
qed

lemma *static-to-dynamic-overriding*:
  assumes *stat-override*: $G\vdash new\ overrides_S\ old$ **and** *wf* : *wf-prog G*
  shows $G\vdash new\ overrides\ old$
proof −
  from *stat-override*
  show *?thesis* (**is** *?Overrides new old*)
  proof (*induct*)
    case (*Direct new old superNew*)
    then have *stat-override*: $G\vdash new\ overrides_S\ old$
      by (*rule stat-overridesR.Direct*)
    from *stat-override wf*
    have *resTy-widen*: $G\vdash resTy\ new \preceq resTy\ old$ **and**
      not-static-old: $\neg$ *is-static old*
      by (*auto dest*: *wf-prog-stat-overridesD*)
    have *not-private-new*: *accmodi new* $\neq$ *Private*
    proof −
      from *stat-override*
      have *accmodi old* $\neq$ *Private*
        by (*rule no-Private-stat-override*)
      moreover
      from *stat-override wf*
      have *accmodi old* $\leq$ *accmodi new*
        by (*auto dest*: *wf-prog-stat-overridesD*)
      ultimately

```
    show ?thesis
      by (auto dest: acc-modi-bottom)
  qed
  with Direct resTy-widen not-static-old
  show ?Overrides new old
    by (auto intro: overridesR.Direct stat-override-declclasses-relation)
next
  case (Indirect new inter old)
  then show ?Overrides new old
    by (blast intro: overridesR.Indirect)
qed
qed


lemma non-Package-instance-method-inheritance:
  assumes old-inheritable: G⊢Method old inheritable-in (pid C) and
          accmodi-old: accmodi old ≠ Package and
      instance-method: ¬ is-static old and
               subcls: G⊢C ≺C declclass old and
         old-declared: G⊢Method old declared-in (declclass old) and
                   wf: wf-prog G
  shows G⊢Method old member-of C ∨
   (∃ new. G⊢ new overridesS old ∧ G⊢Method new member-of C)
proof −
  from wf have ws: ws-prog G by auto
  from old-declared have iscls-declC-old: is-class G (declclass old)
    by (auto simp add: declared-in-def cdeclaredmethd-def)
  from subcls have  iscls-C: is-class G C
    by (blast dest:  subcls-is-class)
  from iscls-C ws old-inheritable subcls
  show ?thesis (is ?P C old)
  proof (induct rule: ws-class-induct')
    case Object
    assume G⊢Object≺C declclass old
    then show ?P Object old
      by blast
  next
    case (Subcls C c)
    assume cls-C: class G C = Some c and
       neq-C-Obj: C ≠ Object and
             hyp: [[G ⊢Method old inheritable-in pid (super c);
                   G⊢super c≺C declclass old]] ⟹ ?P (super c) old and
     inheritable: G ⊢Method old inheritable-in pid C and
        subclsC: G⊢C≺C declclass old
    from cls-C neq-C-Obj
    have super: G⊢C ≺C1 super c
      by (rule subcls1I)
    from wf cls-C neq-C-Obj
    have accessible-super: G⊢(Class (super c)) accessible-in (pid C)
      by (auto dest: wf-prog-cdecl wf-cdecl-supD is-acc-classD)
    {
      fix old
      assume    member-super: G⊢Method old member-of (super c)
      assume     inheritable: G ⊢Method old inheritable-in pid C
      assume instance-method: ¬ is-static old
      from member-super
      have old-declared: G⊢Method old declared-in (declclass old)
       by (cases old) (auto dest: member-of-declC)
      have ?P C old
```

```
    proof (cases G⊢mid (msig old) undeclared-in C)
      case True
      with inheritable super accessible-super member-super
      have G⊢Method old member-of C
        by (cases old) (auto intro: members.Inherited)
      then show ?thesis
        by auto
    next
      case False
      then obtain new-member where
           G⊢new-member declared-in C and
           mid (msig old) = memberid new-member
        by (auto dest: not-undeclared-declared)
      then obtain new where
                new: G⊢Method new declared-in C and
             eq-sig: msig old = msig new and
          declC-new: declclass new = C
        by (cases new-member) auto
      then have member-new: G⊢Method new member-of C
        by (cases new) (auto intro: members.Immediate)
      from declC-new super member-super
      have subcls-new-old: G⊢declclass new ≺C declclass old
        by (auto dest!: member-of-subclseq-declC
                 dest: r-into-trancl intro: trancl-rtrancl-trancl)
      show ?thesis
      proof (cases is-static new)
        case False
        with eq-sig declC-new new old-declared inheritable
             super member-super subcls-new-old
        have G⊢new overridesS old
          by (auto intro!: stat-overridesR.Direct)
        with member-new show ?thesis
          by blast
      next
        case True
        with eq-sig declC-new subcls-new-old new old-declared inheritable
        have G⊢new hides old
          by (auto intro: hidesI)
        with wf
        have is-static old
          by (blast dest: wf-prog-hidesD)
        with instance-method
        show ?thesis
          by (contradiction)
      qed
    qed
  } note hyp-member-super = this
  from subclsC cls-C
  have G⊢(super c)≼C declclass old
    by (rule subcls-superD)
  then
  show ?P C old
  proof (cases rule: subclseq-cases)
    case Eq
    assume super c = declclass old
    with old-declared
    have G⊢Method old member-of (super c)
      by (cases old) (auto intro: members.Immediate)
    with inheritable instance-method
```

show *?thesis*
by (*blast dest*: *hyp-member-super*)
next
case *Subcls*
assume $G \vdash super\ c \prec_C declclass\ old$
moreover
from *inheritable accmodi-old*
have $G \vdash Method\ old\ inheritable\text{-}in\ pid\ (super\ c)$
by (*cases accmodi old*) (*auto simp add*: *inheritable-in-def*)
ultimately
have *?P* (*super c*) *old*
by (*blast dest*: *hyp*)
then show *?thesis*
proof
assume $G \vdash Method\ old\ member\text{-}of\ super\ c$
with *inheritable instance-method*
show *?thesis*
by (*blast dest*: *hyp-member-super*)
next
assume $\exists new.\ G \vdash new\ overrides_S\ old \wedge G \vdash Method\ new\ member\text{-}of\ super\ c$
then obtain *super-new* **where**
super-new-override: $G \vdash super\text{-}new\ overrides_S\ old$ **and**
super-new-member: $G \vdash Method\ super\text{-}new\ member\text{-}of\ super\ c$
by *blast*
from *super-new-override wf*
have $accmodi\ old \leq accmodi\ super\text{-}new$
by (*auto dest*: *wf-prog-stat-overridesD*)
with *inheritable accmodi-old*
have $G \vdash Method\ super\text{-}new\ inheritable\text{-}in\ pid\ C$
by (*auto simp add*: *inheritable-in-def*
split: *acc-modi.splits*
dest: *acc-modi-le-Dests*)
moreover
from *super-new-override*
have $\neg$ *is-static super-new*
by (*auto dest*: *stat-overrides-commonD*)
moreover
note *super-new-member*
ultimately have *?P C super-new*
by (*auto dest*: *hyp-member-super*)
then show *?thesis*
proof
assume $G \vdash Method\ super\text{-}new\ member\text{-}of\ C$
with *super-new-override*
show *?thesis*
by *blast*
next
assume $\exists new.\ G \vdash new\ overrides_S\ super\text{-}new \wedge$
$G \vdash Method\ new\ member\text{-}of\ C$
with *super-new-override* **show** *?thesis*
by (*blast intro*: *stat-overridesR.Indirect*)
qed
qed
qed
qed
qed

lemma *non-Package-instance-method-inheritance-cases*:

assumes *old-inheritable*: $G \vdash Method\ old\ inheritable\text{-}in\ (pid\ C)$ **and**
accmodi-old: $accmodi\ old \neq Package$ **and**
instance-method: $\neg\ is\text{-}static\ old$ **and**
subcls: $G \vdash C \prec_C declclass\ old$ **and**
old-declared: $G \vdash Method\ old\ declared\text{-}in\ (declclass\ old)$ **and**
wf: $wf\text{-}prog\ G$
obtains (*Inheritance*) $G \vdash Method\ old\ member\text{-}of\ C$
| (*Overriding*) *new* **where** $G \vdash new\ overrides_S\ old$ **and** $G \vdash Method\ new\ member\text{-}of\ C$
proof −
from *old-inheritable accmodi-old instance-method subcls old-declared wf Inheritance Overriding*
show *thesis*
by (*auto dest*: *non-Package-instance-method-inheritance*)
qed

lemma *dynamic-to-static-overriding*:
assumes *dyn-override*: $G \vdash new\ overrides\ old$ **and**
accmodi-old: $accmodi\ old \neq Package$ **and**
wf: $wf\text{-}prog\ G$
shows $G \vdash new\ overrides_S\ old$
proof −
from *dyn-override accmodi-old*
show *?thesis* (**is** *?Overrides new old*)
proof (*induct rule*: *overridesR.induct*)
case (*Direct new old*)
assume *new-declared*: $G \vdash Method\ new\ declared\text{-}in\ declclass\ new$
assume *eq-sig-new-old*: $msig\ new = msig\ old$
assume *subcls-new-old*: $G \vdash declclass\ new \prec_C declclass\ old$
assume $G \vdash Method\ old\ inheritable\text{-}in\ pid\ (declclass\ new)$ **and**
$accmodi\ old \neq Package$ **and**
$\neg\ is\text{-}static\ old$ **and**
$G \vdash declclass\ new \prec_C declclass\ old$ **and**
$G \vdash Method\ old\ declared\text{-}in\ declclass\ old$
from *this wf*
show *?Overrides new old*
proof (*cases rule*: *non-Package-instance-method-inheritance-cases*)
case *Inheritance*
assume $G \vdash Method\ old\ member\text{-}of\ declclass\ new$
then have $G \vdash mid\ (msig\ old)\ undeclared\text{-}in\ declclass\ new$
proof *cases*
case *Immediate*
with *subcls-new-old wf* **show** *?thesis*
by (*auto dest*: *subcls-irrefl*)
next
case *Inherited*
then show *?thesis*
by (*cases old*) *auto*
qed
with *eq-sig-new-old new-declared*
show *?thesis*
by (*cases old*,*cases new*) (*auto dest*!: *declared-not-undeclared*)
next
case (*Overriding new'*)
assume *stat-override-new'*: $G \vdash new'\ overrides_S\ old$
then have $msig\ new' = msig\ old$
by (*auto dest*: *stat-overrides-commonD*)
with *eq-sig-new-old* **have** *eq-sig-new-new'*: $msig\ new = msig\ new'$
by *simp*

```
      assume G ⊢Method new' member-of declclass new
      then show ?thesis
      proof (cases)
        case Immediate
        then have declC-new: declclass new' = declclass new
          by auto
        from Immediate
        have G⊢Method new' declared-in declclass new
          by (cases new') auto
        with new-declared eq-sig-new-new' declC-new
        have new=new'
          by (cases new, cases new') (auto dest: unique-declared-in)
        with stat-override-new'
        show ?thesis
          by simp
      next
        case Inherited
        then have G⊢mid (msig new') undeclared-in declclass new
          by (cases new') (auto)
        with eq-sig-new-new' new-declared
        show ?thesis
          by (cases new,cases new') (auto dest!: declared-not-undeclared)
      qed
    qed
  next
    case (Indirect new inter old)
    assume accmodi-old: accmodi old ≠ Package
    assume accmodi old ≠ Package ⟹ G ⊢ inter overrides_S old
    with accmodi-old
    have stat-override-inter-old: G ⊢ inter overrides_S old
      by blast
    moreover
    assume hyp-inter: accmodi inter ≠ Package ⟹ G ⊢ new overrides_S inter
    moreover
    have accmodi inter ≠ Package
    proof −
      from stat-override-inter-old wf
      have accmodi old ≤ accmodi inter
        by (auto dest: wf-prog-stat-overridesD)
      with stat-override-inter-old accmodi-old
      show ?thesis
        by (auto dest!: no-Private-stat-override
                 split: acc-modi.splits
                 dest: acc-modi-le-Dests)
    qed
    ultimately show ?Overrides new old
      by (blast intro: stat-overridesR.Indirect)
  qed
qed


lemma wf-prog-dyn-override-prop:
  assumes dyn-override: G ⊢ new overrides old and
                 wf: wf-prog G
  shows accmodi old ≤ accmodi new
proof (cases accmodi old = Package)
  case True
  note old-Package = this
  show ?thesis
```

proof (*cases accmodi old* $\leq$ *accmodi new*)
case *True* **then show** *?thesis* **.**
next
case *False*
with *old-Package*
have *accmodi new* = *Private*
by (*cases accmodi new*) (*auto simp add*: *le-acc-def less-acc-def*)
with *dyn-override*
show *?thesis*
by (*auto dest*: *overrides-commonD*)
qed
next
case *False*
with *dyn-override wf*
have $G \vdash$ *new overrides*$_S$ *old*
by (*blast intro*: *dynamic-to-static-overriding*)
with *wf*
show *?thesis*
by (*blast dest*: *wf-prog-stat-overridesD*)
qed

lemma *overrides-Package-old*:
assumes *dyn-override*: $G \vdash$ *new overrides old* **and**
accmodi-new: *accmodi new* = *Package* **and**
wf: *wf-prog G*
shows *accmodi old* = *Package*
proof (*cases accmodi old*)
case *Private*
with *dyn-override* **show** *?thesis*
by (*simp add*: *no-Private-override*)
next
case *Package*
then show *?thesis* **.**
next
case *Protected*
with *dyn-override wf*
have $G \vdash$ *new overrides*$_S$ *old*
by (*auto intro*: *dynamic-to-static-overriding*)
with *wf*
have *accmodi old* $\leq$ *accmodi new*
by (*auto dest*: *wf-prog-stat-overridesD*)
with *Protected accmodi-new*
show *?thesis*
by (*simp add*: *less-acc-def le-acc-def*)
next
case *Public*
with *dyn-override wf*
have $G \vdash$ *new overrides*$_S$ *old*
by (*auto intro*: *dynamic-to-static-overriding*)
with *wf*
have *accmodi old* $\leq$ *accmodi new*
by (*auto dest*: *wf-prog-stat-overridesD*)
with *Public accmodi-new*
show *?thesis*
by (*simp add*: *less-acc-def le-acc-def*)
qed

lemma *dyn-override-Package*:
  assumes *dyn-override*: $G \vdash$ *new overrides old* **and**
    accmodi-old: *accmodi old* = *Package* **and**
    accmodi-new: *accmodi new* = *Package* **and**
      wf: *wf-prog G*
  shows *pid* (*declclass old*) = *pid* (*declclass new*)
proof −
  from *dyn-override accmodi-old accmodi-new*
  show *?thesis* (**is** *?EqPid old new*)
  proof (*induct rule*: *overridesR.induct*)
    case (*Direct new old*)
    assume *accmodi old* = *Package*
      $G \vdash$*Method old inheritable-in pid* (*declclass new*)
    then show *pid* (*declclass old*) = *pid* (*declclass new*)
      by (*auto simp add*: *inheritable-in-def*)
  next
    case (*Indirect new inter old*)
    assume *accmodi-old*: *accmodi old* = *Package* **and**
      accmodi-new: *accmodi new* = *Package*
    assume $G \vdash$ *new overrides inter*
    with *accmodi-new wf*
    have *accmodi inter* = *Package*
      by (*auto intro*: *overrides-Package-old*)
    with *Indirect*
    show *pid* (*declclass old*) = *pid* (*declclass new*)
      by *auto*
  qed
qed

lemma *dyn-override-Package-escape*:
  assumes *dyn-override*: $G \vdash$ *new overrides old* **and**
    accmodi-old: *accmodi old* = *Package* **and**
    outside-pack: *pid* (*declclass old*) $\neq$ *pid* (*declclass new*) **and**
      wf: *wf-prog G*
  shows $\exists$ *inter*. $G \vdash$ *new overrides inter* $\wedge$ $G \vdash$ *inter overrides old* $\wedge$
      pid (*declclass old*) = *pid* (*declclass inter*) $\wedge$
      Protected $\leq$ *accmodi inter*
proof −
  from *dyn-override accmodi-old outside-pack*
  show *?thesis* (**is** *?P new old*)
  proof (*induct rule*: *overridesR.induct*)
    case (*Direct new old*)
    assume *accmodi-old*: *accmodi old* = *Package*
    assume *outside-pack*: *pid* (*declclass old*) $\neq$ *pid* (*declclass new*)
    assume $G \vdash$*Method old inheritable-in pid* (*declclass new*)
    with *accmodi-old*
    have *pid* (*declclass old*) = *pid* (*declclass new*)
      by (*simp add*: *inheritable-in-def*)
    with *outside-pack*
    show *?P new old*
      by (*contradiction*)
  next
    case (*Indirect new inter old*)
    assume *accmodi-old*: *accmodi old* = *Package*
    assume *outside-pack*: *pid* (*declclass old*) $\neq$ *pid* (*declclass new*)
    assume *override-new-inter*: $G \vdash$ *new overrides inter*
    assume *override-inter-old*: $G \vdash$ *inter overrides old*
    assume *hyp-new-inter*: $\llbracket$*accmodi inter* = *Package*;

$pid\ (declclass\ inter) \neq pid\ (declclass\ new)]\!]$
$\Longrightarrow\ ?P\ new\ inter$
assume *hyp-inter-old*: $[\![accmodi\ old = Package;$
$pid\ (declclass\ old) \neq pid\ (declclass\ inter)]\!]$
$\Longrightarrow\ ?P\ inter\ old$
show *?P new old*
proof (*cases pid* (*declclass old*) = *pid* (*declclass inter*))
case *True*
note *same-pack-old-inter* = *this*
show *?thesis*
proof (*cases pid* (*declclass inter*) = *pid* (*declclass new*))
case *True*
with *same-pack-old-inter outside-pack*
show *?thesis*
by *auto*
next
case *False*
note *diff-pack-inter-new* = *this*
show *?thesis*
proof (*cases accmodi inter* = *Package*)
case *True*
with *diff-pack-inter-new hyp-new-inter*
obtain *newinter* **where**
over-new-newinter: $G \vdash$ *new overrides newinter* **and**
over-newinter-inter: $G \vdash$ *newinter overrides inter* **and**
eq-pid: *pid* (*declclass inter*) = *pid* (*declclass newinter*) **and**
accmodi-newinter: *Protected* $\leq$ *accmodi newinter*
by *auto*
from *over-newinter-inter override-inter-old*
have $G\vdash$*newinter overrides old*
by (*rule overridesR.Indirect*)
moreover
from *eq-pid same-pack-old-inter*
have *pid* (*declclass old*) = *pid* (*declclass newinter*)
by *simp*
moreover
note *over-new-newinter accmodi-newinter*
ultimately show *?thesis*
by *blast*
next
case *False*
with *override-new-inter*
have *Protected* $\leq$ *accmodi inter*
by (*cases accmodi inter*) (*auto dest*: *no-Private-override*)
with *override-new-inter override-inter-old same-pack-old-inter*
show *?thesis*
by *blast*
qed
qed
next
case *False*
with *accmodi-old hyp-inter-old*
obtain *newinter* **where**
over-inter-newinter: $G \vdash$ *inter overrides newinter* **and**
over-newinter-old: $G \vdash$ *newinter overrides old* **and**
eq-pid: *pid* (*declclass old*) = *pid* (*declclass newinter*) **and**
accmodi-newinter: *Protected* $\leq$ *accmodi newinter*
by *auto*
from *override-new-inter over-inter-newinter*

have $G \vdash$ *new overrides newinter*
by (*rule overridesR.Indirect*)
with *eq-pid over-newinter-old accmodi-newinter*
show *?thesis*
by *blast*
qed
qed
qed

lemmas *class-rec-induct'* = *class-rec.induct* [*of %x y z w. P x y*] **for** *P*

lemma *declclass-widen*[*rule-format*]:
wf-prog G
$\longrightarrow$ ($\forall$ *c m. class G C = Some c* $\longrightarrow$ *methd G C sig = Some m*
$\longrightarrow$ *G*$\vdash$*C* $\preceq_C$ *declclass m*) (**is** *?P G C*)
proof (*induct G C rule*: *class-rec-induct'*, *intro allI impI*)
fix *G C c m*
assume *Hyp*: $\bigwedge$*c. class G C = Some c* $\Longrightarrow$ *ws-prog G* $\Longrightarrow$ *C* $\neq$ *Object*
$\Longrightarrow$ *?P G* (*super c*)
assume *wf*: *wf-prog G* **and** *cls-C*: *class G C = Some c* **and**
m: *methd G C sig = Some m*
show *G*$\vdash$*C*$\preceq_C$ *declclass m*
proof (*cases C=Object*)
case *True*
with *wf m* **show** *?thesis* **by** (*simp add*: *methd-Object-SomeD*)
next
let *?filter=filter-tab* (λ*sig m. G*$\vdash$*C inherits method sig m*)
let *?table = table-of* (*map* (λ(*s, m*). (*s, C, m*)) (*methods c*))
case *False*
with *cls-C wf m*
have *methd-C*: (*?filter* (*methd G* (*super c*)) ++ *?table*) *sig = Some m*
by (*simp add*: *methd-rec*)
show *?thesis*
proof (*cases ?table sig*)
case *None*
from *this methd-C* **have** *?filter* (*methd G* (*super c*)) *sig = Some m*
by *simp*
moreover
from *wf cls-C False* **obtain** *sup* **where** *class G* (*super c*) = *Some sup*
by (*blast dest*: *wf-prog-cdecl wf-cdecl-supD is-acc-class-is-class*)
moreover note *wf False cls-C*
ultimately have *G*$\vdash$*super c* $\preceq_C$ *declclass m*
by (*auto intro*: *Hyp* [*rule-format*])
moreover from *cls-C False* **have** *G*$\vdash$*C* $\prec_C$*1 super c* **by** (*rule subcls1I*)
ultimately show *?thesis* **by** − (*rule rtrancl-into-rtrancl2*)
next
case *Some*
from *this wf False cls-C methd-C* **show** *?thesis* **by** *auto*
qed
qed
qed

lemma *declclass-methd-Object*:
$[\![$*wf-prog G*; *methd G Object sig = Some m*$]\!]$ $\Longrightarrow$ *declclass m = Object*
by *auto*

lemma *methd-declaredD*:
⟦*wf-prog G*; *is-class G C*;*methd G C sig* = *Some m*⟧
⟹ *G*⊢(*mdecl* (*sig*,*mthd m*)) *declared-in* (*declclass m*)
proof −
  assume *wf*: *wf-prog G*
  then have *ws*: *ws-prog G* **..**
  assume *clsC*: *is-class G C*
  from *clsC ws*
  show *methd G C sig* = *Some m*
    ⟹ *G*⊢(*mdecl* (*sig*,*mthd m*)) *declared-in* (*declclass m*)
  proof (*induct C rule*: *ws-class-induct′*)
    case *Object*
    assume *methd G Object sig* = *Some m*
    with *wf* **show** *?thesis*
      by − (*rule method-declared-inI*, *auto*)
  next
    case *Subcls*
    fix *C c*
    assume *clsC*: *class G C* = *Some c*
    and *m*: *methd G C sig* = *Some m*
    and *hyp*: *methd G* (*super c*) *sig* = *Some m* ⟹ *?thesis*
    let *?newMethods* = *table-of* (*map* (λ(*s*, *m*). (*s*, *C*, *m*)) (*methods c*))
    show *?thesis*
    proof (*cases ?newMethods sig*)
      case *None*
      from *None ws clsC m hyp*
      show *?thesis* **by** (*auto intro*: *method-declared-inI simp add*: *methd-rec*)
    next
      case *Some*
      from *Some ws clsC m*
      show *?thesis* **by** (*auto intro*: *method-declared-inI simp add*: *methd-rec*)
    qed
  qed
qed

lemma *methd-rec-Some-cases*:
  assumes *methd-C*: *methd G C sig* = *Some m* **and**
    ws: *ws-prog G* **and**
    clsC: *class G C* = *Some c* **and**
    neq-C-Obj: *C*≠*Object*
  obtains (*NewMethod*) *table-of* (*map* (λ(*s*, *m*). (*s*, *C*, *m*)) (*methods c*)) *sig* = *Some m*
    | (*InheritedMethod*) *G*⊢*C inherits* (*method sig m*) **and** *methd G* (*super c*) *sig* = *Some m*
proof −
  let *?inherited* = *filter-tab* (λ*sig m*. *G*⊢*C inherits method sig m*)
      (*methd G* (*super c*))
  let *?new* = *table-of* (*map* (λ(*s*, *m*). (*s*, *C*, *m*)) (*methods c*))
  from *ws clsC neq-C-Obj methd-C*
  have *methd-unfold*: (*?inherited* ++ *?new*) *sig* = *Some m*
    by (*simp add*: *methd-rec*)
  show *thesis*
  proof (*cases ?new sig*)
    case *None*
    with *methd-unfold* **have** *?inherited sig* = *Some m*
      by (*auto*)
    with *InheritedMethod* **show** *?thesis* **by** *blast*
  next
    case *Some*
    with *methd-unfold* **have** *?new sig* = *Some m*

by *auto*
with *NewMethod* **show** *?thesis* **by** *blast*
qed
qed

lemma *methd-member-of*:
assumes *wf*: *wf-prog G*
shows
$[\![$*is-class G C*; *methd G C sig* = *Some m*$]\!] \Longrightarrow G \vdash$*Methd sig m member-of C*
(**is** *?Class C* $\Longrightarrow$ *?Method C* $\Longrightarrow$ *?MemberOf C*)
proof −
from *wf* **have** *ws*: *ws-prog G* **..**
assume *defC*: *is-class G C*
from *defC ws*
show *?Class C* $\Longrightarrow$ *?Method C* $\Longrightarrow$ *?MemberOf C*
proof (*induct rule*: *ws-class-induct'*)
case *Object*
with *wf* **have** *declC*: *Object* = *declclass m*
by (*simp add*: *declclass-methd-Object*)
from *Object wf* **have** $G \vdash$*Methd sig m declared-in Object*
by (*auto intro*: *methd-declaredD simp add*: *declC*)
with *declC*
show *?MemberOf Object*
by (*auto intro*!: *members.Immediate*
simp del: *methd-Object*)
next
case (*Subcls C c*)
assume *clsC*: *class G C* = *Some c* **and**
neq-C-Obj: $C \neq$ *Object*
assume *methd*: *?Method C*
from *methd ws clsC neq-C-Obj*
show *?MemberOf C*
proof (*cases rule*: *methd-rec-Some-cases*)
case *NewMethod*
with *clsC* **show** *?thesis*
by (*auto dest*: *method-declared-inI intro*!: *members.Immediate*)
next
case *InheritedMethod*
then show *?thesis*
by (*blast dest*: *inherits-member*)
qed
qed
qed

lemma *current-methd*:
$[\![$*table-of* (*methods c*) *sig* = *Some new*;
ws-prog G; *class G C* = *Some c*; $C \neq$ *Object*;
methd G (*super c*) *sig* = *Some old*$]\!]$
$\Longrightarrow$ *methd G C sig* = *Some* (*C*,*new*)
by (*auto simp add*: *methd-rec*
intro: *filter-tab-SomeI map-add-find-right table-of-map-SomeI*)

lemma *wf-prog-staticD*:
assumes *wf*: *wf-prog G* **and**
clsC: *class G C* = *Some c* **and**

neq-C-Obj: $C \neq Object$ **and**
old: *methd G* (*super c*) *sig* = *Some old* **and**
accmodi-old: *Protected* $\leq$ *accmodi old* **and**
new: *table-of* (*methods c*) *sig* = *Some new*
shows *is-static new* = *is-static old*
proof –
from *clsC wf*
have *wf-cdecl*: *wf-cdecl G* (*C*,*c*) **by** (*rule wf-prog-cdecl*)
from *wf clsC neq-C-Obj*
have *is-cls-super*: *is-class G* (*super c*)
by (*blast dest*: *wf-prog-acc-superD is-acc-classD*)
from *wf is-cls-super old*
have *old-member-of*: $G \vdash$*Methd sig old member-of* (*super c*)
by (*rule methd-member-of*)
from *old wf is-cls-super*
have *old-declared*: $G \vdash$*Methd sig old declared-in* (*declclass old*)
by (*auto dest*: *methd-declared-in-declclass*)
from *new clsC*
have *new-declared*: $G \vdash$*Methd sig* (*C*,*new*) *declared-in C*
by (*auto intro*: *method-declared-inI*)
note *trancl-rtrancl-tranc* = *trancl-rtrancl-trancl* [*trans*]
from *clsC neq-C-Obj*
have *subcls1-C-super*: $G \vdash C \prec_C 1$ *super c*
by (*rule subcls1I*)
then have $G \vdash C \prec_C$ *super c* **..**
also from *old wf is-cls-super*
have $G \vdash$*super c* $\preceq_C$ (*declclass old*) **by** (*auto dest*: *methd-declC*)
finally have *subcls-C-old*: $G \vdash C \prec_C$ (*declclass old*) **.**
from *accmodi-old*
have *inheritable*: $G \vdash$*Methd sig old inheritable-in pid C*
by (*auto simp add*: *inheritable-in-def*
dest: *acc-modi-le-Dests*)
show *?thesis*
proof (*cases is-static new*)
case *True*
with *subcls-C-old new-declared old-declared inheritable*
have *G*,*sig*$\vdash$(*C*,*new*) *hides old*
by (*auto intro*: *hidesI*)
with *True wf-cdecl neq-C-Obj new*
show *?thesis*
by (*auto dest*: *wf-cdecl-hides-SomeD*)
next
case *False*
with *subcls-C-old new-declared old-declared inheritable subcls1-C-super*
old-member-of
have *G*,*sig*$\vdash$(*C*,*new*) $overrides_S$ *old*
by (*auto intro*: *stat-overridesR.Direct*)
with *False wf-cdecl neq-C-Obj new*
show *?thesis*
by (*auto dest*: *wf-cdecl-overrides-SomeD*)
qed
qed

lemma *inheritable-instance-methd*:
assumes *subclseq-C-D*: $G \vdash C \preceq_C D$ **and**
is-cls-D: *is-class G D* **and**
wf: *wf-prog G* **and**
old: *methd G D sig* = *Some old* **and**

accmodi-old: *Protected* $\leq$ *accmodi old* **and**
not-static-old: $\neg$ *is-static old*
shows
$\exists$ *new. methd G C sig* = *Some new* $\wedge$
(*new* = *old* $\vee$ *G*,*sig*$\vdash$*new overrides*$_S$ *old*)
(**is** ($\exists$ *new.* (*?Constraint C new old*)))
proof −
from *subclseq-C-D is-cls-D*
have *is-cls-C*: *is-class G C* **by** (*rule subcls-is-class2*)
from *wf*
have *ws*: *ws-prog G* **..**
from *is-cls-C ws subclseq-C-D*
show $\exists$ *new. ?Constraint C new old*
proof (*induct rule*: *ws-class-induct'*)
case (*Object co*)
then have *eq-D-Obj*: *D*=*Object* **by** *auto*
with *old*
have *?Constraint Object old old*
by *auto*
with *eq-D-Obj*
show $\exists$ *new. ?Constraint Object new old* **by** *auto*
next
case (*Subcls C c*)
assume *hyp*: *G*$\vdash$*super c*$\preceq_C$ *D* $\Longrightarrow$ $\exists$ *new. ?Constraint* (*super c*) *new old*
assume *clsC*: *class G C* = *Some c*
assume *neq-C-Obj*: *C*$\neq$*Object*
from *clsC wf*
have *wf-cdecl*: *wf-cdecl G* (*C*,*c*)
by (*rule wf-prog-cdecl*)
from *ws clsC neq-C-Obj*
have *is-cls-super*: *is-class G* (*super c*)
by (*auto dest*: *ws-prog-cdeclD*)
from *clsC wf neq-C-Obj*
have *superAccessible*: *G*$\vdash$(*Class* (*super c*)) *accessible-in* (*pid C*) **and**
subcls1-C-super: *G*$\vdash$*C* $\prec_C$*1 super c*
by (*auto dest*: *wf-prog-cdecl wf-cdecl-supD is-acc-classD*
intro: *subcls1I*)
show $\exists$ *new. ?Constraint C new old*
proof (*cases G*$\vdash$*super c*$\preceq_C$ *D*)
case *False*
from *False Subcls*
have *eq-C-D*: *C*=*D*
by (*auto dest*: *subclseq-superD*)
with *old*
have *?Constraint C old old*
by *auto*
with *eq-C-D*
show $\exists$ *new. ?Constraint C new old* **by** *auto*
next
case *True*
with *hyp* **obtain** *super-method*
where *super*: *?Constraint* (*super c*) *super-method old* **by** *blast*
from *super not-static-old*
have *not-static-super*: $\neg$ *is-static super-method*
by (*auto dest!*: *stat-overrides-commonD*)
from *super old wf accmodi-old*
have *accmodi-super-method*: *Protected* $\leq$ *accmodi super-method*
by (*auto dest!*: *wf-prog-stat-overridesD*)
from *super accmodi-old wf*

```
have inheritable: G⊢Methd sig super-method inheritable-in (pid C)
  by (auto dest!: wf-prog-stat-overridesD
                  acc-modi-le-Dests
        simp add: inheritable-in-def)
from super wf is-cls-super
have member: G⊢Methd sig super-method member-of (super c)
  by (auto intro: methd-member-of)
from member
have decl-super-method:
  G⊢Methd sig super-method declared-in (declclass super-method)
  by (auto dest: member-of-declC)
from super subcls1-C-super ws is-cls-super
have subcls-C-super: G⊢C ≺C (declclass super-method)
  by (auto intro: rtrancl-into-trancl2 dest: methd-declC)
show ∃ new. ?Constraint C new old
proof (cases methd G C sig)
  case None
  have methd G (super c) sig = None
  proof −
    from clsC ws None
    have no-new: table-of (methods c) sig = None
      by (auto simp add: methd-rec)
    with clsC
    have undeclared: G⊢mid sig undeclared-in C
      by (auto simp add: undeclared-in-def cdeclaredmethd-def)
    with inheritable member superAccessible subcls1-C-super
    have inherits: G⊢C inherits (method sig super-method)
      by (auto simp add: inherits-def)
    with clsC ws no-new super neq-C-Obj
    have methd G C sig = Some super-method
      by (auto simp add: methd-rec map-add-def intro: filter-tab-SomeI)
    with None show ?thesis
      by simp
  qed
  with super show ?thesis by auto
next
  case (Some new)
  from this ws clsC neq-C-Obj
  show ?thesis
  proof (cases rule: methd-rec-Some-cases)
    case InheritedMethod
    with super Some show ?thesis
      by auto
  next
    case NewMethod
    assume new: table-of (map (λ(s, m). (s, C, m)) (methods c)) sig
              = Some new
    from new
    have declcls-new: declclass new = C
      by auto
    from wf clsC neq-C-Obj super new not-static-super accmodi-super-method
    have not-static-new: ¬ is-static new
      by (auto dest: wf-prog-staticD)
    from clsC new
    have decl-new: G⊢Methd sig new declared-in C
      by (auto simp add: declared-in-def cdeclaredmethd-def)
    from not-static-new decl-new decl-super-method
         member subcls1-C-super inheritable declcls-new subcls-C-super
    have G,sig⊢ new overridesS super-method
```

```
          by (auto intro: stat-overridesR.Direct)
        with super Some
        show ?thesis
          by (auto intro: stat-overridesR.Indirect)
      qed
    qed
  qed
 qed
qed
```

lemma *inheritable-instance-methd-cases*:
```
  assumes subclseq-C-D: G⊢C ≼_C D and
          is-cls-D: is-class G D and
               wf: wf-prog G and
              old: methd G D sig = Some old and
       accmodi-old: Protected ≤ accmodi old and
    not-static-old: ¬ is-static old
  obtains (Inheritance) methd G C sig = Some old
   | (Overriding) new where methd G C sig = Some new and G,sig⊢new overrides_S old
proof −
  from subclseq-C-D is-cls-D wf old accmodi-old not-static-old
  show ?thesis
    by (auto dest: inheritable-instance-methd intro: Inheritance Overriding)
qed
```

lemma *inheritable-instance-methd-props*:
```
  assumes subclseq-C-D: G⊢C ≼_C D and
          is-cls-D: is-class G D and
               wf: wf-prog G and
              old: methd G D sig = Some old and
       accmodi-old: Protected ≤ accmodi old and
    not-static-old: ¬ is-static old
  shows
  ∃ new. methd G C sig = Some new ∧
        ¬ is-static new ∧ G⊢resTy new≼resTy old ∧ accmodi old ≤accmodi new
 (is (∃ new. (?Constraint C new old)))
proof −
  from subclseq-C-D is-cls-D wf old accmodi-old not-static-old
  show ?thesis
  proof (cases rule: inheritable-instance-methd-cases)
    case Inheritance
    with not-static-old accmodi-old show ?thesis by auto
  next
    case (Overriding new)
    then have ¬ is-static new by (auto dest: stat-overrides-commonD)
    with Overriding not-static-old accmodi-old wf
    show ?thesis
      by (auto dest!: wf-prog-stat-overridesD)
  qed
qed
```

lemma *bexI'*: $x \in A \Longrightarrow P\ x \Longrightarrow \exists x{\in}A.\ P\ x$ **by** *blast*

lemma *ballE'*: $\forall x{\in}A.\ P\ x \Longrightarrow (x \notin A \Longrightarrow Q) \Longrightarrow (P\ x \Longrightarrow Q) \Longrightarrow Q$ **by** *blast*

```
lemma subint-widen-imethds:
  assumes irel: G⊢I≼I J
  and wf: wf-prog G
  and is-iface: is-iface G J
  and jm: jm ∈ imethds G J sig
  shows ∃ im ∈ imethds G I sig. is-static im = is-static jm ∧
                                accmodi im = accmodi jm ∧
                                G⊢resTy im≼resTy jm
  using irel jm
proof (induct rule: converse-rtrancl-induct)
    case base
    then show ?case by  (blast elim: bexI′)
  next
    case (step I SI)
    from ‹G⊢I ≺I1 SI›
    obtain i where
      ifI: iface G I = Some i and
       SI: SI ∈ set (isuperIfs i)
      by (blast dest: subint1D)

    let ?newMethods
          = (set-option ∘ table-of (map (λ(sig, mh). (sig, I, mh)) (imethods i)))
    show ?case
    proof (cases ?newMethods sig = {})
      case True
      with ifI SI step wf
      show ?thesis
        by (auto simp add: imethds-rec)
    next
      case False
      from ifI wf False
      have imethds: imethds G I sig = ?newMethods sig
        by (simp add: imethds-rec)
      from False
      obtain im where
        imdef: im ∈ ?newMethods sig
        by (blast)
      with imethds
      have im: im ∈ imethds G I sig
        by (blast)
      with im wf ifI
      obtain
         imStatic: ¬ is-static im and
         imPublic: accmodi im = Public
        by (auto dest!: imethds-wf-mhead)
      from ifI wf
      have wf-I: wf-idecl G (I,i)
        by (rule wf-prog-idecl)
      with SI wf
      obtain si where
         ifSI: iface G SI = Some si and
        wf-SI: wf-idecl G (SI,si)
        by (auto dest!: wf-idecl-supD is-acc-ifaceD
                  dest: wf-prog-idecl)
      from step
      obtain sim::qtname × mhead  where
                      sim: sim ∈ imethds G SI sig and
         eq-static-sim-jm: is-static sim = is-static jm and
```

eq-access-sim-jm: *accmodi sim* = *accmodi jm* **and**
resTy-widen-sim-jm: $G\vdash$*resTy sim*$\preceq$*resTy jm*
by *blast*
with *wf-I SI imdef sim*
have $G\vdash$*resTy im*$\preceq$*resTy sim*
by (*auto dest*!: *wf-idecl-hidings hidings-entailsD*)
with *wf resTy-widen-sim-jm*
have *resTy-widen-im-jm*: $G\vdash$*resTy im*$\preceq$*resTy jm*
by (*blast intro*: *widen-trans*)
from *sim wf ifSI*
obtain
simStatic: $\neg$ *is-static sim* **and**
simPublic: *accmodi sim* = *Public*
by (*auto dest*!: *imethds-wf-mhead*)
from *im*
imStatic simStatic eq-static-sim-jm
imPublic simPublic eq-access-sim-jm
resTy-widen-im-jm
show *?thesis*
by *auto*
qed
qed

lemma *implmt1-methd*:
$\bigwedge$*sig*. $[\![G\vdash C\leadsto 1I;$ *wf-prog G*; *im* $\in$ *imethds G I sig*$]\!] \Longrightarrow$
$\exists$ *cm* $\in$*methd G C sig*: $\neg$ *is-static cm* $\wedge$ $\neg$ *is-static im* $\wedge$
$G\vdash$*resTy cm*$\preceq$*resTy im* $\wedge$
accmodi im = *Public* $\wedge$ *accmodi cm* = *Public*
apply (*drule implmt1D*)
apply *clarify*
apply (*drule* (*2*) *wf-prog-cdecl* [*THEN wf-cdecl-impD*])
apply (*frule* (*1*) *imethds-wf-mhead*)
apply (*simp add*: *is-acc-iface-def*)
apply (*force*)
done

lemma *implmt-methd* [*rule-format* (*no-asm*)]:
$[\![$*wf-prog G*; $G\vdash C\leadsto I]\!] \Longrightarrow$ *is-iface G I* $\longrightarrow$
($\forall$ *im* $\in$*imethds G I* *sig*.
$\exists$ *cm*$\in$*methd G C sig*: $\neg$*is-static cm* $\wedge$ $\neg$ *is-static im* $\wedge$
$G\vdash$*resTy cm*$\preceq$*resTy im* $\wedge$
accmodi im = *Public* $\wedge$ *accmodi cm* = *Public*)
apply (*frule implmt-is-class*)
apply (*erule implmt.induct*)
apply *safe*
apply (*drule* (*2*) *implmt1-methd*)
apply *fast*
apply (*drule* (*1*) *subint-widen-imethds*)
apply *simp*
apply *assumption*

apply *clarify*
apply (*drule* (*2*) *implmt1-methd*)
apply (*force*)
apply (*frule subcls1D*)
apply (*drule* (*1*) *bspec*)
apply *clarify*
apply (*drule* (*3*) *r-into-rtrancl* [*THEN inheritable-instance-methd-props*,
OF - implmt-is-class])
apply *auto*
done

lemma *mheadsD* [*rule-format* (*no-asm*)]:
$emh \in mheads\ G\ S\ t\ sig \longrightarrow wf\text{-}prog\ G \longrightarrow$
$(\exists C\ D\ m.\ t = ClassT\ C \wedge declrefT\ emh = ClassT\ D \wedge$
$accmethd\ G\ S\ C\ sig = Some\ m \wedge$
$(declclass\ m = D) \wedge mhead\ (mthd\ m) = (mhd\ emh)) \vee$
$(\exists I.\ t = IfaceT\ I \wedge ((\exists im.\ im\ \in accimethds\ G\ (pid\ S)\ I\ sig \wedge$
$mthd\ im = mhd\ emh) \vee$
$(\exists m.\ G \vdash Iface\ I\ accessible\text{-}in\ (pid\ S) \wedge accmethd\ G\ S\ Object\ sig = Some\ m \wedge$
$accmodi\ m \neq Private \wedge$
$declrefT\ emh = ClassT\ Object \wedge mhead\ (mthd\ m) = mhd\ emh))) \vee$
$(\exists T\ m.\ t = ArrayT\ T \wedge G \vdash Array\ T\ accessible\text{-}in\ (pid\ S) \wedge$
$accmethd\ G\ S\ Object\ sig = Some\ m \wedge accmodi\ m \neq Private \wedge$
$declrefT\ emh = ClassT\ Object \wedge mhead\ (mthd\ m) = mhd\ emh)$
apply (*rule-tac ref-ty1*=*t* **in** *ref-ty-ty.induct* [*THEN conjunct1*])
apply *auto*
apply (*auto simp add*: *cmheads-def accObjectmheads-def Objectmheads-def*)
apply (*auto dest*!: *accmethd-SomeD*)
done

lemma *mheads-cases*:
assumes $emh \in mheads\ G\ S\ t\ sig$ **and** *wf-prog G*
obtains (*Class-methd*) *C D m* **where**
$t = ClassT\ C\ declrefT\ emh = ClassT\ D\ accmethd\ G\ S\ C\ sig = Some\ m$
$declclass\ m = D\ mhead\ (mthd\ m) = mhd\ emh$
| (*Iface-methd*) *I im* **where** $t = IfaceT\ I$
$im\ \in accimethds\ G\ (pid\ S)\ I\ sig\ mthd\ im = mhd\ emh$
| (*Iface-Object-methd*) *I m* **where**
$t = IfaceT\ I\ G \vdash Iface\ I\ accessible\text{-}in\ (pid\ S)$
$accmethd\ G\ S\ Object\ sig = Some\ m\ accmodi\ m \neq Private$
$declrefT\ emh = ClassT\ Object\ mhead\ (mthd\ m) = mhd\ emh$
| (*Array-Object-methd*) *T m* **where**
$t = ArrayT\ T\ G \vdash Array\ T\ accessible\text{-}in\ (pid\ S)$
$accmethd\ G\ S\ Object\ sig = Some\ m\ accmodi\ m \neq Private$
$declrefT\ emh = ClassT\ Object\ mhead\ (mthd\ m) = mhd\ emh$
using *assms* **by** (*blast dest*!: *mheadsD*)

lemma *declclassD*[*rule-format*]:
$[\![wf\text{-}prog\ G; class\ G\ C = Some\ c;\ methd\ G\ C\ sig = Some\ m;$
$class\ G\ (declclass\ m) = Some\ d]\!]$
$\Longrightarrow table\text{-}of\ (methods\ d)\ sig\ = Some\ (mthd\ m)$
proof −
assume *wf*: *wf-prog G*
then have *ws*: *ws-prog G* **..**
assume *clsC*: $class\ G\ C = Some\ c$
from *clsC ws*

```
  show ⋀ m d. ⟦methd G C sig = Some m; class G (declclass m) = Some d⟧
        ⟹ table-of (methods d) sig = Some (mthd m)
  proof (induct rule: ws-class-induct)
    case Object
    with wf show ?thesis m d by auto
  next
    case (Subcls C c)
    let ?newMethods = table-of (map (λ(s, m). (s, C, m)) (methods c)) sig
    show ?thesis m d
    proof (cases ?newMethods)
      case None
      from None ws Subcls
      show ?thesis by (auto simp add: methd-rec) (rule Subcls)
    next
      case Some
      from Some ws Subcls
      show ?thesis
        by (auto simp add: methd-rec
                 dest: wf-prog-cdecl wf-cdecl-supD is-acc-class-is-class)
    qed
  qed
qed


lemma dynmethd-Object:
  assumes statM: methd G Object sig = Some statM and
        private: accmodi statM = Private and
       is-cls-C: is-class G C and
            wf: wf-prog G
  shows dynmethd G Object C sig = Some statM
proof −
  from is-cls-C wf
  have subclseq: G⊢C ≼C Object
    by (auto intro: subcls-ObjectI)
  from wf have ws: ws-prog G
    by simp
  from wf
  have is-cls-Obj: is-class G Object
    by simp
  from statM subclseq is-cls-Obj ws private
  show ?thesis
  proof (cases rule: dynmethd-cases)
    case Static then show ?thesis .
  next
    case Overrides
    with private show ?thesis
      by (auto dest: no-Private-override)
  qed
qed


lemma wf-imethds-hiding-objmethdsD:
  assumes      old: methd G Object sig = Some old and
          is-if-I: is-iface G I and
              wf: wf-prog G and
      not-private: accmodi old ≠ Private and
             new: new ∈ imethds G I sig
  shows G⊢resTy new≼resTy old ∧ is-static new = is-static old (is ?P new)
proof −
```

```
  from wf have ws: ws-prog G by simp
  {
    fix I i new
    assume ifI: iface G I = Some i
    assume new: table-of (imethods i) sig = Some new
    from ifI new not-private wf old
    have ?P (I,new)
      by (auto dest!: wf-prog-idecl wf-idecl-hiding cond-hiding-entailsD
            simp del: methd-Object)
  } note hyp-newmethod = this
  from is-if-I ws new
  show ?thesis
  proof (induct rule: ws-interface-induct)
    case (Step I i)
    assume ifI: iface G I = Some i
    assume new: new ∈ imethds G I sig
    from Step
    have hyp: ∀ J ∈ set (isuperIfs i). (new ∈ imethds G J sig ⟶ ?P new)
      by auto
    from new ifI ws
    show ?P new
    proof (cases rule: imethds-cases)
      case NewMethod
      with ifI hyp-newmethod
      show ?thesis
        by auto
    next
      case (InheritedMethod J)
      assume J ∈ set (isuperIfs i)
             new ∈ imethds G J sig
      with hyp
      show ?thesis
        by auto
    qed
  qed
qed
```

Which dynamic classes are valid to look up a member of a distinct static type? We have to distinct class members (named static members in Java) from instance members. Class members are global to all Objects of a class, instance members are local to a single Object instance. If a member is equipped with the static modifier it is a class member, else it is an instance member. The following table gives an overview of the current framework. We assume to have a reference with static type statT and a dynamic class dynC. Between both of these types the widening relation holds $G \vdash Class\ dynC \preceq statT$. Unfortunately this ordinary widening relation isn't enough to describe the valid lookup classes, since we must cope the special cases of arrays and interfaces,too. If we statically expect an array or inteface we may lookup a field or a method in Object which isn't covered in the widening relation.

statT field instance method static (class) method

NullT / / / Iface / dynC Object Class dynC dynC dynC Array / Object Object

In most cases we con lookup the member in the dynamic class. But as an interface can't declare new static methods, nor an array can define new methods at all, we have to lookup methods in the base class Object.

The limitation to classes in the field column is artificial and comes out of the typing rule for the field access (see rule *FVar* in the welltyping relation *wt* in theory WellType). I stems out of the fact, that Object indeed has no non private fields. So interfaces and arrays can actually have no fields at all and a field access would be senseless. (In Java interfaces are allowed to declare new fields but in

current Bali not!). So there is no principal reason why we should not allow Objects to declare non private fields. Then we would get the following column:
statT field ————— NullT / Iface Object Class dynC Array Object

primrec *valid-lookup-cls*:: *prog* ⇒ *ref-ty* ⇒ *qtname* ⇒ *bool* ⇒ *bool*
(-,- ⊢ - *valid′-lookup′-cls′-for* - [*61*,*61*,*61*,*61*] *60*)
where
G,*NullT* ⊢ *dynC valid-lookup-cls-for static-membr* = *False*
| *G*,*IfaceT I* ⊢ *dynC valid-lookup-cls-for static-membr*
= (*if static-membr*
then dynC=*Object*
else G⊢*Class dynC*≼ *Iface I*)
| *G*,*ClassT C* ⊢ *dynC valid-lookup-cls-for static-membr* = *G*⊢*Class dynC*≼ *Class C*
| *G*,*ArrayT T* ⊢ *dynC valid-lookup-cls-for static-membr* = (*dynC*=*Object*)

lemma *valid-lookup-cls-is-class*:
assumes *dynC*: *G*,*statT* ⊢ *dynC valid-lookup-cls-for static-membr* **and**
ty-statT: *isrtype G statT* **and**
wf: *wf-prog G*
shows *is-class G dynC*
proof (*cases statT*)
case *NullT*
with *dynC ty-statT* **show** *?thesis*
by (*auto dest*: *widen-NT2*)
next
case (*IfaceT I*)
with *dynC wf* **show** *?thesis*
by (*auto dest*: *implmt-is-class*)
next
case (*ClassT C*)
with *dynC ty-statT* **show** *?thesis*
by (*auto dest*: *subcls-is-class2*)
next
case (*ArrayT T*)
with *dynC wf* **show** *?thesis*
by (*auto*)
qed

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt delloop split-all-tac*)›
setup ‹*map-theory-claset* (*fn ctxt* => *ctxt delSWrapper split-all-tac*)›

lemma *dynamic-mheadsD*:
⟦*emh* ∈ *mheads G S statT sig*;
G,*statT* ⊢ *dynC valid-lookup-cls-for* (*is-static emh*);
isrtype G statT; *wf-prog G*
⟧ ⟹ ∃ *m* ∈ *dynlookup G statT dynC sig*:
is-static m=*is-static emh* ∧ *G*⊢*resTy m*≼*resTy emh*
proof −
assume *emh*: *emh* ∈ *mheads G S statT sig*
and *wf*: *wf-prog G*
and *dynC-Prop*: *G*,*statT* ⊢ *dynC valid-lookup-cls-for* (*is-static emh*)
and *istype*: *isrtype G statT*
from *dynC-Prop istype wf*
obtain *y* **where**
dynC: *class G dynC* = *Some y*
by (*auto dest*: *valid-lookup-cls-is-class*)

from *emh wf* **show** *?thesis*
proof (*cases rule*: *mheads-cases*)
  case *Class-methd*
  fix *statC statDeclC sm*
  assume *statC*: *statT* = *ClassT statC*
  assume *accmethd G S statC sig* = *Some sm*
  then have *sm*: *methd G statC sig* = *Some sm*
    by (*blast dest*: *accmethd-SomeD*)
  assume *eq-mheads*: *mhead* (*mthd sm*) = *mhd emh*
  from *statC*
  have *dynlookup*: *dynlookup G statT dynC sig* = *dynmethd G statC dynC sig*
    by (*simp add*: *dynlookup-def*)
  from *wf statC istype dynC-Prop sm*
  obtain *dm* **where**
    dynmethd G statC dynC sig = *Some dm*
    is-static dm = *is-static sm*
    $G \vdash resTy\ dm \preceq resTy\ sm$
    by (*force dest*!: *ws-dynmethd accmethd-SomeD*)
  with *dynlookup eq-mheads*
  show *?thesis*
    by (*cases emh type*: *prod*) (*auto*)
next
  case *Iface-methd*
  fix *I im*
  assume *statI*: *statT* = *IfaceT I* **and**
    eq-mheads: *mthd im* = *mhd emh* **and**
    $im \in$ *accimethds G* (*pid S*) *I sig*
  then have *im*: $im \in$ *imethds G I sig*
    by (*blast dest*: *accimethdsD*)
  with *istype statI eq-mheads wf*
  have *not-static-emh*: $\neg$ *is-static emh*
    by (*cases emh*) (*auto dest*: *wf-prog-idecl imethds-wf-mhead*)
  from *statI im*
  have *dynlookup*: *dynlookup G statT dynC sig* = *methd G dynC sig*
    by (*auto simp add*: *dynlookup-def dynimethd-def*)
  from *wf dynC-Prop statI istype im not-static-emh*
  obtain *dm* **where**
    methd G dynC sig = *Some dm*
    is-static dm = *is-static im*
    $G \vdash resTy\ (mthd\ dm) \preceq resTy\ (mthd\ im)$
    by (*force dest*: *implmt-methd*)
  with *dynlookup eq-mheads*
  show *?thesis*
    by (*cases emh type*: *prod*) (*auto*)
next
  case *Iface-Object-methd*
  fix *I sm*
  assume *statI*: *statT* = *IfaceT I* **and**
    sm: *accmethd G S Object sig* = *Some sm* **and**
    eq-mheads: *mhead* (*mthd sm*) = *mhd emh* **and**
    nPriv: *accmodi sm* $\neq$ *Private*
   show *?thesis*
   proof (*cases imethds G I sig* = {})
     case *True*
     with *statI*
     have *dynlookup*: *dynlookup G statT dynC sig* = *dynmethd G Object dynC sig*
       by (*simp add*: *dynlookup-def dynimethd-def*)
     from *wf dynC*
     have *subclsObj*: $G \vdash dynC \preceq_C Object$

```
        by (auto intro: subcls-ObjectI)
      from wf dynC dynC-Prop istype sm subclsObj
      obtain dm where
        dynmethd G Object dynC sig = Some dm
        is-static dm = is-static sm
        G⊢resTy (mthd dm)≼resTy (mthd sm)
        by (auto dest!: ws-dynmethd accmethd-SomeD
                intro: class-Object [OF wf] intro: that)
      with dynlookup eq-mheads
      show ?thesis
        by (cases emh type: prod) (auto)
    next
      case False
      with statI
      have dynlookup: dynlookup G statT dynC sig = methd G dynC sig
        by (simp add: dynlookup-def dynimethd-def)
      from istype statI
      have is-iface G I
        by auto
      with wf sm nPriv False
      obtain im where
            im: im ∈ imethds G I sig and
        eq-stat: is-static im = is-static sm and
        resProp: G⊢resTy (mthd im)≼resTy (mthd sm)
        by (auto dest: wf-imethds-hiding-objmethdsD accmethd-SomeD)
      from im wf statI istype eq-stat eq-mheads
      have not-static-sm: ¬ is-static emh
        by (cases emh) (auto dest: wf-prog-idecl imethds-wf-mhead)
      from im wf dynC-Prop dynC istype statI not-static-sm
      obtain dm where
        methd G dynC sig = Some dm
        is-static dm = is-static im
        G⊢resTy (mthd dm)≼resTy (mthd im)
        by (auto dest: implmt-methd)
      with wf eq-stat resProp dynlookup eq-mheads
      show ?thesis
        by (cases emh type: prod) (auto intro: widen-trans)
    qed
  next
    case Array-Object-methd
    fix T sm
    assume statArr: statT = ArrayT T and
              sm: accmethd G S Object sig = Some sm and
        eq-mheads: mhead (mthd sm) = mhd emh
    from statArr dynC-Prop wf
    have dynlookup: dynlookup G statT dynC sig = methd G Object sig
      by (auto simp add: dynlookup-def dynmethd-C-C)
    with sm eq-mheads sm
    show ?thesis
      by (cases emh type: prod) (auto dest: accmethd-SomeD)
  qed
qed
declare split-paired-All [simp] split-paired-Ex [simp]
setup ‹map-theory-claset (fn ctxt => ctxt addSbefore (split-all-tac, split-all-tac))›
setup ‹map-theory-simpset (fn ctxt => ctxt addloop (split-all-tac, split-all-tac))›
```

```
lemma methd-declclass:
⟦class G C = Some c; wf-prog G; methd G C sig = Some m⟧
 ⟹ methd G (declclass m) sig = Some m
proof −
  assume asm: class G C = Some c wf-prog G methd G C sig = Some m
  have wf-prog G  ⟶
          (∀ c m. class G C = Some c ⟶ methd G C sig = Some m
                 ⟶ methd G (declclass m) sig = Some m)      (is ?P G C)
  proof (induct G C rule: class-rec-induct′, intro allI impI)
    fix G C c m
    assume hyp: ⋀c. class G C = Some c ⟹ ws-prog G ⟹ C ≠ Object ⟹
                    ?P G (super c)
    assume wf: wf-prog G and cls-C: class G C = Some c and
            m: methd G C sig = Some m
    show methd G (declclass m) sig = Some m
    proof (cases C=Object)
      case True
      with wf m show ?thesis by (auto intro: table-of-map-SomeI)
    next
      let ?filter=filter-tab (λsig m. G⊢C inherits method sig m)
      let ?table = table-of (map (λ(s, m). (s, C, m)) (methods c))
      case False
      with cls-C wf m
      have methd-C: (?filter (methd G (super c)) ++ ?table) sig = Some m
        by (simp add: methd-rec)
      show ?thesis
      proof (cases ?table sig)
        case None
        from this methd-C have ?filter (methd G (super c)) sig = Some m
          by simp
        moreover
        from wf cls-C False obtain sup where class G (super c) = Some sup
          by (blast dest: wf-prog-cdecl wf-cdecl-supD is-acc-class-is-class)
        moreover note wf False cls-C
        ultimately show ?thesis by (auto intro: hyp [rule-format])
      next
        case Some
        from this methd-C m show ?thesis by auto
      qed
    qed
  qed
  with asm show ?thesis by auto
qed


lemma dynmethd-declclass:
 ⟦dynmethd G statC dynC sig = Some m;
   wf-prog G; is-class G statC
  ⟧ ⟹ methd G (declclass m) sig = Some m
by (auto dest: dynmethd-declC)


lemma dynlookup-declC:
 ⟦dynlookup G statT dynC sig = Some m; wf-prog G;
   is-class G dynC;isrtype G statT
  ⟧ ⟹ G⊢dynC ≼C (declclass m) ∧ is-class G (declclass m)
by (cases statT)
```

```
  (auto simp add: dynlookup-def dynimethd-def
          dest: methd-declC dynmethd-declC)


lemma dynlookup-Array-declclassD [simp]:
⟦dynlookup G (ArrayT T) Object sig = Some dm;wf-prog G⟧
 ⟹ declclass dm = Object
proof -
  assume dynL: dynlookup G (ArrayT T) Object sig = Some dm
  assume wf: wf-prog G
  from wf have ws: ws-prog G by auto
  from wf have is-cls-Obj: is-class G Object by auto
  from dynL wf
  show ?thesis
    by (auto simp add: dynlookup-def dynmethd-C-C [OF is-cls-Obj ws]
               dest: methd-Object-SomeD)
qed


declare split-paired-All [simp del] split-paired-Ex [simp del]
setup ‹map-theory-simpset (fn ctxt => ctxt delloop split-all-tac)›
setup ‹map-theory-claset (fn ctxt => ctxt delSWrapper split-all-tac)›


lemma wt-is-type: E,dt⊨v::T ⟹  wf-prog (prg E) ⟶
  dt=empty-dt ⟶ (case T of
                  Inl T ⇒ is-type (prg E) T
                | Inr Ts ⇒ Ball (set Ts) (is-type (prg E)))
apply (unfold empty-dt-def)
apply (erule wt.induct)
apply (auto split del: if-split-asm simp del: snd-conv
         simp add: is-acc-class-def is-acc-type-def)
apply    (erule typeof-empty-is-type)
apply   (frule (1) wf-prog-cdecl [THEN wf-cdecl-supD],
      force simp del: snd-conv, clarsimp simp add: is-acc-class-def)
apply  (drule (1) max-spec2mheads [THEN conjunct1, THEN mheadsD])
apply  (drule-tac [2] accfield-fields)
apply  (frule class-Object)
apply  (auto dest: accmethd-rT-is-type
              imethds-wf-mhead [THEN conjunct1, THEN rT-is-acc-type]
          dest!:accimethdsD
          simp del: class-Object
          simp add: is-acc-type-def
    )
done
declare split-paired-All [simp] split-paired-Ex [simp]
setup ‹map-theory-claset (fn ctxt => ctxt addSbefore (split-all-tac, split-all-tac))›
setup ‹map-theory-simpset (fn ctxt => ctxt addloop (split-all-tac, split-all-tac))›


lemma ty-expr-is-type:
⟦E⊢e::-T; wf-prog (prg E)⟧ ⟹ is-type (prg E) T
by (auto dest!: wt-is-type)

lemma ty-var-is-type:
⟦E⊢v::=T; wf-prog (prg E)⟧ ⟹ is-type (prg E) T
by (auto dest!: wt-is-type)

lemma ty-exprs-is-type:
```

$[\![E\vdash es::\doteq Ts;\ wf\text{-}prog\ (prg\ E)]\!] \Longrightarrow Ball\ (set\ Ts)\ (is\text{-}type\ (prg\ E))$
by (*auto dest*!: *wt-is-type*)

lemma *static-mheadsD*:
$[\![\ emh \in mheads\ G\ S\ t\ sig;\ wf\text{-}prog\ G;\ E\vdash e::-RefT\ t;\ prg\ E{=}G\ ;$
$\quad invmode\ (mhd\ emh)\ e \neq IntVir$
$\ ]\!] \Longrightarrow \exists\, m.\ (\quad (\exists\ \ C.\ t = ClassT\ C \wedge accmethd\ G\ S\ C\ sig = Some\ m)$
$\qquad\qquad \vee (\forall\ \ C.\ t \neq ClassT\ C \wedge accmethd\ G\ S\ Object\ sig = Some\ m\ )) \wedge$
$\qquad declrefT\ emh = ClassT\ (declclass\ m) \wedge\ \ mhead\ (mthd\ m) = (mhd\ emh)$
apply (*subgoal-tac is-static emh* ∨ *e* = *Super*)
defer apply (*force simp add*: *invmode-def*)
apply (*frule ty-expr-is-type*)
apply *simp*
apply (*case-tac is-static emh*)
apply (*frule* (*1*) *mheadsD*)
apply *clarsimp*
apply *safe*
apply *blast*
apply (*auto dest*!: *imethds-wf-mhead*
accmethd-SomeD
accimethdsD
simp add: *accObjectmheads-def Objectmheads-def*)

apply (*erule wt-elim-cases*)
apply (*force simp add*: *cmheads-def*)
done

lemma *wt-MethdI*:
$[\![methd\ G\ C\ sig = Some\ m;\ wf\text{-}prog\ G;$
$\quad class\ G\ C = Some\ c]\!] \Longrightarrow$
$\ \exists\, T.\ (\!|prg{=}G,cls{=}(declclass\ m),$
$\qquad lcl{=}callee\text{-}lcl\ (declclass\ m)\ sig\ (mthd\ m)|\!)\vdash\ Methd\ C\ sig::-T\ \wedge\ G\vdash T\preceq resTy\ m$
apply (*frule* (*2*) *methd-wf-mdecl*, *clarify*)
apply (*force dest*!: *wf-mdecl-bodyD intro*!: *wt.Methd*)
done

2 accessibility concerns

lemma *mheads-type-accessible*:
$[\![emh \in mheads\ G\ S\ T\ sig;\ wf\text{-}prog\ G]\!]$
$\Longrightarrow G\vdash RefT\ T\ accessible\text{-}in\ (pid\ S)$
by (*erule mheads-cases*)
(*auto dest*: *accmethd-SomeD accessible-from-commonD accimethdsD*)

lemma *static-to-dynamic-accessible-from-aux*:
$[\![G\vdash m\ of\ C\ accessible\text{-}from\ accC;wf\text{-}prog\ G]\!]$
$\Longrightarrow G\vdash m\ in\ C\ dyn\text{-}accessible\text{-}from\ accC$
proof (*induct rule*: *accessible-fromR.induct*)
qed (*auto intro*: *dyn-accessible-fromR.intros*
member-of-to-member-in
static-to-dynamic-overriding)

lemma *static-to-dynamic-accessible-from*:
assumes *stat-acc*: *G*⊢*m of statC accessible-from accC* **and**

subclseq: $G\vdash dynC \preceq_C statC$ **and**
wf: *wf-prog G*
shows $G\vdash m$ *in dynC dyn-accessible-from accC*
proof −
from *stat-acc subclseq*
show *?thesis* (**is** *?Dyn-accessible m*)
proof (*induct rule*: *accessible-fromR.induct*)
case (*Immediate m statC*)
then show *?Dyn-accessible m*
by (*blast intro*: *dyn-accessible-fromR.Immediate*
member-inI
permits-acc-inheritance)
next
case (*Overriding m - -*)
with *wf* **show** *?Dyn-accessible m*
by (*blast intro*: *dyn-accessible-fromR.Overriding*
member-inI
static-to-dynamic-overriding
rtrancl-trancl-trancl
static-to-dynamic-accessible-from-aux)
qed
qed

lemma *static-to-dynamic-accessible-from-static*:
assumes *stat-acc*: $G\vdash m$ *of statC accessible-from accC* **and**
static: *is-static m* **and**
wf: *wf-prog G*
shows $G\vdash m$ *in* (*declclass m*) *dyn-accessible-from accC*
proof −
from *stat-acc wf*
have $G\vdash m$ *in statC dyn-accessible-from accC*
by (*auto intro*: *static-to-dynamic-accessible-from*)
from *this static*
show *?thesis*
by (*rule dyn-accessible-from-static-declC*)
qed

lemma *dynmethd-member-in*:
assumes *m*: *dynmethd G statC dynC sig* = *Some m* **and**
iscls-statC: *is-class G statC* **and**
wf: *wf-prog G*
shows $G\vdash$*Methd sig m member-in dynC*
proof −
from *m*
have *subclseq*: $G\vdash dynC \preceq_C statC$
by (*auto simp add*: *dynmethd-def*)
from *subclseq iscls-statC*
have *iscls-dynC*: *is-class G dynC*
by (*rule subcls-is-class2*)
from *iscls-dynC iscls-statC wf m*
have $G\vdash dynC \preceq_C$ (*declclass m*) ∧ *is-class G* (*declclass m*) ∧
methd G (*declclass m*) *sig* = *Some m*
by − (*drule dynmethd-declC*, *auto*)
with *wf*
show *?thesis*
by (*auto intro*: *member-inI dest*: *methd-member-of*)
qed

lemma *dynmethd-access-prop*:
assumes *statM*: *methd G statC sig = Some statM* **and**
stat-acc: *G⊢Methd sig statM of statC accessible-from accC* **and**
dynM: *dynmethd G statC dynC sig = Some dynM* **and**
wf: *wf-prog G*
shows *G⊢Methd sig dynM in dynC dyn-accessible-from accC*
proof −
from *wf* **have** *ws*: *ws-prog G* **..**
from *dynM*
have *subclseq*: $G \vdash dynC \preceq_C statC$
by (*auto simp add*: *dynmethd-def*)
from *stat-acc*
have *is-cls-statC*: *is-class G statC*
by (*auto dest*: *accessible-from-commonD member-of-is-classD*)
with *subclseq*
have *is-cls-dynC*: *is-class G dynC*
by (*rule subcls-is-class2*)
from *is-cls-statC statM wf*
have *member-statC*: *G⊢Methd sig statM member-of statC*
by (*auto intro*: *methd-member-of*)
from *stat-acc*
have *statC-acc*: *G⊢Class statC accessible-in* (*pid accC*)
by (*auto dest*: *accessible-from-commonD*)
from *statM subclseq is-cls-statC ws*
show *?thesis*
proof (*cases rule*: *dynmethd-cases*)
case *Static*
assume *dynmethd*: *dynmethd G statC dynC sig = Some statM*
with *dynM* **have** *eq-dynM-statM*: *dynM=statM*
by *simp*
with *stat-acc subclseq wf*
show *?thesis*
by (*auto intro*: *static-to-dynamic-accessible-from*)
next
case (*Overrides newM*)
assume *dynmethd*: *dynmethd G statC dynC sig = Some newM*
assume *override*: *G,sig⊢newM overrides statM*
assume *neq*: *newM≠statM*
from *dynmethd dynM*
have *eq-dynM-newM*: *dynM=newM*
by *simp*
from *dynmethd eq-dynM-newM wf is-cls-statC*
have *G⊢Methd sig dynM member-in dynC*
by (*auto intro*: *dynmethd-member-in*)
moreover
from *subclseq*
have $G \vdash dynC \prec_C statC$
proof (*cases rule*: *subclseq-cases*)
case *Eq*
assume *dynC=statC*
moreover
from *is-cls-statC* **obtain** *c*
where *class G statC = Some c*
by *auto*
moreover
note *statM ws dynmethd*
ultimately

```
      have newM=statM
        by (auto simp add: dynmethd-C-C)
      with neq show ?thesis
        by (contradiction)
    next
      case Subcls then show ?thesis .
    qed
    moreover
    from stat-acc wf
    have G⊢Methd sig statM in statC dyn-accessible-from accC
      by (blast intro: static-to-dynamic-accessible-from)
    moreover
    note override eq-dynM-newM
    ultimately show ?thesis
      by (cases dynM,cases statM) (auto intro: dyn-accessible-fromR.Overriding)
  qed
qed


lemma implmt-methd-access:
  fixes accC::qtname
  assumes iface-methd: imethds G I sig ≠ {} and
          implements: G⊢dynC⇝I and
              isif-I: is-iface G I and
                  wf: wf-prog G
  shows ∃ dynM. methd G dynC sig = Some dynM ∧
            G⊢Methd sig dynM in dynC dyn-accessible-from accC
proof −
  from implements
  have iscls-dynC: is-class G dynC by (rule implmt-is-class)
  from iface-methd
  obtain im
    where im ∈ imethds G I sig
    by auto
  with wf implements isif-I
  obtain dynM
    where dynM: methd G dynC sig = Some dynM and
          pub: accmodi dynM = Public
    by (blast dest: implmt-methd)
  with iscls-dynC wf
  have G⊢Methd sig dynM in dynC dyn-accessible-from accC
    by (auto intro!: dyn-accessible-fromR.Immediate
             intro: methd-member-of member-of-to-member-in
                    simp add: permits-acc-def)
  with dynM
  show ?thesis
    by blast
qed

corollary implmt-dynimethd-access:
  fixes accC::qtname
  assumes iface-methd: imethds G I sig ≠ {} and
          implements: G⊢dynC⇝I and
              isif-I: is-iface G I and
                  wf: wf-prog G
  shows ∃ dynM. dynimethd G I dynC sig = Some dynM ∧
            G⊢Methd sig dynM in dynC dyn-accessible-from accC
proof −
  from iface-methd
```

have *dynimethd G I dynC sig = methd G dynC sig*
by (*simp add*: *dynimethd-def*)
with *iface-methd implements isif-I wf*
show *?thesis*
by (*simp only*:)
(*blast intro*: *implmt-methd-access*)
qed

lemma *dynlookup-access-prop*:
assumes *emh*: *emh* $\in$ *mheads G accC statT sig* **and**
dynM: *dynlookup G statT dynC sig = Some dynM* **and**
dynC-prop: *G*,*statT* $\vdash$ *dynC valid-lookup-cls-for is-static emh* **and**
isT-statT: *isrtype G statT* **and**
wf: *wf-prog G*
shows *G* $\vdash$*Methd sig dynM in dynC dyn-accessible-from accC*
proof −
from *emh wf*
have *statT-acc*: *G*$\vdash$*RefT statT accessible-in* (*pid accC*)
by (*rule mheads-type-accessible*)
from *dynC-prop isT-statT wf*
have *iscls-dynC*: *is-class G dynC*
by (*rule valid-lookup-cls-is-class*)
from *emh dynC-prop isT-statT wf dynM*
have *eq-static*: *is-static emh = is-static dynM*
by (*auto dest*: *dynamic-mheadsD*)
from *emh wf* **show** *?thesis*
proof (*cases rule*: *mheads-cases*)
case (*Class-methd statC - statM*)
assume *statT*: *statT = ClassT statC*
assume *accmethd G accC statC sig = Some statM*
then have *statM*: *methd G statC sig = Some statM* **and**
stat-acc: *G*$\vdash$*Methd sig statM of statC accessible-from accC*
by (*auto dest*: *accmethd-SomeD*)
from *dynM statT*
have *dynM'*: *dynmethd G statC dynC sig = Some dynM*
by (*simp add*: *dynlookup-def*)
from *statM stat-acc wf dynM'*
show *?thesis*
by (*auto dest!*: *dynmethd-access-prop*)
next
case (*Iface-methd I im*)
then have *iface-methd*: *imethds G I sig* $\neq$ {} **and**
statT-acc: *G*$\vdash$*RefT statT accessible-in* (*pid accC*)
by (*auto dest*: *accimethdsD*)
assume *statT*: *statT = IfaceT I*
assume *im*: *im* $\in$ *accimethds G* (*pid accC*) *I sig*
assume *eq-mhds*: *mthd im = mhd emh*
from *dynM statT*
have *dynM'*: *dynimethd G I dynC sig = Some dynM*
by (*simp add*: *dynlookup-def*)
from *isT-statT statT*
have *isif-I*: *is-iface G I*
by *simp*
show *?thesis*
proof (*cases is-static emh*)
case *False*
with *statT dynC-prop*
have *widen-dynC*: *G*$\vdash$*Class dynC* $\preceq$ *RefT statT*

by *simp*
from *statT widen-dynC*
have *implmnt*: $G\vdash dynC \leadsto I$
by *auto*
from *eq-static False*
have *not-static-dynM*: ¬ *is-static dynM*
by *simp*
from *iface-methd implmnt isif-I wf dynM'*
show *?thesis*
by − (*drule implmt-dynimethd-access*, *auto*)
next
case *True*
assume *is-static emh*
moreover
from *wf isT-statT statT im*
have ¬ *is-static im*
by (*auto dest*: *accimethdsD wf-prog-idecl imethds-wf-mhead*)
moreover note *eq-mhds*
ultimately show *?thesis*
by (*cases emh*) *auto*
qed
next
case (*Iface-Object-methd I statM*)
assume *statT*: *statT* = *IfaceT I*
assume *accmethd G accC Object sig* = *Some statM*
then have *statM*: *methd G Object sig* = *Some statM* **and**
stat-acc: $G\vdash$*Methd sig statM of Object accessible-from accC*
by (*auto dest*: *accmethd-SomeD*)
assume *not-Private-statM*: *accmodi statM* ≠ *Private*
assume *eq-mhds*: *mhead* (*mthd statM*) = *mhd emh*
from *iscls-dynC wf*
have *widen-dynC-Obj*: $G\vdash dynC \preceq_C Object$
by (*auto intro*: *subcls-ObjectI*)
show *?thesis*
proof (*cases imethds G I sig* = {})
case *True*
from *dynM statT True*
have *dynM'*: *dynmethd G Object dynC sig* = *Some dynM*
by (*simp add*: *dynlookup-def dynimethd-def*)
from *statT*
have $G\vdash RefT\ statT \preceq Class\ Object$
by *auto*
with *statM statT-acc stat-acc widen-dynC-Obj statT isT-statT*
wf dynM' eq-static dynC-prop
show *?thesis*
by − (*drule dynmethd-access-prop*,*force*+)
next
case *False*
then obtain *im* **where**
im: *im* ∈ *imethds G I sig*
by *auto*
have *not-static-emh*: ¬ *is-static emh*
proof −
from *im statM statT isT-statT wf not-Private-statM*
have *is-static im* = *is-static statM*
by (*fastforce dest*: *wf-imethds-hiding-objmethdsD*)
with *wf isT-statT statT im*
have ¬ *is-static statM*
by (*auto dest*: *wf-prog-idecl imethds-wf-mhead*)

```
        with eq-mhds
        show ?thesis
          by (cases emh) auto
      qed
      with statT dynC-prop
      have implmnt: G⊢dynC↝I
        by simp
      with isT-statT statT
      have isif-I: is-iface G I
        by simp
      from dynM statT
      have dynM': dynimethd G I dynC sig = Some dynM
        by (simp add: dynlookup-def)
      from False implmnt isif-I wf dynM'
      show ?thesis
        by − (drule implmt-dynimethd-access, auto)
    qed
  next
    case (Array-Object-methd T statM)
    assume statT: statT = ArrayT T
    assume accmethd G accC Object sig = Some statM
    then have    statM: methd G Object sig = Some statM and
              stat-acc: G⊢Methd sig statM of Object accessible-from accC
      by (auto dest: accmethd-SomeD)
    from statT dynC-prop
    have dynC-Obj: dynC = Object
      by simp
    then
    have widen-dynC-Obj: G⊢Class dynC ≼ Class Object
      by simp
    from dynM statT
    have dynM': dynmethd G Object dynC sig = Some dynM
      by (simp add: dynlookup-def)
    from statM statT-acc stat-acc dynM' wf widen-dynC-Obj
         statT isT-statT
    show ?thesis
      by − (drule dynmethd-access-prop, simp+)
  qed
qed


lemma dynlookup-access:
  assumes emh: emh ∈ mheads G accC statT sig and
    dynC-prop: G,statT ⊢ dynC valid-lookup-cls-for (is-static emh)  and
    isT-statT: isrtype G statT and
           wf: wf-prog G
  shows ∃ dynM. dynlookup G statT dynC sig = Some dynM ∧
            G⊢Methd sig dynM in dynC dyn-accessible-from accC
proof −
  from dynC-prop isT-statT wf
  have is-cls-dynC: is-class G dynC
    by (auto dest: valid-lookup-cls-is-class)
  with emh wf dynC-prop isT-statT
  obtain dynM where
    dynlookup G statT dynC sig = Some dynM
    by − (drule dynamic-mheadsD,auto)
  with  emh dynC-prop isT-statT wf
  show ?thesis
    by (blast intro: dynlookup-access-prop)
```

qed

lemma *stat-overrides-Package-old*:
assumes *stat-override*: $G \vdash new\ overrides_S\ old$ **and**
accmodi-new: $accmodi\ new = Package$ **and**
wf: *wf-prog G*
shows $accmodi\ old = Package$
proof −
from *stat-override wf*
have $accmodi\ old \leq accmodi\ new$
by (*auto dest*: *wf-prog-stat-overridesD*)
with *stat-override accmodi-new* **show** *?thesis*
by (*cases accmodi old*) (*auto dest*: *no-Private-stat-override*
dest: *acc-modi-le-Dests*)
qed

Properties of dynamic accessibility

lemma *dyn-accessible-Private*:
assumes *dyn-acc*: $G \vdash m\ in\ C\ dyn\text{-}accessible\text{-}from\ accC$ **and**
priv: $accmodi\ m = Private$
shows $accC = declclass\ m$
proof −
from *dyn-acc priv*
show *?thesis*
proof (*induct*)
case (*Immediate m C*)
from ‹$G \vdash m\ in\ C\ permits\text{-}acc\text{-}from\ accC$› **and** ‹$accmodi\ m = Private$›
show *?case*
by (*simp add*: *permits-acc-def*)
next
case *Overriding*
then show *?case*
by (*auto dest*!: *overrides-commonD*)
qed
qed

dyn-accessible-Package only works with the *wf-prog* assumption. Without it. it is easy to leaf the Package!

lemma *dyn-accessible-Package*:
$[\![G \vdash m\ in\ C\ dyn\text{-}accessible\text{-}from\ accC;\ accmodi\ m = Package;$
$wf\text{-}prog\ G]\!]$
$\Longrightarrow pid\ accC = pid\ (declclass\ m)$
proof −
assume *wf*: *wf-prog G*
assume *accessible*: $G \vdash m\ in\ C\ dyn\text{-}accessible\text{-}from\ accC$
then show $accmodi\ m = Package$
$\Longrightarrow pid\ accC = pid\ (declclass\ m)$
(**is** $?Pack\ m \Longrightarrow\ ?P\ m$)
proof (*induct rule*: *dyn-accessible-fromR.induct*)
case (*Immediate m C*)
assume $G \vdash m\ member\text{-}in\ C$
$G \vdash m\ in\ C\ permits\text{-}acc\text{-}from\ accC$
$accmodi\ m = Package$
then show *?P m*
by (*auto simp add*: *permits-acc-def*)
next
case (*Overriding new declC newm old Sup C*)

assume *member-new*: $G \vdash$ *new member-in C* **and**
new: *new* = (*declC*, *mdecl newm*) **and**
override: $G \vdash$ (*declC*, *newm*) *overrides old* **and**
subcls-C-Sup: $G \vdash C \prec_C$ *Sup* **and**
acc-old: $G \vdash$ *methdMembr old in Sup dyn-accessible-from accC* **and**
hyp: *?Pack* (*methdMembr old*) $\Longrightarrow$ *?P* (*methdMembr old*) **and**
accmodi-new: *accmodi new* = *Package*
from *override accmodi-new new wf*
have *accmodi-old*: *accmodi old* = *Package*
by (*auto dest*: *overrides-Package-old*)
with *hyp*
have *P-sup*: *?P* (*methdMembr old*)
by (*simp*)
from *wf override new accmodi-old accmodi-new*
have *eq-pid-new-old*: *pid* (*declclass new*) = *pid* (*declclass old*)
by (*auto dest*: *dyn-override-Package*)
with *eq-pid-new-old P-sup* **show** *?P new*
by *auto*
qed
qed

For fields we don't need the wellformedness of the program, since there is no overriding

lemma *dyn-accessible-field-Package*:
assumes *dyn-acc*: $G \vdash$ *f in C dyn-accessible-from accC* **and**
pack: *accmodi f* = *Package* **and**
field: *is-field f*
shows *pid accC* = *pid* (*declclass f*)
proof −
from *dyn-acc pack field*
show *?thesis*
proof (*induct*)
case (*Immediate f C*)
from ‹$G \vdash$ *f in C permits-acc-from accC*› **and** ‹*accmodi f* = *Package*›
show *?case*
by (*simp add*: *permits-acc-def*)
next
case *Overriding*
then show *?case* **by** (*simp add*: *is-field-def*)
qed
qed

dyn-accessible-instance-field-Protected only works for fields since methods can break the package bounds due to overriding

lemma *dyn-accessible-instance-field-Protected*:
assumes *dyn-acc*: $G \vdash$ *f in C dyn-accessible-from accC* **and**
prot: *accmodi f* = *Protected* **and**
field: *is-field f* **and**
instance-field: ¬ *is-static f* **and**
outside: *pid* (*declclass f*) ≠ *pid accC*
shows $G \vdash C \preceq_C$ *accC*
proof −
from *dyn-acc prot field instance-field outside*
show *?thesis*
proof (*induct*)
case (*Immediate f C*)
note ‹$G \vdash$ *f in C permits-acc-from accC*›
moreover
assume *accmodi f* = *Protected* **and** *is-field f* **and** ¬ *is-static f* **and**

pid (*declclass f*) $\neq$ *pid accC*
ultimately
show $G\vdash$ $C \preceq_C$ *accC*
by (*auto simp add*: *permits-acc-def*)
next
case *Overriding*
then show *?case* **by** (*simp add*: *is-field-def*)
qed
qed

lemma *dyn-accessible-static-field-Protected*:
assumes *dyn-acc*: $G \vdash$ *f in C dyn-accessible-from accC* **and**
prot: *accmodi f* = *Protected* **and**
field: *is-field f* **and**
static-field: *is-static f* **and**
outside: *pid* (*declclass f*) $\neq$ *pid accC*
shows $G\vdash$ *accC* $\preceq_C$ *declclass f* $\wedge$ $G\vdash C \preceq_C$ *declclass f*
proof −
from *dyn-acc prot field static-field outside*
show *?thesis*
proof (*induct*)
case (*Immediate f C*)
assume *accmodi f* = *Protected* **and** *is-field f* **and** *is-static f* **and**
pid (*declclass f*) $\neq$ *pid accC*
moreover
note ‹$G \vdash$ *f in C permits-acc-from accC*›
ultimately
have $G\vdash$ *accC* $\preceq_C$ *declclass f*
by (*auto simp add*: *permits-acc-def*)
moreover
from ‹$G \vdash$ *f member-in C*›
have $G\vdash C \preceq_C$ *declclass f*
by (*rule member-in-class-relation*)
ultimately show *?case*
by *blast*
next
case *Overriding*
then show *?case* **by** (*simp add*: *is-field-def*)
qed
qed

end

Chapter 14

State

1 State for evaluation of Java expressions and statements

theory *State*
imports *DeclConcepts*
begin

design issues:

- all kinds of objects (class instances, arrays, and class objects) are handeled via a general object abstraction
- the heap and the map for class objects are combined into a single table (*recall* (*loc*, *obj*) *table* × (*qtname*, *obj*) *table* ~= (*loc* + *qtname*, *obj*) *table*)

objects

datatype *obj-tag* = — tag for generic object
  CInst qtname — class instance
  | *Arr ty int* — array with component type and length
  — | CStat qtname the tag is irrelevant for a class object, i.e. the static fields of a class, since its type is given already by the reference to it (see below)

type-synonym *vn* = *fspec* + *int* — variable name
record *obj* =
  tag :: *obj-tag* — generalized object
  values :: (*vn*, *val*) *table*

translations
  (*type*) *fspec* <= (*type*) *vname* × *qtname*
  (*type*) *vn* <= (*type*) *fspec* + *int*
  (*type*) *obj* <= (*type*) (|*tag*::*obj-tag*, *values*::*vn* ⇒ *val option*|)
  (*type*) *obj* <= (*type*) (|*tag*::*obj-tag*, *values*::*vn* ⇒ *val option*,...::'*a*|)

definition
  the-Arr :: *obj option* ⇒ *ty* × *int* × (*vn*, *val*) *table*
  where *the-Arr obj* = (*SOME* (*T*,*k*,*t*). *obj* = *Some* (|*tag*=*Arr T k*,*values*=*t*|))

lemma *the-Arr-Arr* [*simp*]: *the-Arr* (*Some* (|*tag*=*Arr T k*,*values*=*cs*|)) = (*T*,*k*,*cs*)
apply (*auto simp*: *the-Arr-def*)
done

lemma *the-Arr-Arr1* [*simp*,*intro*,*dest*]:
$[\![tag\ obj = Arr\ T\ k]\!] \Longrightarrow the\text{-}Arr\ (Some\ obj) = (T,k,values\ obj)$
apply (*auto simp add*: *the-Arr-def*)
done

definition
$upd\text{-}obj :: vn \Rightarrow val \Rightarrow obj \Rightarrow obj$
where $upd\text{-}obj\ n\ v = (\lambda obj.\ obj\ (\!|values{:=}(values\ obj)(n{\mapsto}v)|\!))$

lemma *upd-obj-def2* [*simp*]:
$upd\text{-}obj\ n\ v\ obj = obj\ (\!|values{:=}(values\ obj)(n{\mapsto}v)|\!)$
apply (*auto simp*: *upd-obj-def*)
done

definition
$obj\text{-}ty :: obj \Rightarrow ty$ **where**
$obj\text{-}ty\ obj = (case\ tag\ obj\ of$
$\quad CInst\ C \Rightarrow Class\ C$
$\quad |\ Arr\ T\ k \Rightarrow T.[])$

lemma *obj-ty-eq* [*intro*!]: $obj\text{-}ty\ (\!|tag{=}oi,values{=}x|\!) = obj\text{-}ty\ (\!|tag{=}oi,values{=}y|\!)$
by (*simp add*: *obj-ty-def*)

lemma *obj-ty-eq1* [*intro*!,*dest*]:
$tag\ obj = tag\ obj' \Longrightarrow obj\text{-}ty\ obj = obj\text{-}ty\ obj'$
by (*simp add*: *obj-ty-def*)

lemma *obj-ty-cong* [*simp*]:
$obj\text{-}ty\ (obj\ (\!|values{:=}vs|\!)) = obj\text{-}ty\ obj$
by *auto*

lemma *obj-ty-CInst* [*simp*]:
$obj\text{-}ty\ (\!|tag{=}CInst\ C,values{=}vs|\!) = Class\ C$
by (*simp add*: *obj-ty-def*)

lemma *obj-ty-CInst1* [*simp*,*intro*!,*dest*]:
$[\![tag\ obj = CInst\ C]\!] \Longrightarrow obj\text{-}ty\ obj = Class\ C$
by (*simp add*: *obj-ty-def*)

lemma *obj-ty-Arr* [*simp*]:
$obj\text{-}ty\ (\!|tag{=}Arr\ T\ i,values{=}vs|\!) = T.[]$
by (*simp add*: *obj-ty-def*)

lemma *obj-ty-Arr1* [*simp*,*intro*!,*dest*]:
$[\![tag\ obj = Arr\ T\ i]\!] \Longrightarrow obj\text{-}ty\ obj = T.[]$
by (*simp add*: *obj-ty-def*)

lemma *obj-ty-widenD*:
$G{\vdash}obj\text{-}ty\ obj{\preceq}RefT\ t \Longrightarrow (\exists\,C.\ tag\ obj = CInst\ C) \lor (\exists\,T\ k.\ tag\ obj = Arr\ T\ k)$

apply (*unfold obj-ty-def*)
apply (*auto split*: *obj-tag.split-asm*)
done

definition
obj-class :: *obj* ⇒ *qtname* **where**
obj-class obj = (*case tag obj of*
CInst C ⇒ *C*
| *Arr T k* ⇒ *Object*)

lemma *obj-class-CInst* [*simp*]: *obj-class* (|*tag*=*CInst C*,*values*=*vs*|) = *C*
by (*auto simp*: *obj-class-def*)

lemma *obj-class-CInst1* [*simp*,*intro*!,*dest*]:
tag obj = *CInst C* ⟹ *obj-class obj* = *C*
by (*auto simp*: *obj-class-def*)

lemma *obj-class-Arr* [*simp*]: *obj-class* (|*tag*=*Arr T k*,*values*=*vs*|) = *Object*
by (*auto simp*: *obj-class-def*)

lemma *obj-class-Arr1* [*simp*,*intro*!,*dest*]:
tag obj = *Arr T k* ⟹ *obj-class obj* = *Object*
by (*auto simp*: *obj-class-def*)

lemma *obj-ty-obj-class*: $G \vdash$*obj-ty obj*$\preceq$ *Class statC* = $G \vdash$*obj-class obj* $\preceq_C$ *statC*
apply (*case-tac tag obj*)
apply (*auto simp add*: *obj-ty-def obj-class-def*)
apply (*case-tac statC* = *Object*)
apply (*auto dest*: *widen-Array-Class*)
done

object references

type-synonym *oref* = *loc* + *qtname* — generalized object reference
syntax
Heap :: *loc* ⇒ *oref*
Stat :: *qtname* ⇒ *oref*

translations
Heap => *CONST Inl*
Stat => *CONST Inr*
(*type*) *oref* <= (*type*) *loc* + *qtname*

definition
fields-table :: *prog* ⇒ *qtname* ⇒ (*fspec* ⇒ *field* ⇒ *bool*) ⇒ (*fspec*, *ty*) *table* **where**
fields-table G C P =
map-option type ∘ *table-of* (*filter* (*case-prod P*) (*DeclConcepts.fields G C*))

lemma *fields-table-SomeI*:
⟦*table-of* (*DeclConcepts.fields G C*) *n* = *Some f*; *P n f*⟧
⟹ *fields-table G C P n* = *Some* (*type f*)
apply (*unfold fields-table-def*)

apply *clarsimp*
apply (*rule exI*)
apply (*rule conjI*)
apply (*erule map-of-filter-in*)
apply *assumption*
apply *simp*
done

lemma *fields-table-SomeD′*: *fields-table G C P fn = Some T* $\Longrightarrow$
$\exists f.$ (*fn,f*)$\in$*set*(*DeclConcepts.fields G C*) $\wedge$ *type f = T*
apply (*unfold fields-table-def*)
apply *clarsimp*
apply (*drule map-of-SomeD*)
apply *auto*
done

lemma *fields-table-SomeD*:
$[\![$*fields-table G C P fn = Some T*; *unique* (*DeclConcepts.fields G C*)$]\!]$ $\Longrightarrow$
$\exists f.$ *table-of* (*DeclConcepts.fields G C*) *fn = Some f* $\wedge$ *type f = T*
apply (*unfold fields-table-def*)
apply *clarsimp*
apply (*rule exI*)
apply (*rule conjI*)
apply (*erule table-of-filter-unique-SomeD*)
apply *assumption*
apply *simp*
done

definition
in-bounds :: *int* $\Rightarrow$ *int* $\Rightarrow$ *bool* ((-/ *in′-bounds* -) [*50*, *51*] *50*)
where *i in-bounds k* = (*0* $\leq$ *i* $\wedge$ *i* $<$ *k*)

definition
arr-comps :: *′a* $\Rightarrow$ *int* $\Rightarrow$ *int* $\Rightarrow$ *′a option*
where *arr-comps T k* = ($\lambda i.$ *if i in-bounds k then Some T else None*)

definition
var-tys :: *prog* $\Rightarrow$ *obj-tag* $\Rightarrow$ *oref* $\Rightarrow$ (*vn*, *ty*) *table* **where**
var-tys G oi r =
(*case r of*
Heap a $\Rightarrow$ (*case oi of*
CInst C $\Rightarrow$ *fields-table G C* ($\lambda n\ f.$ $\neg$*static f*) (+) *Map.empty*
| *Arr T k* $\Rightarrow$ *Map.empty* (+) *arr-comps T k*)
| *Stat C* $\Rightarrow$ *fields-table G C* ($\lambda fn\ f.$ *declclassf fn = C* $\wedge$ *static f*)
(+) *Map.empty*)

lemma *var-tys-Some-eq*:
var-tys G oi r n = Some T
= (*case r of*
Inl a $\Rightarrow$ (*case oi of*
CInst C $\Rightarrow$ ($\exists$ *nt. n = Inl nt* $\wedge$ *fields-table G C* ($\lambda n\ f.$
$\neg$*static f*) *nt = Some T*)
| *Arr t k* $\Rightarrow$ ($\exists$ *i. n = Inr i* $\wedge$ *i in-bounds k* $\wedge$ *t = T*))
| *Inr C* $\Rightarrow$ ($\exists$ *nt. n = Inl nt* $\wedge$
fields-table G C ($\lambda fn\ f.$ *declclassf fn = C* $\wedge$ *static f*) *nt*

```
                    = Some T))
apply (unfold var-tys-def arr-comps-def)
apply (force split: sum.split-asm sum.split obj-tag.split)
done
```

stores

```
type-synonym globs                  — global variables: heap and static variables
        = (oref , obj) table
type-synonym heap
        = (loc  , obj) table
```

```
translations
 (type) globs  <= (type) (oref , obj) table
 (type) heap   <= (type) (loc  , obj) table
```

```
datatype st =
         st globs locals
```

2 access

```
definition
  globs :: st ⇒ globs
  where globs = case-st (λg l. g)
```

```
definition
  locals :: st ⇒ locals
  where locals = case-st (λg l. l)
```

```
definition heap :: st ⇒ heap where
 heap s = globs s ∘ Heap
```

```
lemma globs-def2 [simp]:  globs (st g l) = g
by (simp add: globs-def)
```

```
lemma locals-def2 [simp]: locals (st g l) = l
by (simp add: locals-def)
```

```
lemma heap-def2 [simp]:  heap s a=globs s (Heap a)
by (simp add: heap-def)
```

```
abbreviation val-this :: st ⇒ val
  where val-this s == the (locals s This)
```

```
abbreviation lookup-obj :: st ⇒ val ⇒ obj
  where lookup-obj s a' == the (heap s (the-Addr a'))
```

3 memory allocation

```
definition
  new-Addr :: heap ⇒ loc option where
  new-Addr h = (if (∀ a. h a ≠ None) then None else Some (SOME a. h a = None))
```

lemma *new-AddrD*: *new-Addr h* = *Some a* $\Longrightarrow$ *h a* = *None*
apply (*auto simp add*: *new-Addr-def*)
apply (*erule someI*)
done

lemma *new-AddrD2*: *new-Addr h* = *Some a* $\Longrightarrow \forall b.$ *h b* $\neq$ *None* $\longrightarrow b \neq a$
apply (*drule new-AddrD*)
apply *auto*
done

lemma *new-Addr-SomeI*: *h a* = *None* $\Longrightarrow \exists b.$ *new-Addr h* = *Some b* $\wedge$ *h b* = *None*
apply (*simp add*: *new-Addr-def*)
apply (*fast intro*: *someI2*)
done

4 initialization

abbreviation *init-vals* :: (*'a*, *ty*) *table* $\Rightarrow$ (*'a*, *val*) *table*
  where *init-vals vs* == *map-option default-val* $\circ$ *vs*

lemma *init-arr-comps-base* [*simp*]: *init-vals* (*arr-comps T 0*) = *Map.empty*
apply (*unfold arr-comps-def in-bounds-def*)
apply (*rule ext*)
apply *auto*
done

lemma *init-arr-comps-step* [*simp*]:
0 < *j* $\Longrightarrow$ *init-vals* (*arr-comps T j*) =
      init-vals (*arr-comps T* (*j* − *1*))(*j* − *1*$\mapsto$*default-val T*)
apply (*unfold arr-comps-def in-bounds-def*)
apply (*rule ext*)
apply *auto*
done

5 update

definition
  gupd :: *oref* $\Rightarrow$ *obj* $\Rightarrow$ *st* $\Rightarrow$ *st* (*gupd'*(-$\mapsto$-') [*10*, *10*] *1000*)
  where *gupd r obj* = *case-st* (λ*g l*. *st* (*g*(*r*$\mapsto$*obj*)) *l*)

definition
  lupd :: *lname* $\Rightarrow$ *val* $\Rightarrow$ *st* $\Rightarrow$ *st* (*lupd'*(-$\mapsto$-') [*10*, *10*] *1000*)
  where *lupd vn v* = *case-st* (λ*g l*. *st g* (*l*(*vn*$\mapsto$*v*)))

definition
  upd-gobj :: *oref* $\Rightarrow$ *vn* $\Rightarrow$ *val* $\Rightarrow$ *st* $\Rightarrow$ *st*
  where *upd-gobj r n v* = *case-st* (λ*g l*. *st* (*chg-map* (*upd-obj n v*) *r g*) *l*)

definition
  set-locals :: *locals* $\Rightarrow$ *st* $\Rightarrow$ *st*
  where *set-locals l* = *case-st* (λ*g l'*. *st g l*)

definition

init-obj :: *prog* ⇒ *obj-tag* ⇒ *oref* ⇒ *st* ⇒ *st*
where *init-obj G oi r* = *gupd*(*r*↦⦇*tag*=*oi*, *values*=*init-vals* (*var-tys G oi r*)⦈)

abbreviation
init-class-obj :: *prog* ⇒ *qtname* ⇒ *st* ⇒ *st*
where *init-class-obj G C* == *init-obj G undefined* (*Inr C*)

lemma *gupd-def2* [*simp*]: *gupd*(*r*↦*obj*) (*st g l*) = *st* (*g*(*r*↦*obj*)) *l*
apply (*unfold gupd-def*)
apply (*simp* (*no-asm*))
done

lemma *lupd-def2* [*simp*]: *lupd*(*vn*↦*v*) (*st g l*) = *st g* (*l*(*vn*↦*v*))
apply (*unfold lupd-def*)
apply (*simp* (*no-asm*))
done

lemma *globs-gupd* [*simp*]: *globs* (*gupd*(*r*↦*obj*) *s*) = *globs s*(*r*↦*obj*)
apply (*induct s*)
by (*simp add*: *gupd-def*)

lemma *globs-lupd* [*simp*]: *globs* (*lupd*(*vn*↦*v*) *s*) = *globs* *s*
apply (*induct s*)
by (*simp add*: *lupd-def*)

lemma *locals-gupd* [*simp*]: *locals* (*gupd*(*r*↦*obj*) *s*) = *locals s*
apply (*induct s*)
by (*simp add*: *gupd-def*)

lemma *locals-lupd* [*simp*]: *locals* (*lupd*(*vn*↦*v*) *s*) = *locals s*(*vn*↦*v*)
apply (*induct s*)
by (*simp add*: *lupd-def*)

lemma *globs-upd-gobj-new* [*rule-format* (*no-asm*), *simp*]:
globs s r = *None* ⟶ *globs* (*upd-gobj r n v s*) = *globs s*
apply (*unfold upd-gobj-def*)
apply (*induct s*)
apply *auto*
done

lemma *globs-upd-gobj-upd* [*rule-format* (*no-asm*), *simp*]:
globs s r=*Some obj*⟶ *globs* (*upd-gobj r n v s*) = *globs s*(*r*↦*upd-obj n v obj*)
apply (*unfold upd-gobj-def*)
apply (*induct s*)
apply *auto*
done

lemma *locals-upd-gobj* [*simp*]: *locals* (*upd-gobj r n v s*) = *locals s*
apply (*induct s*)
by (*simp add*: *upd-gobj-def*)

lemma *globs-init-obj* [*simp*]: *globs* (*init-obj G oi r s*) *t* =
(*if t=r then Some* (|*tag=oi,values=init-vals* (*var-tys G oi r*)|) *else globs s t*)
apply (*unfold init-obj-def*)
apply (*simp* (*no-asm*))
done

lemma *locals-init-obj* [*simp*]: *locals* (*init-obj G oi r s*) = *locals s*
by (*simp add*: *init-obj-def*)

lemma *surjective-st* [*simp*]: *st* (*globs s*) (*locals s*) = *s*
apply (*induct s*)
by *auto*

lemma *surjective-st-init-obj*:
st (*globs* (*init-obj G oi r s*)) (*locals s*) = *init-obj G oi r s*
apply (*subst locals-init-obj* [*THEN sym*])
apply (*rule surjective-st*)
done

lemma *heap-heap-upd* [*simp*]:
heap (*st* (*g*(*Inl a*↦*obj*)) *l*) = *heap* (*st g l*)(*a*↦*obj*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-stat-upd* [*simp*]: *heap* (*st* (*g*(*Inr C*↦*obj*)) *l*) = *heap* (*st g l*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-local-upd* [*simp*]: *heap* (*st g* (*l*(*vn*↦*v*))) = *heap* (*st g l*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-gupd-Heap* [*simp*]: *heap* (*gupd*(*Heap a*↦*obj*) *s*) = *heap s*(*a*↦*obj*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-gupd-Stat* [*simp*]: *heap* (*gupd*(*Stat C*↦*obj*) *s*) = *heap s*
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-lupd* [*simp*]: *heap* (*lupd*(*vn*↦*v*) *s*) = *heap s*
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-upd-gobj-Stat* [*simp*]: *heap* (*upd-gobj* (*Stat C*) *n v s*) = *heap s*
apply (*rule ext*)
apply (*simp* (*no-asm*))
apply (*case-tac globs s* (*Stat C*))
apply *auto*
done

lemma *set-locals-def2* [*simp*]: *set-locals l* (*st g l'*) = *st g l*
apply (*unfold set-locals-def*)
apply (*simp* (*no-asm*))
done

lemma *set-locals-id* [*simp*]: *set-locals* (*locals s*) *s* = *s*
apply (*unfold set-locals-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *set-set-locals* [*simp*]: *set-locals l* (*set-locals l' s*) = *set-locals l s*
apply (*unfold set-locals-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *locals-set-locals* [*simp*]: *locals* (*set-locals l s*) = *l*
apply (*unfold set-locals-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *globs-set-locals* [*simp*]: *globs* (*set-locals l s*) = *globs s*
apply (*unfold set-locals-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *heap-set-locals* [*simp*]: *heap* (*set-locals l s*) = *heap s*
apply (*unfold heap-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

abrupt completion

primrec *the-Xcpt* :: *abrupt* ⇒ *xcpt*
where *the-Xcpt* (*Xcpt x*) = *x*

primrec *the-Jump* :: *abrupt* => *jump*
where *the-Jump* (*Jump j*) = *j*

primrec *the-Loc* :: *xcpt* ⇒ *loc*
where *the-Loc* (*Loc a*) = *a*

primrec *the-Std* :: *xcpt* ⇒ *xname*
  where *the-Std* (*Std x*) = *x*

definition
  abrupt-if :: *bool* ⇒ *abopt* ⇒ *abopt* ⇒ *abopt*
  where *abrupt-if c x′ x* = (*if c* ∧ (*x* = *None*) *then x′ else x*)

lemma *abrupt-if-True-None* [*simp*]: *abrupt-if True x None* = *x*
by (*simp add*: *abrupt-if-def*)

lemma *abrupt-if-True-not-None* [*simp*]: *x* ≠ *None* ⟹ *abrupt-if True x y* ≠ *None*
by (*simp add*: *abrupt-if-def*)

lemma *abrupt-if-False* [*simp*]: *abrupt-if False x y* = *y*
by (*simp add*: *abrupt-if-def*)

lemma *abrupt-if-Some* [*simp*]: *abrupt-if c x* (*Some y*) = *Some y*
by (*simp add*: *abrupt-if-def*)

lemma *abrupt-if-not-None* [*simp*]: *y* ≠ *None* ⟹ *abrupt-if c x y* = *y*
apply (*simp add*: *abrupt-if-def*)
by *auto*

lemma *split-abrupt-if*:
P (*abrupt-if c x′ x*) =
      ((*c* ∧ *x* = *None* ⟶ *P x′*) ∧ (¬ (*c* ∧ *x* = *None*) ⟶ *P x*))
apply (*unfold abrupt-if-def*)
apply (*split if-split*)
apply *auto*
done

abbreviation *raise-if* :: *bool* ⇒ *xname* ⇒ *abopt* ⇒ *abopt*
  where *raise-if c xn* == *abrupt-if c* (*Some* (*Xcpt* (*Std xn*)))

abbreviation *np* :: *val* ⇒ *abopt* ⇒ *abopt*
  where *np v* == *raise-if* (*v* = *Null*) *NullPointer*

abbreviation *check-neg* :: *val* ⇒ *abopt* ⇒ *abopt*
  where *check-neg i′* == *raise-if* (*the-Intg i′*<*0*) *NegArrSize*

abbreviation *error-if* :: *bool* ⇒ *error* ⇒ *abopt* ⇒ *abopt*
  where *error-if c e* == *abrupt-if c* (*Some* (*Error e*))

lemma *raise-if-None* [*simp*]: (*raise-if c x y* = *None*) = (¬*c* ∧ *y* = *None*)
apply (*simp add*: *abrupt-if-def*)
by *auto*
declare *raise-if-None* [*THEN iffD1*, *dest*!]

lemma *if-raise-if-None* [*simp*]:

$((if\ b\ then\ y\ else\ raise\text{-}if\ c\ x\ y) = None) = ((c \longrightarrow b) \wedge y = None)$
apply (*simp add*: *abrupt-if-def*)
apply *auto*
done

lemma *raise-if-SomeD* [*dest*!]:
$raise\text{-}if\ c\ x\ y = Some\ z \Longrightarrow c \wedge z{=}(Xcpt\ (Std\ x)) \wedge y{=}None \vee (y{=}Some\ z)$
apply (*case-tac y*)
apply (*case-tac c*)
apply (*simp add*: *abrupt-if-def*)
apply (*simp add*: *abrupt-if-def*)
apply *auto*
done

lemma *error-if-None* [*simp*]: $(error\text{-}if\ c\ e\ y = None) = (\neg c \wedge y = None)$
apply (*simp add*: *abrupt-if-def*)
by *auto*
declare *error-if-None* [*THEN iffD1*, *dest*!]

lemma *if-error-if-None* [*simp*]:
$((if\ b\ then\ y\ else\ error\text{-}if\ c\ e\ y) = None) = ((c \longrightarrow b) \wedge y = None)$
apply (*simp add*: *abrupt-if-def*)
apply *auto*
done

lemma *error-if-SomeD* [*dest*!]:
$error\text{-}if\ c\ e\ y = Some\ z \Longrightarrow c \wedge z{=}(Error\ e) \wedge y{=}None \vee (y{=}Some\ z)$
apply (*case-tac y*)
apply (*case-tac c*)
apply (*simp add*: *abrupt-if-def*)
apply (*simp add*: *abrupt-if-def*)
apply *auto*
done

definition
$absorb :: jump \Rightarrow abopt \Rightarrow abopt$
where $absorb\ j\ a = (if\ a{=}Some\ (Jump\ j)\ then\ None\ else\ a)$

lemma *absorb-SomeD* [*dest*!]: $absorb\ j\ a = Some\ x \Longrightarrow a = Some\ x$
by (*auto simp add*: *absorb-def*)

lemma *absorb-same* [*simp*]: $absorb\ j\ (Some\ (Jump\ j)) = None$
by (*auto simp add*: *absorb-def*)

lemma *absorb-other* [*simp*]: $a \neq Some\ (Jump\ j) \Longrightarrow absorb\ j\ a = a$
by (*auto simp add*: *absorb-def*)

lemma *absorb-Some-NoneD*: $absorb\ j\ (Some\ abr) = None \Longrightarrow abr = Jump\ j$
by (*simp add*: *absorb-def*)

lemma *absorb-Some-JumpD*: *absorb j s* = *Some* (*Jump j'*) $\Longrightarrow$ *j'*$\neq$*j*
  by (*simp add*: *absorb-def*)

full program state

type-synonym
  state = *abopt* $\times$ *st*   — state including abruption information

translations
  (*type*) *abopt* <= (*type*) *abrupt option*
  (*type*) *state* <= (*type*) *abopt* $\times$ *st*

abbreviation
  Norm :: *st* $\Rightarrow$ *state*
  where *Norm s* == (*None*, *s*)

abbreviation (*input*)
  abrupt :: *state* $\Rightarrow$ *abopt*
  where *abrupt* == *fst*

abbreviation (*input*)
  store :: *state* $\Rightarrow$ *st*
  where *store* == *snd*

lemma *single-stateE*: $\forall$ *Z*. *Z* = (*s*::*state*) $\Longrightarrow$ *False*
apply (*erule-tac x* = (*Some k*,*y*) **for** *k y* **in** *all-dupE*)
apply (*erule-tac x* = (*None*,*y*) **for** *y* **in** *allE*)
apply *clarify*
done

lemma *state-not-single*: *All* ((=) (*x*::*state*)) $\Longrightarrow$ *R*
apply (*drule-tac x* = (*if abrupt x* = *None then Some x'* *else None*, *y*) **for** *x' y* **in** *spec*)
apply *clarsimp*
done

definition
  normal :: *state* $\Rightarrow$ *bool*
  where *normal* = (λ*s*. *abrupt s* = *None*)

lemma *normal-def2* [*simp*]: *normal s* = (*abrupt s* = *None*)
apply (*unfold normal-def*)
apply (*simp* (*no-asm*))
done

definition
  heap-free :: *nat* $\Rightarrow$ *state* $\Rightarrow$ *bool*
  where *heap-free n* = (λ*s*. *atleast-free* (*heap* (*store s*)) *n*)

lemma *heap-free-def2* [*simp*]: *heap-free n s* = *atleast-free* (*heap* (*store s*)) *n*
apply (*unfold heap-free-def*)
apply *simp*
done

6 update

definition
abupd :: (*abopt* ⇒ *abopt*) ⇒ *state* ⇒ *state*
where *abupd f* = *map-prod f id*

definition
supd :: (*st* ⇒ *st*) ⇒ *state* ⇒ *state*
where *supd* = *map-prod id*

lemma *abupd-def2* [*simp*]: *abupd f* (*x*,*s*) = (*f x*,*s*)
by (*simp add*: *abupd-def*)

lemma *abupd-abrupt-if-False* [*simp*]: ⋀ *s*. *abupd* (*abrupt-if False xo*) *s* = *s*
by *simp*

lemma *supd-def2* [*simp*]: *supd f* (*x*,*s*) = (*x*,*f s*)
by (*simp add*: *supd-def*)

lemma *supd-lupd* [*simp*]:
⋀ *s*. *supd* (*lupd vn v*) *s* = (*abrupt s*,*lupd vn v* (*store s*))
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))
done

lemma *supd-gupd* [*simp*]:
⋀ *s*. *supd* (*gupd r obj*) *s* = (*abrupt s*,*gupd r obj* (*store s*))
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))
done

lemma *supd-init-obj* [*simp*]:
supd (*init-obj G oi r*) *s* = (*abrupt s*,*init-obj G oi r* (*store s*))
apply (*unfold init-obj-def*)
apply (*simp* (*no-asm*))
done

lemma *abupd-store-invariant* [*simp*]: *store* (*abupd f s*) = *store s*
by (*cases s*) *simp*

lemma *supd-abrupt-invariant* [*simp*]: *abrupt* (*supd f s*) = *abrupt s*
by (*cases s*) *simp*

abbreviation *set-lvars* :: *locals* ⇒ *state* ⇒ *state*
where *set-lvars l* == *supd* (*set-locals l*)

abbreviation *restore-lvars* :: *state* ⇒ *state* ⇒ *state*
where *restore-lvars s′ s* == *set-lvars* (*locals* (*store s′*)) *s*

lemma *set-set-lvars* [*simp*]: $\bigwedge$ *s. set-lvars l* (*set-lvars l′ s*) = *set-lvars l s*
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))
done

lemma *set-lvars-id* [*simp*]: $\bigwedge$ *s. set-lvars* (*locals* (*store s*)) *s* = *s*
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))
done

initialisation test

definition
inited :: *qtname* ⇒ *globs* ⇒ *bool*
where *inited C g* = (*g* (*Stat C*) ≠ *None*)

definition
initd :: *qtname* ⇒ *state* ⇒ *bool*
where *initd C* = *inited C* ∘ *globs* ∘ *store*

lemma *not-inited-empty* [*simp*]: ¬*inited C Map.empty*
apply (*unfold inited-def*)
apply (*simp* (*no-asm*))
done

lemma *inited-gupdate* [*simp*]: *inited C* (*g*(*r*↦*obj*)) = (*inited C g* ∨ *r* = *Stat C*)
apply (*unfold inited-def*)
apply (*auto split*: *st.split*)
done

lemma *inited-init-class-obj* [*intro*!]: *inited C* (*globs* (*init-class-obj G C s*))
apply (*unfold inited-def*)
apply (*simp* (*no-asm*))
done

lemma *not-initedD*: ¬ *inited C g* ⟹ *g* (*Stat C*) = *None*
apply (*unfold inited-def*)
apply (*erule notnotD*)
done

lemma *initedD*: *inited C g* ⟹ ∃ *obj*. *g* (*Stat C*) = *Some obj*
apply (*unfold inited-def*)
apply *auto*
done

lemma *initd-def2* [*simp*]: *initd C s* = *inited C* (*globs* (*store s*))
apply (*unfold initd-def*)
apply (*simp* (*no-asm*))
done

error-free

definition
error-free :: *state* ⇒ *bool*
where *error-free s* = (¬ (∃ *err*. *abrupt s* = *Some* (*Error err*)))

lemma *error-free-Norm* [*simp*,*intro*]: *error-free* (*Norm s*)
by (*simp add*: *error-free-def*)

lemma *error-free-normal* [*simp*,*intro*]: *normal s* ⟹ *error-free s*
by (*simp add*: *error-free-def*)

lemma *error-free-Xcpt* [*simp*]: *error-free* (*Some* (*Xcpt x*),*s*)
by (*simp add*: *error-free-def*)

lemma *error-free-Jump* [*simp*,*intro*]: *error-free* (*Some* (*Jump j*),*s*)
by (*simp add*: *error-free-def*)

lemma *error-free-Error* [*simp*]: *error-free* (*Some* (*Error e*),*s*) = *False*
by (*simp add*: *error-free-def*)

lemma *error-free-Some* [*simp*,*intro*]:
¬ (∃ *err*. *x*=*Error err*) ⟹ *error-free* ((*Some x*),*s*)
by (*auto simp add*: *error-free-def*)

lemma *error-free-abupd-absorb* [*simp*,*intro*]:
error-free s ⟹ *error-free* (*abupd* (*absorb j*) *s*)
by (*cases s*)
(*auto simp add*: *error-free-def absorb-def*
split: *if-split-asm*)

lemma *error-free-absorb* [*simp*,*intro*]:
error-free (*a*,*s*) ⟹ *error-free* (*absorb j a*, *s*)
by (*auto simp add*: *error-free-def absorb-def*
split: *if-split-asm*)

lemma *error-free-abrupt-if* [*simp*,*intro*]:
⟦*error-free s*; ¬ (∃ *err*. *x*=*Error err*)⟧
⟹ *error-free* (*abupd* (*abrupt-if p* (*Some x*)) *s*)
by (*cases s*)
(*auto simp add*: *abrupt-if-def*
split: *if-split*)

lemma *error-free-abrupt-if1* [*simp*,*intro*]:
⟦*error-free* (*a*,*s*); ¬ (∃ *err*. *x*=*Error err*)⟧
⟹ *error-free* (*abrupt-if p* (*Some x*) *a*, *s*)
by (*auto simp add*: *abrupt-if-def*
split: *if-split*)

lemma *error-free-abrupt-if-Xcpt* [*simp*,*intro*]:
error-free s
$\Longrightarrow$ *error-free* (*abupd* (*abrupt-if p* (*Some* (*Xcpt x*))) *s*)
by *simp*

lemma *error-free-abrupt-if-Xcpt1* [*simp*,*intro*]:
error-free (*a*,*s*)
$\Longrightarrow$ *error-free* (*abrupt-if p* (*Some* (*Xcpt x*)) *a*, *s*)
by *simp*

lemma *error-free-abrupt-if-Jump* [*simp*,*intro*]:
error-free s
$\Longrightarrow$ *error-free* (*abupd* (*abrupt-if p* (*Some* (*Jump j*))) *s*)
by *simp*

lemma *error-free-abrupt-if-Jump1* [*simp*,*intro*]:
error-free (*a*,*s*)
$\Longrightarrow$ *error-free* (*abrupt-if p* (*Some* (*Jump j*)) *a*, *s*)
by *simp*

lemma *error-free-raise-if* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* (*abupd* (*raise-if p x*) *s*)
by *simp*

lemma *error-free-raise-if1* [*simp*,*intro*]:
error-free (*a*,*s*) $\Longrightarrow$ *error-free* ((*raise-if p x a*), *s*)
by *simp*

lemma *error-free-supd* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* (*supd f s*)
by (*cases s*) (*simp add*: *error-free-def*)

lemma *error-free-supd1* [*simp*,*intro*]:
error-free (*a*,*s*) $\Longrightarrow$ *error-free* (*a*,*f s*)
by (*simp add*: *error-free-def*)

lemma *error-free-set-lvars* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* ((*set-lvars l*) *s*)
by (*cases s*) *simp*

lemma *error-free-set-locals* [*simp*,*intro*]:
error-free (*x*, *s*)
$\Longrightarrow$ *error-free* (*x*, *set-locals l s'*)
by (*simp add*: *error-free-def*)

end

Chapter 15

Eval

1 Operational evaluation (big-step) semantics of Java expressions and statements

theory *Eval* **imports** *State DeclConcepts* **begin**

improvements over Java Specification 1.0:

- dynamic method lookup does not need to consider the return type (cf.15.11.4.4)
- throw raises a NullPointer exception if a null reference is given, and each throw of a standard exception yield a fresh exception object (was not specified)
- if there is not enough memory even to allocate an OutOfMemory exception, evaluation/execution fails, i.e. simply stops (was not specified)
- array assignment checks lhs (and may throw exceptions) before evaluating rhs
- fixed exact positions of class initializations (immediate at first active use)

design issues:

- evaluation vs. (single-step) transition semantics evaluation semantics chosen, because:
  - ++ less verbose and therefore easier to read (and to handle in proofs)
  - \+ more abstract
  - \+ intermediate values (appearing in recursive rules) need not be stored explicitly, e.g. no call body construct or stack of invocation frames containing local variables and return addresses for method calls needed
  - \+ convenient rule induction for subject reduction theorem
  - \- no interleaving (for parallelism) can be described
  - \- stating a property of infinite executions requires the meta-level argument that this property holds for any finite prefixes of it (e.g. stopped using a counter that is decremented to zero and then throwing an exception)
- unified evaluation for variables, expressions, expression lists, statements
- the value entry in statement rules is redundant
- the value entry in rules is irrelevant in case of exceptions, but its full inclusion helps to make the rule structure independent of exception occurrence.
- as irrelevant value entries are ignored, it does not matter if they are unique. For simplicity, (fixed) arbitrary values are preferred over "free" values.

- the rule format is such that the start state may contain an exception.

  ++ faciliates exception handling

  + symmetry

- the rules are defined carefully in order to be applicable even in not type-correct situations (yielding undefined values), e.g. *the-Addr* (*Val* (*Bool* b)) = *undefined*.

  ++ fewer rules

  - less readable because of auxiliary functions like *the-Addr*

  Alternative: "defensive" evaluation throwing some InternalError exception in case of (impossible, for correct programs) type mismatches

- there is exactly one rule per syntactic construct

  + no redundancy in case distinctions

- halloc fails iff there is no free heap address. When there is only one free heap address left, it returns an OutOfMemory exception. In this way it is guaranteed that when an OutOfMemory exception is thrown for the first time, there is a free location on the heap to allocate it.

- the allocation of objects that represent standard exceptions is deferred until execution of any enclosing catch clause, which is transparent to the program.

  - requires an auxiliary execution relation

  ++ avoids copies of allocation code and awkward case distinctions (whether there is enough memory to allocate the exception) in evaluation rules

- unfortunately *new-Addr* is not directly executable because of Hilbert operator.

simplifications:

- local variables are initialized with default values (no definite assignment)
- garbage collection not considered, therefore also no finalizers
- stack overflow and memory overflow during class initialization not modelled
- exceptions in initializations not replaced by ExceptionInInitializerError

type-synonym $vvar = val \times (val \Rightarrow state \Rightarrow state)$
type-synonym $vals = (val,\ vvar,\ val\ list)\ sum3$
translations
  $(type)\ vvar <= (type)\ val \times (val \Rightarrow state \Rightarrow state)$
  $(type)\ vals <= (type)\ (val,\ vvar,\ val\ list)\ sum3$

To avoid redundancy and to reduce the number of rules, there is only one evaluation rule for each syntactic term. This is also true for variables (e.g. see the rules below for *LVar*, *FVar* and *AVar*). So evaluation of a variable must capture both possible further uses: read (rule *Acc*) or write (rule *Ass*) to the variable. Therefor a variable evaluates to a special value *vvar*, which is a pair, consisting of the current value (for later read access) and an update function (for later write access). Because during assignment to an array variable an exception may occur if the types don't match, the update function is very generic: it transforms the full state. This generic update function causes some technical trouble during some proofs (e.g. type safety, correctness of definite assignment). There we need to prove some additional invariant on this update function to prove the assignment correct, since the update function could potentially alter the whole state in an arbitrary manner. This invariant must be carried around through the whole induction. So for future approaches it may be better not to take such a generic update function, but only to store the address and the kind of variable (array (+ element type), local variable or field) for later assignment.

abbreviation
dummy-res :: *vals* ($\diamondsuit$)
where $\diamondsuit$ == *In1 Unit*

abbreviation (*input*)
val-inj-vals ($\lfloor$-$\rfloor_e$ *1000*)
where $\lfloor e \rfloor_e$ == *In1 e*

abbreviation (*input*)
var-inj-vals ($\lfloor$-$\rfloor_v$ *1000*)
where $\lfloor v \rfloor_v$ == *In2 v*

abbreviation (*input*)
lst-inj-vals ($\lfloor$-$\rfloor_l$ *1000*)
where $\lfloor es \rfloor_l$ == *In3 es*

definition *undefined3* :: ($'al$ + $'ar$, $'b$, $'c$) *sum3* $\Rightarrow$ *vals* **where**
undefined3 = *case-sum3* (*In1* $\circ$ *case-sum* (λx. *undefined*) (λx. *Unit*))
(λx. *In2 undefined*) (λx. *In3 undefined*)

lemma [*simp*]: *undefined3* (*In1l x*) = *In1 undefined*
by (*simp add*: *undefined3-def*)

lemma [*simp*]: *undefined3* (*In1r x*) = $\diamondsuit$
by (*simp add*: *undefined3-def*)

lemma [*simp*]: *undefined3* (*In2 x*) = *In2 undefined*
by (*simp add*: *undefined3-def*)

lemma [*simp*]: *undefined3* (*In3 x*) = *In3 undefined*
by (*simp add*: *undefined3-def*)

exception throwing and catching

definition
throw :: *val* $\Rightarrow$ *abopt* $\Rightarrow$ *abopt* **where**
throw a' *x* = *abrupt-if True* (*Some* (*Xcpt* (*Loc* (*the-Addr* a')))) (*np* a' *x*)

lemma *throw-def2*:
throw a' *x* = *abrupt-if True* (*Some* (*Xcpt* (*Loc* (*the-Addr* a')))) (*np* a' *x*)
apply (*unfold throw-def*)
apply (*simp* (*no-asm*))
done

definition
fits :: *prog* $\Rightarrow$ *st* $\Rightarrow$ *val* $\Rightarrow$ *ty* $\Rightarrow$ *bool* (-,-$\vdash$- *fits* -[*61*,*61*,*61*,*61*]*60*)
where $G,s\vdash a'$ *fits* T = (($\exists$ *rt*. T=*RefT rt*) $\longrightarrow$ a'=*Null* $\vee$ $G\vdash$*obj-ty*(*lookup-obj s* a')$\preceq T$)

lemma *fits-Null* [*simp*]: $G,s\vdash$*Null fits* T

by (*simp add*: *fits-def*)

lemma *fits-Addr-RefT* [*simp*]:
$G{,}s{\vdash}Addr\ a\ fits\ RefT\ t = G{\vdash}obj\text{-}ty\ (the\ (heap\ s\ a)){\preceq}RefT\ t$
by (*simp add*: *fits-def*)

lemma *fitsD*: $\bigwedge X.\ G{,}s{\vdash}a'\ fits\ T \Longrightarrow (\exists\,pt.\ T = PrimT\ pt) \vee$
$(\exists\,t.\ T = RefT\ t) \wedge a' = Null \vee$
$(\exists\,t.\ T = RefT\ t) \wedge a' \neq Null \wedge\ \ G{\vdash}obj\text{-}ty\ (lookup\text{-}obj\ s\ a'){\preceq}T$
apply (*unfold fits-def*)
apply (*case-tac* $\exists\,pt.\ T = PrimT\ pt$)
apply *simp-all*
apply (*case-tac T*)
defer
apply (*case-tac* $a' = Null$)
apply *simp-all*
done

definition
catch :: $prog \Rightarrow state \Rightarrow qtname \Rightarrow bool$ (-,-$\vdash$*catch* -[*61*,*61*,*61*]*60*) **where**
$G{,}s{\vdash}catch\ C = (\exists\,xc.\ abrupt\ s{=}Some\ (Xcpt\ xc) \wedge$
$G{,}store\ s{\vdash}Addr\ (the\text{-}Loc\ xc)\ fits\ Class\ C)$

lemma *catch-Norm* [*simp*]: $\neg G{,}Norm\ s{\vdash}catch\ tn$
apply (*unfold catch-def*)
apply (*simp* (*no-asm*))
done

lemma *catch-XcptLoc* [*simp*]:
$G{,}(Some\ (Xcpt\ (Loc\ a)){,}s){\vdash}catch\ C = G{,}s{\vdash}Addr\ a\ fits\ Class\ C$
apply (*unfold catch-def*)
apply (*simp* (*no-asm*))
done

lemma *catch-Jump* [*simp*]: $\neg G{,}(Some\ (Jump\ j){,}s){\vdash}catch\ tn$
apply (*unfold catch-def*)
apply (*simp* (*no-asm*))
done

lemma *catch-Error* [*simp*]: $\neg G{,}(Some\ (Error\ e){,}s){\vdash}catch\ tn$
apply (*unfold catch-def*)
apply (*simp* (*no-asm*))
done

definition
new-xcpt-var :: $vname \Rightarrow state \Rightarrow state$ **where**
$new\text{-}xcpt\text{-}var\ vn = (\lambda(x{,}s).\ Norm\ (lupd(VName\ vn{\mapsto}Addr\ (the\text{-}Loc\ (the\text{-}Xcpt\ (the\ x))))\ s))$

lemma *new-xcpt-var-def2* [*simp*]:
$new\text{-}xcpt\text{-}var\ vn\ (x{,}s) =$
$Norm\ (lupd(VName\ vn{\mapsto}Addr\ (the\text{-}Loc\ (the\text{-}Xcpt\ (the\ x))))\ s)$

apply (*unfold new-xcpt-var-def*)
apply (*simp* (*no-asm*))
done

misc

definition
assign :: $('a \Rightarrow state \Rightarrow state) \Rightarrow 'a \Rightarrow state \Rightarrow state$ **where**
assign f v = $(\lambda(x,s).\ let\ (x',s') = (if\ x = None\ then\ f\ v\ else\ id)\ (x,s)$
$in\ (x', if\ x' = None\ then\ s'\ else\ s))$

lemma *assign-Norm-Norm* [*simp*]:
$f\ v\ (Norm\ s) = Norm\ s' \Longrightarrow assign\ f\ v\ (Norm\ s) = Norm\ s'$
by (*simp add*: *assign-def Let-def*)

lemma *assign-Norm-Some* [*simp*]:
$[\![abrupt\ (f\ v\ (Norm\ s)) = Some\ y]\!]$
$\Longrightarrow assign\ f\ v\ (Norm\ s) = (Some\ y,s)$
by (*simp add*: *assign-def Let-def split-beta*)

lemma *assign-Some* [*simp*]:
$assign\ f\ v\ (Some\ x,s) = (Some\ x,s)$
by (*simp add*: *assign-def Let-def split-beta*)

lemma *assign-Some1* [*simp*]: $\neg\ normal\ s \Longrightarrow assign\ f\ v\ s = s$
by (*auto simp add*: *assign-def Let-def split-beta*)

lemma *assign-supd* [*simp*]:
$assign\ (\lambda v.\ supd\ (f\ v))\ v\ (x,s)$
$= (x,\ if\ x = None\ then\ f\ v\ s\ else\ s)$
apply *auto*
done

lemma *assign-raise-if* [*simp*]:
$assign\ (\lambda v\ (x,s).\ ((raise\text{-}if\ (b\ s\ v)\ xcpt)\ x,\ f\ v\ s))\ v\ (x,\ s) =$
$(raise\text{-}if\ (b\ s\ v)\ xcpt\ x,\ if\ x{=}None \wedge \neg b\ s\ v\ then\ f\ v\ s\ else\ s)$
apply (*case-tac* $x = None$)
apply *auto*
done

definition
init-comp-ty :: $ty \Rightarrow stmt$
where *init-comp-ty* $T = (if\ (\exists\, C.\ T = Class\ C)\ then\ Init\ (the\text{-}Class\ T)\ else\ Skip)$

lemma *init-comp-ty-PrimT* [*simp*]: *init-comp-ty* (*PrimT pt*) = *Skip*
apply (*unfold init-comp-ty-def*)
apply (*simp* (*no-asm*))
done

definition
invocation-class :: *inv-mode* ⇒ *st* ⇒ *val* ⇒ *ref-ty* ⇒ *qtname* **where**
invocation-class m s a′ statT =
(*case m of*
Static ⇒ *if* (∃ *statC*. *statT* = *ClassT statC*)
then the-Class (*RefT statT*)
else Object
| *SuperM* ⇒ *the-Class* (*RefT statT*)
| *IntVir* ⇒ *obj-class* (*lookup-obj s a′*))

definition
invocation-declclass :: *prog* ⇒ *inv-mode* ⇒ *st* ⇒ *val* ⇒ *ref-ty* ⇒ *sig* ⇒ *qtname* **where**
invocation-declclass G m s a′ statT sig =
declclass (*the* (*dynlookup G statT*
(*invocation-class m s a′ statT*)
sig))

lemma *invocation-class-IntVir* [*simp*]:
invocation-class IntVir s a′ statT = *obj-class* (*lookup-obj s a′*)
by (*simp add*: *invocation-class-def*)

lemma *dynclass-SuperM* [*simp*]:
invocation-class SuperM s a′ statT = *the-Class* (*RefT statT*)
by (*simp add*: *invocation-class-def*)

lemma *invocation-class-Static* [*simp*]:
invocation-class Static s a′ statT = (*if* (∃ *statC*. *statT* = *ClassT statC*)
then the-Class (*RefT statT*)
else Object)
by (*simp add*: *invocation-class-def*)

definition
init-lvars :: *prog* ⇒ *qtname* ⇒ *sig* ⇒ *inv-mode* ⇒ *val* ⇒ *val list* ⇒ *state* ⇒ *state*
where
init-lvars G C sig mode a′ pvs =
(λ(*x*,*s*).
let m = *mthd* (*the* (*methd G C sig*));
l = λ *k*.
(*case k of*
EName e
⇒ (*case e of*
VNam v ⇒ (*Map.empty* ((*pars m*)[↦]*pvs*)) *v*
| *Res* ⇒ *None*)
| *This*
⇒ (*if mode*=*Static then None else Some a′*))
in set-lvars l (*if mode* = *Static then x else np a′ x*,*s*))

lemma *init-lvars-def2*: — better suited for simplification

init-lvars G C sig mode a′ pvs (x,s) =
  set-lvars
    (λ k.
      (case k of
        EName e
          ⇒ (case e of
              VNam v
              ⇒ (Map.empty ((pars (mthd (the (methd G C sig))))[↦]pvs)) v
            | Res ⇒ None)
      | This
          ⇒ (if mode=Static then None else Some a′)))
    (if mode = Static then x else np a′ x,s)
apply *(unfold init-lvars-def)*
apply *(simp (no-asm) add: Let-def)*
done

definition
  body :: prog ⇒ qtname ⇒ sig ⇒ expr **where**
 body G C sig =
    (let m = the (methd G C sig)
     in Body (declclass m) (stmt (mbody (mthd m))))

lemma *body-def2*: — better suited for simplification
body G C sig = Body (declclass (the (methd G C sig)))
                    (stmt (mbody (mthd (the (methd G C sig)))))
apply *(unfold body-def Let-def)*
apply *auto*
done

variables

definition
  lvar :: lname ⇒ st ⇒ vvar
  where *lvar vn s = (the (locals s vn), λv. supd (lupd(vn↦v)))*

definition
  fvar :: qtname ⇒ bool ⇒ vname ⇒ val ⇒ state ⇒ vvar × state **where**
 fvar C stat fn a′ s =
   (let (oref,xf) = if stat then (Stat C,id)
                            else (Heap (the-Addr a′),np a′);
                  n = Inl (fn,C);
                  f = (λv. supd (upd-gobj oref n v))
    in ((the (values (the (globs (store s) oref)) n),f),abupd xf s))

definition
  avar :: prog ⇒ val ⇒ val ⇒ state ⇒ vvar × state **where**
  avar G i′ a′ s =
    (let oref = Heap (the-Addr a′);
           i = the-Intg i′;
           n = Inr i;
      (T,k,cs) = the-Arr (globs (store s) oref);
           f = (λv (x,s). (raise-if (¬G,s⊢v fits T)
                                              ArrStore x
                              ,upd-gobj oref n v s))
     in ((the (cs n),f),abupd (raise-if (¬i in-bounds k) IndOutBound ∘ np a′) s))

lemma *fvar-def2*: — better suited for simplification

```
fvar C stat fn a′ s =
  ((the
     (values
      (the (globs (store s) (if stat then Stat C else Heap (the-Addr a′))))
      (Inl (fn,C)))
   ,(λv. supd (upd-gobj (if stat then Stat C else Heap (the-Addr a′))
                        (Inl (fn,C))
                        v)))
  ,abupd (if stat then id else np a′) s)
```

apply (*unfold fvar-def*)
apply (*simp* (*no-asm*) *add*: *Let-def split-beta*)
done

lemma *avar-def2*: — better suited for simplification

```
avar G i′ a′ s =
  ((the ((snd(snd(the-Arr (globs (store s) (Heap (the-Addr a′))))))
          (Inr (the-Intg i′)))
   ,(λv (x,s′).  (raise-if (¬G,s′⊢v fits (fst(the-Arr (globs (store s)
                                                    (Heap (the-Addr a′))))))
                            ArrStore x
                  ,upd-gobj  (Heap (the-Addr a′))
                             (Inr (the-Intg i′)) v s′)))
  ,abupd (raise-if (¬(the-Intg i′) in-bounds (fst(snd(the-Arr (globs (store s)
                   (Heap (the-Addr a′))))))) IndOutBound ∘ np a′)
          s)
```

apply (*unfold avar-def*)
apply (*simp* (*no-asm*) *add*: *Let-def split-beta*)
done

definition

```
  check-field-access :: prog ⇒ qtname ⇒ qtname ⇒ vname ⇒ bool ⇒ val ⇒ state ⇒ state where
  check-field-access G accC statDeclC fn stat a′ s =
   (let oref = if stat then Stat statDeclC
               else Heap (the-Addr a′);
        dynC = case oref of
                 Heap a ⇒ obj-class (the (globs (store s) oref))
               | Stat C ⇒ C;
        f    = (the (table-of (DeclConcepts.fields G dynC) (fn,statDeclC)))
    in abupd
       (error-if (¬ G⊢Field fn (statDeclC,f) in dynC dyn-accessible-from accC)
                 AccessViolation)
       s)
```

definition

```
  check-method-access :: prog ⇒ qtname ⇒ ref-ty ⇒ inv-mode ⇒  sig ⇒ val ⇒ state ⇒ state where
  check-method-access G accC statT mode sig  a′ s =
   (let invC = invocation-class mode (store s) a′ statT;
      dynM = the (dynlookup G statT invC sig)
    in abupd
       (error-if (¬ G⊢Methd sig dynM in invC dyn-accessible-from accC)
                 AccessViolation)
       s)
```

evaluation judgments

inductive

```
  halloc :: [prog,state,obj-tag,loc,state]⇒bool (-⊢- −halloc -≻-→ -[61,61,61,61,61]60) for G::prog
```

where — allocating objects on the heap, cf. 12.5

Abrupt:
G⊢(*Some x*,*s*) −*halloc oi*≻*undefined*→ (*Some x*,*s*)

| *New*: ⟦*new-Addr* (*heap s*) = *Some a*;
(*x*,*oi′*) = (*if atleast-free* (*heap s*) (*Suc* (*Suc 0*)) *then* (*None*,*oi*)
else (*Some* (*Xcpt* (*Loc a*)),*CInst* (*SXcpt OutOfMemory*)))⟧
⟹
G⊢*Norm s* −*halloc oi*≻*a*→ (*x*,*init-obj G oi′* (*Heap a*) *s*)

inductive *sxalloc* :: [*prog*,*state*,*state*]⇒*bool* (-⊢- −*sxalloc*→ -[*61*,*61*,*61*]*60*) **for** *G*::*prog*
where — allocating exception objects for standard exceptions (other than OutOfMemory)

Norm: G⊢ *Norm* *s* −*sxalloc*→ *Norm* *s*

| *Jmp*: G⊢(*Some* (*Jump j*), *s*) −*sxalloc*→ (*Some* (*Jump j*), *s*)

| *Error*: G⊢(*Some* (*Error e*), *s*) −*sxalloc*→ (*Some* (*Error e*), *s*)

| *XcptL*: G⊢(*Some* (*Xcpt* (*Loc a*)),*s*) −*sxalloc*→ (*Some* (*Xcpt* (*Loc a*)),*s*)

| *SXcpt*: ⟦G⊢*Norm s0* −*halloc* (*CInst* (*SXcpt xn*))≻*a*→ (*x*,*s1*)⟧ ⟹
G⊢(*Some* (*Xcpt* (*Std xn*)),*s0*) −*sxalloc*→ (*Some* (*Xcpt* (*Loc a*)),*s1*)

inductive
eval :: [*prog*,*state*,*term*,*vals*,*state*]⇒*bool* (-⊢- −-≻→ ′(-, -′) [*61*,*61*,*80*,*0*,*0*]*60*)
and *exec* ::[*prog*,*state*,*stmt* ,*state*]⇒*bool*(-⊢- −-→ - [*61*,*61*,*65*, *61*]*60*)
and *evar* ::[*prog*,*state*,*var* ,*vvar*,*state*]⇒*bool*(-⊢- −-=≻-→ -[*61*,*61*,*90*,*61*,*61*]*60*)
and *eval′*::[*prog*,*state*,*expr* ,*val* ,*state*]⇒*bool*(-⊢- −-−≻-→ -[*61*,*61*,*80*,*61*,*61*]*60*)
and *evals*::[*prog*,*state*,*expr list* ,
val list ,*state*]⇒*bool*(-⊢- −-≐≻-→ -[*61*,*61*,*61*,*61*,*61*]*60*)
for *G*::*prog*
where

G⊢*s* −*c* → *s′* ≡ G⊢*s* −*In1r c*≻→ (◇, *s′*)
| G⊢*s* −*e*−≻*v* → *s′* ≡ G⊢*s* −*In1l e*≻→ (*In1 v*, *s′*)
| G⊢*s* −*e*=≻*vf*→ *s′* ≡ G⊢*s* −*In2 e*≻→ (*In2 vf*, *s′*)
| G⊢*s* −*e*≐≻*v* → *s′* ≡ G⊢*s* −*In3 e*≻→ (*In3 v*, *s′*)

— propagation of abrupt completion

— cf. 14.1, 15.5
| *Abrupt*:
G⊢(*Some xc*,*s*) −*t*≻→ (*undefined3 t*, (*Some xc*, *s*))

— execution of statements

— cf. 14.5
| *Skip*: G⊢*Norm s* −*Skip*→ *Norm s*

— cf. 14.7
| *Expr*: ⟦G⊢*Norm s0* −*e*−≻*v*→ *s1*⟧ ⟹
G⊢*Norm s0* −*Expr e*→ *s1*

| *Lab*: ⟦G⊢*Norm s0* −*c* → *s1*⟧ ⟹
G⊢*Norm s0* −*l*• *c*→ *abupd* (*absorb l*) *s1*

— cf. 14.2

$$
\begin{array}{l}
\mid \mathit{Comp}\colon [\![G\vdash \mathit{Norm}\ s0\ -c1\ \rightarrow\ s1;\\
\qquad\quad G\vdash\qquad s1\ -c2\ \rightarrow\ s2]\!] \Longrightarrow\\
\qquad\qquad\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -c1;;\ c2\rightarrow\ s2
\end{array}
$$

— cf. 14.8.2

$$
\begin{array}{l}
\mid \mathit{If}\colon\ [\![G\vdash \mathit{Norm}\ s0\ -e-\succ b\rightarrow\ s1;\\
\qquad\quad G\vdash\qquad s1-(\mathit{if\ the\text{-}Bool}\ b\ \mathit{then}\ c1\ \mathit{else}\ c2)\rightarrow\ s2]\!] \Longrightarrow\\
\qquad\qquad\qquad\quad G\vdash \mathit{Norm}\ s0\ -\mathit{If}(e)\ c1\ \mathit{Else}\ c2\ \rightarrow\ s2
\end{array}
$$

— cf. 14.10, 14.10.1

— A continue jump from the while body c is handled by this rule. If a continue jump with the proper label was invoked inside c this label (Cont l) is deleted out of the abrupt component of the state before the iterative evaluation of the while statement. A break jump is handled by the Lab Statement *Lab l* (*while*...).

$$
\begin{array}{l}
\mid \mathit{Loop}\colon [\![G\vdash \mathit{Norm}\ s0\ -e-\succ b\rightarrow\ s1;\\
\qquad\quad \mathit{if\ the\text{-}Bool}\ b\\
\qquad\qquad \mathit{then}\ (G\vdash s1\ -c\rightarrow\ s2\ \wedge\\
\qquad\qquad\qquad G\vdash(\mathit{abupd}\ (\mathit{absorb}\ (\mathit{Cont}\ l))\ s2)\ -l\bullet\ \mathit{While}(e)\ c\rightarrow\ s3)\\
\qquad\qquad \mathit{else}\ s3\ =\ s1]\!] \Longrightarrow\\
\qquad\qquad\qquad\qquad\quad G\vdash \mathit{Norm}\ s0\ -l\bullet\ \mathit{While}(e)\ c\rightarrow\ s3
\end{array}
$$

$$
\mid \mathit{Jmp}\colon G\vdash \mathit{Norm}\ s\ -\mathit{Jmp}\ j\rightarrow\ (\mathit{Some}\ (\mathit{Jump}\ j),\ s)
$$

— cf. 14.16

$$
\begin{array}{l}
\mid \mathit{Throw}\colon [\![G\vdash \mathit{Norm}\ s0\ -e-\succ a'\rightarrow\ s1]\!] \Longrightarrow\\
\qquad\qquad\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -\mathit{Throw}\ e\rightarrow\ \mathit{abupd}\ (\mathit{throw}\ a')\ s1
\end{array}
$$

— cf. 14.18.1

$$
\begin{array}{l}
\mid \mathit{Try}\colon\ [\![G\vdash \mathit{Norm}\ s0\ -c1\rightarrow\ s1;\ G\vdash s1\ -\mathit{sxalloc}\rightarrow\ s2;\\
\qquad\quad \mathit{if}\ G,s2\vdash \mathit{catch}\ C\ \mathit{then}\ G\vdash \mathit{new\text{-}xcpt\text{-}var}\ vn\ s2\ -c2\rightarrow\ s3\ \mathit{else}\ s3\ =\ s2]\!] \Longrightarrow\\
\qquad\qquad\quad G\vdash \mathit{Norm}\ s0\ -\mathit{Try}\ c1\ \mathit{Catch}(C\ vn)\ c2\rightarrow\ s3
\end{array}
$$

— cf. 14.18.2

$$
\begin{array}{l}
\mid \mathit{Fin}\colon\ [\![G\vdash \mathit{Norm}\ s0\ -c1\rightarrow\ (x1,s1);\\
\qquad\quad G\vdash \mathit{Norm}\ s1\ -c2\rightarrow\ s2;\\
\qquad\quad s3{=}(\mathit{if}\ (\exists\ \mathit{err}.\ x1{=}\mathit{Some}\ (\mathit{Error}\ \mathit{err}))\\
\qquad\qquad \mathit{then}\ (x1,s1)\\
\qquad\qquad \mathit{else}\ \mathit{abupd}\ (\mathit{abrupt\text{-}if}\ (x1{\neq}\mathit{None})\ x1)\ s2)\ ]\!]\\
\qquad\quad \Longrightarrow\\
\qquad\quad G\vdash \mathit{Norm}\ s0\ -c1\ \mathit{Finally}\ c2\rightarrow\ s3
\end{array}
$$

— cf. 12.4.2, 8.5

$$
\begin{array}{l}
\mid \mathit{Init}\colon [\![\mathit{the}\ (\mathit{class}\ G\ C)\ =\ c;\\
\qquad\quad \mathit{if\ inited}\ C\ (\mathit{globs}\ s0)\ \mathit{then}\ s3\ =\ \mathit{Norm}\ s0\\
\qquad\quad \mathit{else}\ (G\vdash \mathit{Norm}\ (\mathit{init\text{-}class\text{-}obj}\ G\ C\ s0)\\
\qquad\qquad\quad -(\mathit{if}\ C\ =\ \mathit{Object}\ \mathit{then}\ \mathit{Skip}\ \mathit{else}\ \mathit{Init}\ (\mathit{super}\ c))\rightarrow\ s1\ \wedge\\
\qquad\qquad G\vdash \mathit{set\text{-}lvars}\ \mathit{Map.empty}\ s1\ -\mathit{init}\ c\rightarrow\ s2\ \wedge\ s3\ =\ \mathit{restore\text{-}lvars}\ s1\ s2)]\!]\\
\qquad\qquad \Longrightarrow\\
\qquad\qquad\quad G\vdash \mathit{Norm}\ s0\ -\mathit{Init}\ C\rightarrow\ s3
\end{array}
$$

— This class initialisation rule is a little bit inaccurate. Look at the exact sequence: (1) The current class object (the static fields) are initialised (*init-class-obj*), (2) the superclasses are initialised, (3) the static initialiser of the current class is invoked. More precisely we should expect another ordering, namely 2 1 3. But we can't just naively toggle 1 and 2. By calling *init-class-obj* before initialising the superclasses, we also implicitly record that we have started to initialise the current class (by setting an value for the class object). This becomes crucial for the completeness proof of the axiomatic semantics *AxCompl.thy*. Static initialisation requires an induction on the number of classes not yet initialised (or to be more precise, classes were the initialisation has not yet begun). So we could first assign a dummy value to the class before superclass initialisation and afterwards set the correct values. But as long as we don't take memory overflow into account when allocating class objects, we can leave things as they are for convenience.

— evaluation of expressions

— cf. 15.8.1, 12.4.1

$$
\begin{array}{l}
\mid \mathit{NewC}\colon [\![G\vdash \mathit{Norm}\ s0\ -\mathit{Init}\ C\rightarrow\ s1;\\
\qquad G\vdash \quad s1\ -\mathit{halloc}\ (\mathit{CInst}\ C)\succ a\rightarrow\ s2]\!]\Longrightarrow\\
\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -\mathit{NewC}\ C-\succ \mathit{Addr}\ a\rightarrow\ s2
\end{array}
$$

— cf. 15.9.1, 12.4.1

$$
\begin{array}{l}
\mid \mathit{NewA}\colon [\![G\vdash \mathit{Norm}\ s0\ -\mathit{init\text{-}comp\text{-}ty}\ T\rightarrow\ s1;\ G\vdash s1\ -e-\succ i'\rightarrow\ s2;\\
\qquad G\vdash \mathit{abupd}\ (\mathit{check\text{-}neg}\ i')\ s2\ -\mathit{halloc}\ (\mathit{Arr}\ T\ (\mathit{the\text{-}Intg}\ i'))\succ a\rightarrow\ s3]\!]\Longrightarrow\\
\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -\mathit{New}\ T[e]-\succ \mathit{Addr}\ a\rightarrow\ s3
\end{array}
$$

— cf. 15.15

$$
\begin{array}{l}
\mid \mathit{Cast}\colon [\![G\vdash \mathit{Norm}\ s0\ -e-\succ v\rightarrow\ s1;\\
\qquad s2 = \mathit{abupd}\ (\mathit{raise\text{-}if}\ (\neg G,\mathit{store}\ s1\vdash v\ \mathit{fits}\ T)\ \mathit{ClassCast})\ s1]\!]\Longrightarrow\\
\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -\mathit{Cast}\ T\ e-\succ v\rightarrow\ s2
\end{array}
$$

— cf. 15.19.2

$$
\begin{array}{l}
\mid \mathit{Inst}\colon [\![G\vdash \mathit{Norm}\ s0\ -e-\succ v\rightarrow\ s1;\\
\qquad b = (v\neq \mathit{Null}\ \wedge\ G,\mathit{store}\ s1\vdash v\ \mathit{fits}\ \mathit{RefT}\ T)]\!]\Longrightarrow\\
\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -e\ \mathit{InstOf}\ T-\succ \mathit{Bool}\ b\rightarrow\ s1
\end{array}
$$

— cf. 15.7.1

$$
\mid \mathit{Lit}\colon\ G\vdash \mathit{Norm}\ s\ -\mathit{Lit}\ v-\succ v\rightarrow\ \mathit{Norm}\ s
$$

$$
\begin{array}{l}
\mid \mathit{UnOp}\colon [\![G\vdash \mathit{Norm}\ s0\ -e-\succ v\rightarrow\ s1]\!]\\
\qquad\Longrightarrow G\vdash \mathit{Norm}\ s0\ -\mathit{UnOp}\ \mathit{unop}\ e-\succ(\mathit{eval\text{-}unop}\ \mathit{unop}\ v)\rightarrow\ s1
\end{array}
$$

$$
\begin{array}{l}
\mid \mathit{BinOp}\colon [\![G\vdash \mathit{Norm}\ s0\ -e1-\succ v1\rightarrow\ s1;\\
\qquad G\vdash s1\ -(\mathit{if}\ \mathit{need\text{-}second\text{-}arg}\ \mathit{binop}\ v1\ \mathit{then}\ (\mathit{In1l}\ e2)\ \mathit{else}\ (\mathit{In1r}\ \mathit{Skip}))\\
\qquad\qquad \succ\rightarrow\ (\mathit{In1}\ v2,\ s2)\\
\qquad]\!]\\
\qquad\Longrightarrow G\vdash \mathit{Norm}\ s0\ -\mathit{BinOp}\ \mathit{binop}\ e1\ e2-\succ(\mathit{eval\text{-}binop}\ \mathit{binop}\ v1\ v2)\rightarrow\ s2
\end{array}
$$

— cf. 15.10.2

$$
\mid \mathit{Super}\colon G\vdash \mathit{Norm}\ s\ -\mathit{Super}-\succ \mathit{val\text{-}this}\ s\rightarrow\ \mathit{Norm}\ s
$$

— cf. 15.2

$$
\begin{array}{l}
\mid \mathit{Acc}\colon\ [\![G\vdash \mathit{Norm}\ s0\ -\mathit{va}{=}\succ(v,f)\rightarrow\ s1]\!]\Longrightarrow\\
\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -\mathit{Acc}\ \mathit{va}-\succ v\rightarrow\ s1
\end{array}
$$

— cf. 15.25.1

$$
\begin{array}{l}
\mid \mathit{Ass}\colon\ [\![G\vdash \mathit{Norm}\ s0\ -\mathit{va}{=}\succ(w,f)\rightarrow\ s1;\\
\qquad G\vdash \quad s1\ -e-\succ v\ \rightarrow\ s2]\!]\Longrightarrow\\
\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -\mathit{va}{:=}e-\succ v\rightarrow\ \mathit{assign}\ f\ v\ s2
\end{array}
$$

— cf. 15.24

$$
\begin{array}{l}
\mid \mathit{Cond}\colon [\![G\vdash \mathit{Norm}\ s0\ -e0-\succ b\rightarrow\ s1;\\
\qquad G\vdash \quad s1\ -(\mathit{if}\ \mathit{the\text{-}Bool}\ b\ \mathit{then}\ e1\ \mathit{else}\ e2)-\succ v\rightarrow\ s2]\!]\Longrightarrow\\
\qquad\qquad\qquad G\vdash \mathit{Norm}\ s0\ -e0\ ?\ e1\ :\ e2-\succ v\rightarrow\ s2
\end{array}
$$

— The interplay of *Call*, *Methd* and *Body*: Method invocation is split up into these three rules:

Call Calculates the target address and evaluates the arguments of the method, and then performs dynamic or static lookup of the method, corresponding to the call mode. Then the *Methd* rule is evaluated on the calculated declaration class of the method invocation.

Methd A syntactic bridge for the folded method body. It is used by the axiomatic semantics to add the proper hypothesis for recursive calls of the method.

Body An extra syntactic entity for the unfolded method body was introduced to properly trigger class initialisation. Without class initialisation we could just evaluate the body statement.

— cf. 15.11.4.1, 15.11.4.2, 15.11.4.4, 15.11.4.5

| *Call*:

$[\![G\vdash Norm\ s0\ -e-\succ a'\rightarrow\ s1;\ G\vdash s1\ -args\dot{=}\succ vs\rightarrow\ s2;$
$D = invocation\text{-}declclass\ G\ mode\ (store\ s2)\ a'\ statT\ (\!|name=mn,parTs=pTs|\!);$
$s3=init\text{-}lvars\ G\ D\ (\!|name=mn,parTs=pTs|\!)\ mode\ a'\ vs\ s2;$
$s3' = check\text{-}method\text{-}access\ G\ accC\ statT\ mode\ (\!|name=mn,parTs=pTs|\!)\ a'\ s3;$
$G\vdash s3'\ -Methd\ D\ (\!|name=mn,parTs=pTs|\!)-\succ v\rightarrow\ s4]\!]$
$\Longrightarrow$
$G\vdash Norm\ s0\ -\{accC,statT,mode\}e\cdot mn(\{pTs\}args)-\succ v\rightarrow\ (restore\text{-}lvars\ s2\ s4)$

— The accessibility check is after *init-lvars*, to keep it simple. *init-lvars* already tests for the absence of a null-pointer reference in case of an instance method invocation.

| *Methd*: $[\![G\vdash Norm\ s0\ -body\ G\ D\ sig-\succ v\rightarrow\ s1]\!] \Longrightarrow$
$G\vdash Norm\ s0\ -Methd\ D\ sig-\succ v\rightarrow\ s1$

| *Body*: $[\![G\vdash Norm\ s0\ -Init\ D\rightarrow\ s1;\ G\vdash s1\ -c\rightarrow\ s2;$
$s3 = (if\ (\exists\ l.\ abrupt\ s2 = Some\ (Jump\ (Break\ l))\ \vee$
$abrupt\ s2 = Some\ (Jump\ (Cont\ l)))$
$then\ abupd\ (\lambda\ x.\ Some\ (Error\ CrossMethodJump))\ s2$
$else\ s2)]\!] \Longrightarrow$
$G\vdash Norm\ s0\ -Body\ D\ c-\succ the\ (locals\ (store\ s2)\ Result)$
$\rightarrow abupd\ (absorb\ Ret)\ s3$

— cf. 14.15, 12.4.1

— We filter out a break/continue in *s2*, so that we can proof definite assignment correct, without the need of conformance of the state. By this the different parts of the typesafety proof can be disentangled a little.

— evaluation of variables

— cf. 15.13.1, 15.7.2

| *LVar*: $G\vdash Norm\ s\ -LVar\ vn=\succ lvar\ vn\ s\rightarrow\ Norm\ s$

— cf. 15.10.1, 12.4.1

| *FVar*: $[\![G\vdash Norm\ s0\ -Init\ statDeclC\rightarrow\ s1;\ G\vdash s1\ -e-\succ a\rightarrow\ s2;$
$(v,s2') = fvar\ statDeclC\ stat\ fn\ a\ s2;$
$s3 = check\text{-}field\text{-}access\ G\ accC\ statDeclC\ fn\ stat\ a\ s2'\ ]\!] \Longrightarrow$
$G\vdash Norm\ s0\ -\{accC,statDeclC,stat\}e..fn=\succ v\rightarrow\ s3$

— The accessibility check is after *fvar*, to keep it simple. *fvar* already tests for the absence of a null-pointer reference in case of an instance field

— cf. 15.12.1, 15.25.1

| *AVar*: $[\![G\vdash\ Norm\ s0\ -e1-\succ a\rightarrow\ s1;\ G\vdash s1\ -e2-\succ i\rightarrow\ s2;$
$(v,s2') = avar\ G\ i\ a\ s2]\!] \Longrightarrow$
$G\vdash Norm\ s0\ -e1.[e2]=\succ v\rightarrow\ s2'$

— evaluation of expression lists

— cf. 15.11.4.2

| *Nil*:

$$G\vdash Norm\ s0\ -[]\dot{=}\succ[]\rightarrow\ Norm\ s0$$

— cf. 15.6.4

| *Cons*: $[\![G\vdash Norm\ s0\ -e\ -\succ\ v\ \rightarrow\ s1;$
$G\vdash\quad s1\ -es\dot{=}\succ vs\rightarrow\ s2]\!] \Longrightarrow$
$G\vdash Norm\ s0\ -e\#es\dot{=}\succ v\#vs\rightarrow\ s2$

```
ML ‹
ML-Thms.bind-thm (eval-induct, rearrange-prems
[0,1,2,8,4,30,31,27,15,16,
 17,18,19,20,21,3,5,25,26,23,6,
 7,11,9,13,14,12,22,10,28,
 29,24] @{thm eval.induct})
›


declare if-split      [split del] if-split-asm      [split del]
        option.split [split del] option.split-asm [split del]
inductive-cases halloc-elim-cases:
  G⊢(Some xc,s) −halloc oi≻a→ s′
  G⊢(Norm    s) −halloc oi≻a→ s′

inductive-cases sxalloc-elim-cases:
        G⊢ Norm                 s  −sxalloc→ s′
        G⊢(Some (Jump j),s) −sxalloc→ s′
        G⊢(Some (Error e),s) −sxalloc→ s′
        G⊢(Some (Xcpt (Loc a )),s) −sxalloc→ s′
        G⊢(Some (Xcpt (Std xn)),s) −sxalloc→ s′
inductive-cases sxalloc-cases: G⊢s −sxalloc→ s′


lemma sxalloc-elim-cases2: ⟦G⊢s −sxalloc→ s′;
       ⋀s.   ⟦s′ = Norm s⟧ ⟹ P;
       ⋀j s. ⟦s′ = (Some (Jump j),s)⟧ ⟹ P;
       ⋀e s. ⟦s′ = (Some (Error e),s)⟧ ⟹ P;
       ⋀a s. ⟦s′ = (Some (Xcpt (Loc a)),s)⟧ ⟹ P
      ⟧ ⟹ P
apply cut-tac
apply (erule sxalloc-cases)
apply blast+
done

declare not-None-eq [simp del]
declare split-paired-All [simp del] split-paired-Ex [simp del]
setup ‹map-theory-simpset (fn ctxt => ctxt delloop split-all-tac)›

inductive-cases eval-cases: G⊢s −t≻→ (v, s′)

inductive-cases eval-elim-cases [cases set]:
        G⊢(Some xc,s) −t                          ≻→ (v, s′)
        G⊢Norm s −In1r Skip                        ≻→ (x, s′)
        G⊢Norm s −In1r (Jmp j)                     ≻→ (x, s′)
        G⊢Norm s −In1r (l· c)                      ≻→ (x, s′)
        G⊢Norm s −In3  ([])                        ≻→ (v, s′)
        G⊢Norm s −In3  (e#es)                      ≻→ (v, s′)
        G⊢Norm s −In1l (Lit w)                     ≻→ (v, s′)
        G⊢Norm s −In1l (UnOp unop e)               ≻→ (v, s′)
        G⊢Norm s −In1l (BinOp binop e1 e2)         ≻→ (v, s′)
        G⊢Norm s −In2  (LVar vn)                   ≻→ (v, s′)
        G⊢Norm s −In1l (Cast T e)                  ≻→ (v, s′)
        G⊢Norm s −In1l (e InstOf T)                ≻→ (v, s′)
        G⊢Norm s −In1l (Super)                     ≻→ (v, s′)
        G⊢Norm s −In1l (Acc va)                    ≻→ (v, s′)
```

$G\vdash Norm\ s\ -In1r\ (Expr\ e) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (c1;;\ c2) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (Methd\ C\ sig) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (Body\ D\ c) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (e0\ ?\ e1\ :\ e2) \succ\rightarrow (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (If(e)\ c1\ Else\ c2) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (l\bullet\ While(e)\ c) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (c1\ Finally\ c2) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (Throw\ e) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (NewC\ C) \succ\rightarrow (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (New\ T[e]) \succ\rightarrow (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Ass\ va\ e) \succ\rightarrow (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (Try\ c1\ Catch(tn\ vn)\ c2) \succ\rightarrow (x,\ s')$
$G\vdash Norm\ s\ -In2\ (\{accC,statDeclC,stat\}e..fn) \succ\rightarrow (v,\ s')$
$G\vdash Norm\ s\ -In2\ (e1.[e2]) \succ\rightarrow (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (\{accC,statT,mode\}e\cdot mn(\{pT\}p)) \succ\rightarrow (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (Init\ C) \succ\rightarrow (x,\ s')$

declare *not-None-eq* [*simp*]
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
declaration ‹*K* (*Simplifier.map-ss* (*fn ss* => *ss addloop* (*split-all-tac*, *split-all-tac*)))›
declare *if-split* [*split*] *if-split-asm* [*split*]
option.split [*split*] *option.split-asm* [*split*]

lemma *eval-Inj-elim*:
$G\vdash s\ -t\succ\rightarrow (w,s')$
$\Longrightarrow case\ t\ of$
$In1\ ec \Rightarrow (case\ ec\ of$
$Inl\ e \Rightarrow (\exists v.\ w = In1\ v)$
$\mid Inr\ c \Rightarrow w = \diamondsuit)$
$\mid In2\ e \Rightarrow (\exists v.\ w = In2\ v)$
$\mid In3\ e \Rightarrow (\exists v.\ w = In3\ v)$
apply (*erule eval-cases*)
apply *auto*
apply (*induct-tac t*)
apply (*rename-tac a*, *induct-tac a*)
apply *auto*
done

The following simplification procedures set up the proper injections of terms and their corresponding values in the evaluation relation: E.g. an expression (injection *In1l* into terms) always evaluates to ordinary values (injection *In1* into generalised values *vals*).

lemma *eval-expr-eq*: $G\vdash s\ -In1l\ t\succ\rightarrow (w,\ s') = (\exists v.\ w{=}In1\ v \wedge G\vdash s\ -t-\succ v \rightarrow s')$
by (*auto*, *frule eval-Inj-elim*, *auto*)

lemma *eval-var-eq*: $G\vdash s\ -In2\ t\succ\rightarrow (w,\ s') = (\exists vf.\ w{=}In2\ vf \wedge G\vdash s\ -t{=}\succ vf\rightarrow s')$
by (*auto*, *frule eval-Inj-elim*, *auto*)

lemma *eval-exprs-eq*: $G\vdash s\ -In3\ t\succ\rightarrow (w,\ s') = (\exists vs.\ w{=}In3\ vs \wedge G\vdash s\ -t\doteq\succ vs\rightarrow s')$
by (*auto*, *frule eval-Inj-elim*, *auto*)

lemma *eval-stmt-eq*: $G\vdash s\ -In1r\ t\succ\rightarrow (w,\ s') = (w{=}\diamondsuit \wedge G\vdash s\ -t \rightarrow s')$
by (*auto*, *frule eval-Inj-elim*, *auto*, *frule eval-Inj-elim*, *auto*)

simproc-setup *eval-expr* ($G\vdash s\ -In1l\ t\succ\rightarrow (w,\ s')$) = ‹

```
  fn _ => fn _ => fn ct =>
    (case Thm.term_of ct of
      (_ $ _ $ _ $ _ $ (Const _ $ _) $ _) => NONE
    | _ => SOME (mk_meta_eq @{thm eval_expr_eq}))›
```

simproc-setup *eval-var* ($G \vdash s -In2\ t \succ\rightarrow (w, s')$) = ‹

```
  fn _ => fn _ => fn ct =>
    (case Thm.term_of ct of
      (_ $ _ $ _ $ _ $ (Const _ $ _) $ _) => NONE
    | _ => SOME (mk_meta_eq @{thm eval_var_eq}))›
```

simproc-setup *eval-exprs* ($G \vdash s -In3\ t \succ\rightarrow (w, s')$) = ‹

```
  fn _ => fn _ => fn ct =>
    (case Thm.term_of ct of
      (_ $ _ $ _ $ _ $ (Const _ $ _) $ _) => NONE
    | _ => SOME (mk_meta_eq @{thm eval_exprs_eq}))›
```

simproc-setup *eval-stmt* ($G \vdash s -In1r\ t \succ\rightarrow (w, s')$) = ‹

```
  fn _ => fn _ => fn ct =>
    (case Thm.term_of ct of
      (_ $ _ $ _ $ _ $ (Const _ $ _) $ _) => NONE
    | _ => SOME (mk_meta_eq @{thm eval_stmt_eq}))›
```

ML ‹
ML-Thms.bind-thms (*AbruptIs*, *sum3-instantiate* ***context*** @{*thm eval.Abrupt*})
›

declare *halloc.Abrupt* [*intro*!] *eval.Abrupt* [*intro*!] *AbruptIs* [*intro*!]

Callee,*InsInitE*, *InsInitV*, *FinA* are only used in smallstep semantics, not in the bigstep semantics. So their is no valid evaluation of these terms

lemma *eval-Callee*: $G \vdash Norm\ s - Callee\ l\ e - \succ v \rightarrow s' = False$
proof −
  { **fix** $s\ t\ v\ s'$
    assume *eval*: $G \vdash s -t \succ\rightarrow (v,s')$ **and**
         normal: *normal s* **and**
         callee: $t = In1l\ (Callee\ l\ e)$
    then have *False* **by** *induct auto*
  }
  then show *?thesis*
    by (*cases* s') *fastforce*
qed

lemma *eval-InsInitE*: $G \vdash Norm\ s - InsInitE\ c\ e - \succ v \rightarrow s' = False$
proof −
  { **fix** $s\ t\ v\ s'$
    assume *eval*: $G \vdash s -t \succ\rightarrow (v,s')$ **and**
         normal: *normal s* **and**
         callee: $t = In1l\ (InsInitE\ c\ e)$
    then have *False* **by** *induct auto*
  }
  then show *?thesis*
    by (*cases* s') *fastforce*
qed

lemma *eval-InsInitV*: $G \vdash Norm\ s - InsInitV\ c\ w = \succ v \rightarrow s' = False$

proof –
 { **fix** *s t v s′*
  assume *eval*: $G\vdash s -t\succ\rightarrow (v,s')$ **and**
   normal: *normal s* **and**
   callee: $t{=}In2\ (InsInitV\ c\ w)$
  then have *False* **by** *induct auto*
 }
 then show *?thesis*
  by (*cases s′*) *fastforce*
qed

lemma *eval-FinA*: $G\vdash Norm\ s-FinA\ a\ c\rightarrow\ s' = False$
proof –
 { **fix** *s t v s′*
  assume *eval*: $G\vdash s -t\succ\rightarrow (v,s')$ **and**
   normal: *normal s* **and**
   callee: $t{=}In1r\ (FinA\ a\ c)$
  then have *False* **by** *induct auto*
 }
 then show *?thesis*
  by (*cases s′*) *fastforce*
qed

lemma *eval-no-abrupt-lemma*:
 $\bigwedge s\ s'.\ G\vdash s -t\succ\rightarrow (w,s') \Longrightarrow normal\ s' \longrightarrow normal\ s$
by (*erule eval-cases*, *auto*)

lemma *eval-no-abrupt*:
 $G\vdash(x,s) -t\succ\rightarrow (w,Norm\ s') =$
  $(x = None \wedge G\vdash Norm\ s -t\succ\rightarrow (w,Norm\ s'))$
apply *auto*
apply (*frule eval-no-abrupt-lemma*, *auto*)+
done

simproc-setup *eval-no-abrupt* $(G\vdash(x,s) -e\succ\rightarrow (w,Norm\ s')) =$ ‹
 fn - => *fn* - => *fn ct* =>
  (*case Thm.term-of ct of*
   (- \$ - \$ (*Const* (***const-name*** ‹*Pair*›, -) \$ (*Const* (***const-name*** ‹*None*›, -)) \$ -) \$ - \$ - \$ -) => *NONE*
  | - => *SOME* (*mk-meta-eq* @{*thm eval-no-abrupt*}))
›

lemma *eval-abrupt-lemma*:
 $G\vdash s -t\succ\rightarrow (v,s') \Longrightarrow abrupt\ s{=}Some\ xc \longrightarrow s'{=}\ s \wedge v = undefined3\ t$
by (*erule eval-cases*, *auto*)

lemma *eval-abrupt*:
 $G\vdash(Some\ xc,s) -t\succ\rightarrow (w,s') =$
  $(s'{=}(Some\ xc,s) \wedge w{=}undefined3\ t \wedge$
  $G\vdash(Some\ xc,s) -t\succ\rightarrow (undefined3\ t,(Some\ xc,s)))$
apply *auto*
apply (*frule eval-abrupt-lemma*, *auto*)+
done

simproc-setup *eval-abrupt* ($G\vdash(Some\ xc,s)\ -e\succ\rightarrow\ (w,s')$) = ‹
  fn - => *fn* - => *fn ct* =>
    (*case Thm.term-of ct of*
      (- $ - $ - $ - $ - $ (*Const* (***const-name*** ‹*Pair*›, -) $ (*Const* (***const-name*** ‹*Some*›, -) $ -)$ -)) =>
NONE
    | - => *SOME* (*mk-meta-eq* @{*thm eval-abrupt*}))
›

lemma *LitI*: $G\vdash s\ -Lit\ v-\succ(if\ normal\ s\ then\ v\ else\ undefined)\rightarrow\ s$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Lit*)

lemma *SkipI* [*intro*!]: $G\vdash s\ -Skip\rightarrow\ s$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Skip*)

lemma *ExprI*: $G\vdash s\ -e-\succ v\rightarrow\ s' \Longrightarrow G\vdash s\ -Expr\ e\rightarrow\ s'$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Expr*)

lemma *CompI*: $[\![G\vdash s\ -c1\rightarrow\ s1;\ G\vdash s1\ -c2\rightarrow\ s2]\!] \Longrightarrow G\vdash s\ -c1;;\ c2\rightarrow\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Comp*)

lemma *CondI*:
  $\bigwedge s1.\ [\![G\vdash s\ -e-\succ b\rightarrow\ s1;\ G\vdash s1\ -(if\ the\text{-}Bool\ b\ then\ e1\ else\ e2)-\succ v\rightarrow\ s2]\!] \Longrightarrow$
      $G\vdash s\ -e\ ?\ e1\ :\ e2-\succ(if\ normal\ s1\ then\ v\ else\ undefined)\rightarrow\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Cond*)

lemma *IfI*: $[\![G\vdash s\ -e-\succ v\rightarrow\ s1;\ G\vdash s1\ -(if\ the\text{-}Bool\ v\ then\ c1\ else\ c2)\rightarrow\ s2]\!]$
      $\Longrightarrow G\vdash s\ -If(e)\ c1\ Else\ c2\rightarrow\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.If*)

lemma *MethdI*: $G\vdash s\ -body\ G\ C\ sig-\succ v\rightarrow\ s'$
      $\Longrightarrow G\vdash s\ -Methd\ C\ sig-\succ v\rightarrow\ s'$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Methd*)

lemma *eval-Call*:
  $[\![G\vdash Norm\ s0\ -e-\succ a'\rightarrow\ s1;\ G\vdash s1\ -ps\doteq\succ pvs\rightarrow\ s2;$
    $D = invocation\text{-}declclass\ G\ mode\ (store\ s2)\ a'\ statT\ (\!|name=mn,parTs=pTs|\!);$
    $s3 = init\text{-}lvars\ G\ D\ (\!|name=mn,parTs=pTs|\!)\ mode\ a'\ pvs\ s2;$
    $s3' = check\text{-}method\text{-}access\ G\ accC\ statT\ mode\ (\!|name=mn,parTs=pTs|\!)\ a'\ s3;$
    $G\vdash s3'-Methd\ D\ (\!|name=mn,parTs=pTs|\!)-\succ\ v\rightarrow\ s4;$
      $s4' = restore\text{-}lvars\ s2\ s4]\!] \Longrightarrow$
      $G\vdash Norm\ s0\ -\{accC,statT,mode\}e\cdot mn(\{pTs\}ps)-\succ v\rightarrow\ s4'$
apply (*drule eval.Call*, *assumption*)
apply (*rule HOL.refl*)
apply *simp*+

done

lemma *eval-Init*:
$[\![$*if inited C* (*globs s0*) *then s3* = *Norm s0*
else G⊢*Norm* (*init-class-obj G C s0*)
−(*if C* = *Object then Skip else Init* (*super* (*the* (*class G C*))))→ *s1* ∧
G⊢*set-lvars Map.empty s1* −(*init* (*the* (*class G C*)))→ *s2* ∧
s3 = *restore-lvars s1 s2*$]\!]$ ⟹
G⊢*Norm s0* −*Init C*→ *s3*
apply (*rule eval.Init*)
apply *auto*
done

lemma *init-done*: *initd C s* ⟹ *G*⊢*s* −*Init C*→ *s*
apply (*case-tac s*, *simp*)
apply (*case-tac a*)
apply *safe*
apply (*rule eval-Init*)
apply *auto*
done

lemma *eval-StatRef*:
G⊢*s* −*StatRef rt*−≻(*if abrupt s*=*None then Null else undefined*)→ *s*
apply (*case-tac s*, *simp*)
apply (*case-tac a* = *None*)
apply (*auto del*: *eval.Abrupt intro*!: *eval.intros*)
done

lemma *SkipD* [*dest*!]: *G*⊢*s* −*Skip*→ *s′* ⟹ *s′* = *s*
apply (*erule eval-cases*)
by *auto*

lemma *Skip-eq* [*simp*]: *G*⊢*s* −*Skip*→ *s′* = (*s* = *s′*)
by *auto*

lemma *init-retains-locals* [*rule-format* (*no-asm*)]: *G*⊢*s* −*t*≻→ (*w*,*s′*) ⟹
(∀ *C*. *t*=*In1r* (*Init C*) ⟶ *locals* (*store s*) = *locals* (*store s′*))
apply (*erule eval.induct*)
apply (*simp* (*no-asm-use*) *split del*: *if-split-asm option.split-asm*)+
apply *auto*
done

lemma *halloc-xcpt* [*dest*!]:
⋀*s′*. *G*⊢(*Some xc*,*s*) −*halloc oi*≻*a*→ *s′* ⟹ *s′*=(*Some xc*,*s*)
apply (*erule-tac halloc-elim-cases*)
by *auto*

lemma *eval-Methd*:
  $G \vdash s\ -In1l(body\ G\ C\ sig) \succ\rightarrow\ (w,s')$
  $\Longrightarrow G \vdash s\ -In1l(Methd\ C\ sig) \succ\rightarrow\ (w,s')$
apply (*case-tac s*)
apply (*case-tac a*)
apply *clarsimp+*
apply (*erule eval.Methd*)
apply (*drule eval-abrupt-lemma*)
apply *force*
done

lemma *eval-Body*: $[\![G \vdash Norm\ s0\ -Init\ D \rightarrow\ s1;\ G \vdash s1\ -c \rightarrow\ s2;$
$res = the\ (locals\ (store\ s2)\ Result);$
$s3 = (if\ (\exists\ l.\ abrupt\ s2 = Some\ (Jump\ (Break\ l)) \vee$
$abrupt\ s2 = Some\ (Jump\ (Cont\ l)))$
$then\ abupd\ (\lambda\ x.\ Some\ (Error\ CrossMethodJump))\ s2$
$else\ s2);$
$s4 = abupd\ (absorb\ Ret)\ s3]\!] \Longrightarrow$
$G \vdash Norm\ s0\ -Body\ D\ c - \succ res \rightarrow s4$
by (*auto elim*: *eval.Body*)

lemma *eval-binop-arg2-indep*:
$\neg\ need\text{-}second\text{-}arg\ binop\ v1 \Longrightarrow eval\text{-}binop\ binop\ v1\ x = eval\text{-}binop\ binop\ v1\ y$
by (*cases binop*)
  (*simp-all add*: *need-second-arg-def*)

lemma *eval-BinOp-arg2-indepI*:
  assumes *eval-e1*: $G \vdash Norm\ s0\ -e1 - \succ v1 \rightarrow\ s1$ **and**
    no-need: $\neg\ need\text{-}second\text{-}arg\ binop\ v1$
  shows $G \vdash Norm\ s0\ -BinOp\ binop\ e1\ e2 - \succ (eval\text{-}binop\ binop\ v1\ v2) \rightarrow\ s1$
    (**is** *?EvalBinOp v2*)
proof −
  from *eval-e1*
  have *?EvalBinOp Unit*
    by (*rule eval.BinOp*)
      (*simp add*: *no-need*)
  moreover
  from *no-need*
  have $eval\text{-}binop\ binop\ v1\ Unit = eval\text{-}binop\ binop\ v1\ v2$
    by (*simp add*: *eval-binop-arg2-indep*)
  ultimately
  show *?thesis*
    by *simp*
qed

single valued

lemma *unique-halloc* [*rule-format* (*no-asm*)]:
  $G \vdash s\ -halloc\ oi \succ a\ \rightarrow\ s' \Longrightarrow G \vdash s\ -halloc\ oi \succ a'\ \rightarrow\ s'' \longrightarrow a' = a \wedge s'' = s'$
apply (*erule halloc.induct*)
apply (*auto elim!*: *halloc-elim-cases split del*: *if-split if-split-asm*)
apply (*drule trans* [*THEN sym*], *erule sym*)
defer
apply (*drule trans* [*THEN sym*], *erule sym*)
apply *auto*

done

lemma *single-valued-halloc*:
single-valued $\{((s,oi),(a,s')).\ G\vdash s\ -halloc\ oi\succ a\ \to\ s'\}$
apply (*unfold single-valued-def*)
by (*clarsimp*, *drule* (*1*) *unique-halloc*, *auto*)

lemma *unique-sxalloc* [*rule-format* (*no-asm*)]:
$G\vdash s\ -sxalloc\to\ s' \Longrightarrow G\vdash s\ -sxalloc\to\ s'' \longrightarrow s'' = s'$
apply (*erule sxalloc.induct*)
apply (*auto dest*: *unique-halloc elim*!: *sxalloc-elim-cases*
split del: *if-split if-split-asm*)
done

lemma *single-valued-sxalloc*: *single-valued* $\{(s,s').\ G\vdash s\ -sxalloc\to\ s'\}$
apply (*unfold single-valued-def*)
apply (*blast dest*: *unique-sxalloc*)
done

lemma *split-pairD*: $(x,y) = p \Longrightarrow x = fst\ p\ \&\ y = snd\ p$
by *auto*

lemma *unique-eval* [*rule-format* (*no-asm*)]:
$G\vdash s\ -t\succ\to\ (w,\ s') \Longrightarrow (\forall w'\ s''.\ G\vdash s\ -t\succ\to\ (w',\ s'') \longrightarrow w' = w \wedge s'' = s')$
apply (*erule eval-induct*)
apply (*tactic* ‹*ALLGOALS* (*EVERY'*
[*strip-tac* ***context***, *rotate-tac* ~*1*, *eresolve-tac* ***context*** @{*thms eval-elim-cases*}])›)

prefer *28*
apply (*simp* (*no-asm-use*) *only*: *split*: *if-split-asm*)

prefer *30*
apply (*case-tac inited C* (*globs s0*), (*simp only*: *if-True if-False simp-thms*)+)
prefer *26*
apply (*simp* (*no-asm-use*) *only*: *split*: *if-split-asm*, *blast*)

apply (*blast dest*: *unique-sxalloc unique-halloc split-pairD*)+
done

lemma *single-valued-eval*:
single-valued $\{((s,\ t),\ (v,\ s')).\ G\vdash s\ -t\succ\to\ (v,\ s')\}$
apply (*unfold single-valued-def*)
by (*clarify*, *drule* (*1*) *unique-eval*, *auto*)

end

Chapter 16

Example

1 Example Bali program

theory *Example*
imports *Eval WellForm*
begin

The following example Bali program includes:

- class and interface declarations with inheritance, hiding of fields, overriding of methods (with refined result type), array type,
- method call (with dynamic binding), parameter access, return expressions,
- expression statements, sequential composition, literal values, local assignment, local access, field assignment, type cast,
- exception generation and propagation, try and catch statement, throw statement
- instance creation and (default) static initialization

```
package java_lang

public interface HasFoo {
  public Base foo(Base z);
}

public class Base implements HasFoo {
  static boolean arr[] = new boolean[2];
  public HasFoo vee;
  public Base foo(Base z) {
    return z;
  }
}

public class Ext extends Base {
  public int vee;
  public Ext foo(Base z) {
    ((Ext)z).vee = 1;
    return null;
  }
}
```

```
public class Main {
  public static void main(String args[]) throws Throwable {
    Base e = new Ext();
    try {e.foo(null); }
    catch(NullPointerException z) {
      while(Ext.arr[2]) ;
    }
  }
}
```

declare *widen.null* [*intro*]

lemma *wf-fdecl-def2*: $\bigwedge fd.\ wf\text{-}fdecl\ G\ P\ fd = is\text{-}acc\text{-}type\ G\ P\ (type\ (snd\ fd))$
apply (*unfold wf-fdecl-def*)
apply (*simp* (*no-asm*))
done

declare *wf-fdecl-def2* [*iff*]

type and expression names

datatype $tnam' = HasFoo' \mid Base' \mid Ext' \mid Main'$
datatype $vnam' = arr' \mid vee' \mid z' \mid e'$
datatype $label' = lab1'$

axiomatization
  $tnam' :: tnam' \Rightarrow tnam$ **and**
  $vnam' :: vnam' \Rightarrow vname$ **and**
  $label' :: label' \Rightarrow label$
where

  $inj\text{-}tnam'$ [*simp*]: $\bigwedge x\ y.\ (tnam'\ x = tnam'\ y) = (x = y)$ **and**
  $inj\text{-}vnam'$ [*simp*]: $\bigwedge x\ y.\ (vnam'\ x = vnam'\ y) = (x = y)$ **and**
  $inj\text{-}label'$ [*simp*]: $\bigwedge x\ y.\ (label'\ x = label'\ y) = (x = y)$ **and**

  $surj\text{-}tnam'$: $\bigwedge n.\ \exists m.\ n = tnam'\ m$ **and**
  $surj\text{-}vnam'$: $\bigwedge n.\ \exists m.\ n = vnam'\ m$ **and**
  $surj\text{-}label'$: $\bigwedge n.\ \exists m.\ n = label'\ m$

abbreviation
  $HasFoo :: qtname$ **where**
  $HasFoo == (\!|pid=java\text{-}lang,tid=TName\ (tnam'\ HasFoo')|\!)$

abbreviation
  $Base :: qtname$ **where**
  $Base == (\!|pid=java\text{-}lang,tid=TName\ (tnam'\ Base')|\!)$

abbreviation
  $Ext :: qtname$ **where**
  $Ext == (\!|pid=java\text{-}lang,tid=TName\ (tnam'\ Ext')|\!)$

abbreviation
  $Main :: qtname$ **where**
  $Main == (\!|pid=java\text{-}lang,tid=TName\ (tnam'\ Main')|\!)$

abbreviation

arr :: *vname* **where**
arr == (*vnam′ arr′*)

abbreviation
vee :: *vname* **where**
vee == (*vnam′ vee′*)

abbreviation
z :: *vname* **where**
z == (*vnam′ z′*)

abbreviation
e :: *vname* **where**
e == (*vnam′ e′*)

abbreviation
lab1:: *label* **where**
lab1 == *label′ lab1′*

lemma *neq-Base-Object* [*simp*]: *Base*≠*Object*
by (*simp add*: *Object-def*)

lemma *neq-Ext-Object* [*simp*]: *Ext*≠*Object*
by (*simp add*: *Object-def*)

lemma *neq-Main-Object* [*simp*]: *Main*≠*Object*
by (*simp add*: *Object-def*)

lemma *neq-Base-SXcpt* [*simp*]: *Base*≠*SXcpt xn*
by (*simp add*: *SXcpt-def*)

lemma *neq-Ext-SXcpt* [*simp*]: *Ext*≠*SXcpt xn*
by (*simp add*: *SXcpt-def*)

lemma *neq-Main-SXcpt* [*simp*]: *Main*≠*SXcpt xn*
by (*simp add*: *SXcpt-def*)

classes and interfaces

overloading
Object-mdecls ≡ *Object-mdecls*
SXcpt-mdecls ≡ *SXcpt-mdecls*
begin
definition *Object-mdecls* ≡ []
definition *SXcpt-mdecls* ≡ []
end

axiomatization
foo :: *mname*

definition
foo-sig :: *sig*

```
  where foo-sig = ⦇name=foo,parTs=[Class Base]⦈

definition
  foo-mhead :: mhead
  where foo-mhead = ⦇access=Public,static=False,pars=[z],resT=Class Base⦈

definition
  Base-foo :: mdecl
  where Base-foo = (foo-sig, ⦇access=Public,static=False,pars=[z],resT=Class Base,
                        mbody=⦇lcls=[],stmt=Return (!!z)⦈⦈)

definition Ext-foo :: mdecl
  where Ext-foo = (foo-sig,
              ⦇access=Public,static=False,pars=[z],resT=Class Ext,
               mbody=⦇lcls=[]
                    ,stmt=Expr({Ext,Ext,False}Cast (Class Ext) (!!z)..vee :=
                                                  Lit (Intg 1)) ;;
                          Return (Lit Null)⦈
              ⦈)

definition
  arr-viewed-from :: qtname ⇒ qtname ⇒ var
  where arr-viewed-from accC C = {accC,Base,True}StatRef (ClassT C)..arr

definition
  BaseCl :: class where
  BaseCl = ⦇access=Public,
          cfields=[(arr, ⦇access=Public,static=True ,type=PrimT Boolean.[]⦈),
                  (vee, ⦇access=Public,static=False,type=Iface HasFoo    ⦈)],
          methods=[Base-foo],
          init=Expr(arr-viewed-from Base Base
                   :=New (PrimT Boolean)[Lit (Intg 2)]),
          super=Object,
          superIfs=[HasFoo]⦈

definition
  ExtCl  :: class where
  ExtCl = ⦇access=Public,
          cfields=[(vee, ⦇access=Public,static=False,type= PrimT Integer⦈)],
          methods=[Ext-foo],
          init=Skip,
          super=Base,
          superIfs=[]⦈

definition
  MainCl :: class where
  MainCl = ⦇access=Public,
          cfields=[],
          methods=[],
          init=Skip,
          super=Object,
          superIfs=[]⦈


definition
  HasFooInt :: iface
  where HasFooInt = ⦇access=Public,imethods=[(foo-sig, foo-mhead)],isuperIfs=[]⦈

definition
```

Ifaces ::*idecl list*
where *Ifaces* = [(*HasFoo*,*HasFooInt*)]

definition
Classes ::*cdecl list*
where *Classes* = [(*Base*,*BaseCl*),(*Ext*,*ExtCl*),(*Main*,*MainCl*)]@*standard-classes*

lemmas *table-classes-defs* =
Classes-def standard-classes-def ObjectC-def SXcptC-def

lemma *table-ifaces* [*simp*]: *table-of Ifaces* = *Map.empty*(*HasFoo*$\mapsto$*HasFooInt*)
apply (*unfold Ifaces-def*)
apply (*simp* (*no-asm*))
done

lemma *table-classes-Object* [*simp*]:
table-of Classes Object = *Some* (|*access*=*Public*,*cfields*=[]
,*methods*=*Object-mdecls*
,*init*=*Skip*,*super*=*undefined*,*superIfs*=[]|)
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*:*Object-def*)
done

lemma *table-classes-SXcpt* [*simp*]:
table-of Classes (*SXcpt xn*)
= *Some* (|*access*=*Public*,*cfields*=[],*methods*=*SXcpt-mdecls*,
init=*Skip*,
super=*if xn* = *Throwable then Object else SXcpt Throwable*,
superIfs=[]|)
apply (*unfold table-classes-defs*)
apply (*induct-tac xn*)
apply (*simp add*: *Object-def SXcpt-def*)+
done

lemma *table-classes-HasFoo* [*simp*]: *table-of Classes HasFoo* = *None*
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*: *Object-def SXcpt-def*)
done

lemma *table-classes-Base* [*simp*]: *table-of Classes Base* = *Some BaseCl*
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*: *Object-def SXcpt-def*)
done

lemma *table-classes-Ext* [*simp*]: *table-of Classes Ext* = *Some ExtCl*
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*: *Object-def SXcpt-def*)
done

lemma *table-classes-Main* [*simp*]: *table-of Classes Main* = *Some MainCl*
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*: *Object-def SXcpt-def*)

done

program

abbreviation
tprg :: *prog* **where**
tprg == (|*ifaces*=*Ifaces*,*classes*=*Classes*|)

definition
test :: (*ty*)*list* ⇒ *stmt* **where**
test pTs = (*e*:==*NewC Ext*;;
Try Expr({*Main*,*ClassT Base*,*IntVir*}!!*e*·*foo*({*pTs*}[*Lit Null*]))
Catch((*SXcpt NullPointer*) *z*)
(*lab1*· *While*(*Acc*
(*Acc* (*arr-viewed-from Main Ext*).[*Lit* (*Intg 2*)])) *Skip*))

well-structuredness

lemma *not-Object-subcls-any* [*elim*!]: (*Object*, *C*) ∈ (*subcls1 tprg*)$^+$ ⟹ *R*
apply (*auto dest*!: *tranclD subcls1D*)
done

lemma *not-Throwable-subcls-SXcpt* [*elim*!]:
(*SXcpt Throwable*, *SXcpt xn*) ∈ (*subcls1 tprg*)$^+$ ⟹ *R*
apply (*auto dest*!: *tranclD subcls1D*)
apply (*simp add*: *Object-def SXcpt-def*)
done

lemma *not-SXcpt-n-subcls-SXcpt-n* [*elim*!]:
(*SXcpt xn*, *SXcpt xn*) ∈ (*subcls1 tprg*)$^+$ ⟹ *R*
apply (*auto dest*!: *tranclD subcls1D*)
apply (*drule rtranclD*)
apply *auto*
done

lemma *not-Base-subcls-Ext* [*elim*!]: (*Base*, *Ext*) ∈ (*subcls1 tprg*)$^+$ ⟹ *R*
apply (*auto dest*!: *tranclD subcls1D simp add*: *BaseCl-def*)
done

lemma *not-TName-n-subcls-TName-n* [*rule-format* (*no-asm*), *elim*!]:
((|*pid*=*java-lang*,*tid*=*TName tn*|), (|*pid*=*java-lang*,*tid*=*TName tn*|))
∈ (*subcls1 tprg*)$^+$ ⟶ *R*
apply (*rule-tac n1* = *tn* **in** *surj-tnam′* [*THEN exE*])
apply (*erule ssubst*)
apply (*rule tnam′.induct*)
apply *safe*
apply (*auto dest*!: *tranclD subcls1D simp add*: *BaseCl-def ExtCl-def MainCl-def*)
apply (*drule rtranclD*)
apply *auto*
done

lemma *ws-idecl-HasFoo*: *ws-idecl tprg HasFoo* []
apply (*unfold ws-idecl-def*)

apply (*simp* (*no-asm*))
done

lemma *ws-cdecl-Object*: *ws-cdecl tprg Object any*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-Throwable*: *ws-cdecl tprg* (*SXcpt Throwable*) *Object*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-SXcpt*: *ws-cdecl tprg* (*SXcpt xn*) (*SXcpt Throwable*)
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-Base*: *ws-cdecl tprg Base Object*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-Ext*: *ws-cdecl tprg Ext Base*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-Main*: *ws-cdecl tprg Main Object*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemmas *ws-cdecls* = *ws-cdecl-SXcpt ws-cdecl-Object ws-cdecl-Throwable*
ws-cdecl-Base ws-cdecl-Ext ws-cdecl-Main

declare *not-Object-subcls-any* [*rule del*]
not-Throwable-subcls-SXcpt [*rule del*]
not-SXcpt-n-subcls-SXcpt-n [*rule del*]
not-Base-subcls-Ext [*rule del*] *not-TName-n-subcls-TName-n* [*rule del*]

lemma *ws-idecl-all*:
G=*tprg* $\Longrightarrow$ ($\forall$ (I,i)$\in$*set Ifaces*. *ws-idecl G I* (*isuperIfs i*))
apply (*simp* (*no-asm*) *add*: *Ifaces-def HasFooInt-def*)
apply (*auto intro*!: *ws-idecl-HasFoo*)
done

lemma *ws-cdecl-all*: G=*tprg* $\Longrightarrow$ ($\forall$ (C,c)$\in$*set Classes*. *ws-cdecl G C* (*super c*))
apply (*simp* (*no-asm*) *add*: *Classes-def BaseCl-def ExtCl-def MainCl-def*)
apply (*auto intro*!: *ws-cdecls simp add*: *standard-classes-def ObjectC-def*
SXcptC-def)

done

lemma *ws-tprg*: *ws-prog tprg*
apply (*unfold ws-prog-def*)
apply (*auto intro*!: *ws-idecl-all ws-cdecl-all*)
done

misc program properties (independent of well-structuredness)

lemma *single-iface* [*simp*]: *is-iface tprg I* = (*I* = *HasFoo*)
apply (*unfold Ifaces-def*)
apply (*simp* (*no-asm*))
done

lemma *empty-subint1* [*simp*]: *subint1 tprg* = {}
apply (*unfold subint1-def Ifaces-def HasFooInt-def*)
apply *auto*
done

lemma *unique-ifaces*: *unique Ifaces*
apply (*unfold Ifaces-def*)
apply (*simp* (*no-asm*))
done

lemma *unique-classes*: *unique Classes*
apply (*unfold table-classes-defs*)
apply (*simp*)
done

lemma *SXcpt-subcls-Throwable* [*simp*]: $tprg \vdash SXcpt\ xn \preceq_C SXcpt\ Throwable$
apply (*rule SXcpt-subcls-Throwable-lemma*)
apply *auto*
done

lemma *Ext-subclseq-Base* [*simp*]: $tprg \vdash Ext \preceq_C Base$
apply (*rule subcls-direct1*)
apply (*simp* (*no-asm*) *add*: *ExtCl-def*)
apply (*simp add*: *Object-def*)
apply (*simp* (*no-asm*))
done

lemma *Ext-subcls-Base* [*simp*]: $tprg \vdash Ext \prec_C Base$
apply (*rule subcls-direct2*)
apply (*simp* (*no-asm*) *add*: *ExtCl-def*)
apply (*simp add*: *Object-def*)
apply (*simp* (*no-asm*))
done

fields and method lookup

lemma *fields-tprg-Object* [*simp*]: *DeclConcepts.fields tprg Object* = []
by (*rule ws-tprg* [*THEN fields-emptyI*], *force*+)

lemma *fields-tprg-Throwable* [*simp*]:
DeclConcepts.fields tprg (*SXcpt Throwable*) = []
by (*rule ws-tprg* [*THEN fields-emptyI*], *force*+)

lemma *fields-tprg-SXcpt* [*simp*]: *DeclConcepts.fields tprg* (*SXcpt xn*) = []
apply (*case-tac xn* = *Throwable*)
apply (*simp* (*no-asm-simp*))
by (*rule ws-tprg* [*THEN fields-emptyI*], *force*+)

lemmas *fields-rec′* = *fields-rec* [*OF - ws-tprg*]

lemma *fields-Base* [*simp*]:
DeclConcepts.fields tprg Base
= [((*arr*,*Base*), (|*access*=*Public*,*static*=*True* ,*type*=*PrimT Boolean*.[]|)),
((*vee*,*Base*), (|*access*=*Public*,*static*=*False*,*type*=*Iface HasFoo* |))]
apply (*subst fields-rec′*)
apply (*auto simp add*: *BaseCl-def*)
done

lemma *fields-Ext* [*simp*]:
DeclConcepts.fields tprg Ext
= [((*vee*,*Ext*), (|*access*=*Public*,*static*=*False*,*type*= *PrimT Integer*|))]
@ *DeclConcepts.fields tprg Base*
apply (*rule trans*)
apply (*rule fields-rec′*)
apply (*auto simp add*: *ExtCl-def Object-def*)
done

lemmas *imethds-rec′* = *imethds-rec* [*OF - ws-tprg*]
lemmas *methd-rec′* = *methd-rec* [*OF - ws-tprg*]

lemma *imethds-HasFoo* [*simp*]:
imethds tprg HasFoo = *set-option* ∘ *Map.empty*(*foo-sig*↦(*HasFoo*, *foo-mhead*))
apply (*rule trans*)
apply (*rule imethds-rec′*)
apply (*auto simp add*: *HasFooInt-def*)
done

lemma *methd-tprg-Object* [*simp*]: *methd tprg Object* = *Map.empty*
apply (*subst methd-rec′*)
apply (*auto simp add*: *Object-mdecls-def*)
done

lemma *methd-Base* [*simp*]:
methd tprg Base = *table-of* [(λ(*s*,*m*). (*s*, *Base*, *m*)) *Base-foo*]
apply (*rule trans*)
apply (*rule methd-rec′*)
apply (*auto simp add*: *BaseCl-def*)
done

lemma *memberid-Base-foo-simp* [*simp*]:
memberid (*mdecl Base-foo*) = *mid foo-sig*
by (*simp add*: *Base-foo-def*)

lemma *memberid-Ext-foo-simp* [*simp*]:
memberid (*mdecl Ext-foo*) = *mid foo-sig*
by (*simp add*: *Ext-foo-def*)

lemma *Base-declares-foo*:
tprg⊢*mdecl Base-foo declared-in Base*
by (*auto simp add*: *declared-in-def cdeclaredmethd-def BaseCl-def Base-foo-def*)

lemma *foo-sig-not-undeclared-in-Base*:
¬ *tprg*⊢*mid foo-sig undeclared-in Base*
proof −
from *Base-declares-foo*
show *?thesis*
by (*auto dest*!: *declared-not-undeclared*)
qed

lemma *Ext-declares-foo*:
tprg⊢*mdecl Ext-foo declared-in Ext*
by (*auto simp add*: *declared-in-def cdeclaredmethd-def ExtCl-def Ext-foo-def*)

lemma *foo-sig-not-undeclared-in-Ext*:
¬ *tprg*⊢*mid foo-sig undeclared-in Ext*
proof −
from *Ext-declares-foo*
show *?thesis*
by (*auto dest*!: *declared-not-undeclared*)
qed

lemma *Base-foo-not-inherited-in-Ext*:
¬ *tprg* ⊢ *Ext inherits* (*Base*,*mdecl Base-foo*)
by (*auto simp add*: *inherits-def foo-sig-not-undeclared-in-Ext*)

lemma *Ext-method-inheritance*:
filter-tab (λ*sig m*. *tprg* ⊢ *Ext inherits method sig m*)
(*Map.empty*(*fst* ((λ(*s*, *m*). (*s*, *Base*, *m*)) *Base-foo*)↦
snd ((λ(*s*, *m*). (*s*, *Base*, *m*)) *Base-foo*)))
= *Map.empty*
proof −
from *Base-foo-not-inherited-in-Ext*
show *?thesis*
by (*auto intro*: *filter-tab-all-False simp add*: *Base-foo-def*)
qed

lemma *methd-Ext* [*simp*]: *methd tprg Ext* =
table-of [(λ(*s*,*m*). (*s*, *Ext*, *m*)) *Ext-foo*]
apply (*rule trans*)

apply (*rule methd-rec'*)
apply (*auto simp add*: *ExtCl-def Object-def Ext-method-inheritance*)
done

accessibility

lemma *classesDefined*:
$[\![$*class tprg C* = *Some c*; *C*≠*Object*$]\!] \Longrightarrow \exists$ *sc*. *class tprg* (*super c*) = *Some sc*
apply (*auto simp add*: *Classes-def standard-classes-def*
BaseCl-def ExtCl-def MainCl-def
SXcptC-def ObjectC-def)
done

lemma *superclassesBase* [*simp*]: *superclasses tprg Base*={*Object*}
proof –
have *ws*: *ws-prog tprg* **by** (*rule ws-tprg*)
then show *?thesis*
by (*auto simp add*: *superclasses-rec BaseCl-def*)
qed

lemma *superclassesExt* [*simp*]: *superclasses tprg Ext*={*Base*,*Object*}
proof –
have *ws*: *ws-prog tprg* **by** (*rule ws-tprg*)
then show *?thesis*
by (*auto simp add*: *superclasses-rec ExtCl-def BaseCl-def*)
qed

lemma *superclassesMain* [*simp*]: *superclasses tprg Main*={*Object*}
proof –
have *ws*: *ws-prog tprg* **by** (*rule ws-tprg*)
then show *?thesis*
by (*auto simp add*: *superclasses-rec MainCl-def*)
qed

lemma *HasFoo-accessible*[*simp*]:*tprg*⊢(*Iface HasFoo*) *accessible-in P*
by (*simp add*: *accessible-in-RefT-simp is-public-def HasFooInt-def*)

lemma *HasFoo-is-acc-iface*[*simp*]: *is-acc-iface tprg P HasFoo*
by (*simp add*: *is-acc-iface-def*)

lemma *HasFoo-is-acc-type*[*simp*]: *is-acc-type tprg P* (*Iface HasFoo*)
by (*simp add*: *is-acc-type-def*)

lemma *Base-accessible*[*simp*]:*tprg*⊢(*Class Base*) *accessible-in P*
by (*simp add*: *accessible-in-RefT-simp is-public-def BaseCl-def*)

lemma *Base-is-acc-class*[*simp*]: *is-acc-class tprg P Base*
by (*simp add*: *is-acc-class-def*)

lemma *Base-is-acc-type*[*simp*]: *is-acc-type tprg P* (*Class Base*)

by (*simp add*: *is-acc-type-def*)

lemma *Ext-accessible*[*simp*]:*tprg*⊢(*Class Ext*) *accessible-in P*
by (*simp add*: *accessible-in-RefT-simp is-public-def ExtCl-def*)

lemma *Ext-is-acc-class*[*simp*]: *is-acc-class tprg P Ext*
by (*simp add*: *is-acc-class-def*)

lemma *Ext-is-acc-type*[*simp*]: *is-acc-type tprg P* (*Class Ext*)
by (*simp add*: *is-acc-type-def*)

lemma *accmethd-tprg-Object* [*simp*]: *accmethd tprg S Object* = *Map.empty*
apply (*unfold accmethd-def*)
apply (*simp*)
done

lemma *snd-special-simp*: *snd* ((λ(*s*, *m*). (*s*, *a*, *m*)) *x*) = (*a*,*snd x*)
by (*cases x*) (*auto*)

lemma *fst-special-simp*: *fst* ((λ(*s*, *m*). (*s*, *a*, *m*)) *x*) = *fst x*
by (*cases x*) (*auto*)

lemma *foo-sig-undeclared-in-Object*:
tprg⊢*mid foo-sig undeclared-in Object*
by (*auto simp add*: *undeclared-in-def cdeclaredmethd-def Object-mdecls-def*)

lemma *unique-sig-Base-foo*:
tprg⊢ *mdecl* (*sig*, *snd Base-foo*) *declared-in Base* ⟹ *sig*=*foo-sig*
by (*auto simp add*: *declared-in-def cdeclaredmethd-def*
Base-foo-def BaseCl-def)

lemma *Base-foo-no-override*:
tprg,*sig*⊢(*Base*,(*snd Base-foo*)) *overrides old* ⟹ *P*
apply (*drule overrides-commonD*)
apply (*clarsimp*)
apply (*frule subclsEval*)
apply (*rule ws-tprg*)
apply (*simp*)
apply (*rule classesDefined*)
apply *assumption*+
apply (*frule unique-sig-Base-foo*)
apply (*auto dest*!: *declared-not-undeclared intro*: *foo-sig-undeclared-in-Object*
dest: *unique-sig-Base-foo*)
done

lemma *Base-foo-no-stat-override*:
tprg,*sig*⊢(*Base*,(*snd Base-foo*)) *overrides*$_S$ *old* ⟹ *P*
apply (*drule stat-overrides-commonD*)
apply (*clarsimp*)

```
apply (frule subclsEval)
apply   (rule ws-tprg)
apply   (simp)
apply   (rule classesDefined)
apply   assumption+
apply (frule unique-sig-Base-foo)
apply (auto dest!: declared-not-undeclared intro: foo-sig-undeclared-in-Object
          dest: unique-sig-Base-foo)
done
```

```
lemma Base-foo-no-hide:
 tprg,sig⊢(Base,(snd Base-foo)) hides old ⟹ P
by (auto dest: hidesD simp add: Base-foo-def member-is-static-simp)
```

```
lemma Ext-foo-no-hide:
 tprg,sig⊢(Ext,(snd Ext-foo)) hides old ⟹ P
by (auto dest: hidesD simp add: Ext-foo-def member-is-static-simp)
```

```
lemma unique-sig-Ext-foo:
 tprg⊢ mdecl (sig, snd Ext-foo) declared-in Ext ⟹ sig=foo-sig
by (auto simp add: declared-in-def cdeclaredmethd-def
                Ext-foo-def ExtCl-def)
```

```
lemma Ext-foo-override:
 tprg,sig⊢(Ext,(snd Ext-foo)) overrides old
  ⟹ old = (Base,(snd Base-foo))
apply (drule overrides-commonD)
apply (clarsimp)
apply (frule subclsEval)
apply   (rule ws-tprg)
apply   (simp)
apply   (rule classesDefined)
apply   assumption+
apply (frule unique-sig-Ext-foo)
apply (case-tac old)
apply (insert Base-declares-foo foo-sig-undeclared-in-Object)
apply (auto simp add: ExtCl-def Ext-foo-def
                  BaseCl-def Base-foo-def Object-mdecls-def
                  split-paired-all
                  member-is-static-simp
         dest: declared-not-undeclared unique-declaration)
done
```

```
lemma Ext-foo-stat-override:
 tprg,sig⊢(Ext,(snd Ext-foo)) overrides_S old
  ⟹ old = (Base,(snd Base-foo))
apply (drule stat-overrides-commonD)
apply (clarsimp)
apply (frule subclsEval)
apply   (rule ws-tprg)
apply   (simp)
apply   (rule classesDefined)
apply   assumption+
```

apply (*frule unique-sig-Ext-foo*)
apply (*case-tac old*)
apply (*insert Base-declares-foo foo-sig-undeclared-in-Object*)
apply (*auto simp add*: *ExtCl-def Ext-foo-def*
BaseCl-def Base-foo-def Object-mdecls-def
split-paired-all
member-is-static-simp
dest: *declared-not-undeclared unique-declaration*)
done

lemma *Base-foo-member-of-Base*:
tprg⊢(*Base*,*mdecl Base-foo*) *member-of Base*
by (*auto intro*!: *members.Immediate Base-declares-foo*)

lemma *Base-foo-member-in-Base*:
tprg⊢(*Base*,*mdecl Base-foo*) *member-in Base*
by (*rule member-of-to-member-in* [*OF Base-foo-member-of-Base*])

lemma *Ext-foo-member-of-Ext*:
tprg⊢(*Ext*,*mdecl Ext-foo*) *member-of Ext*
by (*auto intro*!: *members.Immediate Ext-declares-foo*)

lemma *Ext-foo-member-in-Ext*:
tprg⊢(*Ext*,*mdecl Ext-foo*) *member-in Ext*
by (*rule member-of-to-member-in* [*OF Ext-foo-member-of-Ext*])

lemma *Base-foo-permits-acc*:
tprg ⊢ (*Base*, *mdecl Base-foo*) *in Base permits-acc-from S*
by (*simp add*: *permits-acc-def Base-foo-def*)

lemma *Base-foo-accessible* [*simp*]:
tprg⊢(*Base*,*mdecl Base-foo*) *of Base accessible-from S*
by (*auto intro*: *accessible-fromR.Immediate*
Base-foo-member-of-Base Base-foo-permits-acc)

lemma *Base-foo-dyn-accessible* [*simp*]:
tprg⊢(*Base*,*mdecl Base-foo*) *in Base dyn-accessible-from S*
apply (*rule dyn-accessible-fromR.Immediate*)
apply (*rule Base-foo-member-in-Base*)
apply (*rule Base-foo-permits-acc*)
done

lemma *accmethd-Base* [*simp*]:
accmethd tprg S Base = *methd tprg Base*
apply (*simp add*: *accmethd-def*)
apply (*rule filter-tab-all-True*)
apply (*simp add*: *snd-special-simp fst-special-simp*)
done

lemma *Ext-foo-permits-acc*:

tprg ⊢ (*Ext*, *mdecl Ext-foo*) *in Ext permits-acc-from S*
by (*simp add*: *permits-acc-def Ext-foo-def*)

lemma *Ext-foo-accessible* [*simp*]:
tprg⊢(*Ext*,*mdecl Ext-foo*) *of Ext accessible-from S*
by (*auto intro*: *accessible-fromR.Immediate*
Ext-foo-member-of-Ext Ext-foo-permits-acc)

lemma *Ext-foo-dyn-accessible* [*simp*]:
tprg⊢(*Ext*,*mdecl Ext-foo*) *in Ext dyn-accessible-from S*
apply (*rule dyn-accessible-fromR.Immediate*)
apply (*rule Ext-foo-member-in-Ext*)
apply (*rule Ext-foo-permits-acc*)
done

lemma *Ext-foo-overrides-Base-foo*:
tprg⊢(*Ext*,*Ext-foo*) *overrides* (*Base*,*Base-foo*)
proof (*rule overridesR.Direct*, *simp-all*)
show ¬ *is-static Ext-foo*
by (*simp add*: *member-is-static-simp Ext-foo-def*)
show ¬ *is-static Base-foo*
by (*simp add*: *member-is-static-simp Base-foo-def*)
show *accmodi Ext-foo* ≠ *Private*
by (*simp add*: *Ext-foo-def*)
show *msig* (*Ext*, *Ext-foo*) = *msig* (*Base*, *Base-foo*)
by (*simp add*: *Ext-foo-def Base-foo-def*)
show *tprg*⊢*mdecl Ext-foo declared-in Ext*
by (*auto intro*: *Ext-declares-foo*)
show *tprg*⊢*mdecl Base-foo declared-in Base*
by (*auto intro*: *Base-declares-foo*)
show *tprg* ⊢(*Base*, *mdecl Base-foo*) *inheritable-in java-lang*
by (*simp add*: *inheritable-in-def Base-foo-def*)
show *tprg*⊢*resTy Ext-foo*≼*resTy Base-foo*
by (*simp add*: *Ext-foo-def Base-foo-def mhead-resTy-simp*)
qed

lemma *accmethd-Ext* [*simp*]:
accmethd tprg S Ext = *methd tprg Ext*
apply (*simp add*: *accmethd-def*)
apply (*rule filter-tab-all-True*)
apply (*auto simp add*: *snd-special-simp fst-special-simp*)
done

lemma *cls-Ext*: *class tprg Ext* = *Some ExtCl*
by *simp*

lemma *dynmethd-Ext-foo*:
dynmethd tprg Base Ext ⦇*name* = *foo*, *parTs* = [*Class Base*]⦈
= *Some* (*Ext*,*snd Ext-foo*)
proof −
have *methd tprg Base* ⦇*name* = *foo*, *parTs* = [*Class Base*]⦈
= *Some* (*Base*,*snd Base-foo*) **and**
methd tprg Ext ⦇*name* = *foo*, *parTs* = [*Class Base*]⦈
= *Some* (*Ext*,*snd Ext-foo*)

by (*auto simp add*: *Ext-foo-def Base-foo-def foo-sig-def*)
with *cls-Ext ws-tprg Ext-foo-overrides-Base-foo*
show *?thesis*
by (*auto simp add*: *dynmethd-rec simp add*: *Ext-foo-def Base-foo-def*)
qed

lemma *Base-fields-accessible*[*simp*]:
accfield tprg S Base
= *table-of*((*map* (λ((*n*,*d*),*f*).(*n*,(*d*,*f*)))) (*DeclConcepts.fields tprg Base*))
apply (*auto simp add*: *accfield-def fun-eq-iff Let-def*
accessible-in-RefT-simp
is-public-def
BaseCl-def
permits-acc-def
declared-in-def
cdeclaredfield-def
intro!: *filter-tab-all-True-Some filter-tab-None*
accessible-fromR.Immediate
intro: *members.Immediate*)
done

lemma *arr-member-of-Base*:
tprg⊢(*Base*, *fdecl* (*arr*,
⦇*access* = *Public*, *static* = *True*, *type* = *PrimT Boolean*.[]⦈))
member-of Base
by (*auto intro*: *members.Immediate*
simp add: *declared-in-def cdeclaredfield-def BaseCl-def*)

lemma *arr-member-in-Base*:
tprg⊢(*Base*, *fdecl* (*arr*,
⦇*access* = *Public*, *static* = *True*, *type* = *PrimT Boolean*.[]⦈))
member-in Base
by (*rule member-of-to-member-in* [*OF arr-member-of-Base*])

lemma *arr-member-of-Ext*:
tprg⊢(*Base*, *fdecl* (*arr*,
⦇*access* = *Public*, *static* = *True*, *type* = *PrimT Boolean*.[]⦈))
member-of Ext
apply (*rule members.Inherited*)
apply (*simp add*: *inheritable-in-def*)
apply (*simp add*: *undeclared-in-def cdeclaredfield-def ExtCl-def*)
apply (*auto intro*: *arr-member-of-Base simp add*: *subcls1-def ExtCl-def*)
done

lemma *arr-member-in-Ext*:
tprg⊢(*Base*, *fdecl* (*arr*,
⦇*access* = *Public*, *static* = *True*, *type* = *PrimT Boolean*.[]⦈))
member-in Ext
by (*rule member-of-to-member-in* [*OF arr-member-of-Ext*])

lemma *Ext-fields-accessible*[*simp*]:
accfield tprg S Ext

= *table-of*((*map* (λ((*n*,*d*),*f*).(*n*,(*d*,*f*)))) (*DeclConcepts.fields tprg Ext*))
apply (*auto simp add*: *accfield-def fun-eq-iff Let-def*
accessible-in-RefT-simp
is-public-def
BaseCl-def
ExtCl-def
permits-acc-def
intro!: *filter-tab-all-True-Some filter-tab-None*
accessible-fromR.Immediate)
apply (*auto intro*: *members.Immediate arr-member-of-Ext*
simp add: *declared-in-def cdeclaredfield-def ExtCl-def*)
done

lemma *arr-Base-dyn-accessible* [*simp*]:
tprg⊢(*Base*, *fdecl* (*arr*, ⦇*access*=*Public*,*static*=*True* ,*type*=*PrimT Boolean*.[]⦈))
in Base dyn-accessible-from S
apply (*rule dyn-accessible-fromR.Immediate*)
apply (*rule arr-member-in-Base*)
apply (*simp add*: *permits-acc-def*)
done

lemma *arr-Ext-dyn-accessible*[*simp*]:
tprg⊢(*Base*, *fdecl* (*arr*, ⦇*access*=*Public*,*static*=*True* ,*type*=*PrimT Boolean*.[]⦈))
in Ext dyn-accessible-from S
apply (*rule dyn-accessible-fromR.Immediate*)
apply (*rule arr-member-in-Ext*)
apply (*simp add*: *permits-acc-def*)
done

lemma *array-of-PrimT-acc* [*simp*]:
is-acc-type tprg java-lang (*PrimT t*.[])
apply (*simp add*: *is-acc-type-def accessible-in-RefT-simp*)
done

lemma *PrimT-acc* [*simp*]:
is-acc-type tprg java-lang (*PrimT t*)
apply (*simp add*: *is-acc-type-def accessible-in-RefT-simp*)
done

lemma *Object-acc* [*simp*]:
is-acc-class tprg java-lang Object
apply (*auto simp add*: *is-acc-class-def accessible-in-RefT-simp is-public-def*)
done

well-formedness

lemma *wf-HasFoo*: *wf-idecl tprg* (*HasFoo*, *HasFooInt*)
apply (*unfold wf-idecl-def HasFooInt-def*)
apply (*auto intro*!: *wf-mheadI ws-idecl-HasFoo*
simp add: *foo-sig-def foo-mhead-def mhead-resTy-simp*
member-is-static-simp)
done

```
declare member-is-static-simp [simp]
declare wt.Skip [rule del] wt.Init [rule del]
ML ‹ML-Thms.bind-thms (wt-intros, map (rewrite-rule context @{thms id-def}) @{thms wt.intros})›
lemmas wtIs = wt-Call wt-Super wt-FVar wt-StatRef wt-intros
lemmas daIs = assigned.select-convs da-Skip da-NewC da-Lit da-Super da.intros

lemmas Base-foo-defs = Base-foo-def foo-sig-def foo-mhead-def
lemmas Ext-foo-defs  = Ext-foo-def  foo-sig-def




lemma wf-Base-foo: wf-mdecl tprg Base Base-foo
apply (unfold Base-foo-defs )
apply (auto intro!: wf-mdeclI wf-mheadI intro!: wtIs
            simp add: mhead-resTy-simp)

apply (rule exI)
apply (simp add: parameters-def)
apply (rule conjI)
apply  (rule da.Comp)
apply     (rule da.Expr)
apply     (rule da.AssLVar)
apply       (rule da.AccLVar)
apply         (simp)
apply        (rule assigned.select-convs)
apply       (simp)
apply      (rule assigned.select-convs)
apply     (simp)
apply    (simp)
apply    (rule da.Jmp)
apply      (simp)
apply     (rule assigned.select-convs)
apply    (simp)
apply   (rule assigned.select-convs)
apply  (simp)
apply (simp)
done



lemma wf-Ext-foo: wf-mdecl tprg Ext Ext-foo
apply (unfold Ext-foo-defs )
apply (auto intro!: wf-mdeclI wf-mheadI intro!: wtIs
            simp add: mhead-resTy-simp )
apply  (rule wt.Cast)
prefer 2
apply    simp
apply   (rule-tac [2] narrow.subcls [THEN cast.narrow])
apply   (auto intro!: wtIs)

apply (rule exI)
apply (simp add: parameters-def)
apply (rule conjI)
apply  (rule da.Comp)
apply     (rule da.Expr)
apply     (rule da.Ass)
apply       simp
```

apply (*rule da.FVar*)
apply (*rule da.Cast*)
apply (*rule da.AccLVar*)
apply *simp*
apply (*rule assigned.select-convs*)
apply *simp*
apply (*rule da-Lit*)
apply (*simp*)
apply (*rule da.Comp*)
apply (*rule da.Expr*)
apply (*rule da.AssLVar*)
apply (*rule da.Lit*)
apply (*rule assigned.select-convs*)
apply *simp*
apply (*rule da.Jmp*)
apply *simp*
apply (*rule assigned.select-convs*)
apply *simp*
apply (*rule assigned.select-convs*)
apply (*simp*)
apply (*rule assigned.select-convs*)
apply *simp*
apply *simp*
done

declare *mhead-resTy-simp* [*simp add*]

lemma *wf-BaseC*: *wf-cdecl tprg* (*Base*,*BaseCl*)
apply (*unfold wf-cdecl-def BaseCl-def arr-viewed-from-def*)
apply (*auto intro*!: *wf-Base-foo*)
apply (*auto intro*!: *ws-cdecl-Base simp add*: *Base-foo-def foo-mhead-def*)
apply (*auto intro*!: *wtIs*)

apply (*rule exI*)
apply (*rule da.Expr*)
apply (*rule da.Ass*)
apply (*simp*)
apply (*rule da.FVar*)
apply (*rule da.Cast*)
apply (*rule da-Lit*)
apply *simp*
apply (*rule da.NewA*)
apply (*rule da.Lit*)
apply (*auto simp add*: *Base-foo-defs entails-def Let-def*)
apply (*insert Base-foo-no-stat-override*, *simp add*: *Base-foo-def*,*blast*)+
apply (*insert Base-foo-no-hide* , *simp add*: *Base-foo-def*,*blast*)
done

lemma *wf-ExtC*: *wf-cdecl tprg* (*Ext*,*ExtCl*)
apply (*unfold wf-cdecl-def ExtCl-def*)
apply (*auto intro*!: *wf-Ext-foo ws-cdecl-Ext*)
apply (*auto simp add*: *entails-def snd-special-simp*)
apply (*insert Ext-foo-stat-override*)
apply (*rule exI*,*rule da.Skip*)
apply (*force simp add*: *qmdecl-def Ext-foo-def Base-foo-def*)
apply (*force simp add*: *qmdecl-def Ext-foo-def Base-foo-def*)

apply (*force simp add*: *qmdecl-def Ext-foo-def Base-foo-def*)
apply (*insert Ext-foo-no-hide*)
apply (*simp-all add*: *qmdecl-def*)
apply *blast+*
done

lemma *wf-MainC*: *wf-cdecl tprg* (*Main*,*MainCl*)
apply (*unfold wf-cdecl-def MainCl-def*)
apply (*auto intro*: *ws-cdecl-Main*)
apply (*rule exI*,*rule da.Skip*)
done

lemma *wf-idecl-all*: *p*=*tprg* $\Longrightarrow$ *Ball* (*set Ifaces*) (*wf-idecl p*)
apply (*simp* (*no-asm*) *add*: *Ifaces-def*)
apply (*simp* (*no-asm-simp*))
apply (*rule wf-HasFoo*)
done

lemma *wf-cdecl-all-standard-classes*:
Ball (*set standard-classes*) (*wf-cdecl tprg*)
apply (*unfold standard-classes-def Let-def*
ObjectC-def SXcptC-def Object-mdecls-def SXcpt-mdecls-def)
apply (*simp* (*no-asm*) *add*: *wf-cdecl-def ws-cdecls*)
apply (*auto simp add*:*is-acc-class-def accessible-in-RefT-simp SXcpt-def*
intro: *da.Skip*)
apply (*auto simp add*: *Object-def Classes-def standard-classes-def*
SXcptC-def SXcpt-def)
done

lemma *wf-cdecl-all*: *p*=*tprg* $\Longrightarrow$ *Ball* (*set Classes*) (*wf-cdecl p*)
apply (*simp* (*no-asm*) *add*: *Classes-def*)
apply (*simp* (*no-asm-simp*))
apply (*rule wf-BaseC* [*THEN conjI*])
apply (*rule wf-ExtC* [*THEN conjI*])
apply (*rule wf-MainC* [*THEN conjI*])
apply (*rule wf-cdecl-all-standard-classes*)
done

theorem *wf-tprg*: *wf-prog tprg*
apply (*unfold wf-prog-def Let-def*)
apply (*simp* (*no-asm*) *add*: *unique-ifaces unique-classes*)
apply (*rule conjI*)
apply ((*simp* (*no-asm*) *add*: *Classes-def standard-classes-def*))
apply (*rule conjI*)
apply (*simp add*: *Object-mdecls-def*)
apply *safe*
apply (*cut-tac xn-cases*)
apply (*simp* (*no-asm-simp*) *add*: *Classes-def standard-classes-def*)
apply (*insert wf-idecl-all*)
apply (*insert wf-cdecl-all*)
apply *auto*
done

max spec

lemma *appl-methds-Base-foo*:
appl-methds tprg S (*ClassT Base*) (|*name*=*foo*, *parTs*=[*NT*]|) =
{((*ClassT Base*, (|*access*=*Public*,*static*=*False*,*pars*=[*z*],*resT*=*Class Base*|))
,[*Class Base*])}
apply (*unfold appl-methds-def*)
apply (*simp* (*no-asm*))
apply (*subgoal-tac tprg*$\vdash$*NT*$\preceq$ *Class Base*)
apply (*auto simp add*: *cmheads-def Base-foo-defs*)
done

lemma *max-spec-Base-foo*: *max-spec tprg S* (*ClassT Base*) (|*name*=*foo*,*parTs*=[*NT*]|) =
{((*ClassT Base*, (|*access*=*Public*,*static*=*False*,*pars*=[*z*],*resT*=*Class Base*|))
, [*Class Base*])}
by (*simp add*: *max-spec-def appl-methds-Base-foo Collect-conv-if*)

well-typedness

schematic-goal *wt-test*: (|*prg*=*tprg*,*cls*=*Main*,*lcl*=*Map.empty*(*VName e*$\mapsto$*Class Base*)|)$\vdash$*test ?pTs*::$\surd$
apply (*unfold test-def arr-viewed-from-def*)

apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
prefer *4*
apply (*simp*)
defer
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*simp*)
apply (*rule wtIs*)
apply (*simp*)
apply (*rule max-spec-Base-foo*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)

apply (*rule wtIs*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
apply (*simp*)
apply (*rule wtIs*)
done

definite assignment

schematic-goal *da-test*: $(\!|prg=tprg,cls=Main,lcl=Map.empty(VName\ e \mapsto Class\ Base)|\!)$
$\vdash\{\}\ \gg\langle test\ ?pTs\rangle\gg\ (\!|nrm=\{VName\ e\},brk=\lambda\ l.\ UNIV|\!)$
apply (*unfold test-def arr-viewed-from-def*)
apply (*rule da.Comp*)
apply (*rule da.Expr*)
apply (*rule da.AssLVar*)
apply (*rule da.NewC*)
apply (*rule assigned.select-convs*)
apply (*simp*)
apply (*rule da.Try*)
apply (*rule da.Expr*)
apply (*rule da.Call*)
apply (*rule da.AccLVar*)
apply (*simp*)
apply (*rule assigned.select-convs*)
apply (*simp*)
apply (*rule da.Cons*)
apply (*rule da.Lit*)
apply (*rule da.Nil*)
apply (*rule da.Loop*)
apply (*rule da.Acc*)
apply (*simp*)
apply (*rule da.AVar*)
apply (*rule da.Acc*)
apply *simp*
apply (*rule da.FVar*)
apply (*rule da.Cast*)
apply (*rule da.Lit*)
apply (*rule da.Lit*)
apply (*rule da-Skip*)
apply (*simp*)
apply (*simp*,*rule assigned.select-convs*)
apply (*simp*)
apply (*simp*,*rule assigned.select-convs*)
apply (*simp*)
apply *simp*
apply *simp*
apply (*simp add*: *intersect-ts-def*)
done

execution

lemma *alloc-one*: $\bigwedge a\ obj.\ [\![the\ (new\text{-}Addr\ h) = a;\ atleast\text{-}free\ h\ (Suc\ n)]\!] \Longrightarrow$
$new\text{-}Addr\ h = Some\ a \wedge atleast\text{-}free\ (h(a \mapsto obj))\ n$
apply (*frule atleast-free-SucD*)
apply (*drule atleast-free-Suc* [*THEN iffD1*])

apply *clarsimp*
apply (*frule new-Addr-SomeI*)
apply *force*
done

declare *fvar-def2* [*simp*] *avar-def2* [*simp*] *init-lvars-def2* [*simp*]
declare *init-obj-def* [*simp*] *var-tys-def* [*simp*] *fields-table-def* [*simp*]
declare *BaseCl-def* [*simp*] *ExtCl-def* [*simp*] *Ext-foo-def* [*simp*]
  Base-foo-defs [*simp*]

ML ‹*ML-Thms.bind-thms* (*eval-intros*, *map*
  (*simplify* (***context*** *delsimps* @{*thms Skip-eq*} *addsimps* @{*thms lvar-def*}) *o*
  rewrite-rule ***context*** [@{*thm assign-def*}, @{*thm Let-def*}]) @{*thms eval.intros*})›
lemmas *eval-Is* = *eval-Init eval-StatRef AbruptIs eval-intros*

axiomatization
  a :: *loc* **and**
  b :: *loc* **and**
  c :: *loc*

abbreviation *one* == *Suc 0*
abbreviation *two* == *Suc one*
abbreviation *three* == *Suc two*
abbreviation *four* == *Suc three*

abbreviation
  obj-a == (|*tag*=*Arr* (*PrimT Boolean*) *2*
    ,*values*= *Map.empty*(*Inr 0*↦*Bool False*)(*Inr 1*↦*Bool False*)|)

abbreviation
  obj-b == (|*tag*=*CInst Ext*
    ,*values*=(*Map.empty*(*Inl* (*vee*, *Base*)↦*Null*)
      (*Inl* (*vee*, *Ext*)↦*Intg 0*))|)

abbreviation
  obj-c == (|*tag*=*CInst* (*SXcpt NullPointer*),*values*=*CONST Map.empty*|)

abbreviation *arr-N* == *Map.empty*(*Inl* (*arr*, *Base*)↦*Null*)
abbreviation *arr-a* == *Map.empty*(*Inl* (*arr*, *Base*)↦*Addr a*)

abbreviation
  globs1 == *Map.empty*(*Inr Ext* ↦(|*tag*=*undefined*, *values*=*Map.empty*|))
    (*Inr Base* ↦(|*tag*=*undefined*, *values*=*arr-N*|))
    (*Inr Object*↦(|*tag*=*undefined*, *values*=*Map.empty*|))

abbreviation
  globs2 == *Map.empty*(*Inr Ext* ↦(|*tag*=*undefined*, *values*=*Map.empty*|))
    (*Inr Object*↦(|*tag*=*undefined*, *values*=*Map.empty*|))
    (*Inl a*↦*obj-a*)
    (*Inr Base* ↦(|*tag*=*undefined*, *values*=*arr-a*|))

abbreviation *globs3* == *globs2*(*Inl b*↦*obj-b*)
abbreviation *globs8* == *globs3*(*Inl c*↦*obj-c*)
abbreviation *locs3* == *Map.empty*(*VName e*↦*Addr b*)
abbreviation *locs8* == *locs3*(*VName z*↦*Addr c*)

abbreviation *s0* == *st Map.empty Map.empty*
abbreviation *s0'* == *Norm s0*
abbreviation *s1* == *st globs1 Map.empty*

abbreviation *s1′* == *Norm s1*
abbreviation *s2* == *st globs2 Map.empty*
abbreviation *s2′* == *Norm s2*
abbreviation *s3* == *st globs3 locs3*
abbreviation *s3′* == *Norm s3*
abbreviation *s7′* == (*Some* (*Xcpt* (*Std NullPointer*)), *s3*)
abbreviation *s8* == *st globs8 locs8*
abbreviation *s8′* == *Norm s8*
abbreviation *s9′* == (*Some* (*Xcpt* (*Std IndOutBound*)), *s8*)

declare *prod.inject* [*simp del*]
schematic-goal *exec-test*:
⟦*the* (*new-Addr* (*heap s1*)) = *a*;
the (*new-Addr* (*heap ?s2*)) = *b*;
the (*new-Addr* (*heap ?s3*)) = *c*⟧ ⟹
atleast-free (*heap s0*) *four* ⟹
tprg⊢*s0′* −*test* [*Class Base*]→ *?s9′*
apply (*unfold test-def arr-viewed-from-def*)

apply (*simp* (*no-asm-use*))
apply (*drule* (*1*) *alloc-one*,*clarsimp*)
apply (*rule eval-Is*)
apply (*erule-tac V* = *the* (*new-Addr* -) = *c* **in** *thin-rl*)
apply (*erule-tac* [*2*] *V* = *new-Addr* - = *Some a* **in** *thin-rl*)
apply (*erule-tac* [*2*] *V* = *atleast-free* - *four* **in** *thin-rl*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)

apply (*erule-tac V* = *the* (*new-Addr* -) = *b* **in** *thin-rl*)
apply (*erule-tac V* = *atleast-free* - *three* **in** *thin-rl*)
apply (*erule-tac* [*2*] *V* = *atleast-free* - *four* **in** *thin-rl*)
apply (*erule-tac* [*2*] *V* = *new-Addr* - = *Some a* **in** *thin-rl*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule conjI*)
prefer *2* **apply** (*rule conjI HOL.refl*)+
apply (*rule eval-Is*)
apply (*simp add*: *arr-viewed-from-def*)
apply (*rule conjI*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*simp*)
apply (*rule conjI*, *rule-tac* [*2*] *HOL.refl*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule init-done*, *simp*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*simp add*: *check-field-access-def Let-def*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule halloc.New*)
apply (*simp* (*no-asm-simp*))

apply (*drule atleast-free-weaken*,*drule atleast-free-weaken*)
apply (*simp* (*no-asm-simp*))
apply (*simp add*: *upd-gobj-def*)

apply (*rule halloc.New*)
apply (*drule alloc-one*)
prefer *2* **apply** *fast*
apply (*simp* (*no-asm-simp*))
apply (*drule atleast-free-weaken*)
apply *force*
apply (*simp*)
apply (*drule alloc-one*)
apply (*simp* (*no-asm-simp*))
apply *clarsimp*
apply (*erule-tac* V = *atleast-free - three* **in** *thin-rl*)
apply (*drule-tac* x = a **in** *new-AddrD2* [*THEN spec*])
apply (*simp* (*no-asm-use*))
apply (*rule eval-Is*)
apply (*rule eval-Is*)

apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*simp*)
apply (*subgoal-tac*
tprg⊢(*Ext*,*mdecl Ext-foo*) *in Ext dyn-accessible-from Main*)
apply (*simp add*: *check-method-access-def Let-def*
invocation-declclass-def dynlookup-def dynmethd-Ext-foo)
apply (*rule Ext-foo-dyn-accessible*)
apply (*rule eval-Is*)
apply (*simp add*: *body-def Let-def*)
apply (*rule eval-Is*)
apply (*rule init-done*, *simp*)
apply (*simp add*: *invocation-declclass-def dynlookup-def dynmethd-Ext-foo*)
apply (*simp add*: *invocation-declclass-def dynlookup-def dynmethd-Ext-foo*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule init-done*, *simp*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*simp split del*: *if-split*)
apply (*simp add*: *check-field-access-def Let-def*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule conjI*)
apply (*simp*)
apply (*rule eval-Is*)
apply (*simp*)

apply *simp*
apply (*rule sxalloc.intros*)

apply (*rule halloc.New*)
apply (*erule alloc-one* [*THEN conjunct1*])
apply (*simp* (*no-asm-simp*))
apply (*simp* (*no-asm-simp*))
apply (*simp add*: *gupd-def lupd-def obj-ty-def split del*: *if-split*)
apply (*drule alloc-one* [*THEN conjunct1*])
apply (*simp* (*no-asm-simp*))
apply (*erule-tac V* = *atleast-free - two* **in** *thin-rl*)
apply (*drule-tac x* = *a* **in** *new-AddrD2* [*THEN spec*])
apply *simp*
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule init-done*, *simp*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*simp add*: *check-field-access-def Let-def*)
apply (*rule eval-Is*)
apply (*simp* (*no-asm-simp*))
apply (*auto simp add*: *in-bounds-def*)
done
declare *prod.inject* [*simp*]

end

Chapter 17

Conform

1 Conformance notions for the type soundness proof for Java

theory *Conform* **imports** *State* **begin**

design issues:

- lconf allows for (arbitrary) inaccessible values
- "conforms" does not directly imply that the dynamic types of all objects on the heap are indeed existing classes. Yet this can be inferred for all referenced objs.

type-synonym $env' = prog \times (lname,\ ty)\ table$

extension of global store

definition $gext :: st \Rightarrow st \Rightarrow bool$ (-≤|- [71,71] 70) **where**
$s{\le}|s' \equiv \forall r.\ \forall obj{\in}globs\ s\ r:\ \exists obj'{\in}globs\ s'\ r:\ tag\ obj'{=}\ tag\ obj$

For the the proof of type soundness we will need the property that during execution, objects are not lost and moreover retain the values of their tags. So the object store grows conservatively. Note that if we considered garbage collection, we would have to restrict this property to accessible objects.

lemma *gext-objD*:
$[\![s{\le}|s';\ globs\ s\ r = Some\ obj]\!]$
$\Longrightarrow \exists obj'.\ globs\ s'\ r = Some\ obj' \wedge tag\ obj' = tag\ obj$
apply (*simp only*: *gext-def*)
by *force*

lemma *rev-gext-objD*:
$[\![globs\ s\ r = Some\ obj;\ s{\le}|s']\!]$
$\Longrightarrow \exists obj'.\ globs\ s'\ r = Some\ obj' \wedge tag\ obj' = tag\ obj$
by (*auto elim*: *gext-objD*)

lemma *init-class-obj-inited*:
$init\text{-}class\text{-}obj\ G\ C\ s1{\le}|s2 \Longrightarrow inited\ C\ (globs\ s2)$
apply (*unfold inited-def init-obj-def*)
apply (*auto dest*!: *gext-objD*)
done

lemma *gext-refl* [*intro*!, *simp*]: $s{\le}|s$
apply (*unfold gext-def*)
apply (*fast del*: *fst-splitE*)

done

lemma *gext-gupd* [*simp*, *elim*!]: $\bigwedge s.\ globs\ s\ r = None \Longrightarrow s{\le}|gupd(r{\mapsto}x)s$
by (*auto simp*: *gext-def*)

lemma *gext-new* [*simp*, *elim*!]: $\bigwedge s.\ globs\ s\ r = None \Longrightarrow s{\le}|init\text{-}obj\ G\ oi\ r\ s$
apply (*simp only*: *init-obj-def*)
apply (*erule-tac gext-gupd*)
done

lemma *gext-trans* [*elim*]: $\bigwedge X.\ [\![s{\le}|s';\ s'{\le}|s'']\!] \Longrightarrow s{\le}|s''$
by (*force simp*: *gext-def*)

lemma *gext-upd-gobj* [*intro*!]: $s{\le}|upd\text{-}gobj\ r\ n\ v\ s$
apply (*simp only*: *gext-def*)
apply *auto*
apply (*case-tac ra = r*)
apply *auto*
apply (*case-tac globs s r = None*)
apply *auto*
done

lemma *gext-cong1* [*simp*]: $set\text{-}locals\ l\ s1{\le}|s2 = s1{\le}|s2$
by (*auto simp*: *gext-def*)

lemma *gext-cong2* [*simp*]: $s1{\le}|set\text{-}locals\ l\ s2 = s1{\le}|s2$
by (*auto simp*: *gext-def*)

lemma *gext-lupd1* [*simp*]: $lupd(vn{\mapsto}v)s1{\le}|s2 = s1{\le}|s2$
by (*auto simp*: *gext-def*)

lemma *gext-lupd2* [*simp*]: $s1{\le}|lupd(vn{\mapsto}v)s2 = s1{\le}|s2$
by (*auto simp*: *gext-def*)

lemma *inited-gext*: $[\![inited\ C\ (globs\ s);\ s{\le}|s']\!] \Longrightarrow inited\ C\ (globs\ s')$
apply (*unfold inited-def*)
apply (*auto dest*: *gext-objD*)
done

value conformance

definition *conf* :: $prog \Rightarrow st \Rightarrow val \Rightarrow ty \Rightarrow bool$ (-,-⊢-::≼- [71,71,71,71] 70)
  where $G,s{\vdash}v{::}{\preceq}T = (\exists\,T'{\in}typeof\ (\lambda a.\ map\text{-}option\ obj\text{-}ty\ (heap\ s\ a))\ v{:}G{\vdash}T'{\preceq}T)$

lemma *conf-cong* [*simp*]: $G,set\text{-}locals\ l\ s{\vdash}v{::}{\preceq}T = G,s{\vdash}v{::}{\preceq}T$
by (*auto simp*: *conf-def*)

lemma *conf-lupd* [*simp*]: $G,lupd(vn\mapsto va)s\vdash v::\preceq T\ =\ G,s\vdash v::\preceq T$
by (*auto simp*: *conf-def*)

lemma *conf-PrimT* [*simp*]: $\forall\, dt.\ typeof\ dt\ v = Some\ (PrimT\ t) \Longrightarrow G,s\vdash v::\preceq PrimT\ t$
apply (*simp add*: *conf-def*)
done

lemma *conf-Boolean*: $G,s\vdash v::\preceq PrimT\ Boolean \Longrightarrow \exists\ b.\ v{=}Bool\ b$
by (*cases v*)
  (*auto simp*: *conf-def obj-ty-def*
    dest: *widen-Boolean2*
    split: *obj-tag.splits*)

lemma *conf-litval* [*rule-format* (*no-asm*)]:
  $typeof\ (\lambda a.\ None)\ v = Some\ T \longrightarrow G,s\vdash v::\preceq T$
apply (*unfold conf-def*)
apply (*rule val.induct*)
apply *auto*
done

lemma *conf-Null* [*simp*]: $G,s\vdash Null::\preceq T\ =\ G\vdash NT\preceq T$
by (*simp add*: *conf-def*)

lemma *conf-Addr*:
  $G,s\vdash Addr\ a::\preceq T = (\exists\, obj.\ heap\ s\ a = Some\ obj \wedge G\vdash obj\text{-}ty\ obj\preceq T)$
by (*auto simp*: *conf-def*)

lemma *conf-AddrI*: $[\![heap\ s\ a = Some\ obj;\ G\vdash obj\text{-}ty\ obj\preceq T]\!] \Longrightarrow G,s\vdash Addr\ a::\preceq T$
apply (*rule conf-Addr* [*THEN iffD2*])
by *fast*

lemma *defval-conf* [*rule-format* (*no-asm*), *elim*]:
  $is\text{-}type\ G\ T \longrightarrow G,s\vdash default\text{-}val\ T::\preceq T$
apply (*unfold conf-def*)
apply (*induct T*)
apply (*auto intro*: *prim-ty.induct*)
done

lemma *conf-widen* [*rule-format* (*no-asm*), *elim*]:
  $G\vdash T\preceq T' \Longrightarrow G,s\vdash x::\preceq T \longrightarrow ws\text{-}prog\ G \longrightarrow G,s\vdash x::\preceq T'$
apply (*unfold conf-def*)
apply (*rule val.induct*)
apply (*auto elim*: *ws-widen-trans*)
done

lemma *conf-gext* [*rule-format* (*no-asm*), *elim*]:
  $G,s\vdash v::\preceq T \longrightarrow s{\le}|s' \longrightarrow G,s'\vdash v::\preceq T$
apply (*unfold gext-def conf-def*)

apply (*rule val.induct*)
apply *force+*
done

lemma *conf-list-widen* [*rule-format* (*no-asm*)]:
ws-prog G $\Longrightarrow$
$\forall$ *Ts Ts'*. *list-all2* (*conf G s*) *vs Ts*
$\longrightarrow$ $G\vdash Ts[\preceq]$ *Ts'* $\longrightarrow$ *list-all2* (*conf G s*) *vs Ts'*
apply (*unfold widens-def*)
apply (*rule list-all2-trans*)
apply *auto*
done

lemma *conf-RefTD* [*rule-format* (*no-asm*)]:
$G,s\vdash a'::\preceq$*RefT T*
$\longrightarrow$ a' = *Null* $\vee$ ($\exists$ *a obj T'*. a' = *Addr a* $\wedge$ *heap s a* = *Some obj* $\wedge$
obj-ty obj = T' $\wedge$ $G\vdash T'\preceq$*RefT T*)
apply (*unfold conf-def*)
apply (*induct-tac a'*)
apply (*auto dest*: *widen-PrimT*)
done

value list conformance

definition
lconf :: *prog* $\Rightarrow$ *st* $\Rightarrow$ ('*a*, *val*) *table* $\Rightarrow$ ('*a*, *ty*) *table* $\Rightarrow$ *bool* (-,-$\vdash$-[::$\preceq$]- [*71*,*71*,*71*,*71*] *70*)
where $G,s\vdash vs[::\preceq]Ts$ = ($\forall$ *n*. $\forall$ $T\in$*Ts n*: $\exists$ $v\in$*vs n*: $G,s\vdash v::\preceq T$)

lemma *lconfD*: $[\![G,s\vdash vs[::\preceq]Ts$; *Ts n* = *Some T*$]\!]$ $\Longrightarrow$ $G,s\vdash$(*the* (*vs n*))$::\preceq T$
by (*force simp*: *lconf-def*)

lemma *lconf-cong* [*simp*]: $\bigwedge$*s*. *G*,*set-locals x s*$\vdash l[::\preceq]L$ = $G,s\vdash l[::\preceq]L$
by (*auto simp*: *lconf-def*)

lemma *lconf-lupd* [*simp*]: *G*,*lupd*(*vn*$\mapsto$*v*)*s*$\vdash l[::\preceq]L$ = $G,s\vdash l[::\preceq]L$
by (*auto simp*: *lconf-def*)

lemma *lconf-new*: $[\![$*L vn* = *None*; $G,s\vdash l[::\preceq]L]\!]$ $\Longrightarrow$ $G,s\vdash l(vn\mapsto v)[::\preceq]L$
by (*auto simp*: *lconf-def*)

lemma *lconf-upd*: $[\![G,s\vdash l[::\preceq]L$; $G,s\vdash v::\preceq T$; *L vn* = *Some T*$]\!]$ $\Longrightarrow$
$G,s\vdash l(vn\mapsto v)[::\preceq]L$
by (*auto simp*: *lconf-def*)

lemma *lconf-ext*: $[\![G,s\vdash l[::\preceq]L$; $G,s\vdash v::\preceq T]\!]$ $\Longrightarrow$ $G,s\vdash l(vn\mapsto v)[::\preceq]L(vn\mapsto T)$
by (*auto simp*: *lconf-def*)

lemma *lconf-map-sum* [*simp*]:
$G,s\vdash l1\ (+)\ l2[::\preceq]L1\ (+)\ L2 = (G,s\vdash l1[::\preceq]L1 \wedge G,s\vdash l2[::\preceq]L2)$
apply (*unfold lconf-def*)
apply *safe*
apply (*case-tac* [*3*] *n*)
apply (*force split*: *sum.split*)+
done

lemma *lconf-ext-list* [*rule-format* (*no-asm*)]:
$\bigwedge X.\ [\![G,s\vdash l[::\preceq]L]\!] \Longrightarrow$
$\forall vs\ Ts.\ distinct\ vns \longrightarrow length\ Ts = length\ vns$
$\longrightarrow list\text{-}all2\ (conf\ G\ s)\ vs\ Ts \longrightarrow G,s\vdash l(vns[\mapsto]vs)[::\preceq]L(vns[\mapsto]Ts)$
apply (*unfold lconf-def*)
apply (*induct-tac vns*)
apply *clarsimp*
apply *clarify*
apply (*frule list-all2-lengthD*)
apply (*clarsimp*)
done

lemma *lconf-deallocL*: $[\![G,s\vdash l[::\preceq]L(vn\mapsto T);\ L\ vn = None]\!] \Longrightarrow G,s\vdash l[::\preceq]L$
apply (*simp only*: *lconf-def*)
apply *safe*
apply (*drule spec*)
apply (*drule ospec*)
apply *auto*
done

lemma *lconf-gext* [*elim*]: $[\![G,s\vdash l[::\preceq]L;\ s\leq|s']\!] \Longrightarrow G,s'\vdash l[::\preceq]L$
apply (*simp only*: *lconf-def*)
apply *fast*
done

lemma *lconf-empty* [*simp*, *intro*!]: $G,s\vdash vs[::\preceq]Map.empty$
apply (*unfold lconf-def*)
apply *force*
done

lemma *lconf-init-vals* [*intro*!]:
$\forall n.\ \forall T\in fs\ n{:}is\text{-}type\ G\ T \Longrightarrow G,s\vdash init\text{-}vals\ fs[::\preceq]fs$
apply (*unfold lconf-def*)
apply *force*
done

weak value list conformance

Only if the value is defined it has to conform to its type. This is the contribution of the definite assignment analysis to the notion of conformance. The definite assignment analysis ensures that the program only attempts to access local variables that actually have a defined value in the state. So conformance must only ensure that the defined values are of the right type, and not also that the value is defined.

definition
wlconf :: *prog* ⇒ *st* ⇒ ('*a*, *val*) *table* ⇒ ('*a*, *ty*) *table* ⇒ *bool* (-,-⊢-[∼::≼]- [*71*,*71*,*71*,*71*] *70*)
where *G*,*s*⊢*vs*[∼::≼]*Ts* = (∀ *n*. ∀ *T*∈*Ts n*: ∀ *v*∈*vs n*: *G*,*s*⊢*v*::≼*T*)

lemma *wlconfD*: ⟦*G*,*s*⊢*vs*[∼::≼]*Ts*; *Ts n* = *Some T*; *vs n* = *Some v*⟧ ⟹ *G*,*s*⊢*v*::≼*T*
by (*auto simp*: *wlconf-def*)

lemma *wlconf-cong* [*simp*]: ⋀*s*. *G*,*set-locals x s*⊢*l*[∼::≼]*L* = *G*,*s*⊢*l*[∼::≼]*L*
by (*auto simp*: *wlconf-def*)

lemma *wlconf-lupd* [*simp*]: *G*,*lupd*(*vn*↦*v*)*s*⊢*l*[∼::≼]*L* = *G*,*s*⊢*l*[∼::≼]*L*
by (*auto simp*: *wlconf-def*)

lemma *wlconf-upd*: ⟦*G*,*s*⊢*l*[∼::≼]*L*; *G*,*s*⊢*v*::≼*T*; *L vn* = *Some T*⟧ ⟹
G,*s*⊢*l*(*vn*↦*v*)[∼::≼]*L*
by (*auto simp*: *wlconf-def*)

lemma *wlconf-ext*: ⟦*G*,*s*⊢*l*[∼::≼]*L*; *G*,*s*⊢*v*::≼*T*⟧ ⟹ *G*,*s*⊢*l*(*vn*↦*v*)[∼::≼]*L*(*vn*↦*T*)
by (*auto simp*: *wlconf-def*)

lemma *wlconf-map-sum* [*simp*]:
G,*s*⊢*l1* (+) *l2*[∼::≼]*L1* (+) *L2* = (*G*,*s*⊢*l1*[∼::≼]*L1* ∧ *G*,*s*⊢*l2*[∼::≼]*L2*)
apply (*unfold wlconf-def*)
apply *safe*
apply (*case-tac* [*3*] *n*)
apply (*force split*: *sum.split*)+
done

lemma *wlconf-ext-list* [*rule-format* (*no-asm*)]:
⋀*X*. ⟦*G*,*s*⊢*l*[∼::≼]*L*⟧ ⟹
∀ *vs Ts*. *distinct vns* ⟶ *length Ts* = *length vns*
⟶ *list-all2* (*conf G s*) *vs Ts* ⟶ *G*,*s*⊢*l*(*vns*[↦]*vs*)[∼::≼]*L*(*vns*[↦]*Ts*)
apply (*unfold wlconf-def*)
apply (*induct-tac vns*)
apply *clarsimp*
apply *clarify*
apply (*frule list-all2-lengthD*)
apply *clarsimp*
done

lemma *wlconf-deallocL*: ⟦*G*,*s*⊢*l*[∼::≼]*L*(*vn*↦*T*); *L vn* = *None*⟧ ⟹ *G*,*s*⊢*l*[∼::≼]*L*
apply (*simp only*: *wlconf-def*)
apply *safe*
apply (*drule spec*)
apply (*drule ospec*)
defer
apply (*drule ospec*)
apply *auto*

done

lemma *wlconf-gext* [*elim*]: $[\![G,s\vdash l[\sim::\preceq]L;\ s\leq|s']\!] \Longrightarrow G,s'\vdash l[\sim::\preceq]L$
apply (*simp only*: *wlconf-def*)
apply *fast*
done

lemma *wlconf-empty* [*simp*, *intro*!]: $G,s\vdash vs[\sim::\preceq]Map.empty$
apply (*unfold wlconf-def*)
apply *force*
done

lemma *wlconf-empty-vals*: $G,s\vdash Map.empty[\sim::\preceq]ts$
by (*simp add*: *wlconf-def*)

lemma *wlconf-init-vals* [*intro*!]:
$\forall n.\ \forall T\in fs\ n{:}is\text{-}type\ G\ T \Longrightarrow G,s\vdash init\text{-}vals\ fs[\sim::\preceq]fs$
apply (*unfold wlconf-def*)
apply *force*
done

lemma *lconf-wlconf*:
$G,s\vdash l[::\preceq]L \Longrightarrow G,s\vdash l[\sim::\preceq]L$
by (*force simp add*: *lconf-def wlconf-def*)

object conformance

definition
oconf :: *prog* ⇒ *st* ⇒ *obj* ⇒ *oref* ⇒ *bool* (-,-⊢-::≼√- [*71*,*71*,*71*,*71*] *70*) **where**
$(G,s\vdash obj::\preceq\surd r) = (G,s\vdash values\ obj[::\preceq]var\text{-}tys\ G\ (tag\ obj)\ r \wedge$
$(case\ r\ of$
$Heap\ a \Rightarrow is\text{-}type\ G\ (obj\text{-}ty\ obj)$
$|\ Stat\ C \Rightarrow True))$

lemma *oconf-is-type*: $G,s\vdash obj::\preceq\surd Heap\ a \Longrightarrow is\text{-}type\ G\ (obj\text{-}ty\ obj)$
by (*auto simp*: *oconf-def Let-def*)

lemma *oconf-lconf*: $G,s\vdash obj::\preceq\surd r \Longrightarrow G,s\vdash values\ obj[::\preceq]var\text{-}tys\ G\ (tag\ obj)\ r$
by (*simp add*: *oconf-def*)

lemma *oconf-cong* [*simp*]: $G,set\text{-}locals\ l\ s\vdash obj::\preceq\surd r = G,s\vdash obj::\preceq\surd r$
by (*auto simp*: *oconf-def Let-def*)

lemma *oconf-init-obj-lemma*:
$[\![\bigwedge C\ c.\ class\ G\ C = Some\ c \Longrightarrow unique\ (DeclConcepts.fields\ G\ C);$
$\bigwedge C\ c\ f\ fld.\ [\![class\ G\ C = Some\ c;$
$table\text{-}of\ (DeclConcepts.fields\ G\ C)\ f = Some\ fld\ ]\!]$
$\Longrightarrow is\text{-}type\ G\ (type\ fld);$

(*case r of*
  Heap a ⇒ *is-type G* (*obj-ty obj*)
| *Stat C* ⇒ *is-class G C*)
⟧ ⟹ *G*,*s*⊢*obj* (|*values*:=*init-vals* (*var-tys G* (*tag obj*) *r*)|)::≼√*r*
apply (*auto simp add*: *oconf-def*)
apply (*drule-tac var-tys-Some-eq* [*THEN iffD1*])
defer
apply (*subst obj-ty-cong*)
apply (*auto dest*!: *fields-table-SomeD split*: *sum.split-asm obj-tag.split-asm*)
done

state conformance

definition
conforms :: *state* ⇒ *env′* ⇒ *bool* (-::≼- [*71*,*71*] *70*) **where**
xs::≼*E* =
(*let* (*G*, *L*) = *E*; *s* = *snd xs*; *l* = *locals s in*
(∀ *r*. ∀ *obj*∈*globs s r*: *G*,*s*⊢*obj* ::≼√*r*) ∧ *G*,*s*⊢*l* [∼::≼]*L* ∧
(∀ *a*. *fst xs*=*Some*(*Xcpt* (*Loc a*)) ⟶ *G*,*s*⊢*Addr a*::≼ *Class* (*SXcpt Throwable*)) ∧
(*fst xs*=*Some*(*Jump Ret*) ⟶ *l Result* ≠ *None*))

conforms

lemma *conforms-globsD*:
⟦(*x*, *s*)::≼(*G*, *L*); *globs s r* = *Some obj*⟧ ⟹ *G*,*s*⊢*obj*::≼√*r*
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-localD*: (*x*, *s*)::≼(*G*, *L*) ⟹ *G*,*s*⊢*locals s*[∼::≼]*L*
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-XcptLocD*: ⟦(*x*, *s*)::≼(*G*, *L*); *x* = *Some* (*Xcpt* (*Loc a*))⟧ ⟹
G,*s*⊢*Addr a*::≼ *Class* (*SXcpt Throwable*)
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-RetD*: ⟦(*x*, *s*)::≼(*G*, *L*); *x* = *Some* (*Jump Ret*)⟧ ⟹
(*locals s*) *Result* ≠ *None*
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-RefTD*:
⟦*G*,*s*⊢*a′*::≼*RefT t*; *a′* ≠ *Null*; (*x*,*s*) ::≼(*G*, *L*)⟧ ⟹
∃ *a obj*. *a′* = *Addr a* ∧ *globs s* (*Inl a*) = *Some obj* ∧
G⊢*obj-ty obj*≼*RefT t* ∧ *is-type G* (*obj-ty obj*)
apply (*drule-tac conf-RefTD*)
apply *clarsimp*
apply (*rule conforms-globsD* [*THEN oconf-is-type*])
apply *auto*
done

lemma *conforms-Jump* [*iff*]:
j=*Ret* ⟶ *locals s Result* ≠ *None*
⟹ ((*Some* (*Jump j*), *s*)::≼(*G*, *L*)) = (*Norm s*::≼(*G*, *L*))
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-StdXcpt* [*iff*]:
$((Some\ (Xcpt\ (Std\ xn)),\ s)::\preceq(G,\ L)) = (Norm\ s::\preceq(G,\ L))$
by (*auto simp*: *conforms-def*)

lemma *conforms-Err* [*iff*]:
$((Some\ (Error\ e),\ s)::\preceq(G,\ L)) = (Norm\ s::\preceq(G,\ L))$
by (*auto simp*: *conforms-def*)

lemma *conforms-raise-if* [*iff*]:
$((raise\text{-}if\ c\ xn\ x,\ s)::\preceq(G,\ L)) = ((x,\ s)::\preceq(G,\ L))$
by (*auto simp*: *abrupt-if-def*)

lemma *conforms-error-if* [*iff*]:
$((error\text{-}if\ c\ err\ x,\ s)::\preceq(G,\ L)) = ((x,\ s)::\preceq(G,\ L))$
by (*auto simp*: *abrupt-if-def*)

lemma *conforms-NormI*: $(x,\ s)::\preceq(G,\ L) \Longrightarrow Norm\ s::\preceq(G,\ L)$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-absorb* [*rule-format*]:
$(a,\ b)::\preceq(G,\ L) \longrightarrow (absorb\ j\ a,\ b)::\preceq(G,\ L)$
apply (*rule impI*)
apply (*case-tac a*)
apply (*case-tac absorb j a*)
apply *auto*
apply (*rename-tac a'*)
apply (*case-tac absorb j (Some a'),auto*)
apply (*erule conforms-NormI*)
done

lemma *conformsI*: $[\![\forall r.\ \forall obj{\in}globs\ s\ r:\ G,s{\vdash}obj::\preceq\surd r;$
$G,s{\vdash}locals\ s[\sim::\preceq]L;$
$\forall a.\ x = Some\ (Xcpt\ (Loc\ a)) \longrightarrow G,s{\vdash}Addr\ a::\preceq\ Class\ (SXcpt\ Throwable);$
$x = Some\ (Jump\ Ret){\longrightarrow}\ locals\ s\ Result \neq None]\!] \Longrightarrow$
$(x,\ s)::\preceq(G,\ L)$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-xconf*: $[\![(x,\ s)::\preceq(G,L);$
$\forall a.\ x' = Some\ (Xcpt\ (Loc\ a)) \longrightarrow G,s{\vdash}Addr\ a::\preceq\ Class\ (SXcpt\ Throwable);$
$x' = Some\ (Jump\ Ret) \longrightarrow locals\ s\ Result \neq None]\!] \Longrightarrow$
$(x',s)::\preceq(G,L)$
by (*fast intro*: *conformsI elim*: *conforms-globsD conforms-localD*)

lemma *conforms-lupd*:
$[\![(x,\ s)::\preceq(G,\ L);\ L\ vn = Some\ T;\ G,s{\vdash}v::\preceq T]\!] \Longrightarrow (x,\ lupd(vn{\mapsto}v)s)::\preceq(G,\ L)$
by (*force intro*: *conformsI wlconf-upd dest*: *conforms-globsD conforms-localD*
conforms-XcptLocD conforms-RetD
simp: *oconf-def*)

lemmas *conforms-allocL-aux* = *conforms-localD* [*THEN wlconf-ext*]

lemma *conforms-allocL*:
$[\![(x,\ s)::\preceq(G,\ L);\ G,s\vdash v::\preceq T]\!] \Longrightarrow (x,\ lupd(vn\mapsto v)s)::\preceq(G,\ L(vn\mapsto T))$
by (*force intro*: *conformsI dest*: *conforms-globsD conforms-RetD*
elim: *conforms-XcptLocD conforms-allocL-aux*
simp: *oconf-def*)

lemmas *conforms-deallocL-aux* = *conforms-localD* [*THEN wlconf-deallocL*]

lemma *conforms-deallocL*: $\bigwedge s.[\![s::\preceq(G,\ L(vn\mapsto T));\ L\ vn = None]\!] \Longrightarrow s::\preceq(G,L)$
by (*fast intro*: *conformsI dest*: *conforms-globsD conforms-RetD*
elim: *conforms-XcptLocD conforms-deallocL-aux*)

lemma *conforms-gext*: $[\![(x,\ s)::\preceq(G,L);\ s\leq|s';$
$\forall r.\ \forall obj\in globs\ s'\ r:\ G,s'\vdash obj::\preceq\surd r;$
$locals\ s'{=}locals\ s]\!] \Longrightarrow (x,s')::\preceq(G,L)$
apply (*rule conformsI*)
apply *assumption*
apply (*drule conforms-localD*) **apply** *force*
apply (*intro strip*)
apply (*drule* (*1*) *conforms-XcptLocD*) **apply** *force*
apply (*intro strip*)
apply (*drule* (*1*) *conforms-RetD*) **apply** *force*
done

lemma *conforms-xgext*:
$[\![(x\ ,s)::\preceq(G,L);\ (x',\ s')::\preceq(G,\ L);\ s'\leq|s;dom\ (locals\ s') \subseteq dom\ (locals\ s)]\!]$
$\Longrightarrow (x',s)::\preceq(G,L)$
apply (*erule-tac conforms-xconf*)
apply (*fast dest*: *conforms-XcptLocD*)
apply (*intro strip*)
apply (*drule* (*1*) *conforms-RetD*)
apply (*auto dest*: *domI*)
done

lemma *conforms-gupd*: $\bigwedge obj.\ [\![(x,\ s)::\preceq(G,\ L);\ G,s\vdash obj::\preceq\surd r;\ s\leq|gupd(r\mapsto obj)s]\!]$
$\Longrightarrow\ (x,\ gupd(r\mapsto obj)s)::\preceq(G,\ L)$
apply (*rule conforms-gext*)
apply *auto*
apply (*force dest*: *conforms-globsD simp add*: *oconf-def*)+
done

lemma *conforms-upd-gobj*: $[\![(x,s)::\preceq(G,\ L);\ globs\ s\ r = Some\ obj;$
$var\text{-}tys\ G\ (tag\ obj)\ r\ n = Some\ T;\ G,s\vdash v::\preceq T]\!] \Longrightarrow (x,upd\text{-}gobj\ r\ n\ v\ s)::\preceq(G,L)$
apply (*rule conforms-gext*)
apply *auto*
apply (*drule* (*1*) *conforms-globsD*)
apply (*simp add*: *oconf-def*)
apply *safe*
apply (*rule lconf-upd*)
apply *auto*

apply (*simp only*: *obj-ty-cong*)
apply (*force dest*: *conforms-globsD intro*!: *lconf-upd*
simp add: *oconf-def cong del*: *old.sum.case-cong-weak*)
done

lemma *conforms-set-locals*:
$[\![(x,s){::}\preceq(G,\ L'); \ G,s\vdash l[\sim{::}\preceq]L; \ x{=}Some\ (Jump\ Ret) \longrightarrow l\ Result \neq None]\!]$
$\Longrightarrow (x,set\text{-}locals\ l\ s){::}\preceq(G,L)$
apply (*rule conformsI*)
apply (*intro strip*)
apply *simp*
apply (*drule* (*2*) *conforms-globsD*)
apply *simp*
apply (*intro strip*)
apply (*drule* (*1*) *conforms-XcptLocD*)
apply *simp*
apply (*intro strip*)
apply (*drule* (*1*) *conforms-RetD*)
apply *simp*
done

lemma *conforms-locals*:
$[\![(a,b){::}\preceq(G,\ L); \ L\ x = Some\ T; locals\ b\ x \neq None]\!]$
$\Longrightarrow G,b\vdash the\ (locals\ b\ x){::}\preceq T$
apply (*force simp*: *conforms-def Let-def wlconf-def*)
done

lemma *conforms-return*:
$\bigwedge s'.\ [\![(x,s){::}\preceq(G,\ L); \ (x',s'){::}\preceq(G,\ L'); \ s{\le}|s'; x'{\neq}Some\ (Jump\ Ret)]\!] \Longrightarrow$
$(x',set\text{-}locals\ (locals\ s)\ s'){::}\preceq(G,\ L)$
apply (*rule conforms-xconf*)
prefer *2* **apply** (*force dest*: *conforms-XcptLocD*)
apply (*erule conforms-gext*)
apply (*force dest*: *conforms-globsD*)+
done

end

Chapter 18

DefiniteAssignmentCorrect

1 Correctness of Definite Assignment

theory *DefiniteAssignmentCorrect* **imports** *WellForm Eval* **begin**

declare [[*simproc del*: *wt-expr wt-var wt-exprs wt-stmt*]]

lemma *sxalloc-no-jump*:
  assumes *sxalloc*: $G \vdash s0$ $-sxalloc\rightarrow$ $s1$ **and**
      no-jmp: *abrupt s0* $\neq$ *Some* (*Jump j*)
   shows *abrupt s1* $\neq$ *Some* (*Jump j*)
using *sxalloc no-jmp*
by *cases simp-all*

lemma *sxalloc-no-jump'*:
  assumes *sxalloc*: $G \vdash s0$ $-sxalloc\rightarrow$ $s1$ **and**
      jump: *abrupt s1* = *Some* (*Jump j*)
 shows *abrupt s0* = *Some* (*Jump j*)
using *sxalloc jump*
by *cases simp-all*

lemma *halloc-no-jump*:
  assumes *halloc*: $G \vdash s0$ $-halloc\ oi \succ a \rightarrow$ $s1$ **and**
      no-jmp: *abrupt s0* $\neq$ *Some* (*Jump j*)
   shows *abrupt s1* $\neq$ *Some* (*Jump j*)
using *halloc no-jmp*
by *cases simp-all*

lemma *halloc-no-jump'*:
  assumes *halloc*: $G \vdash s0$ $-halloc\ oi \succ a \rightarrow$ $s1$ **and**
      jump: *abrupt s1* = *Some* (*Jump j*)
  shows *abrupt s0* = *Some* (*Jump j*)
using *halloc jump*
by *cases simp-all*

lemma *Body-no-jump*:
   assumes *eval*: $G \vdash s0$ $-Body\ D\ c - \succ v \rightarrow s1$ **and**
      jump: *abrupt s0* $\neq$ *Some* (*Jump j*)
   shows *abrupt s1* $\neq$ *Some* (*Jump j*)
proof (*cases normal s0*)

```
  case True
  with eval obtain s0′ where eval′: G⊢Norm s0′ −Body D c−≻v→s1 and
                         s0: s0 = Norm s0′
    by (cases s0) simp
  from eval′ obtain s2 where
     s1: s1 = abupd (absorb Ret)
            (if ∃ l. abrupt s2 = Some (Jump (Break l)) ∨
                    abrupt s2 = Some (Jump (Cont l))
             then abupd (λx. Some (Error CrossMethodJump)) s2 else s2)
    by cases simp
  show ?thesis
  proof (cases ∃ l. abrupt s2 = Some (Jump (Break l)) ∨
                  abrupt s2 = Some (Jump (Cont l)))
    case True
    with s1 have abrupt s1 = Some (Error CrossMethodJump)
      by (cases s2) simp
    thus ?thesis by simp
  next
    case False
    with s1 have s1=abupd (absorb Ret) s2
      by simp
    with False show ?thesis
      by (cases s2,cases j) (auto simp add: absorb-def)
  qed
next
  case False
  with eval obtain s0′ abr where G⊢(Some abr,s0′) −Body D c−≻v→s1
                           s0 = (Some abr, s0′)
    by (cases s0) fastforce
  with this jump
  show ?thesis
    by (cases) (simp)
qed


lemma Methd-no-jump:
   assumes eval: G⊢s0 −Methd D sig−≻v→ s1 and
          jump: abrupt s0 ≠ Some (Jump j)
   shows abrupt s1 ≠ Some (Jump j)
proof (cases normal s0)
  case True
   with eval obtain s0′ where G⊢Norm s0′ −Methd D sig−≻v→s1
                          s0 = Norm s0′
    by (cases s0) simp
  then obtain D′ body where G⊢s0 −Body D′ body−≻v→s1
    by (cases) (simp add: body-def2)
  from this jump
  show ?thesis
    by (rule Body-no-jump)
next
  case False
  with eval obtain s0′ abr where G⊢(Some abr,s0′) −Methd D sig−≻v→s1
                           s0 = (Some abr, s0′)
    by (cases s0) fastforce
  with this jump
  show ?thesis
    by (cases) (simp)
qed
```

lemma *jumpNestingOkS-mono*:
  assumes *jumpNestingOk-l'*: *jumpNestingOkS jmps' c*
    and *subset*: $jmps' \subseteq jmps$
 shows *jumpNestingOkS jmps c*
proof −
  have *True* **and** *True* **and**
   $\bigwedge$ *jmps' jmps*. $[\![$*jumpNestingOkS jmps' c*; $jmps' \subseteq jmps]\!] \Longrightarrow$ *jumpNestingOkS jmps c*
 proof (*induct rule*: *var.induct expr.induct stmt.induct*)
  case (*Lab j c jmps' jmps*)
  note *jmpOk* = ‹*jumpNestingOkS jmps'* (*j*• *c*)›
  note *jmps* = ‹$jmps' \subseteq jmps$›
  with *jmpOk* **have** *jumpNestingOkS* ($\{j\} \cup jmps'$) *c* **by** *simp*
  moreover from *jmps* **have** ($\{j\} \cup jmps'$) $\subseteq$ ($\{j\} \cup jmps$) **by** *auto*
  ultimately
  have *jumpNestingOkS* ($\{j\} \cup jmps$) *c*
   by (*rule Lab.hyps*)
  thus *?case*
   by *simp*
 next
  case (*Jmp j jmps' jmps*)
  thus *?case*
   by (*cases j*) *auto*
 next
  case (*Comp c1 c2 jmps' jmps*)
  from *Comp.prems*
  have *jumpNestingOkS jmps c1* **by** − (*rule Comp.hyps*,*auto*)
  moreover from *Comp.prems*
  have *jumpNestingOkS jmps c2* **by** − (*rule Comp.hyps*,*auto*)
  ultimately show *?case*
   by *simp*
 next
  case (*If' e c1 c2 jmps' jmps*)
  from *If'.prems*
  have *jumpNestingOkS jmps c1* **by** − (*rule If'.hyps*,*auto*)
  moreover from *If'.prems*
  have *jumpNestingOkS jmps c2* **by** − (*rule If'.hyps*,*auto*)
  ultimately show *?case*
   by *simp*
 next
  case (*Loop l e c jmps' jmps*)
  from ‹*jumpNestingOkS jmps'* (*l*• *While*(*e*) *c*)›
  have *jumpNestingOkS* ($\{Cont\ l\} \cup jmps'$) *c* **by** *simp*
  moreover
  from ‹$jmps' \subseteq jmps$›
  have $\{Cont\ l\} \cup jmps' \subseteq \{Cont\ l\} \cup jmps$ **by** *auto*
  ultimately
  have *jumpNestingOkS* ($\{Cont\ l\} \cup jmps$) *c*
   by (*rule Loop.hyps*)
  thus *?case* **by** *simp*
 next
  case (*TryC c1 C vn c2 jmps' jmps*)
  from *TryC.prems*
  have *jumpNestingOkS jmps c1* **by** − (*rule TryC.hyps*,*auto*)
  moreover from *TryC.prems*
  have *jumpNestingOkS jmps c2* **by** − (*rule TryC.hyps*,*auto*)
  ultimately show *?case*
   by *simp*
 next

```
    case (Fin c1 c2 jmps′ jmps)
    from Fin.prems
    have jumpNestingOkS jmps c1 by − (rule Fin.hyps,auto)
    moreover from Fin.prems
    have jumpNestingOkS jmps c2 by − (rule Fin.hyps,auto)
    ultimately show ?case
      by simp
  qed (simp-all)
  with jumpNestingOk-l′ subset
  show ?thesis
    by iprover
qed

corollary jumpNestingOk-mono:
  assumes jmpOk: jumpNestingOk jmps′ t
     and subset: jmps′ ⊆ jmps
  shows  jumpNestingOk jmps t
proof (cases t)
  case (In1 expr-stmt)
  show ?thesis
  proof (cases expr-stmt)
    case (Inl e)
    with In1 show ?thesis by simp
  next
    case (Inr s)
    with In1 jmpOk subset show ?thesis by (auto intro: jumpNestingOkS-mono)
  qed
qed (simp-all)


lemma assign-abrupt-propagation:
 assumes f-ok: abrupt (f n s) ≠ x
   and     ass: abrupt (assign f n s) = x
  shows abrupt s = x
proof (cases x)
  case None
  with ass show ?thesis
    by (cases s) (simp add: assign-def Let-def)
next
  case (Some xcpt)
  from f-ok
  obtain xf sf where f n s = (xf,sf)
    by (cases f n s)
  with Some ass f-ok show ?thesis
    by (cases s) (simp add: assign-def Let-def)
qed


lemma wt-init-comp-ty′:
is-acc-type (prg Env) (pid (cls Env)) T =⇒ Env⊢init-comp-ty T::√
apply (unfold init-comp-ty-def)
apply (clarsimp simp add: accessible-in-RefT-simp
                          is-acc-type-def is-acc-class-def)
done


lemma fvar-upd-no-jump:
      assumes upd: upd = snd (fst (fvar statDeclC stat fn a s′))
        and noJmp: abrupt s ≠ Some (Jump j)
```

    shows *abrupt* (*upd val s*) $\neq$ *Some* (*Jump j*)
proof (*cases stat*)
  case *True*
  with *noJmp upd*
  show *?thesis*
    by (*cases s*) (*simp add*: *fvar-def2*)
next
  case *False*
  with *noJmp upd*
  show *?thesis*
    by (*cases s*) (*simp add*: *fvar-def2*)
qed

lemma *avar-state-no-jump*:
  assumes *jmp*: *abrupt* (*snd* (*avar G i a s*)) = *Some* (*Jump j*)
  shows *abrupt s* = *Some* (*Jump j*)
proof (*cases normal s*)
  case *True* **with** *jmp* **show** *?thesis* **by** (*auto simp add*: *avar-def2 abrupt-if-def*)
next
  case *False* **with** *jmp* **show** *?thesis* **by** (*auto simp add*: *avar-def2 abrupt-if-def*)
qed

lemma *avar-upd-no-jump*:
    assumes *upd*: *upd* = *snd* (*fst* (*avar G i a s'*))
    and *noJmp*: *abrupt s* $\neq$ *Some* (*Jump j*)
    shows *abrupt* (*upd val s*) $\neq$ *Some* (*Jump j*)
using *upd noJmp*
by (*cases s*) (*simp add*: *avar-def2 abrupt-if-def*)

The next theorem expresses: If jumps (breaks, continues, returns) are nested correctly, we won't find an unexpected jump in the result state of the evaluation. For exeample, a break can't leave its enclosing loop, an return cant leave its enclosing method. To proove this, the method call is critical. Allthough the wellformedness of the whole program guarantees that the jumps (breaks,continues and returns) are nested correctly in all method bodies, the call rule alone does not guarantee that I will call a method or even a class that is part of the program due to dynamic binding! To be able to enshure this we need a kind of conformance of the state, like in the typesafety proof. But then we will redo the typesafty proof here. It would be nice if we could find an easy precondition that will guarantee that all calls will actually call classes and methods of the current program, which can be instantiated in the typesafty proof later on. To fix this problem, I have instrumented the semantic definition of a call to filter out any breaks in the state and to throw an error instead.

To get an induction hypothesis which is strong enough to perform the proof, we can't just assume *jumpNestingOk* for the empty set and conlcude, that no jump at all will be in the resulting state, because the set is altered by the statements *Lab* and *While*.

The wellformedness of the program is used to enshure that for all classinitialisations and methods the nesting of jumps is wellformed, too.

theorem *jumpNestingOk-eval*:
  assumes *eval*: $G\vdash s0 -t\succ\rightarrow (v,s1)$
    and *jmpOk*: *jumpNestingOk jmps t*
    and *wt*: $Env\vdash t::T$
    and *wf*: *wf-prog G*
    and *G*: *prg Env* = *G*
    and *no-jmp*: $\forall j.$ *abrupt s0* = *Some* (*Jump j*) $\longrightarrow j \in$ *jmps*
          (**is** *?Jmp jmps s0*)

```
  shows  (∀j. fst s1 = Some (Jump j) ⟶ j ∈ jmps) ∧
              (normal s1 ⟶
               (∀ w upd. v=In2 (w,upd)
                ⟶  (∀ s j val.
                      abrupt s ≠ Some (Jump j) ⟶
                      abrupt (upd val s) ≠ Some (Jump j))))
       (is ?Jmp jmps s1 ∧ ?Upd v s1)
proof -
  let ?HypObj = λ t s0 s1 v.
       (∀ jmps T Env.
          ?Jmp jmps s0 ⟶ jumpNestingOk jmps t ⟶ Env⊢t::T ⟶ prg Env=G⟶
          ?Jmp jmps s1 ∧ ?Upd v s1)
```

— Variable *?HypObj* is the following goal spelled in terms of the object logic, instead of the meta logic. It is needed in some cases of the induction were, the atomize-rulify process of induct does not work fine, because the eval rules mix up object and meta logic. See for example the case for the loop.

```
  from eval
  have ⋀ jmps T Env. ⟦?Jmp jmps s0; jumpNestingOk jmps t; Env⊢t::T;prg Env=G⟧
          ⟹ ?Jmp jmps s1 ∧ ?Upd v s1
       (is PROP ?Hyp t s0 s1 v)
```

— We need to abstract over *jmps* since *jmps* are extended during analysis of *Lab*. Also we need to abstract over *T* and *Env* since they are altered in various typing judgements.

```
  proof (induct)
    case Abrupt thus ?case by simp
  next
    case Skip thus ?case by simp
  next
    case Expr thus ?case by (elim wt-elim-cases) simp
  next
    case (Lab s0 c s1 jmp jmps T Env)
    note jmpOK = ‹jumpNestingOk jmps (In1r (jmp· c))›
    note G = ‹prg Env = G›
    have wt-c: Env⊢c::√
      using Lab.prems by (elim wt-elim-cases)
    {
      fix j
      assume ab-s1: abrupt (abupd (absorb jmp) s1) = Some (Jump j)
      have j∈jmps
      proof -
        from ab-s1 have jmp-s1: abrupt s1 = Some (Jump j)
          by (cases s1) (simp add: absorb-def)
        note hyp-c = ‹PROP ?Hyp (In1r c) (Norm s0) s1 ◇›
        from ab-s1 have j ≠ jmp
          by (cases s1) (simp add: absorb-def)
        moreover have j ∈ {jmp} ∪ jmps
        proof -
          from jmpOK
          have jumpNestingOk ({jmp} ∪ jmps) (In1r c) by simp
          with wt-c jmp-s1 G hyp-c
          show ?thesis
            by - (rule hyp-c [THEN conjunct1,rule-format],simp)
        qed
        ultimately show ?thesis
          by simp
      qed
    }
    thus ?case by simp
  next
    case (Comp s0 c1 s1 c2 s2 jmps T Env)
    note jmpOk = ‹jumpNestingOk jmps (In1r (c1;; c2))›
```

```
    note G = ‹prg Env = G›
    from Comp.prems obtain
      wt-c1: Env⊢c1::√ and wt-c2: Env⊢c2::√
      by (elim wt-elim-cases)
    {
      fix j
      assume abr-s2: abrupt s2 = Some (Jump j)
      have j∈jmps
      proof −
        have jmp: ?Jmp jmps s1
        proof −
          note hyp-c1 = ‹PROP ?Hyp (In1r c1) (Norm s0) s1 ◇›
          with wt-c1 jmpOk G
          show ?thesis by simp
        qed
        moreover note hyp-c2 = ‹PROP ?Hyp (In1r c2) s1 s2 (◇::vals)›
        have jmpOk': jumpNestingOk jmps (In1r c2) using jmpOk by simp
        moreover note wt-c2 G abr-s2
        ultimately show j ∈ jmps
          by (rule hyp-c2 [THEN conjunct1,rule-format (no-asm)])
      qed
    } thus ?case by simp
  next
    case (If s0 e b s1 c1 c2 s2 jmps T Env)
    note jmpOk = ‹jumpNestingOk jmps (In1r (If(e) c1 Else c2))›
    note G = ‹prg Env = G›
    from If.prems obtain
              wt-e: Env⊢e::−PrimT Boolean and
      wt-then-else: Env⊢(if the-Bool b then c1 else c2)::√
      by (elim wt-elim-cases) simp
    {
      fix j
      assume jmp: abrupt s2 = Some (Jump j)
      have j∈jmps
      proof −
        note ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 b)›
        with wt-e G have ?Jmp jmps s1
          by simp
        moreover note hyp-then-else =
          ‹PROP ?Hyp (In1r (if the-Bool b then c1 else c2)) s1 s2 ◇›
        have jumpNestingOk jmps (In1r (if the-Bool b then c1 else c2))
          using jmpOk by (cases the-Bool b) simp-all
        moreover note wt-then-else G jmp
        ultimately show j∈ jmps
          by (rule hyp-then-else [THEN conjunct1,rule-format (no-asm)])
      qed
    }
    thus ?case by simp
  next
    case (Loop s0 e b s1 c s2 l s3 jmps T Env)
    note jmpOk = ‹jumpNestingOk jmps (In1r (l· While(e) c))›
    note G = ‹prg Env = G›
    note wt = ‹Env⊢In1r (l· While(e) c)::T›
    then obtain
              wt-e: Env⊢e::−PrimT Boolean and
              wt-c: Env⊢c::√
      by (elim wt-elim-cases)
    {
      fix j
```

```
assume jmp: abrupt s3 = Some (Jump j)
have j∈jmps
proof -
  note ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 b)›
  with wt-e G have jmp-s1: ?Jmp jmps s1
    by simp
  show ?thesis
  proof (cases the-Bool b)
    case False
    from Loop.hyps
    have s3=s1
      by (simp (no-asm-use) only: if-False False)
    with jmp-s1 jmp have j ∈ jmps by simp
    thus ?thesis by simp
  next
    case True
    from Loop.hyps

    have ?HypObj (In1r c) s1 s2 (◇::vals)
      apply (simp (no-asm-use) only: True if-True )
      apply (erule conjE)+
      apply assumption
      done
    note hyp-c = this [rule-format (no-asm)]
    moreover from jmpOk have jumpNestingOk ({Cont l} ∪ jmps) (In1r c)
      by simp
    moreover from jmp-s1 have ?Jmp ({Cont l} ∪ jmps) s1 by simp
    ultimately have jmp-s2: ?Jmp ({Cont l} ∪ jmps) s2
      using wt-c G by iprover
    have ?Jmp jmps (abupd (absorb (Cont l)) s2)
    proof -
      {
        fix j'
        assume abs: abrupt (abupd (absorb (Cont l)) s2)=Some (Jump j')
        have j' ∈ jmps
        proof (cases j' = Cont l)
          case True
          with abs show ?thesis
            by (cases s2) (simp add: absorb-def)
        next
          case False
          with abs have abrupt s2 = Some (Jump j')
            by (cases s2) (simp add: absorb-def)
          with jmp-s2 False show ?thesis
            by simp
        qed
      }
      thus ?thesis by simp
    qed
    moreover
    from Loop.hyps
    have ?HypObj (In1r (l· While(e) c))
                 (abupd (absorb (Cont l)) s2) s3 (◇::vals)
      apply (simp (no-asm-use) only: True if-True)
      apply (erule conjE)+
      apply assumption
      done
    note hyp-w = this [rule-format (no-asm)]
    note jmpOk wt G jmp
```

ultimately show *j*∈ *jmps*
by (*rule hyp-w* [*THEN conjunct1*,*rule-format* (*no-asm*)])
qed
qed
}
thus *?case* **by** *simp*
next
case (*Jmp s j jmps T Env*) **thus** *?case* **by** *simp*
next
case (*Throw s0 e a s1 jmps T Env*)
note *jmpOk* = ‹*jumpNestingOk jmps* (*In1r* (*Throw e*))›
note *G* = ‹*prg Env* = *G*›
from *Throw.prems* **obtain** *Te* **where**
wt-e: *Env*⊢*e*::−*Te*
by (*elim wt-elim-cases*)
{
fix *j*
assume *jmp*: *abrupt* (*abupd* (*throw a*) *s1*) = *Some* (*Jump j*)
have *j*∈*jmps*
proof −
from ‹*PROP ?Hyp* (*In1l e*) (*Norm s0*) *s1* (*In1 a*)›
have *?Jmp jmps s1* **using** *wt-e G* **by** *simp*
moreover
from *jmp*
have *abrupt s1* = *Some* (*Jump j*)
by (*cases s1*) (*simp add*: *throw-def abrupt-if-def*)
ultimately show *j* ∈ *jmps* **by** *simp*
qed
}
thus *?case* **by** *simp*
next
case (*Try s0 c1 s1 s2 C vn c2 s3 jmps T Env*)
note *jmpOk* = ‹*jumpNestingOk jmps* (*In1r* (*Try c1 Catch*(*C vn*) *c2*))›
note *G* = ‹*prg Env* = *G*›
from *Try.prems* **obtain**
wt-c1: *Env*⊢*c1*::√ **and**
wt-c2: *Env*⦇*lcl* := *lcl Env*(*VName vn*↦*Class C*)⦈⊢*c2*::√
by (*elim wt-elim-cases*)
{
fix *j*
assume *jmp*: *abrupt s3* = *Some* (*Jump j*)
have *j*∈*jmps*
proof −
note ‹*PROP ?Hyp* (*In1r c1*) (*Norm s0*) *s1* (◇::*vals*)›
with *jmpOk wt-c1 G*
have *jmp-s1*: *?Jmp jmps s1* **by** *simp*
note *s2* = ‹*G*⊢*s1* −*sxalloc*→ *s2*›
show *j* ∈ *jmps*
proof (*cases G*,*s2*⊢*catch C*)
case *False*
from *Try.hyps* **have** *s3*=*s2*
by (*simp* (*no-asm-use*) *only*: *False if-False*)
with *jmp* **have** *abrupt s1* = *Some* (*Jump j*)
using *sxalloc-no-jump*′ [*OF s2*] **by** *simp*
with *jmp-s1*
show *?thesis* **by** *simp*
next
case *True*
with *Try.hyps*

```
        have ?HypObj (In1r c2) (new-xcpt-var vn s2) s3 (◇::vals)
          apply (simp (no-asm-use) only: True if-True simp-thms)
          apply (erule conjE)+
          apply assumption
          done
        note hyp-c2 = this [rule-format (no-asm)]
        from jmp-s1 sxalloc-no-jump' [OF s2]
        have ?Jmp jmps s2
          by simp
        hence ?Jmp jmps (new-xcpt-var vn s2)
          by (cases s2) simp
        moreover have jumpNestingOk jmps (In1r c2) using jmpOk by simp
        moreover note wt-c2
        moreover from G
        have prg (Env⦇lcl := lcl Env(VName vn↦Class C)⦈) = G
          by simp
        moreover note jmp
        ultimately show ?thesis
          by (rule hyp-c2 [THEN conjunct1,rule-format (no-asm)])
      qed
    qed
  }
  thus ?case by simp
next
  case (Fin s0 c1 x1 s1 c2 s2 s3 jmps T Env)
  note jmpOk = ‹jumpNestingOk jmps (In1r (c1 Finally c2))›
  note G = ‹prg Env = G›
  from Fin.prems obtain
    wt-c1: Env⊢c1::√ and wt-c2: Env⊢c2::√
    by (elim wt-elim-cases)
  {
    fix j
    assume jmp: abrupt s3 = Some (Jump j)
    have j ∈ jmps
    proof (cases x1=Some (Jump j))
      case True
      note hyp-c1 = ‹PROP ?Hyp (In1r c1) (Norm s0) (x1,s1) ◇›
      with True jmpOk wt-c1 G show ?thesis
        by − (rule hyp-c1 [THEN conjunct1,rule-format (no-asm)],simp-all)
    next
      case False
      note hyp-c2 = ‹PROP ?Hyp (In1r c2) (Norm s1) s2 ◇›
      note ‹s3 = (if ∃ err. x1 = Some (Error err) then (x1, s1)
                else abupd (abrupt-if (x1 ≠ None) x1) s2)›
      with False jmp have abrupt s2 = Some (Jump j)
        by (cases s2) (simp add: abrupt-if-def)
      with jmpOk wt-c2 G show ?thesis
        by − (rule hyp-c2 [THEN conjunct1,rule-format (no-asm)],simp-all)
    qed
  }
  thus ?case by simp
next
  case (Init C c s0 s3 s1 s2 jmps T Env)
  note ‹jumpNestingOk jmps (In1r (Init C))›
  note G = ‹prg Env = G›
  note ‹the (class G C) = c›
  with Init.prems have c: class G C = Some c
    by (elim wt-elim-cases) auto
  {
```

```
      fix j
      assume jmp: abrupt s3 = (Some (Jump j))
      have j∈jmps
      proof (cases inited C (globs s0))
        case True
        with Init.hyps have s3=Norm s0
          by simp
        with jmp
        have False by simp thus ?thesis ..
      next
        case False
        from wf c G
        have wf-cdecl: wf-cdecl G (C,c)
          by (simp add: wf-prog-cdecl)
        from Init.hyps
        have ?HypObj (In1r (if C = Object then Skip else Init (super c)))
                          (Norm ((init-class-obj G C) s0)) s1 (◇::vals)
          apply (simp (no-asm-use) only: False if-False simp-thms)
          apply (erule conjE)+
          apply assumption
          done
        note hyp-s1 = this [rule-format (no-asm)]
        from wf-cdecl G have
          wt-super: Env⊢(if C = Object then Skip else Init (super c))::√
          by (cases C=Object)
             (auto dest: wf-cdecl-supD is-acc-classD)
        from hyp-s1 [OF - - wt-super G]
        have ?Jmp jmps s1
          by simp
        hence jmp-s1: ?Jmp jmps ((set-lvars Map.empty) s1) by (cases s1) simp
        from False Init.hyps
        have ?HypObj (In1r (init c)) ((set-lvars Map.empty) s1) s2 (◇::vals)
          apply (simp (no-asm-use) only: False if-False simp-thms)
          apply (erule conjE)+
          apply assumption
          done
        note hyp-init-c = this [rule-format (no-asm)]
        from wf-cdecl
        have wt-init-c: ⦇prg = G, cls = C, lcl = Map.empty⦈⊢init c::√
          by (rule wf-cdecl-wt-init)
        from wf-cdecl have jumpNestingOkS {} (init c)
          by (cases rule: wf-cdeclE)
        hence jumpNestingOkS jmps (init c)
          by (rule jumpNestingOkS-mono) simp
        moreover
        have abrupt s2 = Some (Jump j)
        proof −
          from False Init.hyps
          have s3 = (set-lvars (locals (store s1))) s2  by simp
          with jmp show ?thesis by (cases s2) simp
        qed
        ultimately show ?thesis
          using hyp-init-c [OF jmp-s1 - wt-init-c]
          by simp
      qed
    }
    thus ?case by simp
  next
    case (NewC s0 C s1 a s2 jmps T Env)
```

```
    {
      fix j
      assume jmp: abrupt s2 = Some (Jump j)
      have j∈jmps
      proof −
        note ‹prg Env = G›
        moreover note hyp-init = ‹PROP ?Hyp (In1r (Init C)) (Norm s0) s1 ♢›
        moreover from wf NewC.prems
        have Env⊢(Init C)::√
          by (elim wt-elim-cases) (drule is-acc-classD,simp)
        moreover
        have abrupt s1 = Some (Jump j)
        proof −
          from ‹G⊢s1 −halloc CInst C≻a→ s2› and jmp show ?thesis
            by (rule halloc-no-jump')
        qed
        ultimately show j ∈ jmps
          by − (rule hyp-init [THEN conjunct1,rule-format (no-asm)],auto)
      qed
    }
    thus ?case by simp
  next
    case (NewA s0 elT s1 e i s2 a s3 jmps T Env)
    {
      fix j
      assume jmp: abrupt s3 = Some (Jump j)
      have j∈jmps
      proof −
        note G = ‹prg Env = G›
        from NewA.prems
        obtain wt-init: Env⊢init-comp-ty elT::√ and
               wt-size: Env⊢e::−PrimT Integer
          by (elim wt-elim-cases) (auto dest:  wt-init-comp-ty')
        note ‹PROP ?Hyp (In1r (init-comp-ty elT)) (Norm s0) s1 ♢›
        with wt-init G
        have ?Jmp jmps s1
          by (simp add: init-comp-ty-def)
        moreover
        note hyp-e = ‹PROP ?Hyp (In1l e) s1 s2 (In1 i)›
        have abrupt s2 = Some (Jump j)
        proof −
          note ‹G⊢abupd (check-neg i) s2−halloc Arr elT (the-Intg i)≻a→ s3›
          moreover note jmp
          ultimately
          have abrupt (abupd (check-neg i) s2) = Some (Jump j)
            by (rule halloc-no-jump')
          thus ?thesis by (cases s2) auto
        qed
        ultimately show j∈jmps using wt-size G
          by − (rule hyp-e [THEN conjunct1,rule-format (no-asm)],simp-all)
      qed
    }
    thus ?case by simp
  next
    case (Cast s0 e v s1 s2 cT jmps T Env)
    {
      fix j
      assume jmp: abrupt s2 = Some (Jump j)
      have j∈jmps
```

```
      proof -
        note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 v)›
        note ‹prg Env = G›
        moreover from Cast.prems
        obtain eT where Env⊢e::−eT by (elim wt-elim-cases)
        moreover
        have abrupt s1 = Some (Jump j)
        proof -
          note ‹s2 = abupd (raise-if (¬ G,snd s1⊢v fits cT) ClassCast) s1›
          moreover note jmp
          ultimately show ?thesis by (cases s1) (simp add: abrupt-if-def)
        qed
        ultimately show ?thesis
          by - (rule hyp-e [THEN conjunct1,rule-format (no-asm)], simp-all)
      qed
    }
    thus ?case by simp
  next
    case (Inst s0 e v s1 b eT jmps T Env)
    {
      fix j
      assume jmp: abrupt s1 = Some (Jump j)
      have j∈jmps
      proof -
        note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 v)›
        note ‹prg Env = G›
        moreover from Inst.prems
        obtain eT where Env⊢e::−eT by (elim wt-elim-cases)
        moreover note jmp
        ultimately show j∈jmps
          by - (rule hyp-e [THEN conjunct1,rule-format (no-asm)], simp-all)
      qed
    }
    thus ?case by simp
  next
    case Lit thus ?case by simp
  next
    case (UnOp s0 e v s1 unop jmps T Env)
    {
      fix j
      assume jmp: abrupt s1 = Some (Jump j)
      have j∈jmps
      proof -
        note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 v)›
        note ‹prg Env = G›
        moreover from UnOp.prems
        obtain eT where Env⊢e::−eT by (elim wt-elim-cases)
        moreover note jmp
        ultimately show  j∈jmps
          by - (rule hyp-e [THEN conjunct1,rule-format (no-asm)], simp-all)
      qed
    }
    thus ?case by simp
  next
    case (BinOp s0 e1 v1 s1 binop e2 v2 s2 jmps T Env)
    {
      fix j
      assume jmp: abrupt s2 = Some (Jump j)
      have j∈jmps
```

```
    proof −
      note G = ‹prg Env = G›
      from BinOp.prems
      obtain e1T e2T where
        wt-e1: Env⊢e1::−e1T and
        wt-e2: Env⊢e2::−e2T
        by (elim wt-elim-cases)
      note ‹PROP ?Hyp (In1l e1) (Norm s0) s1 (In1 v1)›
      with G wt-e1 have jmp-s1: ?Jmp jmps s1 by simp
      note hyp-e2 =
        ‹PROP ?Hyp (if need-second-arg binop v1 then In1l e2 else In1r Skip)
                   s1 s2 (In1 v2)›
      show j∈jmps
      proof (cases need-second-arg binop v1)
        case True with jmp-s1 wt-e2 jmp G
        show ?thesis
          by − (rule hyp-e2 [THEN conjunct1,rule-format (no-asm)],simp-all)
      next
        case False with jmp-s1 jmp G
        show ?thesis
          by − (rule hyp-e2 [THEN conjunct1,rule-format (no-asm)],auto)
      qed
    qed
  }
  thus ?case by simp
next
  case Super thus ?case by simp
next
  case (Acc s0 va v f s1 jmps T Env)
  {
    fix j
    assume jmp: abrupt s1 = Some (Jump j)
    have j∈jmps
    proof −
      note hyp-va = ‹PROP ?Hyp (In2 va) (Norm s0) s1 (In2 (v,f))›
      note ‹prg Env = G›
      moreover from Acc.prems
      obtain vT where Env⊢va::=vT by (elim wt-elim-cases)
      moreover note jmp
      ultimately show j∈jmps
        by − (rule hyp-va [THEN conjunct1,rule-format (no-asm)], simp-all)
    qed
  }
  thus ?case by simp
next
  case (Ass s0 va w f s1 e v s2 jmps T Env)
  note G = ‹prg Env = G›
  from Ass.prems
  obtain vT eT where
    wt-va: Env⊢va::=vT and
     wt-e: Env⊢e::−eT
    by (elim wt-elim-cases)
  note hyp-v = ‹PROP ?Hyp (In2 va) (Norm s0) s1 (In2 (w,f))›
  note hyp-e = ‹PROP ?Hyp (In1l e) s1 s2 (In1 v)›
  {
    fix j
    assume jmp: abrupt (assign f v s2) = Some (Jump j)
    have j∈jmps
    proof −
```

```
      have abrupt s2 = Some (Jump j)
      proof (cases normal s2)
        case True
        from ‹G⊢s1 −e−≻v→ s2› and True have nrm-s1: normal s1
          by (rule eval-no-abrupt-lemma [rule-format])
        with nrm-s1 wt-va G True
        have abrupt (f v s2) ≠ Some (Jump j)
          using hyp-v [THEN conjunct2,rule-format (no-asm)]
          by simp
        from this jmp
        show ?thesis
          by (rule assign-abrupt-propagation)
      next
        case False with jmp
        show ?thesis by (cases s2) (simp add: assign-def Let-def)
      qed
      moreover from wt-va G
      have ?Jmp jmps s1
        by − (rule hyp-v [THEN conjunct1],simp-all)
      ultimately show ?thesis using G
        by − (rule hyp-e [THEN conjunct1,rule-format (no-asm)], simp-all, rule wt-e)
    qed
  }
  thus ?case by simp
next
  case (Cond s0 e0 b s1 e1 e2 v s2 jmps T Env)
  note G = ‹prg Env = G›
  note hyp-e0 = ‹PROP ?Hyp (In1l e0) (Norm s0) s1 (In1 b)›
  note hyp-e1-e2 = ‹PROP ?Hyp (In1l (if the-Bool b then e1 else e2)) s1 s2 (In1 v)›
  from Cond.prems
  obtain e1T e2T
    where wt-e0: Env⊢e0::−PrimT Boolean
     and  wt-e1: Env⊢e1::−e1T
     and  wt-e2: Env⊢e2::−e2T
    by (elim wt-elim-cases)
  {
    fix j
    assume jmp: abrupt s2 = Some (Jump j)
    have j∈jmps
    proof −
      from wt-e0 G
      have jmp-s1: ?Jmp jmps s1
        by − (rule hyp-e0 [THEN conjunct1],simp-all)
      show ?thesis
      proof (cases the-Bool b)
        case True
        with jmp-s1 wt-e1 G jmp
        show ?thesis
          by−(rule hyp-e1-e2 [THEN conjunct1,rule-format (no-asm)],simp-all)
      next
        case False
        with jmp-s1 wt-e2 G jmp
        show ?thesis
          by−(rule hyp-e1-e2 [THEN conjunct1,rule-format (no-asm)],simp-all)
      qed
    qed
  }
  thus ?case by simp
next
```

```
  case (Call s0 e a s1 args vs s2 D mode statT mn pTs s3 s3' accC v s4
           jmps T Env)
  note G = ‹prg Env = G›
  from Call.prems
  obtain eT argsT
    where wt-e: Env⊢e::−eT and wt-args: Env⊢args::≐argsT
    by (elim wt-elim-cases)
  {
    fix j
    assume jmp: abrupt ((set-lvars (locals (store s2))) s4)
                  = Some (Jump j)
    have j∈jmps
    proof −
      note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 a)›
      from wt-e G
      have jmp-s1: ?Jmp jmps s1
        by − (rule hyp-e [THEN conjunct1],simp-all)
      note hyp-args = ‹PROP ?Hyp (In3 args) s1 s2 (In3 vs)›
      have abrupt s2 = Some (Jump j)
      proof −
        note ‹G⊢s3' −Methd D ⦇name = mn, parTs = pTs⦈−≻v→ s4›
        moreover
        from jmp have abrupt s4 = Some (Jump j)
          by (cases s4) simp
        ultimately have abrupt s3' = Some (Jump j)
          by − (rule ccontr,drule (1) Methd-no-jump,simp)
        moreover note ‹s3' = check-method-access G accC statT mode
                          ⦇name = mn, parTs = pTs⦈ a s3›
        ultimately have abrupt s3 = Some (Jump j)
          by (cases s3)
             (simp add: check-method-access-def abrupt-if-def Let-def)
        moreover
        note ‹s3 = init-lvars G D ⦇name=mn, parTs=pTs⦈ mode a vs s2›
        ultimately show ?thesis
          by (cases s2) (auto simp add: init-lvars-def2)
      qed
      with jmp-s1 wt-args G
      show ?thesis
        by − (rule hyp-args [THEN conjunct1,rule-format (no-asm)], simp-all)
    qed
  }
  thus ?case by simp
next
  case (Methd s0 D sig v s1 jmps T Env)
  from ‹G⊢Norm s0 −body G D sig−≻v→ s1›
  have G⊢Norm s0 −Methd D sig−≻v→ s1
    by (rule eval.Methd)
  hence ⋀ j. abrupt s1 ≠ Some (Jump j)
    by (rule Methd-no-jump) simp
  thus ?case by simp
next
  case (Body s0 D s1 c s2 s3 jmps T Env)
  have G⊢Norm s0 −Body D c−≻the (locals (store s2) Result)
         → abupd (absorb Ret) s3
    by (rule eval.Body) (rule Body)+
  hence ⋀ j. abrupt (abupd (absorb Ret) s3) ≠ Some (Jump j)
    by (rule Body-no-jump) simp
  thus ?case by simp
next
```

```
  case LVar
  thus ?case by (simp add: lvar-def Let-def)
next
  case (FVar s0 statDeclC s1 e a s2 v s2' stat fn s3 accC jmps T Env)
  note G = ‹prg Env = G›
  from wf FVar.prems
  obtain  statC f where
    wt-e: Env⊢e::−Class statC and
    accfield: accfield (prg Env) accC statC fn = Some (statDeclC,f)
    by  (elim wt-elim-cases) simp
  have wt-init: Env⊢Init statDeclC::√
  proof −
    from wf wt-e G
    have is-class (prg Env) statC
      by (auto dest: ty-expr-is-type type-is-class)
    with wf accfield G
    have is-class (prg Env) statDeclC
      by (auto dest!: accfield-fields dest: fields-declC)
    thus ?thesis
      by simp
  qed
  note fvar = ‹(v, s2') = fvar statDeclC stat fn a s2›
  {
    fix j
    assume jmp: abrupt s3 = Some (Jump j)
    have j∈jmps
    proof −
      note hyp-init = ‹PROP ?Hyp (In1r (Init statDeclC)) (Norm s0) s1 ◇›
      from G wt-init
      have ?Jmp jmps s1
        by − (rule hyp-init [THEN conjunct1],auto)
      moreover
      note hyp-e = ‹PROP ?Hyp (In1l e) s1 s2 (In1 a)›
      have abrupt s2 = Some (Jump j)
      proof −
        note ‹s3 = check-field-access G accC statDeclC fn stat a s2'›
        with jmp have abrupt s2' = Some (Jump j)
          by (cases s2')
             (simp add: check-field-access-def abrupt-if-def Let-def)
       with fvar show abrupt s2 =  Some (Jump j)
          by (cases s2) (simp add: fvar-def2 abrupt-if-def)
      qed
      ultimately show ?thesis
        using G wt-e
        by − (rule hyp-e [THEN conjunct1, rule-format (no-asm)],simp-all)
    qed
  }
  moreover
  from fvar obtain upd w
    where upd: upd = snd (fst (fvar statDeclC stat fn a s2)) and
            v: v=(w,upd)
    by (cases fvar statDeclC stat fn a s2)
      (simp, use surjective-pairing in blast)
  {
    fix j val fix s::state
    assume normal s3
    assume no-jmp: abrupt s ≠ Some (Jump j)
    with upd
    have abrupt (upd val s) ≠ Some (Jump j)
```

```
      by (rule fvar-upd-no-jump)
    }
    ultimately show ?case using v by simp
  next
    case (AVar s0 e1 a s1 e2 i s2 v s2' jmps T Env)
    note G = ‹prg Env = G›
    from AVar.prems
    obtain  e1T e2T where
      wt-e1: Env⊢e1::−e1T and wt-e2: Env⊢e2::−e2T
      by  (elim wt-elim-cases) simp
    note avar = ‹(v, s2') = avar G i a s2›
    {
      fix j
      assume jmp: abrupt s2' = Some (Jump j)
      have j∈jmps
      proof −
        note hyp-e1 = ‹PROP ?Hyp (In1l e1) (Norm s0) s1 (In1 a)›
        from G wt-e1
        have ?Jmp jmps s1
          by − (rule hyp-e1 [THEN conjunct1], auto)
        moreover
        note hyp-e2 = ‹PROP ?Hyp (In1l e2) s1 s2 (In1 i)›
        have abrupt s2 = Some (Jump j)
        proof −
          from avar have s2' = snd (avar G i a s2)
            by (cases avar G i a s2) simp
          with jmp show ?thesis by − (rule avar-state-no-jump,simp)
        qed
        ultimately show ?thesis
          using wt-e2 G
          by − (rule hyp-e2 [THEN conjunct1, rule-format (no-asm)],simp-all)
      qed
    }
    moreover
    from avar obtain upd w
      where upd: upd = snd (fst (avar G i a s2)) and
            v: v=(w,upd)
      by (cases avar G i a s2)
        (simp, use surjective-pairing in blast)
    {
      fix j val fix s::state
      assume normal s2'
      assume no-jmp: abrupt s ≠ Some (Jump j)
      with upd
      have abrupt (upd val s) ≠ Some (Jump j)
        by (rule avar-upd-no-jump)
    }
    ultimately show ?case using v by simp
  next
    case Nil thus ?case by simp
  next
    case (Cons s0 e v s1 es vs s2 jmps T Env)
    note G = ‹prg Env = G›
    from Cons.prems obtain eT esT
      where wt-e: Env⊢e::−eT and wt-e2: Env⊢es::≐esT
      by  (elim wt-elim-cases) simp
    {
      fix j
      assume jmp: abrupt s2 = Some (Jump j)
```

```
      have j∈jmps
      proof –
        note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 v)›
        from G wt-e
        have ?Jmp jmps s1
          by – (rule hyp-e [THEN conjunct1],simp-all)
        moreover
        note hyp-es = ‹PROP ?Hyp (In3 es) s1 s2 (In3 vs)›
        ultimately show ?thesis
          using wt-e2 G jmp
          by – (rule hyp-es [THEN conjunct1, rule-format (no-asm)],
                (assumption|simp (no-asm-simp))+)
      qed
    }
    thus ?case by simp
  qed
  note generalized = this
  from no-jmp jmpOk wt G
  show ?thesis
    by (rule generalized)
qed

lemmas jumpNestingOk-evalE = jumpNestingOk-eval [THEN conjE,rule-format]


lemma jumpNestingOk-eval-no-jump:
 assumes   eval: prg Env⊢ s0 −t≻→ (v,s1) and
           jmpOk: jumpNestingOk {} t and
          no-jmp: abrupt s0 ≠ Some (Jump j) and
              wt: Env⊢t::T and
              wf: wf-prog (prg Env)
 shows abrupt s1 ≠ Some (Jump j) ∧
       (normal s1 ⟶ v=In2 (w,upd)
        ⟶ abrupt s ≠ Some (Jump j′)
        ⟶ abrupt (upd val s) ≠ Some (Jump j′))
proof (cases ∃ j′. abrupt s0 = Some (Jump j′))
  case True
  then obtain j′ where jmp: abrupt s0 = Some (Jump j′) ..
  with no-jmp have j′≠j by simp
  with eval jmp have s1=s0 by auto
  with no-jmp jmp show ?thesis by simp
next
  case False
  obtain G where G: prg Env = G
    by (cases Env) simp
  from G eval have G⊢ s0 −t≻→ (v,s1) by simp
  moreover note jmpOk wt
  moreover from G wf have wf-prog G by simp
  moreover note G
  moreover from False have ⋀ j. abrupt s0 = Some (Jump j) ⟹ j ∈ {}
    by simp
  ultimately show ?thesis
    apply (rule jumpNestingOk-evalE)
    apply assumption
    apply simp
    apply fastforce
    done
qed
```

lemmas *jumpNestingOk-eval-no-jumpE*
  = *jumpNestingOk-eval-no-jump* [*THEN conjE*,*rule-format*]

corollary *eval-expression-no-jump*:
 assumes *eval*: $prg\ Env \vdash s0\ -e-\succ v \rightarrow\ s1$ **and**
  no-jmp: $abrupt\ s0 \neq Some\ (Jump\ j)$ **and**
  wt: $Env \vdash e{::}-T$ **and**
  wf: *wf-prog* (*prg Env*)
 shows $abrupt\ s1 \neq Some\ (Jump\ j)$
using *eval - no-jmp wt wf*
by (*rule jumpNestingOk-eval-no-jumpE*, *simp-all*)

corollary *eval-var-no-jump*:
 assumes *eval*: $prg\ Env \vdash s0\ -var{=}\succ(w,upd) \rightarrow\ s1$ **and**
  no-jmp: $abrupt\ s0 \neq Some\ (Jump\ j)$ **and**
  wt: $Env \vdash var{::}{=}T$ **and**
  wf: *wf-prog* (*prg Env*)
 shows $abrupt\ s1 \neq Some\ (Jump\ j) \wedge$
  $(normal\ s1 \longrightarrow$
   $(abrupt\ s \neq Some\ (Jump\ j')$
   $\longrightarrow abrupt\ (upd\ val\ s) \neq Some\ (Jump\ j')))$
apply (*rule-tac upd=upd* **and** *val=val* **and** *s=s* **and** *w=w* **and** $j'{=}j'$
  in *jumpNestingOk-eval-no-jumpE* [*OF eval - no-jmp wt wf*])
by *simp-all*

lemmas *eval-var-no-jumpE* = *eval-var-no-jump* [*THEN conjE*,*rule-format*]

corollary *eval-statement-no-jump*:
 assumes *eval*: $prg\ Env \vdash s0\ -c \rightarrow\ s1$ **and**
  jmpOk: *jumpNestingOkS* {} *c* **and**
  no-jmp: $abrupt\ s0 \neq Some\ (Jump\ j)$ **and**
  wt: $Env \vdash c{::}\surd$ **and**
  wf: *wf-prog* (*prg Env*)
 shows $abrupt\ s1 \neq Some\ (Jump\ j)$
using *eval - no-jmp wt wf*
by (*rule jumpNestingOk-eval-no-jumpE*) (*simp-all add*: *jmpOk*)

corollary *eval-expression-list-no-jump*:
 assumes *eval*: $prg\ Env \vdash s0\ -es\doteq\succ v \rightarrow\ s1$ **and**
  no-jmp: $abrupt\ s0 \neq Some\ (Jump\ j)$ **and**
  wt: $Env \vdash es{::}\doteq T$ **and**
  wf: *wf-prog* (*prg Env*)
 shows $abrupt\ s1 \neq Some\ (Jump\ j)$
using *eval - no-jmp wt wf*
by (*rule jumpNestingOk-eval-no-jumpE*, *simp-all*)

lemma *union-subseteq-elim* [*elim*]: $[\![A \cup B \subseteq C;\ [\![A \subseteq C;\ B \subseteq C]\!] \Longrightarrow P]\!] \Longrightarrow P$
 by *blast*

lemma *dom-locals-halloc-mono*:
 assumes *halloc*: $G \vdash s0 - halloc\ oi \succ a \rightarrow s1$
 shows $dom\ (locals\ (store\ s0)) \subseteq dom\ (locals\ (store\ s1))$
proof –
 from *halloc* **show** *?thesis*
  by *cases simp-all*

qed

lemma *dom-locals-sxalloc-mono*:
  assumes *sxalloc*: $G \vdash s0 -sxalloc\rightarrow s1$
  shows *dom* (*locals* (*store s0*)) $\subseteq$ *dom* (*locals* (*store s1*))
proof −
  from *sxalloc* **show** *?thesis*
  proof (*cases*)
    case *Norm* **thus** *?thesis* **by** *simp*
  next
    case *Jmp* **thus** *?thesis* **by** *simp*
  next
    case *Error* **thus** *?thesis* **by** *simp*
  next
    case *XcptL* **thus** *?thesis* **by** *simp*
  next
    case *SXcpt* **thus** *?thesis*
      by − (*drule dom-locals-halloc-mono*,*simp*)
  qed
qed

lemma *dom-locals-assign-mono*:
 assumes *f-ok*: *dom* (*locals* (*store s*)) $\subseteq$ *dom* (*locals* (*store* (*f n s*)))
   shows *dom* (*locals* (*store s*)) $\subseteq$ *dom* (*locals* (*store* (*assign f n s*)))
proof (*cases normal s*)
  case *False* **thus** *?thesis*
    by (*cases s*) (*auto simp add*: *assign-def Let-def*)
next
  case *True*
  then obtain s' **where** s': $s = (None, s')$
    by *auto*
  moreover
  obtain *x1 s1* **where** *f n s* = (*x1*,*s1*)
    by (*cases f n s*)
  ultimately
  show *?thesis*
    using *f-ok*
    by (*simp add*: *assign-def Let-def*)
qed

lemma *dom-locals-lvar-mono*:
 dom (*locals* (*store s*)) $\subseteq$ *dom* (*locals* (*store* (*snd* (*lvar vn s'*) *val s*)))
by (*simp add*: *lvar-def*) *blast*

lemma *dom-locals-fvar-vvar-mono*:
dom (*locals* (*store s*))
  $\subseteq$ *dom* (*locals* (*store* (*snd* (*fst* (*fvar statDeclC stat fn a s'*)) *val s*)))
proof (*cases stat*)
  case *True*
  thus *?thesis*
    by (*cases s*) (*simp add*: *fvar-def2*)

next
  case *False*
  thus *?thesis*
    by (*cases s*) (*simp add*: *fvar-def2*)
qed

lemma *dom-locals-fvar-mono*:
dom (*locals* (*store s*))
  ⊆ *dom* (*locals* (*store* (*snd* (*fvar statDeclC stat fn a s*))))
proof (*cases stat*)
  case *True*
  thus *?thesis*
    by (*cases s*) (*simp add*: *fvar-def2*)
next
  case *False*
  thus *?thesis*
    by (*cases s*) (*simp add*: *fvar-def2*)
qed

lemma *dom-locals-avar-vvar-mono*:
dom (*locals* (*store s*))
  ⊆ *dom* (*locals* (*store* (*snd* (*fst* (*avar G i a s'*)) *val s*)))
by (*cases s*, *simp add*: *avar-def2*)

lemma *dom-locals-avar-mono*:
dom (*locals* (*store s*))
  ⊆ *dom* (*locals* (*store* (*snd* (*avar G i a s*))))
by (*cases s*, *simp add*: *avar-def2*)

Since assignments are modelled as functions from states to states, we must take into account these functions. They appear only in the assignment rule and as result from evaluating a variable. Thats why we need the complicated second part of the conjunction in the goal. The reason for the very generic way to treat assignments was the aim to omit redundancy. There is only one evaluation rule for each kind of variable (locals, fields, arrays). These rules are used for both accessing variables and updating variables. Thats why the evaluation rules for variables result in a pair consisting of a value and an update function. Of course we could also think of a pair of a value and a reference in the store, instead of the generic update function. But as only array updates can cause a special exception (if the types mismatch) and not array reads we then have to introduce two different rules to handle array reads and updates

lemma *dom-locals-eval-mono*:
  assumes *eval*: *G*⊢ *s0* −*t*≻→ (*v*,*s1*)
  shows *dom* (*locals* (*store s0*)) ⊆ *dom* (*locals* (*store s1*)) ∧
        (∀ *vv*. *v*=*In2 vv* ∧ *normal s1*
          ⟶ (∀ *s val*. *dom* (*locals* (*store s*))
                    ⊆ *dom* (*locals* (*store* ((*snd vv*) *val s*)))))
proof −
  from *eval* **show** *?thesis*
  proof (*induct*)
    case *Abrupt* **thus** *?case* **by** *simp*
  next
    case *Skip* **thus** *?case* **by** *simp*
  next
    case *Expr* **thus** *?case* **by** *simp*
  next

```
    case Lab thus ?case by simp
  next
    case (Comp s0 c1 s1 c2 s2)
    from Comp.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from Comp.hyps
    have ... ⊆ dom (locals (store s2))
      by simp
    finally show ?case by simp
  next
    case (If s0 e b s1 c1 c2 s2)
    from If.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from If.hyps
    have ... ⊆ dom (locals (store s2))
      by simp
    finally show ?case by simp
  next
    case (Loop s0 e b s1 c s2 l s3)
    show ?case
    proof (cases the-Bool b)
      case True
      with Loop.hyps
      obtain
        s0-s1:
        dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1)) and
        s1-s2: dom (locals (store s1)) ⊆ dom (locals (store s2)) and
        s2-s3: dom (locals (store s2)) ⊆ dom (locals (store s3))
        by simp
      note s0-s1 also note s1-s2 also note s2-s3
      finally show ?thesis
        by simp
    next
      case False
      with Loop.hyps show ?thesis
        by simp
    qed
  next
    case Jmp thus ?case by simp
  next
    case Throw thus ?case by simp
  next
    case (Try s0 c1 s1 s2 C vn c2 s3)
    then
    have s0-s1: dom (locals (store ((Norm s0)::state)))
                  ⊆ dom (locals (store s1)) by simp
    from ‹G⊢s1 −sxalloc→ s2›
    have s1-s2: dom (locals (store s1)) ⊆ dom (locals (store s2))
      by (rule dom-locals-sxalloc-mono)
    thus ?case
    proof (cases G,s2⊢catch C)
      case True
      note s0-s1 also note s1-s2
      also
      from True Try.hyps
```

```
      have dom (locals (store (new-xcpt-var vn s2)))
              ⊆ dom (locals (store s3))
        by simp
      hence dom (locals (store s2)) ⊆ dom (locals (store s3))
        by (cases s2, simp )
      finally show ?thesis by simp
    next
      case False
      note s0-s1 also note s1-s2
      finally
      show ?thesis
        using False Try.hyps by simp
    qed
  next
    case (Fin s0 c1 x1 s1 c2 s2 s3)
    show ?case
    proof (cases ∃ err. x1 = Some (Error err))
      case True
      with Fin.hyps show ?thesis
        by simp
    next
      case False
      from Fin.hyps
      have dom (locals (store ((Norm s0)::state)))
             ⊆ dom (locals (store (x1, s1)))
        by simp
      hence dom (locals (store ((Norm s0)::state)))
              ⊆ dom (locals (store ((Norm s1)::state)))
        by simp
      also
      from Fin.hyps
      have ... ⊆ dom (locals (store s2))
        by simp
      finally show ?thesis
        using Fin.hyps by simp
    qed
  next
    case (Init C c s0 s3 s1 s2)
    show ?case
    proof (cases inited C (globs s0))
      case True
      with Init.hyps show ?thesis by simp
    next
      case False
      with Init.hyps
      obtain s0-s1: dom (locals (store (Norm ((init-class-obj G C) s0))))
                      ⊆ dom (locals (store s1)) and
             s3: s3 = (set-lvars (locals (snd s1))) s2
        by simp
      from s0-s1
      have dom (locals (store (Norm s0))) ⊆ dom (locals (store s1))
        by (cases s0) simp
      with s3
      have dom (locals (store (Norm s0))) ⊆ dom (locals (store s3))
        by (cases s2) simp
      thus ?thesis by simp
    qed
  next
    case (NewC s0 C s1 a s2)
```

```
    note halloc = ‹G⊢s1 −halloc CInst C≻a→ s2›
    from NewC.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from halloc
    have ... ⊆ dom (locals (store s2)) by (rule dom-locals-halloc-mono)
    finally show ?case by simp
  next
    case (NewA s0 T s1 e i s2 a s3)
    note halloc = ‹G⊢abupd (check-neg i) s2 −halloc Arr T (the-Intg i)≻a→ s3›
    from NewA.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from NewA.hyps
    have ... ⊆ dom (locals (store s2)) by simp
    also
    from halloc
    have ... ⊆  dom (locals (store s3))
      by (rule dom-locals-halloc-mono [elim-format]) simp
    finally show ?case by simp
  next
    case Cast thus ?case by simp
  next
    case Inst thus ?case by simp
  next
    case Lit thus ?case by simp
  next
    case UnOp thus ?case by simp
  next
    case (BinOp s0 e1 v1 s1 binop e2 v2 s2)
    from BinOp.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from BinOp.hyps
    have ... ⊆ dom (locals (store s2)) by simp
    finally show ?case by simp
  next
    case Super thus ?case by simp
  next
    case Acc thus ?case by simp
  next
    case (Ass s0 va w f s1 e v s2)
    from Ass.hyps
    have s0-s1:
      dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    show ?case
    proof (cases normal s1)
      case True
      with Ass.hyps
      have ass-ok:
        ⋀ s val. dom (locals (store s)) ⊆ dom (locals (store (f val s)))
        by simp
      note s0-s1
      also
      from Ass.hyps
```

have *dom* (*locals* (*store s1*)) ⊆ *dom* (*locals* (*store s2*))
  by *simp*
also
from *ass-ok*
have ... ⊆ *dom* (*locals* (*store* (*assign f v s2*)))
  by (*rule dom-locals-assign-mono* [**where** *f* = *f*])
finally show *?thesis* **by** *simp*
next
case *False*
with ‹*G*⊢*s1* −*e*−≻*v*→ *s2*›
have *s2=s1*
  by *auto*
with *s0-s1 False*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*)))
    ⊆ *dom* (*locals* (*store* (*assign f v s2*)))
  by *simp*
thus *?thesis*
  by *simp*
qed
next
case (*Cond s0 e0 b s1 e1 e2 v s2*)
from *Cond.hyps*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
  by *simp*
also
from *Cond.hyps*
have ... ⊆ *dom* (*locals* (*store s2*))
  by *simp*
finally show *?case* **by** *simp*
next
case (*Call s0 e a′ s1 args vs s2 D mode statT mn pTs s3 s3′ accC v s4*)
note *s3* = ‹*s3* = *init-lvars G D* (|*name* = *mn*, *parTs* = *pTs*|) *mode a′ vs s2*›
from *Call.hyps*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
  by *simp*
also
from *Call.hyps*
have ... ⊆ *dom* (*locals* (*store s2*))
  by *simp*
also
have ... ⊆ *dom* (*locals* (*store* ((*set-lvars* (*locals* (*store s2*))) *s4*)))
  by (*cases s4*) *simp*
finally show *?case* **by** *simp*
next
case *Methd* **thus** *?case* **by** *simp*
next
case (*Body s0 D s1 c s2 s3*)
from *Body.hyps*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
  by *simp*
also
from *Body.hyps*
have ... ⊆ *dom* (*locals* (*store s2*))
  by *simp*
also
have ... ⊆ *dom* (*locals* (*store* (*abupd* (*absorb Ret*) *s2*)))
  by *simp*
also
have ... ⊆ *dom* (*locals* (*store* (*abupd* (*absorb Ret*) *s3*)))

```
  proof –
    from ‹s3 =
       (if ∃ l. abrupt s2 = Some (Jump (Break l)) ∨
              abrupt s2 = Some (Jump (Cont l))
          then abupd (λx. Some (Error CrossMethodJump)) s2 else s2)›
    show ?thesis
      by simp
  qed
  finally show ?case by simp
next
  case LVar
  thus ?case
    using dom-locals-lvar-mono
    by simp
next
  case (FVar s0 statDeclC s1 e a s2 v s2' stat fn s3 accC)
  from FVar.hyps
  obtain s2': s2' = snd (fvar statDeclC stat fn a s2) and
          v: v = fst (fvar statDeclC stat fn a s2)
    by (cases fvar statDeclC stat fn a s2 ) simp
  from v
  have ∀ s val. dom (locals (store s))
                    ⊆ dom (locals (store (snd v val s))) (is ?V-ok)
    by (simp add: dom-locals-fvar-vvar-mono)
  hence v-ok: (∀ vv. In2 v = In2 vv ∧ normal s3 ⟶ ?V-ok)
    by – (intro strip, simp)
  note s3 = ‹s3 = check-field-access G accC statDeclC fn stat a s2'›
  from FVar.hyps
  have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
    by simp
  also
  from FVar.hyps
  have ... ⊆ dom (locals (store s2))
    by simp
  also
  from s2'
  have ... ⊆ dom (locals (store s2'))
    by (simp add: dom-locals-fvar-mono)
  also
  from s3
  have ... ⊆ dom (locals (store s3))
    by (simp add: check-field-access-def Let-def)
  finally
  show ?case
    using v-ok
    by simp
next
  case (AVar s0 e1 a s1 e2 i s2 v s2')
  from AVar.hyps
  obtain s2': s2' = snd (avar G i a s2) and
          v: v   = fst (avar G i a s2)
    by (cases avar G i a s2) simp
  from v
  have ∀ s val. dom (locals (store s))
                    ⊆ dom (locals (store (snd v val s))) (is ?V-ok)
    by (simp add: dom-locals-avar-vvar-mono)
  hence v-ok: (∀ vv. In2 v = In2 vv ∧ normal s2' ⟶ ?V-ok)
    by – (intro strip, simp)
  from AVar.hyps
```

have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
  by *simp*
also
from *AVar.hyps*
have ... ⊆ *dom* (*locals* (*store s2*))
  by *simp*
also
from *s2'*
have ... ⊆ *dom* (*locals* (*store s2'*))
  by (*simp add*: *dom-locals-avar-mono*)
finally
show *?case* **using** *v-ok* **by** *simp*
next
  case *Nil* **thus** *?case* **by** *simp*
next
  case (*Cons s0 e v s1 es vs s2*)
  from *Cons.hyps*
  have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
    by *simp*
  also
  from *Cons.hyps*
  have ... ⊆ *dom* (*locals* (*store s2*))
    by *simp*
  finally show *?case* **by** *simp*
qed
qed

lemma *dom-locals-eval-mono-elim*:
  assumes *eval*: $G\vdash s0 \ -t\succ\rightarrow (v,s1)$
  obtains *dom* (*locals* (*store s0*)) ⊆ *dom* (*locals* (*store s1*)) **and**
  $\bigwedge$ *vv s val*. ⟦*v=In2 vv*; *normal s1*⟧
    ⟹ *dom* (*locals* (*store s*))
      ⊆ *dom* (*locals* (*store* ((*snd vv*) *val s*)))
  using *eval* **by** (*rule dom-locals-eval-mono* [*THEN conjE*]) (*rule that*, *auto*)

lemma *halloc-no-abrupt*:
  assumes *halloc*: $G\vdash s0-halloc\ oi\succ a\rightarrow s1$ **and**
    normal: *normal s1*
  shows *normal s0*
proof −
  from *halloc normal* **show** *?thesis*
    by *cases simp-all*
qed

lemma *sxalloc-mono-no-abrupt*:
  assumes *sxalloc*: $G\vdash s0-sxalloc\rightarrow s1$ **and**
    normal: *normal s1*
  shows *normal s0*
proof −
  from *sxalloc normal* **show** *?thesis*
    by *cases simp-all*
qed

lemma *union-subseteqI*: $[\![A \cup B \subseteq C;\ A' \subseteq A;\ B' \subseteq B]\!] \implies A' \cup B' \subseteq C$
  by *blast*

lemma *union-subseteqIl*: $[\![A \cup B \subseteq C;\ A' \subseteq A]\!] \implies A' \cup B \subseteq C$
  by *blast*

lemma *union-subseteqIr*: $[\![A \cup B \subseteq C;\ B' \subseteq B]\!] \implies A \cup B' \subseteq C$
  by *blast*

lemma *subseteq-union-transl* [*trans*]: $[\![A \subseteq B;\ B \cup C \subseteq D]\!] \Longrightarrow A \cup C \subseteq D$
  by *blast*

lemma *subseteq-union-transr* [*trans*]: $[\![A \subseteq B;\ C \cup B \subseteq D]\!] \Longrightarrow A \cup C \subseteq D$
  by *blast*

lemma *union-subseteq-weaken*: $[\![A \cup B \subseteq C;\ [\![A \subseteq C;\ B \subseteq C]\!] \Longrightarrow P\ ]\!] \Longrightarrow P$
  by *blast*

lemma *assigns-good-approx*:
  assumes
      eval: $G\vdash\ s0\ -t\succ\rightarrow\ (v,s1)$ **and**
    normal: *normal s1*
  shows *assigns t* $\subseteq$ *dom* (*locals* (*store s1*))
proof −
  from *eval normal* **show** *?thesis*
  proof (*induct*)
    case *Abrupt* **thus** *?case* **by** *simp*
  next — For statements its trivial, since then *assigns t* = {}
    case *Skip* **show** *?case* **by** *simp*
  next
    case *Expr* **show** *?case* **by** *simp*
  next
    case *Lab* **show** *?case* **by** *simp*
  next
    case *Comp* **show** *?case* **by** *simp*
  next
    case *If* **show** *?case* **by** *simp*
  next
    case *Loop* **show** *?case* **by** *simp*
  next
    case *Jmp* **show** *?case* **by** *simp*
  next
    case *Throw* **show** *?case* **by** *simp*
  next
    case *Try* **show** *?case* **by** *simp*
  next
    case *Fin* **show** *?case* **by** *simp*
  next
    case *Init* **show** *?case* **by** *simp*
  next
    case *NewC* **show** *?case* **by** *simp*
  next
    case (*NewA s0 T s1 e i s2 a s3*)
    note *halloc* = ‹$G\vdash$*abupd* (*check-neg i*) *s2* −*halloc Arr T* (*the-Intg i*)$\succ a\rightarrow$ *s3*›
    have *assigns* (*In1l e*) $\subseteq$ *dom* (*locals* (*store s2*))

```
    proof −
      from NewA
      have normal (abupd (check-neg i) s2)
        by − (erule halloc-no-abrupt [rule-format])
      hence normal s2 by (cases s2) simp
      with NewA.hyps
      show ?thesis by iprover
    qed
    also
    from halloc
    have ... ⊆  dom (locals (store s3))
      by (rule dom-locals-halloc-mono [elim-format]) simp
    finally show ?case by simp
  next
    case (Cast s0 e v s1 s2 T)
    hence normal s1 by (cases s1,simp)
    with Cast.hyps
    have assigns (In1l e) ⊆ dom (locals (store s1))
      by simp
    also
    from Cast.hyps
    have ... ⊆ dom (locals (store s2))
      by simp
    finally
    show ?case
      by simp
  next
    case Inst thus ?case by simp
  next
    case Lit thus ?case by simp
  next
    case UnOp thus ?case by simp
  next
    case (BinOp s0 e1 v1 s1 binop e2 v2 s2)
    hence normal s1 by − (erule eval-no-abrupt-lemma [rule-format])
    with BinOp.hyps
    have assigns (In1l e1) ⊆ dom (locals (store s1))
      by iprover
    also
    have ...  ⊆ dom (locals (store s2))
    proof −
      note ‹G⊢s1 −(if need-second-arg binop v1 then In1l e2
                    else In1r Skip)≻→ (In1 v2, s2)›
      thus ?thesis
        by (rule dom-locals-eval-mono-elim)
    qed
    finally have s2: assigns (In1l e1) ⊆ dom (locals (store s2)) .
    show ?case
    proof (cases binop=CondAnd ∨ binop=CondOr)
      case True
      with s2 show ?thesis by simp
    next
      case False
      with BinOp
      have assigns (In1l e2) ⊆ dom (locals (store s2))
        by (simp add: need-second-arg-def)
      with s2
      show ?thesis using False by simp
    qed
```

```
next
  case Super thus ?case by simp
next
  case Acc thus ?case by simp
next
  case (Ass s0 va w f s1 e v s2)
  note nrm-ass-s2 = ‹normal (assign f v s2)›
  hence nrm-s2: normal s2
    by (cases s2, simp add: assign-def Let-def)
  with Ass.hyps
  have nrm-s1: normal s1
    by − (erule eval-no-abrupt-lemma [rule-format])
  with Ass.hyps
  have assigns (In2 va) ⊆ dom (locals (store s1))
    by iprover
  also
  from Ass.hyps
  have ... ⊆ dom (locals (store s2))
    by − (erule dom-locals-eval-mono-elim)
  also
  from nrm-s2 Ass.hyps
  have assigns (In1l e) ⊆ dom (locals (store s2))
    by iprover
  ultimately
  have assigns (In2 va) ∪ assigns (In1l e) ⊆  dom (locals (store s2))
    by (rule Un-least)
  also
  from Ass.hyps nrm-s1
  have ...  ⊆ dom (locals (store (f v s2)))
    by − (erule dom-locals-eval-mono-elim, cases s2,simp)
  then
  have dom (locals (store s2))  ⊆ dom (locals (store (assign f v s2)))
    by (rule dom-locals-assign-mono)
  finally
  have va-e:  assigns (In2 va) ∪ assigns (In1l e)
              ⊆ dom (locals (snd (assign f v s2))) .
  show ?case
  proof (cases ∃ n. va = LVar n)
    case False
    with va-e show ?thesis
      by (simp add: Un-assoc)
  next
    case True
    then obtain n where va: va = LVar n
      by blast
    with Ass.hyps
    have G⊢Norm s0 −LVar n=≻(w,f)→ s1
      by simp
    hence (w,f) = lvar n s0
      by (rule eval-elim-cases) simp
    with nrm-ass-s2
    have n ∈ dom (locals (store (assign f v s2)))
      by (cases s2) (simp add: assign-def Let-def lvar-def)
    with va-e True va
    show ?thesis by (simp add: Un-assoc)
  qed
next
  case (Cond s0 e0 b s1 e1 e2 v s2)
  hence normal s1
```

```
    by − (erule eval-no-abrupt-lemma [rule-format])
  with Cond.hyps
  have assigns (In1l e0) ⊆ dom (locals (store s1))
    by iprover
  also from Cond.hyps
  have . . . ⊆ dom (locals (store s2))
    by − (erule dom-locals-eval-mono-elim)
  finally have e0: assigns (In1l e0) ⊆ dom (locals (store s2)) .
  show ?case
  proof (cases the-Bool b)
    case True
    with Cond
    have assigns (In1l e1) ⊆ dom (locals (store s2))
      by simp
    hence assigns (In1l e1) ∩ assigns (In1l e2) ⊆ . . .
      by blast
    with e0
    have assigns (In1l e0) ∪ assigns (In1l e1) ∩ assigns (In1l e2)
          ⊆ dom (locals (store s2))
      by (rule Un-least)
    thus ?thesis using True by simp
  next
    case False
    with Cond
    have  assigns (In1l e2) ⊆ dom (locals (store s2))
      by simp
    hence assigns (In1l e1) ∩ assigns (In1l e2) ⊆ . . .
      by blast
    with e0
    have assigns (In1l e0) ∪ assigns (In1l e1) ∩ assigns (In1l e2)
          ⊆ dom (locals (store s2))
      by (rule Un-least)
    thus ?thesis using False by simp
  qed
next
  case (Call s0 e a′ s1 args vs s2 D mode statT mn pTs s3 s3′ accC v s4)
  have nrm-s2: normal s2
  proof −
    from ‹normal ((set-lvars (locals (snd s2))) s4)›
    have normal-s4: normal s4 by simp
    hence normal s3′ using Call.hyps
      by − (erule eval-no-abrupt-lemma [rule-format])
    moreover note
     ‹s3′ = check-method-access G accC statT mode (|name=mn, parTs=pTs|) a′ s3›
    ultimately have normal s3
      by (cases s3) (simp add: check-method-access-def Let-def)
    moreover
    note s3 = ‹s3 = init-lvars G D (|name = mn, parTs = pTs|) mode a′ vs s2›
    ultimately show normal s2
      by (cases s2) (simp add: init-lvars-def2)
  qed
  hence normal s1 using Call.hyps
    by − (erule eval-no-abrupt-lemma [rule-format])
  with Call.hyps
  have assigns (In1l e) ⊆ dom (locals (store s1))
    by iprover
  also from Call.hyps
  have . . . ⊆  dom (locals (store s2))
    by − (erule dom-locals-eval-mono-elim)
```

```
  also
  from nrm-s2 Call.hyps
  have assigns (In3 args) ⊆ dom (locals (store s2))
    by iprover
  ultimately have assigns (In1l e) ∪ assigns (In3 args) ⊆ ...
    by (rule Un-least)
  also
  have ... ⊆ dom (locals (store ((set-lvars (locals (store s2))) s4)))
    by (cases s4) simp
  finally show ?case
    by simp
next
  case Methd thus ?case by simp
next
  case Body thus ?case by simp
next
  case LVar thus ?case by simp
next
  case (FVar s0 statDeclC s1 e a s2 v s2' stat fn s3 accC)
  note s3 = ‹s3 = check-field-access G accC statDeclC fn stat a s2'›
  note avar = ‹(v, s2') = fvar statDeclC stat fn a s2›
  have nrm-s2: normal s2
  proof -
    note ‹normal s3›
    with s3 have normal s2'
      by (cases s2') (simp add: check-field-access-def Let-def)
    with avar show normal s2
      by (cases s2) (simp add: fvar-def2)
  qed
  with FVar.hyps
  have assigns (In1l e) ⊆ dom (locals (store s2))
    by iprover
  also
  have ... ⊆  dom (locals (store s2'))
  proof -
    from avar
    have s2' = snd (fvar statDeclC stat fn a s2)
      by (cases fvar statDeclC stat fn a s2) simp
    thus ?thesis
      by simp (rule dom-locals-fvar-mono)
  qed
  also from s3
  have ... ⊆  dom (locals (store s3))
    by (cases s2') (simp add: check-field-access-def Let-def)
  finally show ?case
    by simp
next
  case (AVar s0 e1 a s1 e2 i s2 v s2')
  note avar = ‹(v, s2') = avar G i a s2›
  have nrm-s2: normal s2
  proof -
    from avar and ‹normal s2'›
    show ?thesis by (cases s2) (simp add: avar-def2)
  qed
  with AVar.hyps
  have normal s1
    by - (erule eval-no-abrupt-lemma [rule-format])
  with AVar.hyps
  have assigns (In1l e1) ⊆ dom (locals (store s1))
```

```
      by iprover
    also from AVar.hyps
    have ... ⊆ dom (locals (store s2))
      by − (erule dom-locals-eval-mono-elim)
    also
    from AVar.hyps nrm-s2
    have assigns (In1l e2) ⊆ dom (locals (store s2))
      by iprover
    ultimately
    have assigns (In1l e1) ∪ assigns (In1l e2) ⊆ ...
      by (rule Un-least)
    also
    have dom (locals (store s2)) ⊆ dom (locals (store s2'))
    proof −
      from avar have s2' = snd (avar G i a s2)
        by (cases avar G i a s2) simp
      thus ?thesis
        by simp (rule dom-locals-avar-mono)
    qed
    finally
    show ?case
      by simp
  next
    case Nil show ?case by simp
  next
    case (Cons s0 e v s1 es vs s2)
    have assigns (In1l e) ⊆ dom (locals (store s1))
    proof −
      from Cons
      have normal s1 by − (erule eval-no-abrupt-lemma [rule-format])
      with Cons.hyps show ?thesis by iprover
    qed
    also from Cons.hyps
    have ... ⊆ dom (locals (store s2))
      by − (erule dom-locals-eval-mono-elim)
    also from Cons
    have assigns (In3 es) ⊆ dom (locals (store s2))
      by iprover
    ultimately
    have assigns (In1l e) ∪ assigns (In3 es) ⊆ dom (locals (store s2))
      by (rule Un-least)
    thus ?case
      by simp
  qed
qed

corollary assignsE-good-approx:
  assumes
      eval: prg Env⊢ s0 −e−≻v→ s1 and
    normal: normal s1
  shows assignsE e ⊆ dom (locals (store s1))
proof −
from eval normal show ?thesis
  by (rule assigns-good-approx [elim-format]) simp
qed

corollary assignsV-good-approx:
  assumes
     eval: prg Env⊢ s0 −v=≻vf→ s1 and
```

```
  normal: normal s1
 shows assignsV v ⊆ dom (locals (store s1))
proof −
from eval normal show ?thesis
 by (rule assigns-good-approx [elim-format]) simp
qed

corollary assignsEs-good-approx:
 assumes
     eval: prg Env⊢ s0 −es≐≻vs→ s1 and
   normal: normal s1
 shows assignsEs es ⊆ dom (locals (store s1))
proof −
from eval normal show ?thesis
 by (rule assigns-good-approx [elim-format]) simp
qed


lemma constVal-eval:
 assumes const: constVal e = Some c and
          eval: G⊢Norm s0 −e−≻v→ s
 shows v = c ∧ normal s
proof −
 have  True and
        ⋀ c v s0 s. ⟦ constVal e = Some c; G⊢Norm s0 −e−≻v→ s⟧
                      ⟹ v = c ∧ normal s
       and True
 proof (induct rule: var.induct expr.induct stmt.induct)
   case NewC hence False by simp thus ?case ..
 next
   case NewA hence False by simp thus ?case ..
 next
   case Cast hence False by simp thus ?case ..
 next
   case Inst hence False by simp thus ?case ..
 next
   case (Lit val c v s0 s)
   note ‹constVal (Lit val) = Some c›
   moreover
   from ‹G⊢Norm s0 −Lit val−≻v→ s›
   obtain v=val and normal s
     by cases simp
   ultimately show v=c ∧ normal s by simp
 next
   case (UnOp unop e c v s0 s)
   note const = ‹constVal (UnOp unop e) = Some c›
   then obtain ce where ce: constVal e = Some ce by simp
   from ‹G⊢Norm s0 −UnOp unop e−≻v→ s›
   obtain ve where ve: G⊢Norm s0 −e−≻ve→ s and
                    v: v = eval-unop unop ve
     by cases simp
   from ce ve
   obtain eq-ve-ce: ve=ce and nrm-s: normal s
     by (rule UnOp.hyps [elim-format]) iprover
   from eq-ve-ce const ce v
   have v=c
     by simp
   from this nrm-s
   show ?case ..
```

```
next
  case (BinOp binop e1 e2 c v s0 s)
  note const = ‹constVal (BinOp binop e1 e2) = Some c›
  then obtain c1 c2 where c1: constVal e1 = Some c1 and
                          c2: constVal e2 = Some c2 and
                           c: c = eval-binop binop c1 c2
    by simp
  from ‹G⊢Norm s0 −BinOp binop e1 e2−≻v→ s›
  obtain v1 s1 v2
    where v1: G⊢Norm s0 −e1−≻v1→ s1 and
          v2: G⊢s1 −(if need-second-arg binop v1 then In1l e2
                          else In1r Skip)≻→ (In1 v2, s) and
           v: v = eval-binop binop v1 v2
    by cases simp
  from c1 v1
  obtain eq-v1-c1: v1 = c1 and
           nrm-s1: normal s1
    by (rule BinOp.hyps [elim-format]) iprover
  show ?case
  proof (cases need-second-arg binop v1)
    case True
    with v2 nrm-s1  obtain s1'
      where G⊢Norm s1' −e2−≻v2→ s
      by (cases s1) simp
    with c2 obtain v2 = c2 normal s
      by (rule BinOp.hyps [elim-format]) iprover
    with c c1 c2 eq-v1-c1 v
    show ?thesis by simp
  next
    case False
    with nrm-s1 v2
    have s=s1
      by (cases s1) (auto elim!: eval-elim-cases)
    moreover
    from False c v eq-v1-c1
    have v = c
      by (simp add: eval-binop-arg2-indep)
    ultimately
    show ?thesis
      using nrm-s1 by simp
  qed
next
  case Super hence False by simp thus ?case ..
next
  case Acc hence False by simp thus ?case ..
next
  case Ass hence False by simp thus ?case ..
next
  case (Cond b e1 e2 c v s0 s)
  note c = ‹constVal (b ? e1 : e2) = Some c›
  then obtain cb c1 c2 where
    cb: constVal b  = Some cb and
    c1: constVal e1 = Some c1 and
    c2: constVal e2 = Some c2
    by (auto split: bool.splits)
  from ‹G⊢Norm s0 −b ? e1 : e2−≻v→ s›
  obtain vb s1
    where     vb: G⊢Norm s0 −b−≻vb→ s1 and
          eval-v: G⊢s1 −(if the-Bool vb then e1 else e2)−≻v→ s
```

```
      by cases simp
    from cb vb
    obtain eq-vb-cb: vb = cb and nrm-s1: normal s1
      by (rule Cond.hyps [elim-format]) iprover
    show ?case
    proof (cases the-Bool vb)
      case True
      with c cb c1 eq-vb-cb
      have c = c1
        by simp
      moreover
      from True eval-v nrm-s1 obtain s1'
        where G⊢Norm s1' −e1−≻v→ s
        by (cases s1) simp
      with c1 obtain c1 = v normal s
        by (rule Cond.hyps [elim-format]) iprover
      ultimately show ?thesis by simp
    next
      case False
      with c cb c2 eq-vb-cb
      have c = c2
        by simp
      moreover
      from False eval-v nrm-s1 obtain s1'
        where G⊢Norm s1' −e2−≻v→ s
        by (cases s1) simp
      with c2 obtain c2 = v normal s
        by (rule Cond.hyps [elim-format]) iprover
      ultimately show ?thesis by simp
    qed
  next
    case Call hence False by simp thus ?case ..
  qed simp-all
  with const eval
  show ?thesis
    by iprover
qed

lemmas constVal-eval-elim = constVal-eval [THEN conjE]


lemma eval-unop-type:
  typeof dt (eval-unop unop v) = Some (PrimT (unop-type unop))
  by (cases unop) simp-all


lemma eval-binop-type:
  typeof dt (eval-binop binop v1 v2) = Some (PrimT (binop-type binop))
  by (cases binop) simp-all


lemma constVal-Boolean:
 assumes const: constVal e = Some c and
         wt: Env⊢e::−PrimT Boolean
  shows typeof empty-dt c = Some (PrimT Boolean)
proof −
  have True and
      ⋀ c. ⟦constVal e = Some c;Env⊢e::−PrimT Boolean⟧
            ⟹ typeof empty-dt c = Some (PrimT Boolean)
```

```
    and True
proof (induct rule: var.induct expr.induct stmt.induct)
  case NewC hence False by simp thus ?case ..
next
  case NewA hence False by simp thus ?case ..
next
  case Cast hence False by simp thus ?case ..
next
  case Inst hence False by simp thus ?case ..
next
  case (Lit v c)
  from ‹constVal (Lit v) = Some c›
  have c=v by simp
  moreover
  from ‹Env⊢Lit v::−PrimT Boolean›
  have typeof empty-dt v = Some (PrimT Boolean)
    by cases simp
  ultimately show ?case by simp
next
  case (UnOp unop e c)
  from ‹Env⊢UnOp unop e::−PrimT Boolean›
  have Boolean = unop-type unop by cases simp
  moreover
  from ‹constVal (UnOp unop e) = Some c›
  obtain ce where c = eval-unop unop ce by auto
  ultimately show ?case by (simp add: eval-unop-type)
next
  case (BinOp binop e1 e2 c)
  from ‹Env⊢BinOp binop e1 e2::−PrimT Boolean›
  have Boolean = binop-type binop by cases simp
  moreover
  from ‹constVal (BinOp binop e1 e2) = Some c›
  obtain c1 c2 where c = eval-binop binop c1 c2 by auto
  ultimately show ?case by (simp add: eval-binop-type)
next
  case Super hence False by simp thus ?case ..
next
  case Acc hence False by simp thus ?case ..
next
  case Ass hence False by simp thus ?case ..
next
  case (Cond b e1 e2 c)
  note c = ‹constVal (b ? e1 : e2) = Some c›
  then obtain cb c1 c2 where
    cb: constVal b  = Some cb and
    c1: constVal e1 = Some c1 and
    c2: constVal e2 = Some c2
    by (auto split: bool.splits)
  note wt = ‹Env⊢b ? e1 : e2::−PrimT Boolean›
  then
  obtain T1 T2
    where Env⊢b::−PrimT Boolean and
          wt-e1: Env⊢e1::−PrimT Boolean and
          wt-e2: Env⊢e2::−PrimT Boolean
    by cases (auto dest: widen-Boolean2)
  show ?case
  proof (cases the-Bool cb)
    case True
    from c1 wt-e1
```

```
      have typeof empty-dt c1 = Some (PrimT Boolean)
        by (rule Cond.hyps)
      with True c cb c1 show ?thesis by simp
    next
      case False
      from c2 wt-e2
      have typeof empty-dt c2 = Some (PrimT Boolean)
        by (rule Cond.hyps)
      with False c cb c2 show ?thesis by simp
    qed
  next
    case Call hence False by simp thus ?case ..
  qed simp-all
  with const wt
  show ?thesis
    by iprover
qed


lemma assigns-if-good-approx:
  assumes
     eval: prg Env⊢ s0 −e−≻b→ s1   and
   normal: normal s1 and
     bool: Env⊢ e::−PrimT Boolean
  shows assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
proof −
```

— To properly perform induction on the evaluation relation we have to generalize the lemma to terms not only expressions.

```
  { fix t val
   assume eval′: prg Env⊢ s0 −t≻→ (val,s1)
   assume bool′: Env⊢ t::Inl (PrimT Boolean)
   assume expr: ∃ expr. t=In1l expr
   have assigns-if (the-Bool (the-In1 val)) (the-In1l t)
              ⊆ dom (locals (store s1))
   using eval′ normal bool′ expr
   proof (induct)
     case Abrupt thus ?case by simp
   next
     case (NewC s0 C s1 a s2)
     from ‹Env⊢NewC C::−PrimT Boolean›
     have False
       by cases simp
     thus ?case ..
   next
     case (NewA s0 T s1 e i s2 a s3)
     from ‹Env⊢New T[e]::−PrimT Boolean›
     have False
       by cases simp
     thus ?case ..
   next
     case (Cast s0 e b s1 s2 T)
     note s2 = ‹s2 = abupd (raise-if (¬ prg Env,snd s1⊢b fits T) ClassCast) s1›
     have assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
     proof −
       from s2 and ‹normal s2›
       have normal s1
         by (cases s1) simp
       moreover
       from ‹Env⊢Cast T e::−PrimT Boolean›
```

```
      have Env⊢e::− PrimT Boolean
        by cases (auto dest: cast-Boolean2)
      ultimately show ?thesis
        by (rule Cast.hyps [elim-format]) auto
    qed
    also from s2
    have ... ⊆ dom (locals (store s2))
      by simp
    finally show ?case by simp
  next
    case (Inst s0 e v s1 b T)
    from ‹prg Env⊢Norm s0 −e−≻v→ s1› and ‹normal s1›
    have assignsE e ⊆ dom (locals (store s1))
      by (rule assignsE-good-approx)
    thus ?case
      by simp
  next
    case (Lit s v)
    from ‹Env⊢Lit v::−PrimT Boolean›
    have typeof empty-dt v = Some (PrimT Boolean)
      by cases simp
    then obtain b where v=Bool b
      by (cases v) (simp-all add: empty-dt-def)
    thus ?case
      by simp
  next
    case (UnOp s0 e v s1 unop)
    note bool = ‹Env⊢UnOp unop e::−PrimT Boolean›
    hence bool-e: Env⊢e::−PrimT Boolean
      by cases (cases unop,simp-all)
    show ?case
    proof (cases constVal (UnOp unop e))
      case None
      note ‹normal s1›
      moreover note bool-e
      ultimately have assigns-if (the-Bool v) e ⊆ dom (locals (store s1))
        by (rule UnOp.hyps [elim-format]) auto
      moreover
      from bool have unop = UNot
        by cases (cases unop, simp-all)
      moreover note None
      ultimately
      have assigns-if (the-Bool (eval-unop unop v)) (UnOp unop e)
              ⊆ dom (locals (store s1))
        by simp
      thus ?thesis by simp
    next
      case (Some c)
      moreover
      from ‹prg Env⊢Norm s0 −e−≻v→ s1›
      have prg Env⊢Norm s0 −UnOp unop e−≻eval-unop unop v→ s1
        by (rule eval.UnOp)
      with Some
      have eval-unop unop v=c
        by (rule constVal-eval-elim) simp
      moreover
      from Some bool
      obtain b where c=Bool b
        by (rule constVal-Boolean [elim-format])
```

```
        (cases c, simp-all add: empty-dt-def)
    ultimately
    have assigns-if (the-Bool (eval-unop unop v)) (UnOp unop e) = {}
      by simp
    thus ?thesis by simp
  qed
next
  case (BinOp s0 e1 v1 s1 binop e2 v2 s2)
  note bool = ‹Env⊢BinOp binop e1 e2::−PrimT Boolean›
  show ?case
  proof (cases constVal (BinOp binop e1 e2))
    case (Some c)
    moreover
    from BinOp.hyps
    have
      prg Env⊢Norm s0 −BinOp binop e1 e2−≻eval-binop binop v1 v2→ s2
      by − (rule eval.BinOp)
    with Some
    have eval-binop binop v1 v2=c
      by (rule constVal-eval-elim) simp
    moreover
    from Some bool
    obtain b where c = Bool b
      by (rule constVal-Boolean [elim-format])
        (cases c, simp-all add: empty-dt-def)
    ultimately
    have assigns-if (the-Bool (eval-binop binop v1 v2)) (BinOp binop e1 e2)
          = {}
      by simp
    thus ?thesis by simp
  next
    case None
    show ?thesis
    proof (cases binop=CondAnd ∨ binop=CondOr)
      case True
      from bool obtain bool-e1: Env⊢e1::−PrimT Boolean and
                       bool-e2: Env⊢e2::−PrimT Boolean
        using True by cases auto
      have assigns-if (the-Bool v1) e1 ⊆ dom (locals (store s1))
      proof −
        from BinOp have normal s1
          by − (erule eval-no-abrupt-lemma [rule-format])
        from this bool-e1
        show ?thesis
          by (rule BinOp.hyps [elim-format]) auto
      qed
      also
      from BinOp.hyps
      have ... ⊆ dom (locals (store s2))
        by − (erule dom-locals-eval-mono-elim,simp)
      finally
      have e1-s2: assigns-if (the-Bool v1) e1 ⊆ dom (locals (store s2)).
      from True show ?thesis
      proof
        assume condAnd: binop = CondAnd
        show ?thesis
        proof (cases the-Bool (eval-binop binop v1 v2))
          case True
          with condAnd
```

```
      have need-second: need-second-arg binop v1
        by (simp add: need-second-arg-def)
      from ‹normal s2›
      have assigns-if (the-Bool v2) e2 ⊆ dom (locals (store s2))
        by (rule BinOp.hyps [elim-format])
           (simp add: need-second bool-e2)+
      with e1-s2
      have assigns-if (the-Bool v1) e1 ∪ assigns-if (the-Bool v2) e2
             ⊆ dom (locals (store s2))
        by (rule Un-least)
      with True condAnd None show ?thesis
        by simp
    next
      case False
      note binop-False = this
      show ?thesis
      proof (cases need-second-arg binop v1)
        case True
        with binop-False condAnd
        obtain the-Bool v1=True and the-Bool v2 = False
          by (simp add: need-second-arg-def)
        moreover
        from ‹normal s2›
        have assigns-if (the-Bool v2) e2 ⊆ dom (locals (store s2))
          by (rule BinOp.hyps [elim-format]) (simp add: True bool-e2)+
        with e1-s2
        have assigns-if (the-Bool v1) e1 ∪ assigns-if (the-Bool v2) e2
               ⊆ dom (locals (store s2))
          by (rule Un-least)
        moreover note binop-False condAnd None
        ultimately show ?thesis
          by auto
      next
        case False
        with binop-False condAnd
        have the-Bool v1=False
          by (simp add: need-second-arg-def)
        with e1-s2
        show ?thesis
          using binop-False condAnd None by auto
      qed
    qed
  next
    assume condOr: binop = CondOr
    show ?thesis
    proof (cases the-Bool (eval-binop binop v1 v2))
      case False
      with condOr
      have need-second: need-second-arg binop v1
        by (simp add: need-second-arg-def)
      from ‹normal s2›
      have assigns-if (the-Bool v2) e2 ⊆ dom (locals (store s2))
        by (rule BinOp.hyps [elim-format])
           (simp add: need-second bool-e2)+
      with e1-s2
      have assigns-if (the-Bool v1) e1 ∪ assigns-if (the-Bool v2) e2
             ⊆ dom (locals (store s2))
        by (rule Un-least)
      with False condOr None show ?thesis
```

```
          by simp
        next
          case True
          note binop-True = this
          show ?thesis
          proof (cases need-second-arg binop v1)
            case True
            with binop-True condOr
            obtain the-Bool v1=False and the-Bool v2 = True
              by (simp add: need-second-arg-def)
            moreover
            from ‹normal s2›
            have assigns-if (the-Bool v2) e2 ⊆ dom (locals (store s2))
              by (rule BinOp.hyps [elim-format]) (simp add: True bool-e2)+
            with e1-s2
            have assigns-if (the-Bool v1) e1 ∪ assigns-if (the-Bool v2) e2
                   ⊆ dom (locals (store s2))
              by (rule Un-least)
            moreover note binop-True condOr None
            ultimately show ?thesis
              by auto
          next
            case False
            with binop-True condOr
            have the-Bool v1=True
              by (simp add: need-second-arg-def)
            with e1-s2
            show ?thesis
              using binop-True condOr None by auto
          qed
        qed
      qed
    next
      case False
      note ‹¬ (binop = CondAnd ∨ binop = CondOr)›
      from BinOp.hyps
      have
        prg Env⊢Norm s0 −BinOp binop e1 e2−≻eval-binop binop v1 v2→ s2
        by − (rule eval.BinOp)
      moreover note ‹normal s2›
      ultimately
      have assignsE (BinOp binop e1 e2) ⊆ dom (locals (store s2))
        by (rule assignsE-good-approx)
      with False None
      show ?thesis
        by simp
    qed
  qed
next
  case Super
  note ‹Env⊢Super::−PrimT Boolean›
  hence False
    by cases simp
  thus ?case ..
next
  case (Acc s0 va v f s1)
  from ‹prg Env⊢Norm s0 −va=≻(v, f)→ s1› and ‹normal s1›
  have assignsV va ⊆ dom (locals (store s1))
    by (rule assignsV-good-approx)
```

```
  thus ?case by simp
next
  case (Ass s0 va w f s1 e v s2)
  hence prg Env⊢Norm s0 −va := e−≻v→ assign f v s2
    by − (rule eval.Ass)
  moreover note ‹normal (assign f v s2)›
  ultimately
  have assignsE (va := e) ⊆ dom (locals (store (assign f v s2)))
    by (rule assignsE-good-approx)
  thus ?case by simp
next
  case (Cond s0 e0 b s1 e1 e2 v s2)
  from ‹Env⊢e0 ? e1 : e2::−PrimT Boolean›
  obtain wt-e1: Env⊢e1::−PrimT Boolean and
         wt-e2: Env⊢e2::−PrimT Boolean
    by cases (auto dest: widen-Boolean2)
  note eval-e0 = ‹prg Env⊢Norm s0 −e0−≻b→ s1›
  have e0-s2: assignsE e0 ⊆ dom (locals (store s2))
  proof −
    note eval-e0
    moreover
    from Cond.hyps and ‹normal s2› have normal s1
      by − (erule eval-no-abrupt-lemma [rule-format],simp)
    ultimately
    have assignsE e0 ⊆ dom (locals (store s1))
      by (rule assignsE-good-approx)
    also
    from Cond
    have ... ⊆ dom (locals (store s2))
      by − (erule dom-locals-eval-mono [elim-format],simp)
    finally show ?thesis .
  qed
  show ?case
  proof (cases constVal e0)
    case None
    have assigns-if (the-Bool v) e1 ∩  assigns-if (the-Bool v) e2
           ⊆ dom (locals (store s2))
    proof (cases the-Bool b)
      case True
      from ‹normal s2›
      have assigns-if (the-Bool v) e1 ⊆ dom (locals (store s2))
        by (rule Cond.hyps [elim-format]) (simp-all add: wt-e1 True)
      thus ?thesis
        by blast
    next
      case False
      from ‹normal s2›
      have assigns-if (the-Bool v) e2 ⊆ dom (locals (store s2))
        by (rule Cond.hyps [elim-format]) (simp-all add: wt-e2 False)
      thus ?thesis
        by blast
    qed
    with e0-s2
    have assignsE e0 ∪
          (assigns-if (the-Bool v) e1 ∩  assigns-if (the-Bool v) e2)
            ⊆ dom (locals (store s2))
      by (rule Un-least)
    with None show ?thesis
      by simp
```

```
    next
      case (Some c)
      from this eval-e0 have eq-b-c: b=c
        by (rule constVal-eval-elim)
      show ?thesis
      proof (cases the-Bool c)
        case True
        from ‹normal s2›
        have assigns-if (the-Bool v) e1 ⊆ dom (locals (store s2))
          by (rule Cond.hyps [elim-format]) (simp-all add: eq-b-c True wt-e1)
        with e0-s2
        have assignsE e0 ∪ assigns-if (the-Bool v) e1 ⊆ ...
          by (rule Un-least)
        with Some True show ?thesis
          by simp
      next
        case False
        from ‹normal s2›
        have assigns-if (the-Bool v) e2 ⊆ dom (locals (store s2))
          by (rule Cond.hyps [elim-format]) (simp-all add: eq-b-c False wt-e2)
        with e0-s2
        have assignsE e0 ∪ assigns-if (the-Bool v) e2 ⊆ ...
          by (rule Un-least)
        with Some False show ?thesis
          by simp
      qed
    qed
  next
    case (Call s0 e a s1 args vs s2 D mode statT mn pTs s3 s3' accC v s4)
    hence
    prg Env⊢Norm s0 −({accC,statT,mode}e·mn( {pTs}args))−≻v→
                  (set-lvars (locals (store s2)) s4)
      by − (rule eval.Call)
    hence assignsE ({accC,statT,mode}e·mn( {pTs}args))
            ⊆ dom (locals (store ((set-lvars (locals (store s2))) s4)))
      using ‹normal ((set-lvars (locals (snd s2))) s4)›
      by (rule assignsE-good-approx)
    thus ?case by simp
  next
    case Methd show ?case by simp
  next
    case Body show ?case by simp
  qed simp+ — all the statements and variables
 }
 note generalized = this
 from eval bool show ?thesis
   by (rule generalized [elim-format]) simp+
qed


lemma assigns-if-good-approx':
  assumes   eval: G⊢s0 −e−≻b→ s1
     and  normal: normal s1
     and    bool: (|prg=G,cls=C,lcl=L|)⊢e::− (PrimT Boolean)
  shows assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
proof −
  from eval have prg (|prg=G,cls=C,lcl=L|)⊢s0 −e−≻b→ s1 by simp
  from this normal bool show ?thesis
    by (rule assigns-if-good-approx)
```

```
qed

lemma subset-Intl: A ⊆ C ⟹ A ∩ B ⊆ C
  by blast

lemma subset-Intr: B ⊆ C ⟹ A ∩ B ⊆ C
  by blast

lemma da-good-approx:
  assumes  eval: prg Env⊢s0 −t≻→ (v,s1) and
           wt: Env⊢t::T     (is ?Wt Env t T) and
           da: Env⊢ dom (locals (store s0)) »t» A  (is ?Da Env s0 t A) and
           wf: wf-prog (prg Env)
     shows (normal s1 ⟶ (nrm A ⊆  dom (locals (store s1)))) ∧
           (∀ l. abrupt s1 = Some (Jump (Break l)) ∧ normal s0
                      ⟶ (brk A l ⊆ dom (locals (store s1)))) ∧
           (abrupt s1 = Some (Jump Ret) ∧ normal s0
                      ⟶ Result ∈ dom (locals (store s1)))
    (is ?NormalAssigned s1 A ∧ ?BreakAssigned s0 s1 A ∧ ?ResAssigned s0 s1)
proof −
  note inj-term-simps [simp]
  obtain G where G: prg Env = G by (cases Env) simp
  with eval have eval: G⊢s0 −t≻→ (v,s1) by simp
  from G wf have wf: wf-prog G by simp
  let ?HypObj = λ t s0 s1.
      ∀ Env T A. ?Wt Env t T ⟶  ?Da Env s0 t A ⟶ prg Env = G
       ⟶ ?NormalAssigned s1 A ∧ ?BreakAssigned s0 s1 A ∧ ?ResAssigned  s0 s1
  — Goal in object logic variant
  let ?Hyp = λt s0 s1. (⋀ Env T A. ⟦?Wt Env t T; ?Da Env s0 t A; prg Env = G⟧
        ⟹ ?NormalAssigned s1 A ∧ ?BreakAssigned s0 s1 A ∧ ?ResAssigned s0 s1)
  from eval and wt da G
  show ?thesis
  proof (induct arbitrary: Env T A)
    case (Abrupt xc s t Env T A)
    have da: Env⊢ dom (locals s) »t» A using Abrupt.prems by simp
    have ?NormalAssigned (Some xc,s) A
      by simp
    moreover
    have ?BreakAssigned (Some xc,s) (Some xc,s) A
      by simp
    moreover have ?ResAssigned (Some xc,s) (Some xc,s)
      by simp
    ultimately show ?case by (intro conjI)
  next
    case (Skip s Env T A)
    have da: Env⊢ dom (locals (store (Norm s))) »⟨Skip⟩» A
      using Skip.prems by simp
    hence nrm A = dom (locals (store (Norm s)))
      by (rule da-elim-cases) simp
    hence ?NormalAssigned (Norm s) A
      by auto
    moreover
    have ?BreakAssigned (Norm s) (Norm s) A
      by simp
    moreover have ?ResAssigned (Norm s) (Norm s)
```

```
      by simp
    ultimately show ?case by (intro conjI)
  next
    case (Expr s0 e v s1 Env T A)
    from Expr.prems
    show ?NormalAssigned s1 A ∧ ?BreakAssigned (Norm s0) s1 A
           ∧ ?ResAssigned (Norm s0) s1
      by (elim wt-elim-cases da-elim-cases)
         (rule Expr.hyps, auto)
  next
    case (Lab s0 c s1 j Env T A)
    note G = ‹prg Env = G›
    from Lab.prems
    obtain C l where
      da-c: Env⊢ dom (locals (snd (Norm s0))) »⟨c⟩» C and
         A: nrm A = nrm C ∩ (brk C) l brk A = rmlab l (brk C) and
         j: j = Break l
      by − (erule da-elim-cases, simp)
    from Lab.prems
    have wt-c: Env⊢c::√
      by − (erule wt-elim-cases, simp)
    from wt-c da-c G and Lab.hyps
    have norm-c: ?NormalAssigned s1 C and
          brk-c: ?BreakAssigned (Norm s0) s1 C and
          res-c: ?ResAssigned (Norm s0) s1
      by simp-all
    have ?NormalAssigned (abupd (absorb j) s1) A
    proof
      assume normal: normal (abupd (absorb j) s1)
      show nrm A ⊆ dom (locals (store (abupd (absorb j) s1)))
      proof (cases abrupt s1)
        case None
        with norm-c A
        show ?thesis
          by auto
      next
        case Some
        with normal j
        have abrupt s1 = Some (Jump (Break l))
          by (auto dest: absorb-Some-NoneD)
        with brk-c A
        show ?thesis
          by auto
      qed
    qed
    moreover
    have ?BreakAssigned (Norm s0) (abupd (absorb j) s1) A
    proof −
      {
        fix l′
        assume break: abrupt (abupd (absorb j) s1) = Some (Jump (Break l′))
        with j
        have l≠l′
          by (cases s1) (auto dest!: absorb-Some-JumpD)
        hence (rmlab l (brk C)) l′= (brk C) l′
          by (simp)
        with break brk-c A
        have
          (brk A l′ ⊆ dom (locals (store (abupd (absorb j) s1))))
```

```
        by (cases s1) auto
      }
      then show ?thesis
        by simp
    qed
    moreover
    from res-c have ?ResAssigned (Norm s0) (abupd (absorb j) s1)
      by (cases s1) (simp add: absorb-def)
    ultimately show ?case by (intro conjI)
  next
    case (Comp s0 c1 s1 c2 s2 Env T A)
    note G = ‹prg Env = G›
    from Comp.prems
    obtain C1 C2
      where da-c1: Env⊢ dom (locals (snd (Norm s0))) »⟨c1⟩» C1 and
            da-c2: Env⊢ nrm C1 »⟨c2⟩» C2 and
                A: nrm A = nrm C2 brk A = (brk C1) ⇒∩ (brk C2)
      by (elim da-elim-cases) simp
    from Comp.prems
    obtain wt-c1: Env⊢c1::√ and
           wt-c2: Env⊢c2::√
      by (elim wt-elim-cases) simp
    note ‹PROP ?Hyp (In1r c1) (Norm s0) s1›
    with wt-c1 da-c1 G
    obtain nrm-c1: ?NormalAssigned s1 C1 and
           brk-c1: ?BreakAssigned (Norm s0) s1 C1 and
           res-c1: ?ResAssigned (Norm s0) s1
      by simp
    show ?case
    proof (cases normal s1)
      case True
      with nrm-c1 have nrm C1 ⊆ dom (locals (snd s1)) by iprover
      with da-c2 obtain C2'
        where  da-c2': Env⊢ dom (locals (snd s1)) »⟨c2⟩» C2' and
               nrm-c2: nrm C2 ⊆ nrm C2'                    and
               brk-c2: ∀ l. brk C2 l ⊆ brk C2' l
        by (rule da-weakenE) iprover
      note ‹PROP ?Hyp (In1r c2) s1 s2›
      with wt-c2 da-c2' G
      obtain nrm-c2': ?NormalAssigned s2 C2' and
             brk-c2': ?BreakAssigned s1 s2 C2' and
             res-c2 : ?ResAssigned s1 s2
        by simp
      from nrm-c2' nrm-c2 A
      have ?NormalAssigned s2 A
        by blast
      moreover from brk-c2' brk-c2 A
      have ?BreakAssigned s1 s2 A
        by fastforce
      with True
      have ?BreakAssigned (Norm s0) s2 A by simp
      moreover from res-c2 True
      have ?ResAssigned (Norm s0) s2
        by simp
      ultimately show ?thesis by (intro conjI)
    next
      case False
      with ‹G⊢s1 −c2→ s2›
      have eq-s1-s2: s2=s1 by auto
```

```
      with False have ?NormalAssigned s2 A by blast
      moreover
      have ?BreakAssigned (Norm s0) s2 A
      proof (cases ∃ l. abrupt s1 = Some (Jump (Break l)))
        case True
        then obtain l where l: abrupt s1 = Some (Jump (Break l)) ..
        with brk-c1
        have brk C1 l ⊆ dom (locals (store s1))
          by simp
        with A eq-s1-s2
        have brk A l ⊆ dom (locals (store s2))
          by auto
        with l eq-s1-s2
        show ?thesis by simp
      next
        case False
        with eq-s1-s2 show ?thesis by simp
      qed
      moreover from False res-c1 eq-s1-s2
      have ?ResAssigned (Norm s0) s2
        by simp
      ultimately show ?thesis by (intro conjI)
    qed
  next
    case (If s0 e b s1 c1 c2 s2 Env T A)
    note G = ‹prg Env = G›
    with If.hyps have eval-e: prg Env ⊢Norm s0 −e−≻b→ s1 by simp
    from If.prems
    obtain E C1 C2 where
      da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» E and
      da-c1: Env⊢ (dom (locals (store ((Norm s0)::state)))
                 ∪ assigns-if True e) »⟨c1⟩» C1 and
      da-c2: Env⊢ (dom (locals (store ((Norm s0)::state)))
                 ∪ assigns-if False e) »⟨c2⟩» C2 and
      A: nrm A = nrm C1 ∩ nrm C2  brk A = brk C1 ⇒∩ brk C2
      by (elim da-elim-cases)
    from If.prems
    obtain
       wt-e:  Env⊢e::− PrimT Boolean and
       wt-c1: Env⊢c1::√ and
       wt-c2: Env⊢c2::√
      by (elim wt-elim-cases)
    from If.hyps have
     s0-s1:dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by (elim dom-locals-eval-mono-elim)
    show ?case
    proof (cases normal s1)
      case True
      note normal-s1 = this
      show ?thesis
      proof (cases the-Bool b)
        case True
        from eval-e normal-s1 wt-e
        have assigns-if True e ⊆ dom (locals (store s1))
          by (rule assigns-if-good-approx [elim-format]) (simp add: True)
        with s0-s1
        have dom (locals (store ((Norm s0)::state))) ∪ assigns-if True e ⊆ . . .
          by (rule Un-least)
        with da-c1 obtain C1'
```

where *da-c1'*: *Env*⊢ *dom* (*locals* (*store s1*)) »⟨*c1*⟩» *C1'* **and**
nrm-c1: *nrm C1* ⊆ *nrm C1'* **and**
brk-c1: ∀ *l*. *brk C1 l* ⊆ *brk C1' l*
by (*rule da-weakenE*) *iprover*
from *If.hyps True* **have** *PROP ?Hyp* (*In1r c1*) *s1 s2* **by** *simp*
with *wt-c1 da-c1'*
obtain *nrm-c1'*: *?NormalAssigned s2 C1'* **and**
brk-c1': *?BreakAssigned s1 s2 C1'* **and**
res-c1: *?ResAssigned s1 s2*
using *G* **by** *simp*
from *nrm-c1' nrm-c1 A*
have *?NormalAssigned s2 A*
by *blast*
moreover from *brk-c1' brk-c1 A*
have *?BreakAssigned s1 s2 A*
by *fastforce*
with *normal-s1*
have *?BreakAssigned* (*Norm s0*) *s2 A* **by** *simp*
moreover from *res-c1 normal-s1* **have** *?ResAssigned* (*Norm s0*) *s2*
by *simp*
ultimately show *?thesis* **by** (*intro conjI*)
next
case *False*
from *eval-e normal-s1 wt-e*
have *assigns-if False e* ⊆ *dom* (*locals* (*store s1*))
by (*rule assigns-if-good-approx* [*elim-format*]) (*simp add*: *False*)
with *s0-s1*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*)))∪ *assigns-if False e* ⊆ . . .
by (*rule Un-least*)
with *da-c2* **obtain** *C2'*
where *da-c2'*: *Env*⊢ *dom* (*locals* (*store s1*)) »⟨*c2*⟩» *C2'* **and**
nrm-c2: *nrm C2* ⊆ *nrm C2'* **and**
brk-c2: ∀ *l*. *brk C2 l* ⊆ *brk C2' l*
by (*rule da-weakenE*) *iprover*
from *If.hyps False* **have** *PROP ?Hyp* (*In1r c2*) *s1 s2* **by** *simp*
with *wt-c2 da-c2'*
obtain *nrm-c2'*: *?NormalAssigned s2 C2'* **and**
brk-c2': *?BreakAssigned s1 s2 C2'* **and**
res-c2: *?ResAssigned s1 s2*
using *G* **by** *simp*
from *nrm-c2' nrm-c2 A*
have *?NormalAssigned s2 A*
by *blast*
moreover from *brk-c2' brk-c2 A*
have *?BreakAssigned s1 s2 A*
by *fastforce*
with *normal-s1*
have *?BreakAssigned* (*Norm s0*) *s2 A* **by** *simp*
moreover from *res-c2 normal-s1* **have** *?ResAssigned* (*Norm s0*) *s2*
by *simp*
ultimately show *?thesis* **by** (*intro conjI*)
qed
next
case *False*
then obtain *abr* **where** *abr*: *abrupt s1* = *Some abr*
by (*cases s1*) *auto*
moreover
from *eval-e* - *wt-e* **have** ⋀ *j*. *abrupt s1* ≠ *Some* (*Jump j*)
by (*rule eval-expression-no-jump*) (*simp-all add*: *G wf*)

```
    moreover
    have s2 = s1
    proof −
      from abr and ‹G⊢s1 −(if the-Bool b then c1 else c2)→ s2›
      show ?thesis
        by (cases s1) simp
    qed
    ultimately show ?thesis by simp
  qed
next
  case (Loop s0 e b s1 c s2 l s3 Env T A)
  note G = ‹prg Env = G›
  with Loop.hyps have eval-e: prg Env⊢Norm s0 −e−≻b→ s1
    by (simp (no-asm-simp))
  from Loop.prems
  obtain E C where
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» E and
    da-c: Env⊢ (dom (locals (store ((Norm s0)::state)))
                ∪ assigns-if True e) »⟨c⟩» C and
   A: nrm A = nrm C ∩
          (dom (locals (store ((Norm s0)::state))) ∪ assigns-if False e)
      brk A = brk C
    by (elim da-elim-cases)
  from Loop.prems
  obtain
     wt-e: Env⊢e::−PrimT Boolean and
     wt-c: Env⊢c::√
    by (elim wt-elim-cases)
  from wt-e da-e G
  obtain res-s1: ?ResAssigned (Norm s0) s1
    by (elim Loop.hyps [elim-format]) simp+
  from Loop.hyps have
    s0-s1:dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
    by (elim dom-locals-eval-mono-elim)
  show ?case
  proof (cases normal s1)
    case True
    note normal-s1 = this
    show ?thesis
    proof (cases the-Bool b)
      case True
      with Loop.hyps obtain
        eval-c: G⊢s1 −c→ s2 and
        eval-while: G⊢abupd (absorb (Cont l)) s2 −l· While(e) c→ s3
        by simp
      from Loop.hyps True
      have ?HypObj (In1r c) s1 s2 by simp
      note hyp-c = this [rule-format]
      from Loop.hyps True
      have ?HypObj (In1r (l· While(e) c)) (abupd (absorb (Cont l)) s2) s3
        by simp
      note hyp-while = this [rule-format]
      from eval-e normal-s1 wt-e
      have assigns-if True e ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx [elim-format]) (simp add: True)
      with s0-s1
      have dom (locals (store ((Norm s0)::state))) ∪ assigns-if True e ⊆ ...
        by (rule Un-least)
      with da-c obtain C′
```

```
  where da-c′: Env⊢ dom (locals (store s1)) »⟨c⟩» C′ and
     nrm-C-C′: nrm C ⊆ nrm C′                    and
     brk-C-C′: ∀ l. brk C l ⊆ brk C′ l
  by (rule da-weakenE) iprover
from hyp-c wt-c da-c′
obtain nrm-C′: ?NormalAssigned s2 C′ and
  brk-C′: ?BreakAssigned s1 s2 C′ and
  res-s2: ?ResAssigned s1 s2
  using G by simp
show ?thesis
proof (cases normal s2 ∨ abrupt s2 = Some (Jump (Cont l)))
  case True
  from Loop.prems obtain
    wt-while: Env⊢In1r (l· While(e) c)::T and
    da-while: Env⊢ dom (locals (store ((Norm s0)::state)))
                 »⟨l· While(e) c⟩» A
    by simp
  have dom (locals (store ((Norm s0)::state)))
        ⊆ dom (locals (store (abupd (absorb (Cont l)) s2)))
  proof −
    note s0-s1
    also from eval-c
    have dom (locals (store s1)) ⊆ dom (locals (store s2))
      by (rule dom-locals-eval-mono-elim)
    also have ... ⊆ dom (locals (store (abupd (absorb (Cont l)) s2)))
      by simp
    finally show ?thesis .
  qed
  with da-while obtain A′
    where
    da-while′: Env⊢ dom (locals (store (abupd (absorb (Cont l)) s2)))
              »⟨l· While(e) c⟩» A′
    and  nrm-A-A′: nrm A ⊆ nrm A′
    and  brk-A-A′: ∀ l. brk A l ⊆ brk A′ l
    by (rule da-weakenE) simp
  with wt-while hyp-while
  obtain nrm-A′: ?NormalAssigned s3 A′ and
         brk-A′: ?BreakAssigned (abupd (absorb (Cont l)) s2) s3 A′ and
         res-s3: ?ResAssigned (abupd (absorb (Cont l)) s2) s3
    using G by simp
  from nrm-A-A′ nrm-A′
  have ?NormalAssigned s3 A
    by blast
  moreover
  have ?BreakAssigned (Norm s0) s3 A
  proof −
    from brk-A-A′ brk-A′
    have ?BreakAssigned (abupd (absorb (Cont l)) s2) s3 A
      by fastforce
    moreover
    from True have normal (abupd (absorb (Cont l)) s2)
      by (cases s2) auto
    ultimately show ?thesis
      by simp
  qed
  moreover from res-s3 True have ?ResAssigned (Norm s0) s3
    by auto
  ultimately show ?thesis by (intro conjI)
next
```

case *False*
then obtain *abr* **where**
abrupt s2 = *Some abr* **and**
abrupt (*abupd* (*absorb* (*Cont l*)) *s2*) = *Some abr*
by *auto*
with *eval-while*
have *eq-s3-s2*: *s3=s2*
by *auto*
with *nrm-C-C′ nrm-C′ A*
have *?NormalAssigned s3 A*
by *auto*
moreover
from *eq-s3-s2 brk-C-C′ brk-C′ normal-s1 A*
have *?BreakAssigned* (*Norm s0*) *s3 A*
by *fastforce*
moreover
from *eq-s3-s2 res-s2 normal-s1* **have** *?ResAssigned* (*Norm s0*) *s3*
by *simp*
ultimately show *?thesis* **by** (*intro conjI*)
qed
next
case *False*
with *Loop.hyps* **have** *eq-s3-s1*: *s3=s1*
by *simp*
from *eq-s3-s1 res-s1*
have *res-s3*: *?ResAssigned* (*Norm s0*) *s3*
by *simp*
from *eval-e True wt-e*
have *assigns-if False e* $\subseteq$ *dom* (*locals* (*store s1*))
by (*rule assigns-if-good-approx* [*elim-format*]) (*simp add*: *False*)
with *s0-s1*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*)))$\cup$*assigns-if False e* $\subseteq \ldots$
by (*rule Un-least*)
hence *nrm C* $\cap$
(*dom* (*locals* (*store* ((*Norm s0*)::*state*))) $\cup$ *assigns-if False e*)
$\subseteq$ *dom* (*locals* (*store s1*))
by (*rule subset-Intr*)
with *normal-s1 A eq-s3-s1*
have *?NormalAssigned s3 A*
by *simp*
moreover
from *normal-s1 eq-s3-s1*
have *?BreakAssigned* (*Norm s0*) *s3 A*
by *simp*
moreover note *res-s3*
ultimately show *?thesis* **by** (*intro conjI*)
qed
next
case *False*
then obtain *abr* **where** *abr*: *abrupt s1* = *Some abr*
by (*cases s1*) *auto*
moreover
from *eval-e* - *wt-e* **have** *no-jmp*: $\bigwedge$ *j*. *abrupt s1* $\neq$ *Some* (*Jump j*)
by (*rule eval-expression-no-jump*) (*simp-all add*: *wf G*)
moreover
have *eq-s3-s1*: *s3=s1*
proof (*cases the-Bool b*)
case *True*
with *Loop.hyps* **obtain**

```
          eval-c: G⊢s1 −c→ s2 and
          eval-while: G⊢abupd (absorb (Cont l)) s2 −l• While(e) c→ s3
          by simp
        from eval-c abr have s2=s1 by auto
        moreover from calculation no-jmp have abupd (absorb (Cont l)) s2=s2
          by (cases s1) (simp add: absorb-def)
        ultimately show ?thesis
          using eval-while abr
          by auto
      next
        case False
        with Loop.hyps show ?thesis by simp
      qed
      moreover
      from eq-s3-s1 res-s1
      have res-s3: ?ResAssigned (Norm s0) s3
        by simp
      ultimately show ?thesis
        by simp
    qed
  next
    case (Jmp s j Env T A)
    have ?NormalAssigned (Some (Jump j),s) A by simp
    moreover
    from Jmp.prems
    obtain ret: j = Ret ⟶ Result ∈ dom (locals (store (Norm s))) and
           brk: brk A = (case j of
                           Break l ⇒ λ k. if k=l
                                     then dom (locals (store ((Norm s)::state)))
                                     else UNIV
                         | Cont l  ⇒ λ k. UNIV
                         | Ret     ⇒ λ k. UNIV)
      by (elim da-elim-cases) simp
    from brk have ?BreakAssigned (Norm s) (Some (Jump j),s) A
      by simp
    moreover from ret have ?ResAssigned (Norm s) (Some (Jump j),s)
      by simp
    ultimately show ?case by (intro conjI)
  next
    case (Throw s0 e a s1 Env T A)
    note G = ‹prg Env = G›
    from Throw.prems obtain E where
      da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» E
      by (elim da-elim-cases)
    from Throw.prems
      obtain eT where wt-e: Env⊢e::−eT
        by (elim wt-elim-cases)
    have ?NormalAssigned (abupd (throw a) s1) A
      by (cases s1) (simp add: throw-def)
    moreover
    have ?BreakAssigned (Norm s0) (abupd (throw a) s1) A
    proof −
      from G Throw.hyps have eval-e: prg Env⊢Norm s0 −e−≻a→ s1
        by (simp (no-asm-simp))
      from eval-e - wt-e
      have ⋀ l. abrupt s1 ≠ Some (Jump (Break l))
        by (rule eval-expression-no-jump) (simp-all add: wf G)
      hence ⋀ l. abrupt (abupd (throw a) s1) ≠ Some (Jump (Break l))
        by (cases s1) (simp add: throw-def abrupt-if-def)
```

thus *?thesis*
by *simp*
qed
moreover
from *wt-e da-e G* **have** *?ResAssigned* (*Norm s0*) *s1*
by (*elim Throw.hyps* [*elim-format*]) *simp+*
hence *?ResAssigned* (*Norm s0*) (*abupd* (*throw a*) *s1*)
by (*cases s1*) (*simp add*: *throw-def abrupt-if-def*)
ultimately show *?case* **by** (*intro conjI*)
next
case (*Try s0 c1 s1 s2 C vn c2 s3 Env T A*)
note *G* = ‹*prg Env* = *G*›
from *Try.prems* **obtain** *C1 C2* **where**
da-c1: *Env*⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »⟨*c1*⟩» *C1* **and**
da-c2:
Env⦇*lcl* := *lcl Env*(*VName vn*↦*Class C*)⦈
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))) ∪ {*VName vn*}) »⟨*c2*⟩» *C2* **and**
A: *nrm A* = *nrm C1* ∩ *nrm C2 brk A* = *brk C1* ⇒∩ *brk C2*
by (*elim da-elim-cases*) *simp*
from *Try.prems* **obtain**
wt-c1: *Env*⊢*c1*::√ **and**
wt-c2: *Env*⦇*lcl* := *lcl Env*(*VName vn*↦*Class C*)⦈⊢*c2*::√
by (*elim wt-elim-cases*)
have *sxalloc*: *prg Env*⊢*s1* −*sxalloc*→ *s2* **using** *Try.hyps G*
by (*simp* (*no-asm-simp*))
note ‹*PROP ?Hyp* (*In1r c1*) (*Norm s0*) *s1*›
with *wt-c1 da-c1 G*
obtain *nrm-C1*: *?NormalAssigned s1 C1* **and**
brk-C1: *?BreakAssigned* (*Norm s0*) *s1 C1* **and**
res-s1: *?ResAssigned* (*Norm s0*) *s1*
by *simp*
show *?case*
proof (*cases normal s1*)
case *True*
with *nrm-C1* **have** *nrm C1* ∩ *nrm C2* ⊆ *dom* (*locals* (*store s1*))
by *auto*
moreover
have *s3*=*s1*
proof −
from *sxalloc True* **have** *eq-s2-s1*: *s2*=*s1*
by (*cases s1*) (*auto elim*: *sxalloc-elim-cases*)
with *True* **have** ¬ *G*,*s2*⊢*catch C*
by (*simp add*: *catch-def*)
with *Try.hyps* **have** *s3*=*s2*
by *simp*
with *eq-s2-s1* **show** *?thesis* **by** *simp*
qed
ultimately show *?thesis*
using *True A res-s1* **by** *simp*
next
case *False*
note *not-normal-s1* = *this*
show *?thesis*
proof (*cases* ∃ *l*. *abrupt s1* = *Some* (*Jump* (*Break l*)))
case *True*
then obtain *l* **where** *l*: *abrupt s1* = *Some* (*Jump* (*Break l*))
by *auto*
with *brk-C1* **have** (*brk C1* ⇒∩ *brk C2*) *l* ⊆ *dom* (*locals* (*store s1*))
by *auto*

```
      moreover have s3=s1
      proof –
        from sxalloc l have eq-s2-s1: s2=s1
          by (cases s1) (auto elim: sxalloc-elim-cases)
        with l have ¬ G,s2⊢catch C
          by (simp add: catch-def)
        with Try.hyps have s3=s2
          by simp
        with eq-s2-s1 show ?thesis by simp
      qed
      ultimately show ?thesis
        using l A res-s1 by simp
    next
      case False
      note abrupt-no-break = this
      show ?thesis
      proof (cases G,s2⊢catch C)
        case True
        with Try.hyps have ?HypObj (In1r c2) (new-xcpt-var vn s2) s3
          by simp
        note hyp-c2 = this [rule-format]
        have (dom (locals (store ((Norm s0)::state))) ∪ {VName vn})
               ⊆ dom (locals (store (new-xcpt-var vn s2)))
        proof –
          from ‹G⊢Norm s0 −c1→ s1›
          have dom (locals (store ((Norm s0)::state)))
                 ⊆ dom (locals (store s1))
            by (rule dom-locals-eval-mono-elim)
          also
          from sxalloc
          have ... ⊆ dom (locals (store s2))
            by (rule dom-locals-sxalloc-mono)
          also
          have ... ⊆ dom (locals (store (new-xcpt-var vn s2)))
            by (cases s2) (simp add: new-xcpt-var-def, blast)
          also
          have {VName vn} ⊆ ...
            by (cases s2) simp
          ultimately show ?thesis
            by (rule Un-least)
        qed
        with da-c2
        obtain C2' where
          da-C2': Env(|lcl := lcl Env(VName vn↦Class C)|)
                   ⊢ dom (locals (store (new-xcpt-var vn s2))) »⟨c2⟩» C2'
         and nrm-C2': nrm C2 ⊆ nrm C2'
         and brk-C2': ∀ l. brk C2 l ⊆ brk C2' l
          by (rule da-weakenE) simp
        from wt-c2 da-C2' G and hyp-c2
        obtain nrmAss-C2: ?NormalAssigned s3 C2' and
              brkAss-C2: ?BreakAssigned (new-xcpt-var vn s2) s3 C2' and
              resAss-s3: ?ResAssigned (new-xcpt-var vn s2) s3
          by simp
        from nrmAss-C2 nrm-C2' A
        have ?NormalAssigned s3 A
          by auto
        moreover
        have ?BreakAssigned (Norm s0) s3 A
        proof –
```

```
          from brkAss-C2 have ?BreakAssigned (Norm s0) s3 C2'
            by (cases s2) (auto simp add: new-xcpt-var-def)
          with brk-C2' A show ?thesis
            by fastforce
        qed
        moreover
        from resAss-s3 have ?ResAssigned (Norm s0) s3
          by (cases s2) ( simp add: new-xcpt-var-def)
        ultimately show ?thesis by (intro conjI)
      next
        case False
        with Try.hyps
        have eq-s3-s2: s3=s2 by simp
        moreover from sxalloc not-normal-s1 abrupt-no-break
        obtain ¬ normal s2
               ∀ l. abrupt s2 ≠ Some (Jump (Break l))
          by − (rule sxalloc-cases,auto)
        ultimately obtain
          ?NormalAssigned s3 A and ?BreakAssigned (Norm s0) s3 A
          by (cases s2) auto
        moreover have ?ResAssigned (Norm s0) s3
        proof (cases abrupt s1 = Some (Jump Ret))
          case True
          with sxalloc have s2=s1
            by (elim sxalloc-cases) auto
          with res-s1 eq-s3-s2 show ?thesis by simp
        next
          case False
          with sxalloc
          have abrupt s2 ≠ Some (Jump Ret)
            by (rule sxalloc-no-jump)
          with eq-s3-s2 show ?thesis
            by simp
        qed
        ultimately show ?thesis by (intro conjI)
      qed
    qed
  qed
next
  case (Fin s0 c1 x1 s1 c2 s2 s3 Env T A)
  note G = ‹prg Env = G›
  from Fin.prems obtain C1 C2 where
    da-C1: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨c1⟩» C1 and
    da-C2: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨c2⟩» C2 and
    nrm-A: nrm A = nrm C1 ∪ nrm C2 and
    brk-A: brk A = ((brk C1) ⇒∪∀ (nrm C2)) ⇒∩ (brk C2)
    by (elim da-elim-cases) simp
  from Fin.prems obtain
    wt-c1: Env⊢c1::√ and
    wt-c2: Env⊢c2::√
    by (elim wt-elim-cases)
  note ‹PROP ?Hyp (In1r c1) (Norm s0) (x1,s1)›
  with wt-c1 da-C1 G
  obtain nrmAss-C1: ?NormalAssigned (x1,s1) C1 and
         brkAss-C1: ?BreakAssigned (Norm s0) (x1,s1) C1 and
         resAss-s1: ?ResAssigned (Norm s0) (x1,s1)
    by simp
  obtain nrmAss-C2: ?NormalAssigned s2 C2 and
         brkAss-C2: ?BreakAssigned (Norm s1) s2 C2 and
```

```
      resAss-s2: ?ResAssigned (Norm s1) s2
proof -
  from Fin.hyps
  have dom (locals (store ((Norm s0)::state)))
       ⊆ dom (locals (store (x1,s1)))
    by - (rule dom-locals-eval-mono-elim)
  with da-C2 obtain C2'
    where
    da-C2': Env⊢ dom (locals (store (x1,s1))) »⟨c2⟩» C2' and
    nrm-C2': nrm C2 ⊆ nrm C2' and
    brk-C2': ∀ l. brk C2 l ⊆ brk C2' l
    by (rule da-weakenE) simp
  note ‹PROP ?Hyp (In1r c2) (Norm s1) s2›
  with wt-c2 da-C2' G
  obtain nrmAss-C2': ?NormalAssigned s2 C2' and
         brkAss-C2': ?BreakAssigned (Norm s1) s2 C2' and
         resAss-s2': ?ResAssigned (Norm s1) s2
    by simp
  from nrmAss-C2' nrm-C2' have ?NormalAssigned s2 C2
    by blast
  moreover
  from brkAss-C2' brk-C2' have ?BreakAssigned (Norm s1) s2 C2
    by fastforce
  ultimately
  show ?thesis
    using that resAss-s2' by simp
qed
note s3 = ‹s3 = (if ∃ err. x1 = Some (Error err) then (x1, s1)
                 else abupd (abrupt-if (x1 ≠ None) x1) s2)›
have s1-s2: dom (locals s1) ⊆ dom (locals (store s2))
proof -
  from ‹G⊢Norm s1 −c2→ s2›
  show ?thesis
    by (rule dom-locals-eval-mono-elim) simp
qed

have ?NormalAssigned s3 A
proof
  assume normal-s3: normal s3
  show nrm A ⊆ dom (locals (snd s3))
  proof -
    have nrm C1 ⊆ dom (locals (snd s3))
    proof -
      from normal-s3 s3
      have normal (x1,s1)
        by (cases s2) (simp add: abrupt-if-def)
      with normal-s3 nrmAss-C1 s3 s1-s2
      show ?thesis
        by fastforce
    qed
    moreover
    have nrm C2 ⊆ dom (locals (snd s3))
    proof -
      from normal-s3 s3
      have normal s2
        by (cases s2) (simp add: abrupt-if-def)
      with normal-s3 nrmAss-C2 s3 s1-s2
      show ?thesis
        by fastforce
```

```
    qed
    ultimately have nrm C1 ∪ nrm C2 ⊆ . . .
      by (rule Un-least)
    with nrm-A show ?thesis
      by simp
  qed
qed
moreover
{
  fix l assume brk-s3: abrupt s3 = Some (Jump (Break l))
  have brk A l ⊆ dom (locals (store s3))
  proof (cases normal s2)
    case True
    with brk-s3 s3
    have s2-s3: dom (locals (store s2)) ⊆ dom (locals (store s3))
      by simp
    have brk C1 l ⊆ dom (locals (store s3))
    proof −
      from True brk-s3 s3 have x1=Some (Jump (Break l))
        by (cases s2) (simp add: abrupt-if-def)
      with brkAss-C1 s1-s2 s2-s3
      show ?thesis
        by simp
    qed
    moreover from True nrmAss-C2 s2-s3
    have nrm C2 ⊆ dom (locals (store s3))
      by − (rule subset-trans, simp-all)
    ultimately
    have ((brk C1) ⇒∪∀ (nrm C2)) l ⊆ . . .
      by blast
    with brk-A show ?thesis
      by simp blast
  next
    case False
    note not-normal-s2 = this
    have s3=s2
    proof (cases normal (x1,s1))
      case True with not-normal-s2 s3 show ?thesis
        by (cases s2) (simp add: abrupt-if-def)
    next
      case False with not-normal-s2 s3 brk-s3 show ?thesis
        by (cases s2) (simp add: abrupt-if-def)
    qed
    with brkAss-C2 brk-s3
    have brk C2 l ⊆ dom (locals (store s3))
      by simp
    with brk-A show ?thesis
      by simp blast
  qed
}
hence ?BreakAssigned (Norm s0) s3 A
  by simp
moreover
{
  assume abr-s3: abrupt s3 = Some (Jump Ret)
  have Result ∈ dom (locals (store s3))
  proof (cases x1 = Some (Jump Ret))
    case True
    note ret-x1 = this
```

```
      with resAss-s1 have res-s1: Result ∈ dom (locals s1)
        by simp
      moreover have dom (locals (store ((Norm s1)::state)))
                      ⊆ dom (locals (store s2))
        by (rule dom-locals-eval-mono-elim) (rule Fin.hyps)
      ultimately have Result ∈ dom (locals (store s2))
        by − (rule subsetD,auto)
      with res-s1 s3 show ?thesis
        by simp
    next
      case False
      with s3 abr-s3 obtain  abrupt s2 = Some (Jump Ret) and s3=s2
        by (cases s2) (simp add: abrupt-if-def)
      with resAss-s2 show ?thesis
        by simp
    qed
  }
  hence ?ResAssigned (Norm s0) s3
    by simp
  ultimately show ?case by (intro conjI)
next
  case (Init C c s0 s3 s1 s2 Env T A)
  note G = ‹prg Env = G›
  from Init.hyps
  have eval: prg Env⊢ Norm s0 −Init C→ s3
    apply (simp only: G)
    apply (rule eval.Init, assumption)
    apply (cases inited C (globs s0) )
    apply   simp
    apply (simp only: if-False )
    apply (elim conjE,intro conjI,assumption+,simp)
    done
  from Init.prems and ‹the (class G C) = c›
  have c: class G C = Some c
    by (elim wt-elim-cases) auto
  from Init.prems obtain
     nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
    by (elim da-elim-cases) simp
  show ?case
  proof (cases inited C (globs s0))
    case True
    with Init.hyps have s3=Norm s0 by simp
    thus ?thesis
      using nrm-A by simp
  next
    case False
    from Init.hyps False G
    obtain eval-initC:
            prg Env⊢Norm ((init-class-obj G C) s0)
                     −(if C = Object then Skip else Init (super c))→ s1 and
          eval-init: prg Env⊢(set-lvars Map.empty) s1 −init c→ s2 and
          s3: s3=(set-lvars (locals (store s1))) s2
      by simp
    have ?NormalAssigned s3 A
    proof
      show nrm A ⊆ dom (locals (store s3))
      proof −
        from nrm-A have nrm A ⊆ dom (locals (init-class-obj G C s0))
          by simp
```

```
        also from eval-initC have ... ⊆ dom (locals (store s1))
          by (rule dom-locals-eval-mono-elim) simp
        also from s3 have ... ⊆ dom (locals (store s3))
          by (cases s1) (cases s2, simp add: init-lvars-def2)
        finally show ?thesis .
      qed
    qed
    moreover
    from eval
    have ⋀ j. abrupt s3 ≠ Some (Jump j)
      by (rule eval-statement-no-jump) (auto simp add: wf c G)
    then obtain ?BreakAssigned (Norm s0) s3 A
          and ?ResAssigned (Norm s0) s3
      by simp
    ultimately show ?thesis by (intro conjI)
  qed
next
  case (NewC s0 C s1 a s2 Env T A)
  note G = ‹prg Env = G›
  from NewC.prems
  obtain A: nrm A = dom (locals (store ((Norm s0)::state)))
          brk A = (λ l. UNIV)
    by (elim da-elim-cases) simp
  from wf NewC.prems
  have wt-init: Env⊢(Init C)::√
    by  (elim wt-elim-cases) (drule is-acc-classD,simp)
  have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s2))
  proof −
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim) (rule NewC.hyps)
    also
    have ... ⊆ dom (locals (store s2))
      by (rule dom-locals-halloc-mono) (rule NewC.hyps)
    finally show ?thesis .
  qed
  with A have ?NormalAssigned s2 A
    by simp
  moreover
  {
    fix j have abrupt s2 ≠ Some (Jump j)
    proof −
      have eval: prg Env⊢ Norm s0 −NewC C−≻Addr a→ s2
        unfolding G by (rule eval.NewC NewC.hyps)+
      from NewC.prems
      obtain T' where  T=Inl T'
        by (elim wt-elim-cases) simp
      with NewC.prems have Env⊢NewC C::−T'
        by simp
      from eval - this
      show ?thesis
        by (rule eval-expression-no-jump) (simp-all add: G wf)
    qed
  }
  hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (NewA s0 elT s1 e i s2 a s3 Env T A)
  note G = ‹prg Env = G›
```

```
from NewA.prems obtain
  da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» A
  by (elim da-elim-cases)
from NewA.prems obtain
  wt-init: Env⊢init-comp-ty elT::√ and
  wt-size: Env⊢e::−PrimT Integer
  by (elim wt-elim-cases) (auto dest: wt-init-comp-ty')
note halloc = ‹G⊢abupd (check-neg i) s2−halloc Arr elT (the-Intg i)≻a→s3›
have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
  by (rule dom-locals-eval-mono-elim) (rule NewA.hyps)
with da-e obtain A' where
          da-e': Env⊢ dom (locals (store s1)) »⟨e⟩» A'
    and  nrm-A-A': nrm A ⊆ nrm A'
    and  brk-A-A': ∀ l. brk A l ⊆ brk A' l
  by (rule da-weakenE) simp
note ‹PROP ?Hyp (In1l e) s1 s2›
with wt-size da-e' G obtain
  nrmAss-A': ?NormalAssigned s2 A' and
  brkAss-A': ?BreakAssigned s1 s2 A'
  by simp
have s2-s3: dom (locals (store s2)) ⊆ dom (locals (store s3))
proof −
  have dom (locals (store s2))
        ⊆ dom (locals (store (abupd (check-neg i) s2)))
    by (simp)
  also have ... ⊆ dom (locals (store s3))
    by (rule dom-locals-halloc-mono) (rule NewA.hyps)
  finally show ?thesis .
qed
have ?NormalAssigned s3 A
proof
  assume normal-s3: normal s3
  show nrm A ⊆ dom (locals (store s3))
  proof −
    from halloc normal-s3
    have normal (abupd (check-neg i) s2)
      by cases simp-all
    hence normal s2
      by (cases s2) simp
    with nrmAss-A' nrm-A-A' s2-s3 show ?thesis
      by blast
  qed
qed
moreover
{
  fix j have abrupt s3 ≠ Some (Jump j)
  proof −
    have eval: prg Env⊢ Norm s0 −New elT[e]−≻Addr a→ s3
      unfolding G by (rule eval.NewA NewA.hyps)+
    from NewA.prems
    obtain T' where  T=Inl T'
      by (elim wt-elim-cases) simp
    with NewA.prems have Env⊢New elT[e]::−T'
      by simp
    from eval - this
    show ?thesis
      by (rule eval-expression-no-jump) (simp-all add: G wf)
  qed
}
```

```
    hence ?BreakAssigned (Norm s0) s3 A and ?ResAssigned (Norm s0) s3
      by simp-all
    ultimately show ?case by (intro conjI)
  next
    case (Cast s0 e v s1 s2 cT Env T A)
    note G = ‹prg Env = G›
    from Cast.prems obtain
      da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» A
      by (elim da-elim-cases)
    from Cast.prems obtain eT where
      wt-e: Env⊢e::−eT
      by (elim wt-elim-cases)
    note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
    with wt-e da-e G obtain
      nrmAss-A: ?NormalAssigned s1 A and
      brkAss-A: ?BreakAssigned (Norm s0) s1 A
      by simp
    note s2 = ‹s2 = abupd (raise-if (¬ G,snd s1⊢v fits cT) ClassCast) s1›
    hence s1-s2: dom (locals (store s1)) ⊆ dom (locals (store s2))
      by simp
    have ?NormalAssigned s2 A
    proof
      assume normal s2
      with s2 have normal s1
        by (cases s1) simp
      with nrmAss-A s1-s2
      show nrm A ⊆ dom (locals (store s2))
        by blast
    qed
    moreover
    {
      fix j have abrupt s2 ≠ Some (Jump j)
      proof −
        have eval: prg Env⊢ Norm s0 −Cast cT e−≻v→ s2
          unfolding G by (rule eval.Cast Cast.hyps)+
        from Cast.prems
        obtain T' where  T=Inl T'
          by (elim wt-elim-cases) simp
        with Cast.prems have Env⊢Cast cT e::−T'
          by simp
        from eval - this
        show ?thesis
          by (rule eval-expression-no-jump) (simp-all add: G wf)
      qed
    }
    hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
      by simp-all
    ultimately show ?case by (intro conjI)
  next
    case (Inst s0 e v s1 b iT Env T A)
    note G = ‹prg Env = G›
    from Inst.prems obtain
      da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» A
      by (elim da-elim-cases)
    from Inst.prems obtain eT where
      wt-e: Env⊢e::−eT
      by (elim wt-elim-cases)
    note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
    with wt-e da-e G obtain
```

```
    ?NormalAssigned s1 A and
    ?BreakAssigned (Norm s0) s1 A and
    ?ResAssigned (Norm s0) s1
    by simp
  thus ?case by (intro conjI)
next
  case (Lit s v Env T A)
  from Lit.prems
  have nrm A = dom (locals (store ((Norm s)::state)))
    by (elim da-elim-cases) simp
  thus ?case by simp
next
  case (UnOp s0 e v s1 unop Env T A)
  note G = ‹prg Env = G›
  from UnOp.prems obtain
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» A
    by (elim da-elim-cases)
  from UnOp.prems obtain eT where
    wt-e: Env⊢e::−eT
    by (elim wt-elim-cases)
  note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
  with wt-e da-e G obtain
    ?NormalAssigned s1 A and
    ?BreakAssigned (Norm s0) s1 A and
    ?ResAssigned (Norm s0) s1
    by simp
  thus ?case by (intro conjI)
next
  case (BinOp s0 e1 v1 s1 binop e2 v2 s2 Env T A)
  note G = ‹prg Env = G›
  from BinOp.hyps
  have
    eval: prg Env⊢Norm s0 −BinOp binop e1 e2−≻(eval-binop binop v1 v2)→ s2
    by (simp only: G) (rule eval.BinOp)
  have s0-s1: dom (locals (store ((Norm s0)::state)))
              ⊆ dom (locals (store s1))
    by (rule dom-locals-eval-mono-elim) (rule BinOp)
  also have s1-s2: dom (locals (store s1)) ⊆  dom (locals (store s2))
    by (rule dom-locals-eval-mono-elim) (rule BinOp)
  finally
  have s0-s2: dom (locals (store ((Norm s0)::state)))
              ⊆ dom (locals (store s2)) .
  from BinOp.prems obtain e1T e2T
    where wt-e1: Env⊢e1::−e1T
     and  wt-e2: Env⊢e2::−e2T
     and  wt-binop: wt-binop (prg Env) binop e1T e2T
     and  T: T=Inl (PrimT (binop-type binop))
    by (elim wt-elim-cases) simp
  have ?NormalAssigned s2 A
  proof
    assume normal-s2: normal s2
    have   normal-s1: normal s1
      by (rule eval-no-abrupt-lemma [rule-format]) (rule BinOp.hyps, rule normal-s2)
    show nrm A ⊆ dom (locals (store s2))
    proof (cases binop=CondAnd)
      case True
      note CondAnd = this
      from BinOp.prems obtain
         nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
```

$\cup$ (*assigns-if True* (*BinOp CondAnd e1 e2*) $\cap$
assigns-if False (*BinOp CondAnd e1 e2*))
by (*elim da-elim-cases*) (*simp-all add*: *CondAnd*)
from *T BinOp.prems CondAnd*
have *Env*$\vdash$*BinOp binop e1 e2*::$-$*PrimT Boolean*
by (*simp*)
with *eval normal-s2*
have *ass-if*: *assigns-if* (*the-Bool* (*eval-binop binop v1 v2*))
(*BinOp binop e1 e2*)
$\subseteq$ *dom* (*locals* (*store s2*))
by (*rule assigns-if-good-approx*)
have (*assigns-if True* (*BinOp CondAnd e1 e2*) $\cap$
assigns-if False (*BinOp CondAnd e1 e2*)) $\subseteq$ ...
proof (*cases the-Bool* (*eval-binop binop v1 v2*))
case *True*
with *ass-if CondAnd*
have *assigns-if True* (*BinOp CondAnd e1 e2*)
$\subseteq$ *dom* (*locals* (*store s2*))
by *simp*
thus *?thesis* **by** *blast*
next
case *False*
with *ass-if CondAnd*
have *assigns-if False* (*BinOp CondAnd e1 e2*)
$\subseteq$ *dom* (*locals* (*store s2*))
by (*simp only*: *False*)
thus *?thesis* **by** *blast*
qed
with *s0-s2*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*)))
$\cup$ (*assigns-if True* (*BinOp CondAnd e1 e2*) $\cap$
assigns-if False (*BinOp CondAnd e1 e2*)) $\subseteq$ ...
by (*rule Un-least*)
thus *?thesis* **by** (*simp only*: *nrm-A*)
next
case *False*
note *notCondAnd* = *this*
show *?thesis*
proof (*cases binop*=*CondOr*)
case *True*
note *CondOr* = *this*
from *BinOp.prems* **obtain**
nrm-A: *nrm A* = *dom* (*locals* (*store* ((*Norm s0*)::*state*)))
$\cup$ (*assigns-if True* (*BinOp CondOr e1 e2*) $\cap$
assigns-if False (*BinOp CondOr e1 e2*))
by (*elim da-elim-cases*) (*simp-all add*: *CondOr*)
from *T BinOp.prems CondOr*
have *Env*$\vdash$*BinOp binop e1 e2*::$-$*PrimT Boolean*
by (*simp*)
with *eval normal-s2*
have *ass-if*: *assigns-if* (*the-Bool* (*eval-binop binop v1 v2*))
(*BinOp binop e1 e2*)
$\subseteq$ *dom* (*locals* (*store s2*))
by (*rule assigns-if-good-approx*)
have (*assigns-if True* (*BinOp CondOr e1 e2*) $\cap$
assigns-if False (*BinOp CondOr e1 e2*)) $\subseteq$ ...
proof (*cases the-Bool* (*eval-binop binop v1 v2*))
case *True*
with *ass-if CondOr*

```
        have assigns-if True (BinOp CondOr e1 e2)
                ⊆ dom (locals (store s2))
          by (simp)
        thus ?thesis by blast
      next
        case False
        with ass-if CondOr
        have assigns-if False (BinOp CondOr e1 e2)
                ⊆ dom (locals (store s2))
          by (simp)
        thus ?thesis by blast
      qed
      with s0-s2
      have dom (locals (store ((Norm s0)::state)))
              ∪ (assigns-if True (BinOp CondOr e1 e2) ∩
                 assigns-if False (BinOp CondOr e1 e2)) ⊆ . . .
        by (rule Un-least)
      thus ?thesis by (simp only: nrm-A)
    next
      case False
      with notCondAnd obtain notAndOr: binop≠CondAnd binop≠CondOr
        by simp
      from BinOp.prems obtain E1
        where da-e1: Env⊢ dom (locals (snd (Norm s0))) »⟨e1⟩» E1
         and  da-e2: Env⊢ nrm E1 »⟨e2⟩» A
        by (elim da-elim-cases) (simp-all add: notAndOr)
      note ‹PROP ?Hyp (In1l e1) (Norm s0) s1›
      with wt-e1 da-e1 G normal-s1
      obtain ?NormalAssigned s1 E1
        by simp
      with normal-s1 have nrm E1 ⊆ dom (locals (store s1)) by iprover
      with da-e2 obtain A'
        where  da-e2': Env⊢ dom (locals (store s1)) »⟨e2⟩» A' and
             nrm-A-A': nrm A ⊆ nrm A'
        by (rule da-weakenE) iprover
      from notAndOr have need-second-arg binop v1 by simp
      with BinOp.hyps
      have PROP ?Hyp (In1l e2) s1 s2 by simp
      with wt-e2 da-e2' G
      obtain ?NormalAssigned s2 A'
        by simp
      with nrm-A-A' normal-s2
      show nrm A ⊆ dom (locals (store s2))
        by blast
    qed
  qed
qed
moreover
{
  fix j have abrupt s2 ≠ Some (Jump j)
  proof −
    from BinOp.prems T
    have Env⊢In1l (BinOp binop e1 e2)::Inl (PrimT (binop-type binop))
      by simp
    from eval - this
    show ?thesis
      by (rule eval-expression-no-jump) (simp-all add: G wf)
  qed
}
```

```
    hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
      by simp-all
    ultimately show ?case by (intro conjI)
  next
    case (Super s Env T A)
    from Super.prems
    have nrm A = dom (locals (store ((Norm s)::state)))
      by (elim da-elim-cases) simp
    thus ?case by simp
  next
    case (Acc s0 v w upd s1 Env T A)
    show ?case
    proof (cases ∃ vn. v = LVar vn)
      case True
      then obtain vn where vn: v=LVar vn..
      from Acc.prems
      have nrm A = dom (locals (store ((Norm s0)::state)))
        by (simp only: vn) (elim da-elim-cases,simp-all)
      moreover
      from ‹G⊢Norm s0 −v=≻(w, upd)→ s1›
      have s1=Norm s0
        by (simp only: vn) (elim eval-elim-cases,simp)
      ultimately show ?thesis by simp
    next
      case False
      note G = ‹prg Env = G›
      from False Acc.prems
      have da-v: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨v⟩» A
        by (elim da-elim-cases) simp-all
      from Acc.prems obtain vT where
        wt-v: Env⊢v::=vT
        by (elim wt-elim-cases)
      note ‹PROP ?Hyp (In2 v) (Norm s0) s1›
      with wt-v da-v G obtain
        ?NormalAssigned s1 A and
        ?BreakAssigned (Norm s0) s1 A and
        ?ResAssigned (Norm s0) s1
        by simp
      thus ?thesis by (intro conjI)
    qed
  next
    case (Ass s0 var w upd s1 e v s2 Env T A)
    note G = ‹prg Env = G›
    from Ass.prems obtain varT eT where
      wt-var: Env⊢var::=varT and
      wt-e:   Env⊢e::−eT
      by (elim wt-elim-cases) simp
    have eval-var: prg Env⊢Norm s0 −var=≻(w, upd)→ s1
      using Ass.hyps by (simp only: G)
    have ?NormalAssigned (assign upd v s2) A
    proof
      assume normal-ass-s2: normal (assign upd v s2)
      from normal-ass-s2
      have normal-s2: normal s2
        by (cases s2) (simp add: assign-def Let-def)
      hence normal-s1: normal s1
        by − (rule eval-no-abrupt-lemma [rule-format], rule Ass.hyps)
      note hyp-var = ‹PROP ?Hyp (In2 var) (Norm s0) s1›
      note hyp-e = ‹PROP ?Hyp (In1l e) s1 s2›
```

```
show nrm A ⊆ dom (locals (store (assign upd v s2)))
proof (cases ∃ vn. var = LVar vn)
  case True
  then obtain vn where vn: var=LVar vn..
  from Ass.prems obtain E where
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» E and
    nrm-A: nrm A = nrm E ∪ {vn}
    by (elim da-elim-cases) (insert vn,auto)
  obtain E' where
    da-e': Env⊢ dom (locals (store s1)) »⟨e⟩» E' and
    E-E': nrm E ⊆ nrm E'
  proof −
    have dom (locals (store ((Norm s0)::state)))
            ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim) (rule Ass.hyps)
    with da-e show thesis
      by (rule da-weakenE) (rule that)
  qed
  from G eval-var vn
  have eval-lvar: G⊢Norm s0 −LVar vn=≻(w, upd)→ s1
    by simp
  then have upd: upd = snd (lvar vn (store s1))
    by cases (cases lvar vn (store s1),simp)
  have nrm E ⊆ dom (locals (store (assign upd v s2)))
  proof −
    from hyp-e wt-e da-e' G normal-s2
    have nrm E' ⊆ dom (locals (store s2))
      by simp
    also
    from upd
    have dom (locals (store s2))  ⊆ dom (locals (store (upd v s2)))
      by (simp add: lvar-def) blast
    hence dom (locals (store s2))
               ⊆ dom (locals (store (assign upd v s2)))
      by (rule dom-locals-assign-mono)
    finally
    show ?thesis using E-E'
      by blast
  qed
  moreover
  from upd normal-s2
  have {vn} ⊆ dom (locals (store (assign upd v s2)))
    by (auto simp add: assign-def Let-def lvar-def upd split: prod.split)
  ultimately
  show nrm A ⊆ . . .
    by (rule Un-least [elim-format]) (simp add: nrm-A)
next
  case False
  from Ass.prems obtain V where
    da-var: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨var⟩» V and
    da-e:   Env⊢ nrm V »⟨e⟩» A
    by (elim da-elim-cases) (insert False,simp+)
  from hyp-var wt-var da-var G normal-s1
  have nrm V ⊆ dom (locals (store s1))
    by simp
  with da-e obtain A'
    where  da-e': Env⊢ dom (locals (store s1)) »⟨e⟩» A' and
           nrm-A-A': nrm A ⊆ nrm A'
    by (rule da-weakenE) iprover
```

```
      from hyp-e wt-e da-e' G normal-s2
      obtain nrm A' ⊆ dom (locals (store s2))
        by simp
      with nrm-A-A' have nrm A ⊆ ...
        by blast
      also have ... ⊆ dom (locals (store (assign upd v s2)))
      proof −
        from eval-var normal-s1
        have dom (locals (store s2)) ⊆ dom (locals (store (upd v s2)))
          by (cases rule: dom-locals-eval-mono-elim)
             (cases s2, simp)
        thus ?thesis
          by (rule dom-locals-assign-mono)
      qed
      finally show ?thesis .
    qed
  qed
  moreover
  {
    fix j have abrupt (assign upd v s2) ≠ Some (Jump j)
    proof −
      have eval: prg Env⊢Norm s0 −var:=e−≻v→ (assign upd v s2)
        by (simp only: G) (rule eval.Ass Ass.hyps)+
      from Ass.prems
      obtain T' where  T=Inl T'
        by (elim wt-elim-cases) simp
      with Ass.prems have Env⊢var:=e::−T' by simp
      from eval - this
      show ?thesis
        by (rule eval-expression-no-jump) (simp-all add: G wf)
    qed
  }
  hence ?BreakAssigned (Norm s0) (assign upd v s2) A
    and ?ResAssigned (Norm s0) (assign upd v s2)
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (Cond s0 e0 b s1 e1 e2 v s2 Env T A)
  note G = ‹prg Env = G›
  have ?NormalAssigned s2 A
  proof
    assume normal-s2: normal s2
    show nrm A ⊆ dom (locals (store s2))
    proof (cases Env⊢(e0 ? e1 : e2)::−(PrimT Boolean))
      case True
      with Cond.prems
      have nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
                     ∪ (assigns-if True  (e0 ? e1 : e2) ∩
                          assigns-if False (e0 ? e1 : e2))
        by (elim da-elim-cases) simp-all
      have eval: prg Env⊢Norm s0 −(e0 ? e1 : e2)−≻v→ s2
        unfolding G by (rule eval.Cond Cond.hyps)+
      from eval
      have dom (locals (store ((Norm s0)::state)))⊆dom (locals (store s2))
        by (rule dom-locals-eval-mono-elim)
      moreover
      from eval normal-s2 True
      have ass-if: assigns-if (the-Bool v) (e0 ? e1 : e2)
                 ⊆ dom (locals (store s2))
```

```
      by (rule assigns-if-good-approx)
    have assigns-if True  (e0 ? e1:e2) ∩ assigns-if False (e0 ? e1:e2)
           ⊆ dom (locals (store s2))
    proof (cases the-Bool v)
      case True
      from ass-if
      have assigns-if True  (e0 ? e1:e2) ⊆ dom (locals (store s2))
        by (simp only: True)
      thus ?thesis by blast
    next
      case False
      from ass-if
      have assigns-if False  (e0 ? e1:e2) ⊆ dom (locals (store s2))
        by (simp only: False)
      thus ?thesis by blast
    qed
    ultimately show nrm A ⊆ dom (locals (store s2))
      by (simp only: nrm-A) (rule Un-least)
  next
    case False
    with Cond.prems obtain E1 E2 where
     da-e1: Env⊢ (dom (locals (store ((Norm s0)::state)))
                  ∪ assigns-if True e0) »⟨e1⟩» E1 and
     da-e2: Env⊢ (dom (locals (store ((Norm s0)::state)))
                 ∪ assigns-if False e0) »⟨e2⟩» E2 and
     nrm-A: nrm A = nrm E1 ∩ nrm E2
      by (elim da-elim-cases) simp-all
    from Cond.prems obtain e1T e2T where
      wt-e0: Env⊢e0::− PrimT Boolean and
      wt-e1: Env⊢e1::−e1T and
      wt-e2: Env⊢e2::−e2T
      by (elim wt-elim-cases)
    have s0-s1: dom (locals (store ((Norm s0)::state)))
                 ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim) (rule Cond.hyps)
    have eval-e0: prg Env⊢Norm s0 −e0−≻b→ s1
      unfolding G by (rule Cond.hyps)
    have normal-s1: normal s1
      by (rule eval-no-abrupt-lemma [rule-format]) (rule Cond.hyps, rule normal-s2)
    show ?thesis
    proof (cases the-Bool b)
      case True
      from True Cond.hyps have PROP ?Hyp (In1l e1) s1 s2 by simp
      moreover
      from eval-e0 normal-s1 wt-e0
      have assigns-if True e0 ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx [elim-format]) (simp only: True)
      with s0-s1
      have dom (locals (store ((Norm s0)::state)))
             ∪ assigns-if True e0 ⊆ . . .
        by (rule Un-least)
      with da-e1 obtain E1' where
              da-e1': Env⊢ dom (locals (store s1)) »⟨e1⟩» E1' and
          nrm-E1-E1': nrm E1 ⊆ nrm E1'
        by (rule da-weakenE) iprover
      ultimately have nrm E1' ⊆ dom (locals (store s2))
        using wt-e1 G normal-s2 by simp
      with nrm-E1-E1' show ?thesis
        by (simp only: nrm-A) blast
```

```
    next
      case False
      from False Cond.hyps have PROP ?Hyp (In1l e2) s1 s2 by simp
      moreover
      from eval-e0 normal-s1 wt-e0
      have assigns-if False e0 ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx [elim-format]) (simp only: False)
      with s0-s1
      have dom (locals (store ((Norm s0)::state)))
             ∪ assigns-if False e0 ⊆ . . .
        by (rule Un-least)
      with da-e2 obtain E2' where
             da-e2': Env⊢ dom (locals (store s1)) »⟨e2⟩» E2' and
          nrm-E2-E2': nrm E2 ⊆ nrm E2'
        by (rule da-weakenE) iprover
      ultimately have nrm E2' ⊆ dom (locals (store s2))
        using wt-e2 G normal-s2 by simp
      with nrm-E2-E2' show ?thesis
        by (simp only: nrm-A) blast
    qed
  qed
qed
moreover
{
  fix j have abrupt s2 ≠ Some (Jump j)
  proof −
    have eval: prg Env⊢Norm s0 −e0 ? e1 : e2−≻v→ s2
      unfolding G by (rule eval.Cond Cond.hyps)+
    from Cond.prems
    obtain T' where  T=Inl T'
      by (elim wt-elim-cases) simp
    with Cond.prems have Env⊢e0 ? e1 : e2::−T' by simp
    from eval - this
    show ?thesis
      by (rule eval-expression-no-jump) (simp-all add: G wf)
  qed
}
hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
  by simp-all
ultimately show ?case by (intro conjI)
next
case (Call s0 e a s1 args vs s2 D mode statT mn pTs s3 s3' accC v s4
       Env T A)
note G = ‹prg Env = G›
have ?NormalAssigned (restore-lvars s2 s4) A
proof
  assume normal-restore-lvars: normal (restore-lvars s2 s4)
  show nrm A ⊆ dom (locals (store (restore-lvars s2 s4)))
  proof −
    from Call.prems obtain E where
         da-e: Env⊢ (dom (locals (store ((Norm s0)::state))))»⟨e⟩» E and
      da-args: Env⊢ nrm E »⟨args⟩» A
      by (elim da-elim-cases)
    from Call.prems obtain eT argsT where
         wt-e: Env⊢e::−eT and
      wt-args: Env⊢args::≐argsT
      by (elim wt-elim-cases)
    note s3 = ‹s3 = init-lvars G D (|name = mn, parTs = pTs|) mode a vs s2›
    note s3' = ‹s3' = check-method-access G accC statT mode
```

```
                                   (|name=mn,parTs=pTs|) a s3›
    have normal-s2: normal s2
    proof −
      from normal-restore-lvars have normal s4
        by simp
      then have normal s3'
        by − (rule eval-no-abrupt-lemma [rule-format], rule Call.hyps)
      with s3' have normal s3
        by (cases s3) (simp add: check-method-access-def Let-def)
      with s3 show normal s2
        by (cases s2) (simp add: init-lvars-def Let-def)
    qed
    then have normal-s1: normal s1
      by − (rule eval-no-abrupt-lemma [rule-format], rule Call.hyps)
    note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
    with da-e wt-e G normal-s1
    have nrm E ⊆ dom (locals (store s1))
      by simp
    with da-args obtain A' where
      da-args': Env⊢ dom (locals (store s1)) »⟨args⟩» A' and
      nrm-A-A': nrm A ⊆ nrm A'
      by (rule da-weakenE) iprover
    note ‹PROP ?Hyp (In3 args) s1 s2›
    with da-args' wt-args G normal-s2
    have nrm A' ⊆ dom (locals (store s2))
      by simp
    with nrm-A-A' have nrm A ⊆ dom (locals (store s2))
      by blast
    also have ... ⊆ dom (locals (store (restore-lvars s2 s4)))
      by (cases s4) simp
    finally show ?thesis .
  qed
qed
moreover
{
  fix j have abrupt (restore-lvars s2 s4) ≠ Some (Jump j)
  proof −
    have eval: prg Env⊢Norm s0 −({accC,statT,mode}e·mn( {pTs}args))−≻v
                         → (restore-lvars s2 s4)
      unfolding G by (rule eval.Call Call)+
    from Call.prems
    obtain T' where  T=Inl T'
      by (elim wt-elim-cases) simp
    with Call.prems have Env⊢({accC,statT,mode}e·mn( {pTs}args))::−T'
      by simp
    from eval - this
    show ?thesis
      by (rule eval-expression-no-jump) (simp-all add: G wf)
  qed
}
hence ?BreakAssigned (Norm s0) (restore-lvars s2 s4) A
  and ?ResAssigned (Norm s0) (restore-lvars s2 s4)
  by simp-all
ultimately show ?case by (intro conjI)
next
  case (Methd s0 D sig v s1 Env T A)
  note G = ‹prg Env = G›
  from Methd.prems obtain m where
     m:      methd (prg Env) D sig = Some m and
```

```
      da-body: Env⊢(dom (locals (store ((Norm s0)::state))))
                 »⟨Body (declclass m) (stmt (mbody (mthd m)))⟩» A
    by − (erule da-elim-cases)
  from Methd.prems m obtain
    isCls: is-class (prg Env) D and
    wt-body: Env ⊢In1l (Body (declclass m) (stmt (mbody (mthd m))))::T
    by − (erule wt-elim-cases,simp)
  note ‹PROP ?Hyp (In1l (body G D sig)) (Norm s0) s1›
  moreover
  from wt-body have Env⊢In1l (body G D sig)::T
    using isCls m G by (simp add: body-def2)
  moreover
  from da-body have Env⊢(dom (locals (store ((Norm s0)::state))))
                         »⟨body G D sig⟩» A
    using isCls m G by (simp add: body-def2)
  ultimately show ?case
    using G by simp
next
  case (Body s0 D s1 c s2 s3 Env T A)
  note G = ‹prg Env = G›
  from Body.prems
  have nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
    by (elim da-elim-cases) simp
  have eval: prg Env⊢Norm s0 −Body D c−≻the (locals (store s2) Result)
                         →abupd (absorb Ret) s3
    unfolding G by (rule eval.Body Body.hyps)+
  hence nrm A ⊆ dom (locals (store (abupd (absorb Ret) s3)))
    by  (simp only: nrm-A) (rule dom-locals-eval-mono-elim)
  hence ?NormalAssigned (abupd (absorb Ret) s3) A
    by simp
  moreover
  from eval have ⋀ j. abrupt (abupd (absorb Ret) s3) ≠ Some (Jump j)
    by (rule Body-no-jump) simp
  hence ?BreakAssigned (Norm s0) (abupd (absorb Ret) s3) A and
        ?ResAssigned (Norm s0) (abupd (absorb Ret) s3)
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (LVar s vn Env T A)
  from LVar.prems
  have nrm A = dom (locals (store ((Norm s)::state)))
    by (elim da-elim-cases) simp
  thus ?case by simp
next
  case (FVar s0 statDeclC s1 e a s2 v s2' stat fn s3 accC Env T A)
  note G = ‹prg Env = G›
  have ?NormalAssigned s3 A
  proof
    assume normal-s3: normal s3
    show nrm A ⊆ dom (locals (store s3))
    proof −
      note fvar = ‹(v, s2') = fvar statDeclC stat fn a s2› and
        s3 = ‹s3 = check-field-access G accC statDeclC fn stat a s2'›
      from FVar.prems
      have da-e: Env⊢ (dom (locals (store ((Norm s0)::state))))»⟨e⟩» A
        by (elim da-elim-cases)
      from FVar.prems obtain eT where
        wt-e: Env⊢e::−eT
        by (elim wt-elim-cases)
```

```
      have (dom (locals (store ((Norm s0)::state))))
             ⊆ dom (locals (store s1))
        by (rule dom-locals-eval-mono-elim) (rule FVar.hyps)
      with da-e obtain A′ where
        da-e′: Env⊢ dom (locals (store s1)) »⟨e⟩» A′ and
        nrm-A-A′: nrm A ⊆ nrm A′
        by (rule da-weakenE) iprover
      have normal-s2: normal s2
      proof −
        from normal-s3 s3
        have normal s2′
          by (cases s2′) (simp add: check-field-access-def Let-def)
        with fvar
        show normal s2
          by (cases s2) (simp add: fvar-def2)
      qed
      note ‹PROP ?Hyp (In1l e) s1 s2›
      with da-e′ wt-e G normal-s2
      have nrm A′ ⊆ dom (locals (store s2))
        by simp
      with nrm-A-A′ have nrm A ⊆ dom (locals (store s2))
        by blast
      also have . . . ⊆ dom (locals (store s3))
      proof −
        from fvar have s2′ = snd (fvar statDeclC stat fn a s2)
          by (cases fvar statDeclC stat fn a s2) simp
        hence dom (locals (store s2)) ⊆  dom (locals (store s2′))
          by (simp) (rule dom-locals-fvar-mono)
        also from s3 have . . . ⊆ dom (locals (store s3))
          by (cases s2′) (simp add: check-field-access-def Let-def)
        finally show ?thesis .
      qed
      finally show ?thesis .
    qed
  qed
  moreover
  {
    fix j have abrupt s3 ≠ Some (Jump j)
    proof −
      obtain w upd where v: (w,upd)=v
        by (cases v) auto
      have eval: prg Env⊢Norm s0−({accC,statDeclC,stat}e..fn)=≻(w,upd)→s3
        by (simp only: G v) (rule eval.FVar FVar.hyps)+
      from FVar.prems
      obtain T′ where  T=Inl T′
        by (elim wt-elim-cases) simp
      with FVar.prems have Env⊢({accC,statDeclC,stat}e..fn)::=T′
        by simp
      from eval - this
      show ?thesis
        by (rule eval-var-no-jump [THEN conjunct1]) (simp-all add: G wf)
    qed
  }
  hence ?BreakAssigned (Norm s0) s3 A and ?ResAssigned (Norm s0) s3
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (AVar s0 e1 a s1 e2 i s2 v s2′ Env T A)
  note G = ‹prg Env = G›
```

```
have ?NormalAssigned s2' A
proof
  assume normal-s2': normal s2'
  show nrm A ⊆ dom (locals (store s2'))
  proof −
    note avar = ‹(v, s2') = avar G i a s2›
    from AVar.prems obtain E1 where
      da-e1: Env⊢ (dom (locals (store ((Norm s0)::state))))»⟨e1⟩» E1 and
      da-e2: Env⊢ nrm E1 »⟨e2⟩» A
      by (elim da-elim-cases)
    from AVar.prems obtain e1T e2T where
         wt-e1: Env⊢e1::−e1T and
         wt-e2: Env⊢e2::−e2T
      by (elim wt-elim-cases)
    from avar normal-s2'
    have normal-s2: normal s2
      by (cases s2) (simp add: avar-def2)
    hence normal s1
      by − (rule eval-no-abrupt-lemma [rule-format], rule AVar, rule normal-s2)
    moreover note ‹PROP ?Hyp (In1l e1) (Norm s0) s1›
    ultimately have nrm E1 ⊆ dom (locals (store s1))
      using da-e1 wt-e1 G by simp
    with da-e2 obtain A' where
      da-e2': Env⊢ dom (locals (store s1)) »⟨e2⟩» A' and
      nrm-A-A': nrm A ⊆ nrm A'
      by (rule da-weakenE) iprover
    note ‹PROP ?Hyp (In1l e2) s1 s2›
    with da-e2' wt-e2 G normal-s2
    have nrm A' ⊆ dom (locals (store s2))
      by simp
    with nrm-A-A' have nrm A ⊆ dom (locals (store s2))
      by blast
    also have ... ⊆ dom (locals (store s2'))
    proof −
       from avar have s2' = snd (avar G i a s2)
        by (cases (avar G i a s2)) simp
      thus dom (locals (store s2)) ⊆  dom (locals (store s2'))
        by (simp) (rule dom-locals-avar-mono)
    qed
    finally show ?thesis .
  qed
qed
moreover
{
  fix j have abrupt s2' ≠ Some (Jump j)
  proof −
    obtain w upd where v: (w,upd)=v
      by (cases v) auto
    have eval: prg Env⊢Norm s0−(e1.[e2])=≻(w,upd)→s2'
      unfolding G v by (rule eval.AVar AVar.hyps)+
    from AVar.prems
    obtain T' where  T=Inl T'
      by (elim wt-elim-cases) simp
    with AVar.prems have Env⊢(e1.[e2])::=T'
      by simp
    from eval - this
    show ?thesis
      by (rule eval-var-no-jump [THEN conjunct1]) (simp-all add: G wf)
  qed
```

```
    }
    hence ?BreakAssigned (Norm s0) s2' A and ?ResAssigned (Norm s0) s2'
      by simp-all
    ultimately show ?case by (intro conjI)
  next
    case (Nil s0 Env T A)
    from Nil.prems
    have nrm A = dom (locals (store ((Norm s0)::state)))
      by (elim da-elim-cases) simp
    thus ?case by simp
  next
    case (Cons s0 e v s1 es vs s2 Env T A)
    note G = ‹prg Env = G›
    have ?NormalAssigned s2 A
    proof
      assume normal-s2: normal s2
      show nrm A ⊆ dom (locals (store s2))
      proof -
        from Cons.prems obtain E where
          da-e: Env⊢ (dom (locals (store ((Norm s0)::state))))»⟨e⟩» E and
          da-es: Env⊢ nrm E »⟨es⟩» A
          by (elim da-elim-cases)
        from Cons.prems obtain eT esT where
            wt-e: Env⊢e::−eT and
            wt-es: Env⊢es::≐esT
          by (elim wt-elim-cases)
        have normal s1
          by − (rule eval-no-abrupt-lemma [rule-format], rule Cons.hyps, rule normal-s2)
        moreover note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
        ultimately have nrm E ⊆ dom (locals (store s1))
          using da-e wt-e G by simp
        with da-es obtain A' where
          da-es': Env⊢ dom (locals (store s1)) »⟨es⟩» A' and
          nrm-A-A': nrm A ⊆ nrm A'
          by (rule da-weakenE) iprover
        note ‹PROP ?Hyp (In3 es) s1 s2›
        with da-es' wt-es G normal-s2
        have nrm A' ⊆ dom (locals (store s2))
          by simp
        with nrm-A-A' show nrm A ⊆ dom (locals (store s2))
          by blast
      qed
    qed
    moreover
    {
      fix j have abrupt s2 ≠ Some (Jump j)
      proof −
        have eval: prg Env⊢Norm s0−(e # es)≐≻v#vs→s2
          unfolding G by (rule eval.Cons Cons.hyps)+
        from Cons.prems
        obtain T' where  T=Inr T'
          by (elim wt-elim-cases) simp
        with Cons.prems have Env⊢(e # es)::≐T'
          by simp
        from eval - this
        show ?thesis
          by (rule eval-expression-list-no-jump) (simp-all add: G wf)
      qed
    }
```

hence *?BreakAssigned* (*Norm s0*) *s2 A* **and** *?ResAssigned* (*Norm s0*) *s2*
by *simp-all*
ultimately show *?case* **by** (*intro conjI*)
qed
qed

lemma *da-good-approxE*:
assumes
prg Env$\vdash$*s0* $-t\succ\rightarrow$ (*v*, *s1*) **and** *Env*$\vdash$*t*::*T* **and**
Env$\vdash$ *dom* (*locals* (*store s0*)) »*t*» *A* **and** *wf-prog* (*prg Env*)
obtains
normal s1 $\Longrightarrow$ *nrm A* $\subseteq$ *dom* (*locals* (*store s1*)) **and**
$\bigwedge$ *l*. $[\![$*abrupt s1* = *Some* (*Jump* (*Break l*)); *normal s0*$]\!]$
$\Longrightarrow$ *brk A l* $\subseteq$ *dom* (*locals* (*store s1*)) **and**
$[\![$*abrupt s1* = *Some* (*Jump Ret*);*normal s0*$]\!]$$\Longrightarrow$*Result* $\in$ *dom* (*locals* (*store s1*))
using *assms* **by** − (*drule* (*3*) *da-good-approx*, *simp*)

lemma *da-good-approxE'*:
assumes *eval*: *G*$\vdash$*s0* $-t\succ\rightarrow$ (*v*, *s1*)
and *wt*: (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)$\vdash$*t*::*T*
and *da*: (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)$\vdash$ *dom* (*locals* (*store s0*)) »*t*» *A*
and *wf*: *wf-prog G*
obtains *normal s1* $\Longrightarrow$ *nrm A* $\subseteq$ *dom* (*locals* (*store s1*)) **and**
$\bigwedge$ *l*. $[\![$*abrupt s1* = *Some* (*Jump* (*Break l*)); *normal s0*$]\!]$
$\Longrightarrow$ *brk A l* $\subseteq$ *dom* (*locals* (*store s1*)) **and**
$[\![$*abrupt s1* = *Some* (*Jump Ret*);*normal s0*$]\!]$
$\Longrightarrow$ *Result* $\in$ *dom* (*locals* (*store s1*))
proof −
from *eval* **have** *prg* (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)$\vdash$*s0* $-t\succ\rightarrow$ (*v*, *s1*) **by** *simp*
moreover note *wt da*
moreover from *wf* **have** *wf-prog* (*prg* (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)) **by** *simp*
ultimately show *thesis*
using *that* **by** (*rule da-good-approxE*) *iprover*+
qed

declare [[*simproc add*: *wt-expr wt-var wt-exprs wt-stmt*]]

end

Chapter 19

TypeSafe

1 The type soundness proof for Java

theory *TypeSafe*
imports *DefiniteAssignmentCorrect Conform*
begin

error free

lemma *error-free-halloc*:
  assumes *halloc*: $G\vdash s0\ -halloc\ oi\succ a\rightarrow\ s1$ **and**
    error-free-s0: *error-free s0*
  shows *error-free s1*
proof −
  from *halloc error-free-s0*
  obtain *abrupt0 store0 abrupt1 store1*
    where *eqs*: $s0=(abrupt0,store0)$ $s1=(abrupt1,store1)$ **and**
      halloc': $G\vdash(abrupt0,store0)\ -halloc\ oi\succ a\rightarrow\ (abrupt1,store1)$ **and**
      error-free-s0': *error-free* $(abrupt0,store0)$
    by (*cases s0*,*cases s1*) *auto*
  from *halloc' error-free-s0'*
  have *error-free* $(abrupt1,store1)$
  proof (*induct*)
    case *Abrupt*
    then show *?case* **.**
  next
    case *New*
    then show *?case*
      by *auto*
  qed
  with *eqs*
  show *?thesis*
    by *simp*
qed

lemma *error-free-sxalloc*:
  assumes *sxalloc*: $G\vdash s0\ -sxalloc\rightarrow\ s1$ **and** *error-free-s0*: *error-free s0*
  shows *error-free s1*
proof −
  from *sxalloc error-free-s0*
  obtain *abrupt0 store0 abrupt1 store1*
    where *eqs*: $s0=(abrupt0,store0)$ $s1=(abrupt1,store1)$ **and**
      sxalloc': $G\vdash(abrupt0,store0)\ -sxalloc\rightarrow\ (abrupt1,store1)$ **and**
      error-free-s0': *error-free* $(abrupt0,store0)$

by (*cases s0*,*cases s1*) *auto*
from *sxalloc′ error-free-s0′*
have *error-free* (*abrupt1*,*store1*)
proof (*induct*)
qed (*auto*)
with *eqs*
show *?thesis*
by *simp*
qed

lemma *error-free-check-field-access-eq*:
error-free (*check-field-access G accC statDeclC fn stat a s*)
$\Longrightarrow$ (*check-field-access G accC statDeclC fn stat a s*) = *s*
apply (*cases s*)
apply (*auto simp add*: *check-field-access-def Let-def error-free-def*
abrupt-if-def
split: *if-split-asm*)
done

lemma *error-free-check-method-access-eq*:
error-free (*check-method-access G accC statT mode sig a′ s*)
$\Longrightarrow$ (*check-method-access G accC statT mode sig a′ s*) = *s*
apply (*cases s*)
apply (*auto simp add*: *check-method-access-def Let-def error-free-def*
abrupt-if-def)
done

lemma *error-free-FVar-lemma*:
error-free s
$\Longrightarrow$ *error-free* (*abupd* (*if stat then id else np a*) *s*)
by (*case-tac s*) *auto*

lemma *error-free-init-lvars* [*simp*,*intro*]:
error-free s $\Longrightarrow$
error-free (*init-lvars G C sig mode a pvs s*)
by (*cases s*) (*auto simp add*: *init-lvars-def Let-def*)

lemma *error-free-LVar-lemma*:
error-free s $\Longrightarrow$ *error-free* (*assign* (λv. *supd lupd*($vn \mapsto v$)) *w s*)
by (*cases s*) *simp*

lemma *error-free-throw* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* (*abupd* (*throw x*) *s*)
by (*cases s*) (*simp add*: *throw-def*)

result conformance

definition
assign-conforms :: *st* $\Rightarrow$ (*val* $\Rightarrow$ *state* $\Rightarrow$ *state*) $\Rightarrow$ *ty* $\Rightarrow$ *env′* $\Rightarrow$ *bool* (-$\leq$|-$\preceq$-::$\preceq$- [*71*,*71*,*71*,*71*] *70*)
where
$s\leq|f\preceq T::\preceq E$ =
(($\forall s'$ *w*. *Norm* $s'::\preceq E \longrightarrow$ *fst* $E,s'\vdash w::\preceq T \longrightarrow s\leq|s' \longrightarrow$ *assign f w* (*Norm* s')$::\preceq E$) $\wedge$
($\forall s'$ *w*. *error-free* $s' \longrightarrow$ (*error-free* (*assign f w* s'))))

definition
$rconf :: prog \Rightarrow lenv \Rightarrow st \Rightarrow term \Rightarrow vals \Rightarrow tys \Rightarrow bool$ (-,-,-⊢-≻-::≼- [71,71,71,71,71,71] 70)
where
$G,L,s\vdash t\succ v::\preceq T =$
$(case\ T\ of$
$\quad Inl\ T \Rightarrow if\ (\exists\ var.\ t=In2\ var)$
$\qquad then\ (\forall\ n.\ (the\text{-}In2\ t) = LVar\ n$
$\qquad\quad \longrightarrow (fst\ (the\text{-}In2\ v) = the\ (locals\ s\ n)) \wedge$
$\qquad\qquad (locals\ s\ n \neq None \longrightarrow G,s\vdash fst\ (the\text{-}In2\ v)::\preceq T)) \wedge$
$\qquad\quad (\neg\ (\exists\ n.\ the\text{-}In2\ t=LVar\ n) \longrightarrow (G,s\vdash fst\ (the\text{-}In2\ v)::\preceq T))\wedge$
$\qquad\quad (s\leq|snd\ (the\text{-}In2\ v)\preceq T::\preceq(G,L))$
$\qquad else\ G,s\vdash the\text{-}In1\ v::\preceq T$
$\quad |\ Inr\ Ts \Rightarrow list\text{-}all2\ (conf\ G\ s)\ (the\text{-}In3\ v)\ Ts)$

With *rconf* we describe the conformance of the result value of a term. This definition gets rather complicated because of the relations between the injections of the different terms, types and values. The main case distinction is between single values and value lists. In case of value lists, every value has to conform to its type. For single values we have to do a further case distinction, between values of variables $\exists var.\ t = In2\ var$ and ordinary values. Values of variables are modelled as pairs consisting of the current value and an update function which will perform an assignment to the variable. This stems form the decision, that we only have one evaluation rule for each kind of variable. The decision if we read or write to the variable is made by syntactic enclosing rules. So conformance of variable-values must ensure that both the current value and an update will conform to the type. With the introduction of definite assignment of local variables we have to do another case distinction. For the notion of conformance local variables are allowed to be *None*, since the definedness is not ensured by conformance but by definite assignment. Field and array variables must contain a value.

lemma *rconf-In1* [*simp*]:
$G,L,s\vdash In1\ ec\succ In1\ v ::\preceq Inl\ T\ =\ G,s\vdash v::\preceq T$
apply (*unfold rconf-def*)
apply (*simp* (*no-asm*))
done

lemma *rconf-In2-no-LVar* [*simp*]:
$\forall\ n.\ va\neq LVar\ n \Longrightarrow$
$\quad G,L,s\vdash In2\ va\succ In2\ vf::\preceq Inl\ T\ = (G,s\vdash fst\ vf::\preceq T \wedge s\leq|snd\ vf\preceq T::\preceq(G,L))$
apply (*unfold rconf-def*)
apply *auto*
done

lemma *rconf-In2-LVar* [*simp*]:
$va=LVar\ n \Longrightarrow$
$\quad G,L,s\vdash In2\ va\succ In2\ vf::\preceq Inl\ T$
$\quad = ((fst\ vf = the\ (locals\ s\ n)) \wedge$
$\qquad (locals\ s\ n \neq None \longrightarrow G,s\vdash fst\ vf::\preceq T) \wedge s\leq|snd\ vf\preceq T::\preceq(G,L))$
apply (*unfold rconf-def*)
by *simp*

lemma *rconf-In3* [*simp*]:
$G,L,s\vdash In3\ es\succ In3\ vs::\preceq Inr\ Ts = list\text{-}all2\ (\lambda v\ T.\ G,s\vdash v::\preceq T)\ vs\ Ts$
apply (*unfold rconf-def*)
apply (*simp* (*no-asm*))

done

fits and conf

lemma *conf-fits*: $G,s\vdash v{::}\preceq T \Longrightarrow G,s\vdash v$ *fits* T
apply (*unfold fits-def*)
apply *clarify*
apply (*erule contrapos-np*, *simp* (*no-asm-use*))
apply (*drule conf-RefTD*)
apply *auto*
**done*

lemma *fits-conf*:
$[\![G,s\vdash v{::}\preceq T;\ G\vdash T\preceq?\ T';\ G,s\vdash v\ \mathit{fits}\ T';\ \mathit{ws\text{-}prog}\ G]\!] \Longrightarrow G,s\vdash v{::}\preceq T'$
apply (*auto dest*!: *fitsD cast-PrimT2 cast-RefT2*)
apply (*force dest*: *conf-RefTD intro*: *conf-AddrI*)
done

lemma *fits-Array*:
$[\![G,s\vdash v{::}\preceq T;\ G\vdash T'.[]\preceq T.[];\ G,s\vdash v\ \mathit{fits}\ T';\ \mathit{ws\text{-}prog}\ G]\!] \Longrightarrow G,s\vdash v{::}\preceq T'$
apply (*auto dest*!: *fitsD widen-ArrayPrimT widen-ArrayRefT*)
apply (*force dest*: *conf-RefTD intro*: *conf-AddrI*)
done

gext

lemma *halloc-gext*: $\bigwedge s1\ s2.\ G\vdash s1\ -\mathit{halloc}\ oi\succ a\rightarrow\ s2 \Longrightarrow \mathit{snd}\ s1\leq|\mathit{snd}\ s2$
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*erule halloc.induct*)
apply (*auto dest*!: *new-AddrD*)
done

lemma *sxalloc-gext*: $\bigwedge s1\ s2.\ G\vdash s1\ -\mathit{sxalloc}\rightarrow\ s2 \Longrightarrow \mathit{snd}\ s1\leq|\mathit{snd}\ s2$
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*erule sxalloc.induct*)
apply (*auto dest*!: *halloc-gext*)
done

lemma *eval-gext-lemma* [*rule-format* (*no-asm*)]:
$G\vdash s\ -t\succ\rightarrow\ (w,s') \Longrightarrow \mathit{snd}\ s\leq|\mathit{snd}\ s' \wedge (\mathit{case}\ w\ \mathit{of}$
$\quad In1\ v \Rightarrow True$
$\ |\ In2\ vf \Rightarrow \mathit{normal}\ s \longrightarrow (\forall\ v\ x\ s.\ s\leq|\mathit{snd}\ (\mathit{assign}\ (\mathit{snd}\ vf)\ v\ (x,s)))$
$\ |\ In3\ vs \Rightarrow True)$
apply (*erule eval-induct*)
prefer *26*
apply (*case-tac inited C* (*globs s0*), *clarsimp*, *erule thin-rl*)
apply (*auto del*: *conjI dest*!: *not-initedD gext-new sxalloc-gext halloc-gext*
simp add: *lvar-def fvar-def2 avar-def2 init-lvars-def2*
check-field-access-def check-method-access-def Let-def
split del: *if-split-asm split*: *sum3.split*)

apply *force+*
done

lemma *evar-gext-f*:
  G⊢Norm s1 −e=≻vf → s2 ⟹ s≤|snd (assign (snd vf) v (x,s))
apply (*drule eval-gext-lemma* [*THEN conjunct2*])
apply *auto*
done

lemmas *eval-gext* = *eval-gext-lemma* [*THEN conjunct1*]

lemma *eval-gext'*: *G⊢(x1,s1) −t≻→ (w,(x2,s2)) ⟹ s1≤|s2*
apply (*drule eval-gext*)
apply *auto*
done

lemma *init-yields-initd*: *G⊢Norm s1 −Init C→ s2 ⟹ initd C s2*
apply (*erule eval-cases* , *auto split del*: *if-split-asm*)
apply (*case-tac inited C* (*globs s1*))
apply (*clarsimp split del*: *if-split-asm*)+
apply (*drule eval-gext'*)+
apply (*drule init-class-obj-inited*)
apply (*erule inited-gext*)
apply (*simp* (*no-asm-use*))
done

Lemmas

lemma *obj-ty-obj-class1*:
 ⟦*wf-prog G*; *is-type G* (*obj-ty obj*)⟧ ⟹ *is-class G* (*obj-class obj*)
apply (*case-tac tag obj*)
apply (*auto simp add*: *obj-ty-def obj-class-def*)
done

lemma *oconf-init-obj*:
 ⟦*wf-prog G*;
 (*case r of Heap a ⇒ is-type G* (*obj-ty obj*) | *Stat C ⇒ is-class G C*)
 ⟧ ⟹ *G,s⊢obj* (|*values*:=*init-vals* (*var-tys G* (*tag obj*) *r*)|)::≼√*r*
apply (*auto intro*!: *oconf-init-obj-lemma unique-fields*)
done

lemma *conforms-newG*: ⟦*globs s oref* = *None*; (*x*, *s*)::≼(*G*,*L*);
  wf-prog G; *case oref of Heap a ⇒ is-type G* (*obj-ty* (|*tag*=*oi*,*values*=*vs*|))
                        | *Stat C ⇒ is-class G C*⟧ ⟹
  (*x*, *init-obj G oi oref s*)::≼(*G*, *L*)
apply (*unfold init-obj-def*)
apply (*auto elim*!: *conforms-gupd dest*!: *oconf-init-obj*
)
done

lemma *conforms-init-class-obj*:
 ⟦(*x*,*s*)::≼(*G*, *L*); *wf-prog G*; *class G C*=*Some y*; ¬ *inited C* (*globs s*)⟧ ⟹
  (*x*,*init-class-obj G C s*)::≼(*G*, *L*)
apply (*rule not-initedD* [*THEN conforms-newG*])
apply (*auto*)
done

lemma *fst-init-lvars*[*simp*]:
fst (*init-lvars G C sig* (*invmode m e*) *a′ pvs* (*x*,*s*)) =
(*if is-static m then x else* (*np a′*) *x*)
apply (*simp* (*no-asm*) *add*: *init-lvars-def2*)
done

lemma *halloc-conforms*: ⋀*s1*. ⟦*G*⊢*s1* −*halloc oi*≻*a*→ *s2*; *wf-prog G*; *s1*::≼(*G*, *L*);
is-type G (*obj-ty* (|*tag*=*oi*,*values*=*fs*|))⟧ ⟹ *s2*::≼(*G*, *L*)
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*case-tac aa*)
apply (*auto elim*!: *halloc-elim-cases dest*!: *new-AddrD*
intro!: *conforms-newG* [*THEN conforms-xconf*] *conf-AddrI*)
done

lemma *halloc-type-sound*:
⋀*s1*. ⟦*G*⊢*s1* −*halloc oi*≻*a*→ (*x*,*s*); *wf-prog G*; *s1*::≼(*G*, *L*);
T = *obj-ty* (|*tag*=*oi*,*values*=*fs*|); *is-type G T*⟧ ⟹
(*x*,*s*)::≼(*G*, *L*) ∧ (*x* = *None* ⟶ *G*,*s*⊢*Addr a*::≼*T*)
apply (*auto elim*!: *halloc-conforms*)
apply (*case-tac aa*)
apply (*subst obj-ty-eq*)
apply (*auto elim*!: *halloc-elim-cases dest*!: *new-AddrD intro*!: *conf-AddrI*)
done

lemma *sxalloc-type-sound*:
⋀*s1 s2*. ⟦*G*⊢*s1* −*sxalloc*→ *s2*; *wf-prog G*⟧ ⟹
case fst s1 of
None ⇒ *s2* = *s1*
| *Some abr* ⇒ (*case abr of*
Xcpt x ⇒ (∃ *a*. *fst s2* = *Some*(*Xcpt* (*Loc a*)) ∧
(∀ *L*. *s1*::≼(*G*,*L*) ⟶ *s2*::≼(*G*,*L*)))
| *Jump j* ⇒ *s2* = *s1*
| *Error e* ⇒ *s2* = *s1*)
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*erule sxalloc.induct*)
apply *auto*
apply (*rule halloc-conforms* [*THEN conforms-xconf*])
apply (*auto elim*!: *halloc-elim-cases dest*!: *new-AddrD intro*!: *conf-AddrI*)
done

lemma *wt-init-comp-ty*:
is-acc-type G (*pid C*) *T* ⟹ (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*init-comp-ty T*::√
apply (*unfold init-comp-ty-def*)
apply (*clarsimp simp add*: *accessible-in-RefT-simp*
is-acc-type-def is-acc-class-def)
done

declare *fun-upd-same* [*simp*]

declare *fun-upd-apply* [*simp del*]

definition
  DynT-prop :: [*prog*,*inv-mode*,*qtname*,*ref-ty*] ⇒ *bool* (-⊢-→-⪯-[71,71,71,71]70)
where
  $G\vdash mode\rightarrow D\preceq t$ = (*mode* = *IntVir* ⟶ *is-class G D* ∧
                    (*if* (∃ *T*. *t*=*ArrayT T*) *then D*=*Object else G*⊢*Class D*⪯*RefT t*))

lemma *DynT-propI*:
⟦(*x*,*s*)::⪯(*G*, *L*); *G*,*s*⊢*a*′::⪯*RefT statT*; *wf-prog G*; *mode* = *IntVir* ⟶ *a*′ ≠ *Null*⟧
 ⟹ *G*⊢*mode*→*invocation-class mode s a*′ *statT*⪯*statT*
proof (*unfold DynT-prop-def*)
  assume *state-conform*: (*x*,*s*)::⪯(*G*, *L*)
    and   *statT-a*′: *G*,*s*⊢*a*′::⪯*RefT statT*
    and   *wf*: *wf-prog G*
    and   *mode*: *mode* = *IntVir* ⟶ *a*′ ≠ *Null*
  let *?invCls* = (*invocation-class mode s a*′ *statT*)
  let *?IntVir* = *mode* = *IntVir*
  let *?Concl* = λ*invCls*. *is-class G invCls* ∧
            (*if* ∃ *T*. *statT* = *ArrayT T*
               then invCls = *Object*
               else G⊢*Class invCls*⪯*RefT statT*)
  show *?IntVir* ⟶ *?Concl ?invCls*
  proof
   assume *modeIntVir*: *?IntVir*
   with *mode* **have** *not-Null*: *a*′ ≠ *Null* **..**
   from *statT-a*′ *not-Null state-conform*
   obtain *a obj*
    where *obj-props*: *a*′ = *Addr a globs s* (*Inl a*) = *Some obj*
            G⊢*obj-ty obj*⪯*RefT statT is-type G* (*obj-ty obj*)
    by (*blast dest*: *conforms-RefTD*)
   show *?Concl ?invCls*
   proof (*cases tag obj*)
    case *CInst*
    with *modeIntVir obj-props*
    show *?thesis*
     by (*auto dest*!: *widen-Array2*)
   next
    case *Arr*
    from *Arr* **obtain** *T* **where** *obj-ty obj* = *T*.[] **by** *blast*
    moreover from *Arr* **have** *obj-class obj* = *Object*
     by *blast*
    moreover note *modeIntVir obj-props wf*
    ultimately show *?thesis* **by** (*auto dest*!: *widen-Array*)
   qed
  qed
qed

lemma *invocation-methd*:
⟦*wf-prog G*; *statT* ≠ *NullT*;
 (∀ *statC*. *statT* = *ClassT statC* ⟶ *is-class G statC*);
 (∀ *I*. *statT* = *IfaceT I* ⟶ *is-iface G I* ∧ *mode* ≠ *SuperM*);
 (∀ *T*. *statT* = *ArrayT T* ⟶ *mode* ≠ *SuperM*);
 G⊢*mode*→*invocation-class mode s a*′ *statT*⪯*statT*;
 dynlookup G statT (*invocation-class mode s a*′ *statT*) *sig* = *Some m* ⟧
⟹ *methd G* (*invocation-declclass G mode s a*′ *statT sig*) *sig* = *Some m*
proof −
  assume   *wf*: *wf-prog G*
    and *not-NullT*: *statT* ≠ *NullT*

and *statC-prop*: $(\forall\ statC.\ statT = ClassT\ statC \longrightarrow is\text{-}class\ G\ statC)$
and *statI-prop*: $(\forall\ I.\ statT = IfaceT\ I \longrightarrow is\text{-}iface\ G\ I \wedge mode \neq SuperM)$
and *statA-prop*: $(\forall\ \ T.\ statT = ArrayT\ T \longrightarrow mode \neq SuperM)$
and *invC-prop*: $G\vdash mode\rightarrow invocation\text{-}class\ mode\ s\ a'\ statT\preceq statT$
and *dynlookup*: $dynlookup\ G\ statT\ (invocation\text{-}class\ mode\ s\ a'\ statT)\ sig$
$= Some\ m$

show *?thesis*
proof (*cases statT*)
case *NullT*
with *not-NullT* **show** *?thesis* **by** *simp*
next
case *IfaceT*
with *statI-prop* **obtain** *I*
where *statI*: $statT = IfaceT\ I$ **and**
is-iface: *is-iface G I* **and**
not-SuperM: $mode \neq SuperM$ **by** *blast*

show *?thesis*
proof (*cases mode*)
case *Static*
with *wf dynlookup statI is-iface*
show *?thesis*
by (*auto simp add*: *invocation-declclass-def dynlookup-def*
dynimethd-def dynmethd-C-C
intro: *dynmethd-declclass*
dest!: *wf-imethdsD*
dest: *table-of-map-SomeI*)
next
case *SuperM*
with *not-SuperM* **show** *?thesis* **..**
next
case *IntVir*
with *wf dynlookup IfaceT invC-prop* **show** *?thesis*
by (*auto simp add*: *invocation-declclass-def dynlookup-def dynimethd-def*
DynT-prop-def
intro: *methd-declclass dynmethd-declclass*)
qed
next
case *ClassT*
show *?thesis*
proof (*cases mode*)
case *Static*
with *wf ClassT dynlookup statC-prop*
show *?thesis* **by** (*auto simp add*: *invocation-declclass-def dynlookup-def*
intro: *dynmethd-declclass*)
next
case *SuperM*
with *wf ClassT dynlookup statC-prop*
show *?thesis* **by** (*auto simp add*: *invocation-declclass-def dynlookup-def*
intro: *dynmethd-declclass*)
next
case *IntVir*
with *wf ClassT dynlookup statC-prop invC-prop*
show *?thesis*
by (*auto simp add*: *invocation-declclass-def dynlookup-def dynimethd-def*
DynT-prop-def
intro: *dynmethd-declclass*)
qed
next

case *ArrayT*
show *?thesis*
proof (*cases mode*)
case *Static*
with *wf ArrayT dynlookup* **show** *?thesis*
by (*auto simp add*: *invocation-declclass-def dynlookup-def*
dynimethd-def dynmethd-C-C
intro: *dynmethd-declclass*
dest: *table-of-map-SomeI*)
next
case *SuperM*
with *ArrayT statA-prop* **show** *?thesis* **by** *blast*
next
case *IntVir*
with *wf ArrayT dynlookup invC-prop* **show** *?thesis*
by (*auto simp add*: *invocation-declclass-def dynlookup-def dynimethd-def*
DynT-prop-def dynmethd-C-C
intro: *dynmethd-declclass*
dest: *table-of-map-SomeI*)
qed
qed
qed

lemma *DynT-mheadsD*:
⟦*G*⊢*invmode sm e*→*invC*⪯*statT*;
wf-prog G; ⦇*prg*=*G*,*cls*=*C*,*lcl*=*L*⦈⊢*e*::−*RefT statT*;
(*statDeclT*,*sm*) ∈ *mheads G C statT sig*;
invC = *invocation-class* (*invmode sm e*) *s a′ statT*;
declC =*invocation-declclass G* (*invmode sm e*) *s a′ statT sig*
⟧ ⟹
∃ *dm*.
methd G declC sig = *Some dm* ∧ *dynlookup G statT invC sig* = *Some dm* ∧
G⊢*resTy* (*mthd dm*)⪯*resTy sm* ∧
wf-mdecl G declC (*sig*, *mthd dm*) ∧
declC = *declclass dm* ∧
is-static dm = *is-static sm* ∧
is-class G invC ∧ *is-class G declC* ∧ *G*⊢*invC*$\preceq_C$ *declC* ∧
(*if invmode sm e* = *IntVir*
then (∀ *statC*. *statT*=*ClassT statC* ⟶ *G*⊢*invC* $\preceq_C$ *statC*)
else ((∃ *statC*. *statT*=*ClassT statC* ∧ *G*⊢*statC*$\preceq_C$ *declC*)
∨ (∀ *statC*. *statT*≠*ClassT statC* ∧ *declC*=*Object*)) ∧
statDeclT = *ClassT* (*declclass dm*))
proof −
assume *invC-prop*: *G*⊢*invmode sm e*→*invC*⪯*statT*
and *wf*: *wf-prog G*
and *wt-e*: ⦇*prg*=*G*,*cls*=*C*,*lcl*=*L*⦈⊢*e*::−*RefT statT*
and *sm*: (*statDeclT*,*sm*) ∈ *mheads G C statT sig*
and *invC*: *invC* = *invocation-class* (*invmode sm e*) *s a′ statT*
and *declC*: *declC* =
invocation-declclass G (*invmode sm e*) *s a′ statT sig*
from *wt-e wf* **have** *type-statT*: *is-type G* (*RefT statT*)
by (*auto dest*: *ty-expr-is-type*)
from *sm* **have** *not-Null*: *statT* ≠ *NullT* **by** *auto*
from *type-statT*
have *wf-C*: (∀ *statC*. *statT* = *ClassT statC* ⟶ *is-class G statC*)
by (*auto*)
from *type-statT wt-e*
have *wf-I*: (∀*I*. *statT* = *IfaceT I* ⟶ *is-iface G I* ∧

invmode sm e ≠ SuperM)
by (*auto dest*: *invocationTypeExpr-noClassD*)
from *wt-e*
have *wf-A*: (∀ *T*. *statT* = *ArrayT T* ⟶ *invmode sm e* ≠ *SuperM*)
by (*auto dest*: *invocationTypeExpr-noClassD*)
show *?thesis*
proof (*cases invmode sm e* = *IntVir*)
case *True*
with *invC-prop not-Null*
have *invC-prop′*: *is-class G invC* ∧
(*if* (∃ *T*. *statT*=*ArrayT T*) *then invC*=*Object*
else G⊢*Class invC*≼*RefT statT*)
by (*auto simp add*: *DynT-prop-def*)
from *True*
have ¬ *is-static sm*
by (*simp add*: *invmode-IntVir-eq member-is-static-simp*)
with *invC-prop′ not-Null*
have *G*,*statT* ⊢ *invC valid-lookup-cls-for* (*is-static sm*)
by (*cases statT*) *auto*
with *sm wf type-statT* **obtain** *dm* **where**
dm: *dynlookup G statT invC sig* = *Some dm* **and**
resT-dm: *G*⊢*resTy* (*mthd dm*)≼*resTy sm* **and**
static: *is-static dm* = *is-static sm*
by − (*drule dynamic-mheadsD*,*force*+)
with *declC invC not-Null*
have *declC′*: *declC* = (*declclass dm*)
by (*auto simp add*: *invocation-declclass-def*)
with *wf invC declC not-Null wf-C wf-I wf-A invC-prop dm*
have *dm′*: *methd G declC sig* = *Some dm*
by − (*drule invocation-methd*,*auto*)
from *wf dm invC-prop′ declC′ type-statT*
have *declC-prop*: *G*⊢*invC* $\preceq_C$ *declC* ∧ *is-class G declC*
by (*auto dest*: *dynlookup-declC*)
from *wf dm′ declC-prop declC′*
have *wf-dm*: *wf-mdecl G declC* (*sig*,(*mthd dm*))
by (*auto dest*: *methd-wf-mdecl*)
from *invC-prop′*
have *statC-prop*: (∀ *statC*. *statT*=*ClassT statC* ⟶ *G*⊢*invC* $\preceq_C$ *statC*)
by *auto*
from *True dm′ resT-dm wf-dm invC-prop′ declC-prop statC-prop declC′ static dm*
show *?thesis* **by** *auto*
next
case *False*
with *type-statT wf invC not-Null wf-I wf-A*
have *invC-prop′*: *is-class G invC* ∧
((∃ *statC*. *statT*=*ClassT statC* ∧ *invC*=*statC*) ∨
(∀ *statC*. *statT*≠*ClassT statC* ∧ *invC*=*Object*))
by (*case-tac statT*) (*auto simp add*: *invocation-class-def*
split: *inv-mode.splits*)
with *not-Null wf*
have *dynlookup-static*: *dynlookup G statT invC sig* = *methd G invC sig*
by (*case-tac statT*) (*auto simp add*: *dynlookup-def dynmethd-C-C*
dynimethd-def)
from *sm wf wt-e not-Null False invC-prop′* **obtain** *dm* **where**
dm: *methd G invC sig* = *Some dm* **and**
eq-declC-sm-dm:*statDeclT* = *ClassT* (*declclass dm*) **and**
eq-mheads:*sm*=*mhead* (*mthd dm*)
by − (*drule static-mheadsD*, (*force dest*: *accmethd-SomeD*)+)

then have *static*: *is-static dm = is-static sm* **by** − (*auto*)
with *declC invC dynlookup-static dm*
have *declC'*: *declC* = (*declclass dm*)
by (*auto simp add*: *invocation-declclass-def*)
from *invC-prop' wf declC' dm*
have *dm'*: *methd G declC sig = Some dm*
by (*auto intro*: *methd-declclass*)
from *dynlookup-static dm*
have *dm''*: *dynlookup G statT invC sig = Some dm*
by *simp*
from *wf dm invC-prop' declC' type-statT*
have *declC-prop*: $G \vdash invC \preceq_C declC \wedge$ *is-class G declC*
by (*auto dest*: *methd-declC*)
then have *declC-prop1*: $invC{=}Object \longrightarrow declC{=}Object$ **by** *auto*
from *wf dm' declC-prop declC'*
have *wf-dm*: *wf-mdecl G declC* (*sig*,(*mthd dm*))
by (*auto dest*: *methd-wf-mdecl*)
from *invC-prop' declC-prop declC-prop1*
have *statC-prop*: $(\ (\exists\ statC.\ statT{=}ClassT\ statC \wedge G \vdash statC \preceq_C declC)$
$\vee\ (\forall\ statC.\ statT{\neq}ClassT\ statC \wedge declC{=}Object))$
by *auto*
from *False dm' dm'' wf-dm invC-prop' declC-prop statC-prop declC'*
eq-declC-sm-dm eq-mheads static
show *?thesis* **by** *auto*
qed
qed

corollary *DynT-mheadsE* [*consumes 7*]:
— Same as *DynT-mheadsD* but better suited for application in typesafety proof
assumes *invC-compatible*: $G \vdash mode \rightarrow invC \preceq statT$
and *wf*: *wf-prog G*
and *wt-e*: $(\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash e{::}{-}RefT\ statT$
and *mheads*: $(statDeclT,sm) \in$ *mheads G C statT sig*
and *mode*: *mode=invmode sm e*
and *invC*: *invC = invocation-class mode s a' statT*
and *declC*: *declC =invocation-declclass G mode s a' statT sig*
and *dm*: $\bigwedge$ *dm*. $[\![$*methd G declC sig = Some dm*;
dynlookup G statT invC sig = Some dm;
$G \vdash resTy\ (mthd\ dm) \preceq resTy\ sm$;
wf-mdecl G declC (*sig*, *mthd dm*);
declC = declclass dm;
is-static dm = is-static sm;
is-class G invC; *is-class G declC*; $G \vdash invC \preceq_C declC$;
(*if invmode sm e = IntVir*
then $(\forall\ statC.\ statT{=}ClassT\ statC \longrightarrow G \vdash invC \preceq_C statC)$
else $(\ (\exists\ statC.\ statT{=}ClassT\ statC \wedge G \vdash statC \preceq_C declC)$
$\vee (\forall\ statC.\ statT{\neq}ClassT\ statC \wedge declC{=}Object)) \wedge$
statDeclT = ClassT (*declclass dm*))$]\!] \Longrightarrow P$
shows *P*
proof −
from *invC-compatible mode* **have** $G \vdash invmode\ sm\ e \rightarrow invC \preceq statT$ **by** *simp*
moreover note *wf wt-e mheads*
moreover from *invC mode*
have *invC = invocation-class* (*invmode sm e*) *s a' statT* **by** *simp*
moreover from *declC mode*
have *declC =invocation-declclass G* (*invmode sm e*) *s a' statT sig* **by** *simp*
ultimately show *?thesis*
by (*rule DynT-mheadsD* [*THEN exE*,*rule-format*])
(*elim conjE*,*rule dm*)

qed

lemma *DynT-conf*: $[\![G\vdash \mathit{invocation\text{-}class}\ \mathit{mode}\ s\ a'\ \mathit{statT} \preceq_C \mathit{declC};\ \mathit{wf\text{-}prog}\ G;$
$\mathit{isrtype}\ G\ (\mathit{statT});$
$G,s\vdash a'::\preceq \mathit{RefT}\ \mathit{statT};\ \mathit{mode} = \mathit{IntVir} \longrightarrow a' \neq \mathit{Null};$
$\mathit{mode} \neq \mathit{IntVir} \longrightarrow\ \ (\exists\ \mathit{statC}.\ \mathit{statT}{=}\mathit{ClassT}\ \mathit{statC} \wedge G\vdash \mathit{statC}{\preceq_C}\ \mathit{declC})$
$\qquad\qquad \vee\ (\forall\ \mathit{statC}.\ \mathit{statT}{\neq}\mathit{ClassT}\ \mathit{statC} \wedge \mathit{declC}{=}\mathit{Object})]\!]$
$\Longrightarrow G,s\vdash a'::\preceq\ \mathit{Class}\ \mathit{declC}$
apply (*case-tac mode = IntVir*)
apply (*drule conf-RefTD*)
apply (*force intro*!: *conf-AddrI*
simp add: *obj-class-def split*: *obj-tag.split-asm*)
apply *clarsimp*
apply *safe*
apply (*erule* (*1*) *widen.subcls* [*THEN conf-widen*])
apply (*erule wf-ws-prog*)

apply (*frule widen-Object*) **apply** (*erule wf-ws-prog*)
apply (*erule* (*1*) *conf-widen*) **apply** (*erule wf-ws-prog*)
done

lemma *Ass-lemma*:
$[\![\ G\vdash \mathit{Norm}\ s0\ -\mathit{var}{=}{\succ}(w,\ f){\rightarrow}\ \mathit{Norm}\ s1;\ G\vdash \mathit{Norm}\ s1\ -e{-}{\succ}v{\rightarrow}\ \mathit{Norm}\ s2;$
$G,s2\vdash v::\preceq eT;s1{\leq}|s2 \longrightarrow \mathit{assign}\ f\ v\ (\mathit{Norm}\ s2)::\preceq(G,\ L)]\!]$
$\Longrightarrow \mathit{assign}\ f\ v\ (\mathit{Norm}\ s2)::\preceq(G,\ L) \wedge$
$(\mathit{normal}\ (\mathit{assign}\ f\ v\ (\mathit{Norm}\ s2)) \longrightarrow G,\mathit{store}\ (\mathit{assign}\ f\ v\ (\mathit{Norm}\ s2))\vdash v::\preceq eT)$
apply (*drule-tac x* = *None* **and** *s* = *s2* **and** *v* = *v* **in** *evar-gext-f*)
apply (*drule eval-gext′*, *clarsimp*)
apply (*erule conf-gext*)
apply *simp*
done

lemma *Throw-lemma*: $[\![G\vdash tn{\preceq_C}\ \mathit{SXcpt}\ \mathit{Throwable};\ \mathit{wf\text{-}prog}\ G;\ (x1,s1)::\preceq(G,\ L);$
$x1 = \mathit{None} \longrightarrow G,s1\vdash a'::\preceq\ \mathit{Class}\ tn]\!] \Longrightarrow (\mathit{throw}\ a'\ x1,\ s1)::\preceq(G,\ L)$
apply (*auto split*: *split-abrupt-if simp add*: *throw-def2*)
apply (*erule conforms-xconf*)
apply (*frule conf-RefTD*)
apply (*auto elim*: *widen.subcls* [*THEN conf-widen*])
done

lemma *Try-lemma*: $[\![G\vdash \mathit{obj\text{-}ty}\ (\mathit{the}\ (\mathit{globs}\ s1'\ (\mathit{Heap}\ a)))\preceq\ \mathit{Class}\ tn;$
$(\mathit{Some}\ (\mathit{Xcpt}\ (\mathit{Loc}\ a)),\ s1')::\preceq(G,\ L);\ \mathit{wf\text{-}prog}\ G]\!]$
$\Longrightarrow \mathit{Norm}\ (\mathit{lupd}(vn{\mapsto}\mathit{Addr}\ a)\ s1')::\preceq(G,\ L(vn{\mapsto}\mathit{Class}\ tn))$
apply (*rule conforms-allocL*)
apply (*erule conforms-NormI*)
apply (*drule conforms-XcptLocD* [*THEN conf-RefTD*],*rule HOL.refl*)
apply (*auto intro*!: *conf-AddrI*)
done

lemma *Fin-lemma*:
$[\![G\vdash \mathit{Norm}\ s1\ -c2{\rightarrow}\ (x2,s2);\ \mathit{wf\text{-}prog}\ G;\ (\mathit{Some}\ a,\ s1)::\preceq(G,\ L);\ (x2,s2)::\preceq(G,\ L);$
$\mathit{dom}\ (\mathit{locals}\ s1) \subseteq \mathit{dom}\ (\mathit{locals}\ s2)]\!]$
$\Longrightarrow\ (\mathit{abrupt\text{-}if}\ \mathit{True}\ (\mathit{Some}\ a)\ x2,\ s2)::\preceq(G,\ L)$

apply (*auto elim*: *eval-gext′ conforms-xgext split*: *split-abrupt-if*)
done

lemma *FVar-lemma1*:
⟦*table-of* (*DeclConcepts.fields G statC*) (*fn*, *statDeclC*) = *Some f* ;
 x2 = *None* ⟶ *G*,*s2*⊢*a*::≼ *Class statC*; *wf-prog G*; *G*⊢*statC*$\preceq_C$ *statDeclC*;
 statDeclC ≠ *Object*;
 class G statDeclC = *Some y*; (*x2*,*s2*)::≼(*G*, *L*); *s1*≤|*s2*;
 inited statDeclC (*globs s1*);
 (*if static f then id else np a*) *x2* = *None*⟧
 ⟹
 ∃ *obj*. *globs s2* (*if static f then Inr statDeclC else Inl* (*the-Addr a*))
 = *Some obj* ∧
 var-tys G (*tag obj*) (*if static f then Inr statDeclC else Inl*(*the-Addr a*))
 (*Inl*(*fn*,*statDeclC*)) = *Some* (*type f*)
apply (*drule initedD*)
apply (*frule subcls-is-class2*, *simp* (*no-asm-simp*))
apply (*case-tac static f*)
apply *clarsimp*
apply (*drule* (*1*) *rev-gext-objD*, *clarsimp*)
apply (*frule fields-declC*, *erule wf-ws-prog*, *simp* (*no-asm-simp*))
apply (*rule var-tys-Some-eq* [*THEN iffD2*])
apply *clarsimp*
apply (*erule fields-table-SomeI*, *simp* (*no-asm*))
apply *clarsimp*
apply (*drule conf-RefTD*, *clarsimp*, *rule var-tys-Some-eq* [*THEN iffD2*])
apply (*auto dest*!: *widen-Array split*: *obj-tag.split*)
apply (*rule fields-table-SomeI*)
apply (*auto elim*!: *fields-mono subcls-is-class2*)
done

lemma *FVar-lemma2*: *error-free state*
 ⟹ *error-free*
 (*assign*
 (λ*v*. *supd*
 (*upd-gobj*
 (*if static field then Inr statDeclC*
 else Inl (*the-Addr a*))
 (*Inl* (*fn*, *statDeclC*)) *v*))
 w state)
proof −
 assume *error-free*: *error-free state*
 obtain *a s* **where** *state*=(*a*,*s*)
 by (*cases state*)
 with *error-free*
 show *?thesis*
 by (*cases a*) *auto*
qed

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declare *if-split* [*split del*] *if-split-asm* [*split del*]
 option.split [*split del*] *option.split-asm* [*split del*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt delloop split-all-tac*)›
setup ‹*map-theory-claset* (*fn ctxt* => *ctxt delSWrapper split-all-tac*)›

lemma *FVar-lemma*:

⟦((*v*, *f*), *Norm s2′*) = *fvar statDeclC* (*static field*) *fn a* (*x2*, *s2*);
G⊢*statC*≼$_C$ *statDeclC*;
table-of (*DeclConcepts.fields G statC*) (*fn*, *statDeclC*) = *Some field*;
wf-prog G;
x2 = *None* ⟶ *G*,*s2*⊢*a*::≼*Class statC*;
statDeclC ≠ *Object*; *class G statDeclC* = *Some y*;
(*x2*, *s2*)::≼(*G*, *L*); *s1*≤|*s2*; *inited statDeclC* (*globs s1*)⟧ ⟹
G,*s2′*⊢*v*::≼*type field* ∧ *s2′*≤|*f*≼*type field*::≼(*G*, *L*)
apply (*unfold assign-conforms-def*)
apply (*drule sym*)
apply (*clarsimp simp add*: *fvar-def2*)
apply (*drule* (*9*) *FVar-lemma1*)
apply (*clarsimp*)
apply (*drule* (*2*) *conforms-globsD* [*THEN oconf-lconf*, *THEN lconfD*])
apply *clarsimp*
apply (*rule conjI*)
apply *clarsimp*
apply (*drule* (*1*) *rev-gext-objD*)
apply (*force elim*!: *conforms-upd-gobj*)

apply (*blast intro*: *FVar-lemma2*)
done
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
declare *if-split* [*split*] *if-split-asm* [*split*]
option.split [*split*] *option.split-asm* [*split*]
setup ‹*map-theory-claset* (*fn ctxt* => *ctxt addSbefore* (*split-all-tac*, *split-all-tac*))›
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt addloop* (*split-all-tac*, *split-all-tac*))›

lemma *AVar-lemma1*: ⟦*globs s* (*Inl a*) = *Some obj*;*tag obj*=*Arr ty i*;
the-Intg i′ in-bounds i; *wf-prog G*; *G*⊢*ty*.[]≼*Tb*.[]; *Norm s*::≼(*G*, *L*)
⟧ ⟹ *G*,*s*⊢*the* ((*values obj*) (*Inr* (*the-Intg i′*)))::≼*Tb*
apply (*erule widen-Array-Array* [*THEN conf-widen*])
apply (*erule-tac* [*2*] *wf-ws-prog*)
apply (*drule* (*1*) *conforms-globsD* [*THEN oconf-lconf*, *THEN lconfD*])
defer apply *assumption*
apply (*force intro*: *var-tys-Some-eq* [*THEN iffD2*])
done

lemma *obj-split*: ∃ *t vs*. *obj* = (|*tag*=*t*,*values*=*vs*|)
by (*cases obj*) *auto*

lemma *AVar-lemma2*: *error-free state*
⟹ *error-free*
(*assign*
(λ*v* (*x*, *s′*).
((*raise-if* (¬ *G*,*s′*⊢*v fits T*) *ArrStore*) *x*,
upd-gobj (*Inl a*) (*Inr* (*the-Intg i*)) *v s′*))
w state)
proof −
assume *error-free*: *error-free state*
obtain *a s* **where** *state*=(*a*,*s*)
by (*cases state*)
with *error-free*
show *?thesis*
by (*cases a*) *auto*

qed

```
lemma AVar-lemma: ⟦wf-prog G; G⊢(x1, s1) −e2−≻i→ (x2, s2);
  ((v,f), Norm s2') = avar G i a (x2, s2); x1 = None ⟶ G,s1⊢a::≼Ta.[];
  (x2, s2)::≼(G, L); s1≤|s2⟧ ⟹ G,s2'⊢v::≼Ta ∧ s2'≤|f≼Ta::≼(G, L)
apply (unfold assign-conforms-def)
apply (drule sym)
apply (clarsimp simp add: avar-def2)
apply (drule (1) conf-gext)
apply (drule conf-RefTD, clarsimp)
apply (subgoal-tac ∃ t vs. obj = (|tag=t,values=vs|))
defer
apply  (rule obj-split)
apply clarify
apply (frule obj-ty-widenD)
apply (auto dest!: widen-Class)
apply  (force dest: AVar-lemma1)

apply  (force elim!: fits-Array dest: gext-objD
       intro: var-tys-Some-eq [THEN iffD2] conforms-upd-gobj)
done
```

Call

```
lemma conforms-init-lvars-lemma: ⟦wf-prog G;
  wf-mhead G P sig mh;
  list-all2 (conf G s) pvs pTsa; G⊢pTsa[≼](parTs sig)⟧ ⟹
  G,s⊢Map.empty (pars mh[↦]pvs)
      [∼::≼]table-of lvars(pars mh[↦]parTs sig)
apply (unfold wf-mhead-def)
apply clarify
apply (erule (1) wlconf-empty-vals [THEN wlconf-ext-list])
apply (drule wf-ws-prog)
apply (erule (2) conf-list-widen)
done
```

```
lemma lconf-map-lname [simp]:
  G,s⊢(case-lname l1 l2)[::≼](case-lname L1 L2)
   =
  (G,s⊢l1[::≼]L1 ∧ G,s⊢(λx::unit . l2)[::≼](λx::unit. L2))
apply (unfold lconf-def)
apply (auto split: lname.splits)
done
```

```
lemma wlconf-map-lname [simp]:
  G,s⊢(case-lname l1 l2)[∼::≼](case-lname L1 L2)
   =
  (G,s⊢l1[∼::≼]L1 ∧ G,s⊢(λx::unit . l2)[∼::≼](λx::unit. L2))
apply (unfold wlconf-def)
apply (auto split: lname.splits)
done
```

```
lemma lconf-map-ename [simp]:
  G,s⊢(case-ename l1 l2)[::≼](case-ename L1 L2)
```

=
(*G*,*s*⊢*l1*[::≼]*L1* ∧ *G*,*s*⊢(λ*x*::*unit*. *l2*)[::≼](λ*x*::*unit*. *L2*))
apply (*unfold lconf-def*)
apply (*auto split*: *ename.splits*)
done

lemma *wlconf-map-ename* [*simp*]:
G,*s*⊢(*case-ename l1 l2*)[∼::≼](*case-ename L1 L2*)
=
(*G*,*s*⊢*l1*[∼::≼]*L1* ∧ *G*,*s*⊢(λ*x*::*unit*. *l2*)[∼::≼](λ*x*::*unit*. *L2*))
apply (*unfold wlconf-def*)
apply (*auto split*: *ename.splits*)
done

lemma *defval-conf1* [*rule-format* (*no-asm*), *elim*]:
is-type G T ⟶ (∃ *v*∈*Some* (*default-val T*): *G*,*s*⊢*v*::≼*T*)
apply (*unfold conf-def*)
apply (*induct T*)
apply (*auto intro*: *prim-ty.induct*)
done

lemma *np-no-jump*: *x*≠*Some* (*Jump j*) ⟹ (*np a'*) *x* ≠ *Some* (*Jump j*)
by (*auto simp add*: *abrupt-if-def*)

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declare *if-split* [*split del*] *if-split-asm* [*split del*]
option.split [*split del*] *option.split-asm* [*split del*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt delloop split-all-tac*)›
setup ‹*map-theory-claset* (*fn ctxt* => *ctxt delSWrapper split-all-tac*)›

lemma *conforms-init-lvars*:
⟦*wf-mhead G* (*pid declC*) *sig* (*mhead* (*mthd dm*)); *wf-prog G*;
list-all2 (*conf G s*) *pvs pTsa*; *G*⊢*pTsa*[≼](*parTs sig*);
(*x*, *s*)::≼(*G*, *L*);
methd G declC sig = *Some dm*;
isrtype G statT;
G⊢*invC*$\preceq_C$ *declC*;
G,*s*⊢*a'*::≼*RefT statT*;
invmode (*mhd sm*) *e* = *IntVir* ⟶ *a'* ≠ *Null*;
invmode (*mhd sm*) *e* ≠ *IntVir* ⟶
(∃ *statC*. *statT*=*ClassT statC* ∧ *G*⊢*statC*$\preceq_C$ *declC*)
∨ (∀ *statC*. *statT*≠*ClassT statC* ∧ *declC*=*Object*);
invC = *invocation-class* (*invmode* (*mhd sm*) *e*) *s a' statT*;
declC = *invocation-declclass G* (*invmode* (*mhd sm*) *e*) *s a' statT sig*;
x≠*Some* (*Jump Ret*)
⟧ ⟹
init-lvars G declC sig (*invmode* (*mhd sm*) *e*) *a'*
pvs (*x*,*s*)::≼(*G*,λ *k*.
(*case k of*
EName e ⇒ (*case e of*
VNam v
⇒ (*table-of* (*lcls* (*mbody* (*mthd dm*)))
(*pars* (*mthd dm*)[↦]*parTs sig*)) *v*

| *Res* ⇒ *Some* (*resTy* (*mthd dm*)))
| *This* ⇒ *if* (*is-static* (*mthd sm*))
then None else Some (*Class declC*)))
apply (*simp add*: *init-lvars-def2*)
apply (*rule conforms-set-locals*)
apply (*simp* (*no-asm-simp*) *split*: *if-split*)
apply (*drule* (*4*) *DynT-conf*)
apply *clarsimp*

apply (*drule* (*3*) *conforms-init-lvars-lemma*
[**where** *?lvars*=(*lcls* (*mbody* (*mthd dm*)))])
apply (*case-tac dm*,*simp*)
apply (*rule conjI*)
apply (*unfold wlconf-def*, *clarify*)
apply (*clarsimp simp add*: *wf-mhead-def is-acc-type-def*)
apply (*case-tac is-static sm*)
apply *simp*
apply *simp*

apply *simp*
apply (*case-tac is-static sm*)
apply *simp*
apply (*simp add*: *np-no-jump*)
done
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
declare *if-split* [*split*] *if-split-asm* [*split*]
option.split [*split*] *option.split-asm* [*split*]
setup ‹*map-theory-claset* (*fn ctxt* => *ctxt addSbefore* (*split-all-tac*, *split-all-tac*))›
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt addloop* (*split-all-tac*, *split-all-tac*))›

2 accessibility

theorem *dynamic-field-access-ok*:
assumes *wf*: *wf-prog G* **and**
not-Null: ¬ *stat* ⟶ *a*≠*Null* **and**
conform-a: *G*,(*store s*)⊢*a*::≼ *Class statC* **and**
conform-s: *s*::≼(*G*, *L*) **and**
normal-s: *normal s* **and**
wt-e: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*e*::−*Class statC* **and**
f: *accfield G accC statC fn* = *Some f* **and**
dynC: *if stat then dynC*=*declclass f*
else dynC=*obj-class* (*lookup-obj* (*store s*) *a*) **and**
stat: *if stat then* (*is-static f*) *else* (¬ *is-static f*)
shows *table-of* (*DeclConcepts.fields G dynC*) (*fn*,*declclass f*) = *Some* (*fld f*)∧
G⊢*Field fn f in dynC dyn-accessible-from accC*
proof (*cases stat*)
case *True*
with *stat* **have** *static*: (*is-static f*) **by** *simp*
from *True dynC*
have *dynC′*: *dynC*=*declclass f* **by** *simp*
with *f*
have *table-of* (*DeclConcepts.fields G statC*) (*fn*,*declclass f*) = *Some* (*fld f*)
by (*auto simp add*: *accfield-def Let-def intro*!: *table-of-remap-SomeD*)
moreover
from *wt-e wf* **have** *is-class G statC*
by (*auto dest*!: *ty-expr-is-type*)
moreover note *wf dynC′*
ultimately have
table-of (*DeclConcepts.fields G dynC*) (*fn*,*declclass f*) = *Some* (*fld f*)

by (*auto dest*: *fields-declC*)
with *dynC′ f static wf*
show *?thesis*
by (*auto dest*: *static-to-dynamic-accessible-from-static*
dest!: *accfield-accessibleD*)
next
case *False*
with *wf conform-a not-Null conform-s dynC*
obtain *subclseq*: $G\vdash dynC \preceq_C statC$ **and**
is-class G dynC
by (*auto dest*!: *conforms-RefTD* [*of - - - -* (*fst s*) *L*]
dest: *obj-ty-obj-class1*
simp add: *obj-ty-obj-class*)
with *wf f*
have *table-of* (*DeclConcepts.fields G dynC*) (*fn*,*declclass f*) = *Some* (*fld f*)
by (*auto simp add*: *accfield-def Let-def*
dest: *fields-mono*
dest!: *table-of-remap-SomeD*)
moreover
from *f subclseq*
have *G⊢Field fn f in dynC dyn-accessible-from accC*
by (*auto intro*!: *static-to-dynamic-accessible-from wf*
dest: *accfield-accessibleD*)
ultimately show *?thesis*
by *blast*
qed

lemma *error-free-field-access*:
assumes *accfield*: *accfield G accC statC fn* = *Some* (*statDeclC*, *f*) **and**
wt-e: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*e*::−*Class statC* **and**
eval-init: *G⊢Norm s0* −*Init statDeclC*→ *s1* **and**
eval-e: *G⊢s1* −*e*−≻*a*→ *s2* **and**
conf-s2: *s2*::≼(*G*, *L*) **and**
conf-a: *normal s2* ⟹ *G*, *store s2⊢a*::≼*Class statC* **and**
fvar: (*v*,*s2′*)=*fvar statDeclC* (*is-static f*) *fn a s2* **and**
wf: *wf-prog G*
shows *check-field-access G accC statDeclC fn* (*is-static f*) *a s2′* = *s2′*
proof −
from *fvar*
have *store-s2′*: *store s2′*=*store s2*
by (*cases s2*) (*simp add*: *fvar-def2*)
with *fvar conf-s2*
have *conf-s2′*: *s2′*::≼(*G*, *L*)
by (*cases s2*,*cases is-static f*) (*auto simp add*: *fvar-def2*)
from *eval-init*
have *initd-statDeclC-s1*: *initd statDeclC s1*
by (*rule init-yields-initd*)
with *eval-e store-s2′*
have *initd-statDeclC-s2′*: *initd statDeclC s2′*
by (*auto dest*: *eval-gext intro*: *inited-gext*)
show *?thesis*
proof (*cases normal s2′*)
case *False*
then show *?thesis*
by (*auto simp add*: *check-field-access-def Let-def*)
next
case *True*
with *fvar store-s2′*

```
    have not-Null: ¬ (is-static f) ⟶ a≠Null
      by (cases s2) (auto simp add: fvar-def2)
    from True fvar store-s2'
    have normal s2
      by (cases s2,cases is-static f) (auto simp add: fvar-def2)
    with conf-a store-s2'
    have conf-a': G,store s2'⊢a::≼Class statC
      by simp
    from conf-a' conf-s2' True initd-statDeclC-s2'
      dynamic-field-access-ok [OF wf not-Null conf-a' conf-s2'
                                 True wt-e accfield ]
    show ?thesis
      by (cases is-static f)
          (auto dest!: initedD
           simp add: check-field-access-def Let-def)
  qed
qed


lemma call-access-ok:
  assumes invC-prop: G⊢invmode statM e→invC≼statT
      and        wf: wf-prog G
      and      wt-e: (|prg=G,cls=C,lcl=L|)⊢e::−RefT statT
      and     statM: (statDeclT,statM) ∈ mheads G accC statT sig
      and      invC: invC = invocation-class (invmode statM e) s a statT
  shows ∃ dynM. dynlookup G statT invC sig = Some dynM ∧
  G⊢Methd sig dynM in invC dyn-accessible-from accC
proof −
  from wt-e wf have type-statT: is-type G (RefT statT)
    by (auto dest: ty-expr-is-type)
  from statM have not-Null: statT ≠ NullT by auto
  from type-statT wt-e
  have wf-I: (∀ I. statT = IfaceT I ⟶ is-iface G I ∧
                                      invmode statM e ≠ SuperM)
    by (auto dest: invocationTypeExpr-noClassD)
  from wt-e
  have wf-A: (∀      T. statT = ArrayT T ⟶ invmode statM e ≠ SuperM)
    by (auto dest: invocationTypeExpr-noClassD)
  show ?thesis
  proof (cases invmode statM e = IntVir)
    case True
    with invC-prop not-Null
    have invC-prop': is-class G invC ∧
                    (if (∃ T. statT=ArrayT T) then invC=Object
                                          else G⊢Class invC≼RefT statT)
      by (auto simp add: DynT-prop-def)
    with True not-Null
    have G,statT ⊢ invC valid-lookup-cls-for is-static statM
     by (cases statT) (auto simp add: invmode-def)
    with statM type-statT wf
    show ?thesis
      by − (rule dynlookup-access,auto)
  next
    case False
    with type-statT wf invC not-Null wf-I wf-A
    have invC-prop':  is-class G invC ∧
                    ((∃ statC. statT=ClassT statC ∧ invC=statC) ∨
                    (∀ statC. statT≠ClassT statC ∧ invC=Object))
        by (case-tac statT) (auto simp add: invocation-class-def
```

split: *inv-mode.splits*)
with *not-Null wf*
have *dynlookup-static*: *dynlookup G statT invC sig* = *methd G invC sig*
by (*case-tac statT*) (*auto simp add*: *dynlookup-def dynmethd-C-C dynimethd-def*)
from *statM wf wt-e not-Null False invC-prop′* **obtain** *dynM* **where**
accmethd G accC invC sig = *Some dynM*
by (*auto dest*!: *static-mheadsD*)
from *invC-prop′ False not-Null wf-I*
have *G*,*statT* ⊢ *invC valid-lookup-cls-for is-static statM*
by (*cases statT*) (*auto simp add*: *invmode-def*)
with *statM type-statT wf*
show *?thesis*
by − (*rule dynlookup-access*,*auto*)
qed
qed

lemma *error-free-call-access*:
assumes
eval-args: *G*⊢*s1* −*args*≐≻*vs*→ *s2* **and**
wt-e: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*e*::−(*RefT statT*) **and**
statM: *max-spec G accC statT* (|*name* = *mn*, *parTs* = *pTs*|)
= {((*statDeclT*, *statM*), *pTs′*)} **and**
conf-s2: *s2*::≼(*G*, *L*) **and**
conf-a: *normal s1* ⟹ *G*, *store s1*⊢*a*::≼*RefT statT* **and**
invProp: *normal s3* ⟹
G⊢*invmode statM e*→*invC*≼*statT* **and**
s3: *s3*=*init-lvars G invDeclC* (|*name* = *mn*, *parTs* = *pTs′*|)
(*invmode statM e*) *a vs s2* **and**
invC: *invC* = *invocation-class* (*invmode statM e*) (*store s2*) *a statT***and**
invDeclC: *invDeclC* = *invocation-declclass G* (*invmode statM e*) (*store s2*)
a statT (|*name* = *mn*, *parTs* = *pTs′*|) **and**
wf: *wf-prog G*
shows *check-method-access G accC statT* (*invmode statM e*) (|*name*=*mn*,*parTs*=*pTs′*|) *a s3*
= *s3*
proof (*cases normal s2*)
case *False*
with *s3*
have *abrupt s3* = *abrupt s2*
by (*auto simp add*: *init-lvars-def2*)
with *False*
show *?thesis*
by (*auto simp add*: *check-method-access-def Let-def*)
next
case *True*
note *normal-s2* = *True*
with *eval-args*
have *normal-s1*: *normal s1*
by (*cases normal s1*) *auto*
with *conf-a eval-args*
have *conf-a-s2*: *G*, *store s2*⊢*a*::≼*RefT statT*
by (*auto dest*: *eval-gext intro*: *conf-gext*)
show *?thesis*
proof (*cases a*=*Null* ⟶ (*is-static statM*))
case *False*
then obtain ¬ *is-static statM a*=*Null*
by *blast*
with *normal-s2 s3*

  have *abrupt s3 = Some (Xcpt (Std NullPointer))*
    by (*auto simp add*: *init-lvars-def2*)
  then show *?thesis*
    by (*auto simp add*: *check-method-access-def Let-def*)
 next
  case *True*
  from *statM*
  obtain
   statM': $(statDeclT,statM) \in mheads\ G\ accC\ statT\ (|name=mn,parTs=pTs'|)$
   by (*blast dest*: *max-spec2mheads*)
  from *True normal-s2 s3*
  have *normal s3*
   by (*auto simp add*: *init-lvars-def2*)
  then have $G \vdash invmode\ statM\ e \rightarrow invC \preceq statT$
   by (*rule invProp*)
  with *wt-e statM' wf invC*
  obtain *dynM* **where**
   dynM: $dynlookup\ G\ statT\ invC\ (|name=mn,parTs=pTs'|) = Some\ dynM$ **and**
   acc-dynM: $G \vdash Methd\ (|name=mn,parTs=pTs'|)\ dynM$
            in invC dyn-accessible-from accC
   by (*force dest*!: *call-access-ok*)
  moreover
  from *s3 invC*
  have *invC'*: *invC=(invocation-class (invmode statM e) (store s3) a statT)*
   by (*cases s2*,*cases invmode statM e*)
    (*simp add*: *init-lvars-def2 del*: *invmode-Static-eq*)+
  ultimately
  show *?thesis*
   by (*auto simp add*: *check-method-access-def Let-def*)
 qed
qed

lemma *map-upds-eq-length-append-simp*:
 $\bigwedge$ *tab qs. length ps = length qs* $\Longrightarrow$ $tab(ps[\mapsto]qs@zs) = tab(ps[\mapsto]qs)$
proof (*induct ps*)
 case *Nil* **thus** *?case* **by** *simp*
next
 case (*Cons p ps tab qs*)
 from ‹*length* (*p#ps*) = *length qs*›
 obtain *q qs'* **where** *qs*: *qs=q#qs'* **and** *eq-length*: *length ps=length qs'*
  by (*cases qs*) *auto*
 from *eq-length* **have** $(tab(p \mapsto q))(ps[\mapsto]qs'@zs)=(tab(p \mapsto q))(ps[\mapsto]qs')$
  by (*rule Cons.hyps*)
 with *qs* **show** *?case*
  by *simp*
qed

lemma *map-upds-upd-eq-length-simp*:
 $\bigwedge$ *tab qs x y. length ps = length qs*
     $\Longrightarrow tab(ps[\mapsto]qs)(x \mapsto y) = tab(ps@[x][\mapsto]qs@[y])$
proof (*induct ps*)
 case *Nil* **thus** *?case* **by** *simp*
next
 case (*Cons p ps tab qs x y*)
 from ‹*length* (*p#ps*) = *length qs*›
 obtain *q qs'* **where** *qs*: *qs=q#qs'* **and** *eq-length*: *length ps=length qs'*
  by (*cases qs*) *auto*

from *eq-length*
have $(tab(p{\mapsto}q))(ps[{\mapsto}]qs')(x{\mapsto}y) = (tab(p{\mapsto}q))(ps@[x][{\mapsto}]qs'@[y])$
by (*rule Cons.hyps*)
with *qs* **show** *?case*
by *simp*
qed

lemma *map-upd-cong*: $tab{=}tab' \Longrightarrow tab(x{\mapsto}y) = tab'(x{\mapsto}y)$
by *simp*

lemma *map-upd-cong-ext*: $tab\ z{=}tab'\ z \Longrightarrow (tab(x{\mapsto}y))\ z = (tab'(x{\mapsto}y))\ z$
by (*simp add*: *fun-upd-def*)

lemma *map-upds-cong*: $tab{=}tab' \Longrightarrow tab(xs[{\mapsto}]ys) = tab'(xs[{\mapsto}]ys)$
by (*cases xs*) *simp+*

lemma *map-upds-cong-ext*:
$\bigwedge tab\ tab'\ ys.\ tab\ z{=}tab'\ z \Longrightarrow (tab(xs[{\mapsto}]ys))\ z = (tab'(xs[{\mapsto}]ys))\ z$
proof (*induct xs*)
case *Nil* **thus** *?case* **by** *simp*
next
case (*Cons x xs tab tab′ ys*)
note *Hyps* = *this*
show *?case*
proof (*cases ys*)
case *Nil*
with *Hyps*
show *?thesis* **by** *simp*
next
case (*Cons y ys′*)
have $(tab(x{\mapsto}y)(xs[{\mapsto}]ys'))\ z = (tab'(x{\mapsto}y)(xs[{\mapsto}]ys'))\ z$
by (*iprover intro*: *Hyps map-upd-cong-ext*)
with *Cons* **show** *?thesis*
by *simp*
qed
qed

lemma *map-upd-override*: $(tab(x{\mapsto}y))\ x = (tab'(x{\mapsto}y))\ x$
by *simp*

lemma *map-upds-eq-length-suffix*: $\bigwedge tab\ qs.$
$length\ ps = length\ qs \Longrightarrow tab(ps@xs[{\mapsto}]qs) = tab(ps[{\mapsto}]qs)(xs[{\mapsto}][])$
proof (*induct ps*)
case *Nil* **thus** *?case* **by** *simp*
next
case (*Cons p ps tab qs*)
then obtain *q qs′* **where** *qs*: $qs{=}q\#qs'$ **and** *eq-length*: $length\ ps{=}length\ qs'$
by (*cases qs*) *auto*
from *eq-length*
have $tab(p{\mapsto}q)(ps\ @\ xs[{\mapsto}]qs') = tab(p{\mapsto}q)(ps[{\mapsto}]qs')(xs[{\mapsto}][])$
by (*rule Cons.hyps*)
with *qs* **show** *?case*

  by *simp*
qed

lemma *map-upds-upds-eq-length-prefix-simp*:
 $\bigwedge$ *tab qs*. *length ps* = *length qs*
  $\Longrightarrow$ $tab(ps[\mapsto]qs)(xs[\mapsto]ys) = tab(ps@xs[\mapsto]qs@ys)$
proof (*induct ps*)
 case *Nil* **thus** *?case* **by** *simp*
next
 case (*Cons p ps tab qs*)
 then obtain *q qs'* **where** *qs*: *qs=q#qs'* **and** *eq-length*: *length ps=length qs'*
  by (*cases qs*) *auto*
 from *eq-length*
 have $tab(p\mapsto q)(ps[\mapsto]qs')(xs[\mapsto]ys) = tab(p\mapsto q)(ps\ @\ xs[\mapsto](qs'\ @\ ys))$
  by (*rule Cons.hyps*)
 with *qs*
 show *?case* **by** *simp*
qed

lemma *map-upd-cut-irrelevant*:
$[\![(tab(x\mapsto y))\ vn = Some\ el;\ (tab'(x\mapsto y))\ vn = None]\!]$
  $\Longrightarrow$ *tab vn* = *Some el*
by (*cases tab' vn* = *None*) (*simp add*: *fun-upd-def*)+

lemma *map-upd-Some-expand*:
$[\![tab\ vn = Some\ z]\!]$
  $\Longrightarrow \exists\ z.\ (tab(x\mapsto y))\ vn = Some\ z$
by (*simp add*: *fun-upd-def*)

lemma *map-upds-Some-expand*:
$\bigwedge$ *tab ys z*. $[\![tab\ vn = Some\ z]\!]$
  $\Longrightarrow \exists\ z.\ (tab(xs[\mapsto]ys))\ vn = Some\ z$
proof (*induct xs*)
 case *Nil* **thus** *?case* **by** *simp*
next
 case (*Cons x xs tab ys z*)
 note *z* = ‹*tab vn* = *Some z*›
 show *?case*
 proof (*cases ys*)
  case *Nil*
  with *z* **show** *?thesis* **by** *simp*
 next
  case (*Cons y ys'*)
  note *ys* = ‹*ys* = *y#ys'*›
  from *z* **obtain** *z'* **where** $(tab(x\mapsto y))\ vn = Some\ z'$
   by (*rule map-upd-Some-expand* [*of tab*,*elim-format*]) *blast*
  hence $\exists z.\ ((tab(x\mapsto y))(xs[\mapsto]ys'))\ vn = Some\ z$
   by (*rule Cons.hyps*)
  with *ys* **show** *?thesis*
   by *simp*
 qed
qed

lemma *map-upd-Some-swap*:
$(tab(r{\mapsto}w)(u{\mapsto}v))\ vn = Some\ z \Longrightarrow \exists\ z.\ (tab(u{\mapsto}v)(r{\mapsto}w))\ vn = Some\ z$
by (*simp add*: *fun-upd-def*)

lemma *map-upd-None-swap*:
$(tab(r{\mapsto}w)(u{\mapsto}v))\ vn = None \Longrightarrow (tab(u{\mapsto}v)(r{\mapsto}w))\ vn = None$
by (*simp add*: *fun-upd-def*)

lemma *map-eq-upd-eq*: $tab\ vn = tab'\ vn \Longrightarrow (tab(x{\mapsto}y))\ vn = (tab'(x{\mapsto}y))\ vn$
by (*simp add*: *fun-upd-def*)

lemma *map-upd-in-expansion-map-swap*:
$[\![(tab(x{\mapsto}y))\ vn = Some\ z;tab\ vn \neq Some\ z]\!]$
$\quad\Longrightarrow\ (tab'(x{\mapsto}y))\ vn = Some\ z$
by (*simp add*: *fun-upd-def*)

lemma *map-upds-in-expansion-map-swap*:
$\bigwedge tab\ tab'\ ys\ z.\ [\![(tab(xs[{\mapsto}]ys))\ vn = Some\ z;tab\ vn \neq Some\ z]\!]$
$\quad\Longrightarrow\ (tab'(xs[{\mapsto}]ys))\ vn = Some\ z$
proof (*induct xs*)
  case *Nil* **thus** *?case* **by** *simp*
next
  case (*Cons x xs tab tab′ ys z*)
  note *some* = ‹$(tab(x\ \#\ xs[{\mapsto}]ys))\ vn = Some\ z$›
  note *tab-not-z* = ‹$tab\ vn \neq Some\ z$›
  show *?case*
  proof (*cases ys*)
    case *Nil* **with** *some tab-not-z* **show** *?thesis* **by** *simp*
  next
    case (*Cons y tl*)
    note *ys* = ‹$ys = y\#tl$›
    show *?thesis*
    proof (*cases* $(tab(x{\mapsto}y))\ vn \neq Some\ z$)
      case *True*
      with *some ys* **have** $(tab'(x{\mapsto}y)(xs[{\mapsto}]tl))\ vn = Some\ z$
        by (*fastforce intro*: *Cons.hyps*)
      with *ys* **show** *?thesis*
        by *simp*
    next
      case *False*
      hence *tabx-z*: $(tab(x{\mapsto}y))\ vn = Some\ z$ **by** *blast*
      moreover
      from *tabx-z tab-not-z*
      have $(tab'(x{\mapsto}y))\ vn = Some\ z$
        by (*rule map-upd-in-expansion-map-swap*)
      ultimately
      have $(tab(x{\mapsto}y))\ vn =(tab'(x{\mapsto}y))\ vn$
        by *simp*
      hence $(tab(x{\mapsto}y)(xs[{\mapsto}]tl))\ vn = (tab'(x{\mapsto}y)(xs[{\mapsto}]tl))\ vn$
        by (*rule map-upds-cong-ext*)
      with *some ys*
      show *?thesis*
        by *simp*

    qed
  qed
qed

lemma *map-upds-Some-swap*:
 assumes *r-u*: $(tab(r \mapsto w)(u \mapsto v)(xs[\mapsto]ys))\ vn = Some\ z$
  shows $\exists\ z.\ (tab(u \mapsto v)(r \mapsto w)(xs[\mapsto]ys))\ vn = Some\ z$
proof $(cases\ (tab(r \mapsto w)(u \mapsto v))\ vn = Some\ z)$
 case *True*
 then obtain z' **where** $(tab(u \mapsto v)(r \mapsto w))\ vn = Some\ z'$
  by (*rule map-upd-Some-swap* [*elim-format*]) *blast*
 thus $\exists\ z.\ (tab(u \mapsto v)(r \mapsto w)(xs[\mapsto]ys))\ vn = Some\ z$
  by (*rule map-upds-Some-expand*)
next
 case *False*
 with *r-u*
 have $(tab(u \mapsto v)(r \mapsto w)(xs[\mapsto]ys))\ vn = Some\ z$
  by (*rule map-upds-in-expansion-map-swap*)
 thus *?thesis*
  by *simp*
qed

lemma *map-upds-Some-insert*:
 assumes *z*: $(tab(xs[\mapsto]ys))\ vn = Some\ z$
  shows $\exists\ z.\ (tab(u \mapsto v)(xs[\mapsto]ys))\ vn = Some\ z$
proof $(cases\ \exists\ z.\ tab\ vn = Some\ z)$
 case *True*
 then obtain z' **where** $tab\ vn = Some\ z'$ **by** *blast*
 then obtain z'' **where** $(tab(u \mapsto v))\ vn = Some\ z''$
  by (*rule map-upd-Some-expand* [*elim-format*]) *blast*
 thus *?thesis*
  by (*rule map-upds-Some-expand*)
next
 case *False*
 hence $tab\ vn \neq Some\ z$ **by** *simp*
 with *z*
 have $(tab(u \mapsto v)(xs[\mapsto]ys))\ vn = Some\ z$
  by (*rule map-upds-in-expansion-map-swap*)
 thus *?thesis* **..**
qed

lemma *map-upds-None-cut*:
assumes *expand-None*: $(tab(xs[\mapsto]ys))\ vn = None$
 shows $tab\ vn = None$
proof $(cases\ tab\ vn = None)$
 case *True* **thus** *?thesis* **by** *simp*
next
 case *False* **then obtain** *z* **where** $tab\ vn = Some\ z$ **by** *blast*
 then obtain z' **where** $(tab(xs[\mapsto]ys))\ vn = Some\ z'$
  by (*rule map-upds-Some-expand* [**where** *?tab=tab*,*elim-format*]) *blast*
 with *expand-None* **show** *?thesis*
  by *simp*
qed

```
lemma map-upds-cut-irrelevant:
⋀ tab tab′ ys. ⟦(tab(xs[↦]ys)) vn = Some el; (tab′(xs[↦]ys)) vn = None⟧
                  ⟹ tab vn = Some el
proof (induct xs)
  case Nil thus ?case by simp
next
  case (Cons x xs tab tab′ ys)
  note tab-vn = ‹(tab(x # xs[↦]ys)) vn = Some el›
  note tab′-vn = ‹(tab′(x # xs[↦]ys)) vn = None›
  show ?case
  proof (cases ys)
    case Nil
    with tab-vn show ?thesis by simp
  next
    case (Cons y tl)
    note ys = ‹ys=y#tl›
    with tab-vn tab′-vn
    have (tab(x↦y)) vn = Some el
      by − (rule Cons.hyps,auto)
    moreover from tab′-vn ys
    have (tab′(x↦y)(xs[↦]tl)) vn = None
      by simp
    hence (tab′(x↦y)) vn = None
      by (rule map-upds-None-cut)
    ultimately show tab vn = Some el
      by (rule map-upd-cut-irrelevant)
  qed
qed
```

```
lemma dom-vname-split:
 dom (case-lname (case-ename (tab(x↦y)(xs[↦]ys)) a) b)
   = dom (case-lname (case-ename (tab(x↦y)) a) b) ∪
     dom (case-lname (case-ename (tab(xs[↦]ys)) a) b)
  (is ?List x xs y ys = ?Hd x y ∪ ?Tl xs ys)
proof
  show ?List x xs y ys ⊆ ?Hd x y ∪ ?Tl xs ys
  proof
    fix el
    assume el-in-list: el ∈ ?List x xs y ys
    show el ∈  ?Hd x y ∪ ?Tl xs ys
    proof (cases el)
      case This
      with el-in-list show ?thesis by (simp add: dom-def)
    next
      case (EName en)
      show ?thesis
      proof (cases en)
        case Res
        with EName el-in-list show ?thesis by (simp add: dom-def)
      next
        case (VNam vn)
        with EName el-in-list show ?thesis
          by (auto simp add: dom-def dest: map-upds-cut-irrelevant)
      qed
    qed
  qed
next
```

```
  show ?Hd x y ∪ ?Tl xs ys ⊆ ?List x xs y ys
  proof (rule subsetI)
    fix el
    assume el-in-hd-tl: el ∈ ?Hd x y ∪ ?Tl xs ys
    show el ∈ ?List x xs y ys
    proof (cases el)
      case This
      with el-in-hd-tl show ?thesis by (simp add: dom-def)
    next
      case (EName en)
      show ?thesis
      proof (cases en)
        case Res
        with EName el-in-hd-tl show ?thesis by (simp add: dom-def)
      next
        case (VNam vn)
        with EName el-in-hd-tl show ?thesis
          by (auto simp add: dom-def intro: map-upds-Some-expand
                                            map-upds-Some-insert)
      qed
    qed
  qed
qed


lemma dom-map-upd: ⋀ tab. dom (tab(x↦y)) = dom tab ∪ {x}
by (auto simp add: dom-def fun-upd-def)


lemma dom-map-upds: ⋀ tab ys. length xs = length ys
  ⟹ dom (tab(xs[↦]ys)) = dom tab ∪ set xs
proof (induct xs)
  case Nil thus ?case by (simp add: dom-def)
next
  case (Cons x xs tab ys)
  note Hyp = Cons.hyps
  note len = ‹length (x#xs)=length ys›
  show ?case
  proof (cases ys)
    case Nil with len show ?thesis by simp
  next
    case (Cons y tl)
    with len have dom (tab(x↦y)(xs[↦]tl)) = dom (tab(x↦y)) ∪ set xs
      by − (rule Hyp,simp)
    moreover
    have dom (tab(x↦hd ys)) = dom tab ∪ {x}
      by (rule dom-map-upd)
    ultimately
    show ?thesis using Cons
      by simp
  qed
qed


lemma dom-case-ename-None-simp:
 dom (case-ename vname-tab None) = VNam ‘ (dom vname-tab)
  apply (auto simp add: dom-def image-def )
  apply (case-tac x)
  apply auto
```

done

lemma *dom-case-ename-Some-simp*:
dom (*case-ename vname-tab* (*Some a*)) = *VNam* ' (*dom vname-tab*) ∪ {*Res*}
apply (*auto simp add*: *dom-def image-def*)
apply (*case-tac x*)
apply *auto*
done

lemma *dom-case-lname-None-simp*:
dom (*case-lname ename-tab None*) = *EName* ' (*dom ename-tab*)
apply (*auto simp add*: *dom-def image-def*)
apply (*case-tac x*)
apply *auto*
done

lemma *dom-case-lname-Some-simp*:
dom (*case-lname ename-tab* (*Some a*)) = *EName* ' (*dom ename-tab*) ∪ {*This*}
apply (*auto simp add*: *dom-def image-def*)
apply (*case-tac x*)
apply *auto*
done

lemmas *dom-lname-case-ename-simps* =
dom-case-ename-None-simp dom-case-ename-Some-simp
dom-case-lname-None-simp dom-case-lname-Some-simp

lemma *image-comp*:
f ' *g* ' *A* = (*f* ∘ *g*) ' *A*
by (*auto simp add*: *image-def*)

lemma *dom-locals-init-lvars*:
assumes *m*: *m*=(*mthd* (*the* (*methd G C sig*)))
assumes *len*: *length* (*pars m*) = *length pvs*
shows *dom* (*locals* (*store* (*init-lvars G C sig* (*invmode m e*) *a pvs s*)))
= *parameters m*
proof −
from *m*
have *static-m'*: *is-static m* = *static m*
by *simp*
from *len*
have *dom-vnames*: *dom* (*Map.empty*(*pars m*[↦]*pvs*))=*set* (*pars m*)
by (*simp add*: *dom-map-upds*)
show *?thesis*
proof (*cases static m*)
case *True*
with *static-m' dom-vnames m*
show *?thesis*
by (*cases s*) (*simp add*: *init-lvars-def Let-def parameters-def*
dom-lname-case-ename-simps image-comp)
next
case *False*
with *static-m' dom-vnames m*

```
    show ?thesis
      by (cases s) (simp add: init-lvars-def Let-def parameters-def
                               dom-lname-case-ename-simps image-comp)
  qed
qed


lemma da-e2-BinOp:
  assumes da: (|prg=G,cls=accC,lcl=L|)
                ⊢dom (locals (store s0)) »⟨BinOp binop e1 e2⟩ₑ» A
    and wt-e1: (|prg=G,cls=accC,lcl=L|)⊢e1::−e1T
    and wt-e2: (|prg=G,cls=accC,lcl=L|)⊢e2::−e2T
    and wt-binop: wt-binop G binop e1T e2T
    and conf-s0: s0::≼(G,L)
    and normal-s1: normal s1
    and eval-e1: G⊢s0 −e1−≻v1→ s1
    and conf-v1: G,store s1⊢v1::≼e1T
    and wf: wf-prog G
  shows ∃ E2. (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1))
        »(if need-second-arg binop v1 then ⟨e2⟩ₑ else ⟨Skip⟩ₛ)» E2
proof −
  note inj-term-simps [simp]
  from da obtain E1 where
    da-e1: (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨e1⟩ₑ» E1
    by cases simp+
  obtain E2 where
    (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1))
      »(if need-second-arg binop v1 then ⟨e2⟩ₑ else ⟨Skip⟩ₛ)» E2
  proof (cases need-second-arg binop v1)
    case False
    obtain S where
      daSkip: (|prg=G,cls=accC,lcl=L|)
                ⊢ dom (locals (store s1)) »⟨Skip⟩ₛ» S
      by (auto intro: da-Skip [simplified] assigned.select-convs)
    thus ?thesis
      using that by (simp add: False)
  next
    case True
    from eval-e1 have
      s0-s1:dom (locals (store s0)) ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim)
    {
      assume condAnd: binop=CondAnd
      have ?thesis
      proof −
        from da obtain E2' where
          (|prg=G,cls=accC,lcl=L|)
            ⊢ dom (locals (store s0)) ∪ assigns-if True e1 »⟨e2⟩ₑ» E2'
          by cases (simp add: condAnd)+
        moreover
        have dom (locals (store s0))
          ∪ assigns-if True e1 ⊆ dom (locals (store s1))
        proof −
          from condAnd wt-binop have e1T: e1T=PrimT Boolean
            by simp
          with normal-s1 conf-v1 obtain b where v1=Bool b
            by (auto dest: conf-Boolean)
          with True condAnd
```

```
      have v1: v1=Bool True
        by simp
      from eval-e1 normal-s1
      have assigns-if True e1 ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx' [elim-format])
           (insert wt-e1, simp-all add: e1T v1)
      with s0-s1 show ?thesis by (rule Un-least)
    qed
    ultimately
    show ?thesis
      using that by (cases rule: da-weakenE) (simp add: True)
  qed
}
moreover
{
  assume condOr: binop=CondOr
  have ?thesis

  proof −
    from da obtain E2' where
      ⦇prg=G,cls=accC,lcl=L⦈
         ⊢ dom (locals (store s0)) ∪ assigns-if False e1 »⟨e2⟩_e» E2'
      by cases (simp add: condOr)+
    moreover
    have dom (locals (store s0))
              ∪ assigns-if False e1 ⊆ dom (locals (store s1))
    proof −
      from condOr wt-binop have e1T: e1T=PrimT Boolean
        by simp
      with normal-s1 conf-v1 obtain b where v1=Bool b
        by (auto dest: conf-Boolean)
      with True condOr
      have v1: v1=Bool False
        by simp
      from eval-e1 normal-s1
      have assigns-if False e1 ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx' [elim-format])
           (insert wt-e1, simp-all add: e1T v1)
      with s0-s1 show ?thesis by (rule Un-least)
    qed
    ultimately
    show ?thesis
      using that by (rule da-weakenE) (simp add: True)
  qed
}
moreover
{
  assume notAndOr: binop≠CondAnd binop≠CondOr
  have ?thesis
  proof −
    from da notAndOr obtain E1' where
      da-e1: ⦇prg=G,cls=accC,lcl=L⦈
             ⊢ dom (locals (store s0)) »⟨e1⟩_e» E1'
      and da-e2: ⦇prg=G,cls=accC,lcl=L⦈⊢ nrm E1' »In1l e2» A
      by cases simp+
    from eval-e1 wt-e1 da-e1 wf normal-s1
    have nrm E1' ⊆ dom (locals (store s1))
      by (cases rule: da-good-approxE') iprover
    with da-e2 show ?thesis
```

```
      using that by (rule da-weakenE) (simp add: True)
    qed
  }
  ultimately show ?thesis
    by (cases binop) auto
 qed
 thus ?thesis ..
qed
```

main proof of type safety

```
lemma eval-type-sound:
 assumes eval: G⊢s0 −t≻→ (v,s1)
  and    wt: ⦇prg=G,cls=accC,lcl=L⦈⊢t::T
  and    da: ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0))»t»A
  and    wf: wf-prog G
  and conf-s0: s0::≼(G,L)
 shows s1::≼(G,L) ∧ (normal s1 ⟶ G,L,store s1⊢t≻v::≼T) ∧
       (error-free s0 = error-free s1)
proof −
 note inj-term-simps [simp]
 let ?TypeSafeObj = λ s0 s1 t v.
      ∀ L accC T A. s0::≼(G,L) ⟶ ⦇prg=G,cls=accC,lcl=L⦈⊢t::T
                ⟶ ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0))»t»A
                ⟶ s1::≼(G,L) ∧ (normal s1 ⟶ G,L,store s1⊢t≻v::≼T)
                  ∧ (error-free s0 = error-free s1)
 from eval
 have ⋀ L accC T A. ⟦s0::≼(G,L);⦇prg=G,cls=accC,lcl=L⦈⊢t::T;
                ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0))»t»A⟧
     ⟹ s1::≼(G,L) ∧ (normal s1 ⟶ G,L,store s1⊢t≻v::≼T)
       ∧ (error-free s0 = error-free s1)
  (is PROP ?TypeSafe s0 s1 t v
   is ⋀ L accC T A. ?Conform L s0 ⟹ ?WellTyped L accC T t
            ⟹ ?DefAss L accC s0 t A
            ⟹ ?Conform L s1 ∧ ?ValueTyped L T s1 t v ∧
              ?ErrorFree s0 s1)
 proof (induct)
  case (Abrupt xc s t L accC T A)
  from ‹(Some xc, s)::≼(G,L)›
  show (Some xc, s)::≼(G,L) ∧
   (normal (Some xc, s)
   ⟶ G,L,store (Some xc,s)⊢t≻undefined3 t::≼T) ∧
   (error-free (Some xc, s) = error-free (Some xc, s))
    by simp
 next
  case (Skip s L accC T A)
  from ‹Norm s::≼(G, L)› and
   ‹⦇prg = G, cls = accC, lcl = L⦈⊢In1r Skip::T›
  show Norm s::≼(G, L) ∧
        (normal (Norm s) ⟶ G,L,store (Norm s)⊢In1r Skip≻◇::≼T) ∧
        (error-free (Norm s) = error-free (Norm s))
    by simp
 next
  case (Expr s0 e v s1 L accC T A)
  note ‹G⊢Norm s0 −e−≻v→ s1›
  note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  moreover
  note wt = ‹⦇prg = G, cls = accC, lcl = L⦈⊢In1r (Expr e)::T›
```

then obtain eT
  where $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash In1l\ e{::}eT$
  by (*rule wt-elim-cases*) *blast*
moreover
from *Expr.prems* **obtain** E **where**
  $(\!|prg{=}G{,}cls{=}accC,\ lcl{=}L|\!)\vdash dom\ (locals\ (store\ ((Norm\ s0){::}state)))\gg In1l\ e\gg E$
  by (*elim da-elim-cases*) *simp*
ultimately
obtain $s1{::}\preceq(G,\ L)$ **and** *error-free s1*
  by (*rule hyp* [*elim-format*]) *simp*
with *wt*
show $s1{::}\preceq(G,\ L) \wedge$
   $(normal\ s1 \longrightarrow G{,}L{,}store\ s1\vdash In1r\ (Expr\ e)\succ\Diamond{::}\preceq T) \wedge$
   $(error\text{-}free\ (Norm\ s0) = error\text{-}free\ s1)$
  by (*simp*)
next
case (*Lab s0 c s1 l L accC T A*)
note $hyp = \langle PROP\ ?TypeSafe\ (Norm\ s0)\ s1\ (In1r\ c)\ \Diamond\rangle$
note $conf\text{-}s0 = \langle Norm\ s0{::}\preceq(G,\ L)\rangle$
moreover
note $wt = \langle(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash In1r\ (l\bullet\ c){::}T\rangle$
then have $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash c{::}\surd$
  by (*rule wt-elim-cases*) *blast*
moreover from *Lab.prems* **obtain** C **where**
 $(\!|prg{=}G{,}cls{=}accC,\ lcl{=}L|\!)\vdash dom\ (locals\ (store\ ((Norm\ s0){::}state)))\gg In1r\ c\gg C$
  by (*elim da-elim-cases*) *simp*
ultimately
obtain $conf\text{-}s1{:}\ s1{::}\preceq(G,\ L)$ **and**
   $error\text{-}free\text{-}s1{:}\ error\text{-}free\ s1$
  by (*rule hyp* [*elim-format*]) *simp*
from *conf-s1* **have** $abupd\ (absorb\ l)\ s1{::}\preceq(G,\ L)$
  by (*cases s1*) (*auto intro*: *conforms-absorb*)
with *wt error-free-s1*
show $abupd\ (absorb\ l)\ s1{::}\preceq(G,\ L) \wedge$
   $(normal\ (abupd\ (absorb\ l)\ s1)$
   $\longrightarrow G{,}L{,}store\ (abupd\ (absorb\ l)\ s1)\vdash In1r\ (l\bullet\ c)\succ\Diamond{::}\preceq T) \wedge$
   $(error\text{-}free\ (Norm\ s0) = error\text{-}free\ (abupd\ (absorb\ l)\ s1))$
  by (*simp*)
next
case (*Comp s0 c1 s1 c2 s2 L accC T A*)
note $eval\text{-}c1 = \langle G\vdash Norm\ s0\ -c1\rightarrow\ s1\rangle$
note $eval\text{-}c2 = \langle G\vdash s1\ -c2\rightarrow\ s2\rangle$
note $hyp\text{-}c1 = \langle PROP\ ?TypeSafe\ (Norm\ s0)\ s1\ (In1r\ c1)\ \Diamond\rangle$
note $hyp\text{-}c2 = \langle PROP\ ?TypeSafe\ s1\ \ \ s2\ (In1r\ c2)\ \Diamond\rangle$
note $conf\text{-}s0 = \langle Norm\ s0{::}\preceq(G,\ L)\rangle$
note $wt = \langle(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash In1r\ (c1;;\ c2){::}T\rangle$
then obtain $wt\text{-}c1{:}\ (\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash c1{::}\surd$ **and**
   $wt\text{-}c2{:}\ (\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash c2{::}\surd$
  by (*rule wt-elim-cases*) *blast*
from *Comp.prems*
obtain $C1\ C2$
  where $da\text{-}c1{:}\ (\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash$
     $dom\ (locals\ (store\ ((Norm\ s0){::}state)))\ \gg In1r\ c1\gg\ C1$ **and**
   $da\text{-}c2{:}\ (\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash\ \ nrm\ C1\ \gg In1r\ c2\gg\ C2$
  by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-c1 da-c1*
obtain $conf\text{-}s1{:}\ s1{::}\preceq(G,\ L)$ **and**
   $error\text{-}free\text{-}s1{:}\ error\text{-}free\ s1$
  by (*rule hyp-c1* [*elim-format*]) *simp*

```
  show s2::≼(G, L) ∧
        (normal s2 ⟶ G,L,store s2⊢In1r (c1;; c2)≻◇::≼T) ∧
        (error-free (Norm s0) = error-free s2)
  proof (cases normal s1)
    case False
    with eval-c2 have s2=s1 by auto
    with conf-s1 error-free-s1 False wt show ?thesis
      by simp
  next
    case True
    obtain C2' where
      ⦇prg=G, cls=accC, lcl=L⦈⊢ dom (locals (store s1)) »In1r c2» C2'
    proof −
      from eval-c1 wt-c1 da-c1 wf True
      have nrm C1 ⊆ dom (locals (store s1))
        by (cases rule: da-good-approxE') iprover
      with da-c2 show thesis
        by (rule da-weakenE) (rule that)
    qed
    with conf-s1 wt-c2
    obtain s2::≼(G, L) and error-free s2
      by (rule hyp-c2 [elim-format]) (simp add: error-free-s1)
    thus ?thesis
      using wt by simp
  qed
next
  case (If s0 e b s1 c1 c2 s2 L accC T A)
  note eval-e = ‹G⊢Norm s0 −e−≻b→ s1›
  note eval-then-else = ‹G⊢s1 −(if the-Bool b then c1 else c2)→ s2›
  note hyp-e = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 b)›
  note hyp-then-else =
    ‹PROP ?TypeSafe s1 s2 (In1r (if the-Bool b then c1 else c2)) ◇›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹⦇prg = G, cls = accC, lcl = L⦈⊢In1r (If(e) c1 Else c2)::T›
  then obtain
            wt-e: ⦇prg=G, cls=accC, lcl=L⦈⊢e::−PrimT Boolean and
    wt-then-else: ⦇prg=G, cls=accC, lcl=L⦈⊢(if the-Bool b then c1 else c2)::√

    by (rule wt-elim-cases) auto
  from If.prems obtain E C where
    da-e: ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals (store ((Norm s0)::state)))
                                  »In1l e» E and
    da-then-else:
    ⦇prg=G,cls=accC,lcl=L⦈⊢
       (dom (locals (store ((Norm s0)::state))) ∪ assigns-if (the-Bool b) e)
        »In1r (if the-Bool b then c1 else c2)» C

    by (elim da-elim-cases) (cases the-Bool b,auto)
  from conf-s0 wt-e da-e
  obtain conf-s1: s1::≼(G, L) and error-free-s1: error-free s1
    by (rule hyp-e [elim-format]) simp
  show s2::≼(G, L) ∧
         (normal s2 ⟶ G,L,store s2⊢In1r (If(e) c1 Else c2)≻◇::≼T) ∧
         (error-free (Norm s0) = error-free s2)
  proof (cases normal s1)
    case False
    with eval-then-else have s2=s1 by auto
    with conf-s1 error-free-s1 False wt show ?thesis
      by simp
```

next
case *True*
obtain C' **where**
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash$
$(dom\ (locals\ (store\ s1)))\gg In1r\ (if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)\gg\ C'$
proof –
from *eval-e* **have**
$dom\ (locals\ (store\ ((Norm\ s0)::state))) \subseteq dom\ (locals\ (store\ s1))$
by (*rule dom-locals-eval-mono-elim*)
moreover
from *eval-e True wt-e*
have $assigns\text{-}if\ (the\text{-}Bool\ b)\ e \subseteq dom\ (locals\ (store\ s1))$
by (*rule assigns-if-good-approx'*)
ultimately
have $dom\ (locals\ (store\ ((Norm\ s0)::state)))$
$\cup\ assigns\text{-}if\ (the\text{-}Bool\ b)\ e \subseteq dom\ (locals\ (store\ s1))$
by (*rule Un-least*)
with *da-then-else* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *conf-s1 wt-then-else*
obtain $s2::\preceq(G,\ L)$ **and** *error-free s2*
by (*rule hyp-then-else* [*elim-format*]) (*simp add*: *error-free-s1*)
with *wt* **show** *?thesis*
by *simp*
qed

— Note that we don't have to show that *b* really is a boolean value. With *the-Bool* we enforce to get a value of boolean type. So execution will be type safe, even if b would be a string, for example. We might not expect such a behaviour to be called type safe. To remedy the situation we would have to change the evaulation rule, so that it only has a type safe evaluation if we actually get a boolean value for the condition. That b is actually a boolean value is part of *hyp-e*. See also Loop

next
case (*Loop s0 e b s1 c s2 l s3 L accC T A*)
note $eval\text{-}e = \langle G\vdash Norm\ s0\ -e-\succ b\rightarrow\ s1\rangle$
note $hyp\text{-}e = \langle PROP\ ?TypeSafe\ (Norm\ s0)\ s1\ (In1l\ e)\ (In1\ b)\rangle$
note $conf\text{-}s0 = \langle Norm\ s0::\preceq(G,\ L)\rangle$
note $wt = \langle (\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash In1r\ (l\cdot\ While(e)\ c)::T\rangle$
then obtain $wt\text{-}e$: $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash e::-PrimT\ Boolean$ **and**
$wt\text{-}c$: $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash c::\surd$
by (*rule wt-elim-cases*) *blast*
note $da = \langle (\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)$
$\vdash dom\ (locals(store\ ((Norm\ s0)::state)))\ \gg In1r\ (l\cdot\ While(e)\ c)\gg\ A\rangle$
then
obtain *E C* **where**
$da\text{-}e$: $(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)$
$\vdash dom\ (locals\ (store\ ((Norm\ s0)::state)))\ \gg In1l\ e\gg\ E$ **and**
$da\text{-}c$: $(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)$
$\vdash (dom\ (locals\ (store\ ((Norm\ s0)::state)))$
$\cup\ assigns\text{-}if\ True\ e)\ \gg In1r\ c\gg\ C$
by (*rule da-elim-cases*) *simp*
from *conf-s0 wt-e da-e*
obtain $conf\text{-}s1$: $s1::\preceq(G,\ L)$ **and** *error-free-s1*: *error-free s1*
by (*rule hyp-e* [*elim-format*]) *simp*
show $s3::\preceq(G,\ L)\ \wedge$
$(normal\ s3\ \longrightarrow\ G,L,store\ s3\vdash In1r\ (l\cdot\ While(e)\ c)\succ\Diamond::\preceq T)\ \wedge$
$(error\text{-}free\ (Norm\ s0) = error\text{-}free\ s3)$
proof (*cases normal s1*)
case *True*
note *normal-s1* = *this*

```
show ?thesis
proof (cases the-Bool b)
  case True
  with Loop.hyps obtain
    eval-c: G⊢s1 −c→ s2 and
    eval-while: G⊢abupd (absorb (Cont l)) s2 −l· While(e) c→ s3
    by simp
  have ?TypeSafeObj s1 s2 (In1r c) ◇
    using Loop.hyps True by simp
  note hyp-c = this [rule-format]
  have ?TypeSafeObj (abupd (absorb (Cont l)) s2)
    s3 (In1r (l· While(e) c)) ◇
    using Loop.hyps True by simp
  note hyp-w = this [rule-format]
  from eval-e have
    s0-s1: dom (locals (store ((Norm s0)::state)))
            ⊆ dom (locals (store s1))
    by (rule dom-locals-eval-mono-elim)
  obtain C' where
    (|prg=G, cls=accC, lcl=L|)⊢(dom (locals (store s1)))»In1r c» C'
  proof −
    note s0-s1
    moreover
    from eval-e normal-s1 wt-e
    have assigns-if True e ⊆ dom (locals (store s1))
      by (rule assigns-if-good-approx' [elim-format]) (simp add: True)
    ultimately
    have dom (locals (store ((Norm s0)::state)))
          ∪ assigns-if True e ⊆ dom (locals (store s1))
      by (rule Un-least)
    with da-c show thesis
      by (rule da-weakenE) (rule that)
  qed
  with conf-s1 wt-c
  obtain conf-s2: s2::≼(G, L) and error-free-s2: error-free s2
    by (rule hyp-c [elim-format]) (simp add: error-free-s1)
  from error-free-s2
  have error-free-ab-s2: error-free (abupd (absorb (Cont l)) s2)
    by simp
  from conf-s2 have abupd (absorb (Cont l)) s2 ::≼(G, L)
    by (cases s2) (auto intro: conforms-absorb)
  moreover note wt
  moreover
  obtain A' where
    (|prg=G,cls=accC,lcl=L|)⊢
        dom (locals(store (abupd (absorb (Cont l)) s2)))
          »In1r (l· While(e) c)» A'
  proof −
    note s0-s1
    also from eval-c
    have dom (locals (store s1)) ⊆ dom (locals (store s2))
      by (rule dom-locals-eval-mono-elim)
    also have ... ⊆ dom (locals (store (abupd (absorb (Cont l)) s2)))
      by simp
    finally
    have dom (locals (store ((Norm s0)::state))) ⊆ ... .
    with da show thesis
      by (rule da-weakenE) (rule that)
  qed
```

```
      ultimately obtain s3::≼(G, L) and error-free s3
        by (rule hyp-w [elim-format]) (simp add: error-free-ab-s2)
      with wt show ?thesis
        by simp
    next
      case False
      with Loop.hyps have s3=s1 by simp
      with conf-s1 error-free-s1 wt
      show ?thesis
        by simp
    qed
  next
    case False
    have s3=s1
    proof −
      from False obtain abr where abr: abrupt s1 = Some abr
        by (cases s1) auto
      from eval-e - wt-e have no-jmp: ⋀ j. abrupt s1 ≠ Some (Jump j)
        by (rule eval-expression-no-jump
             [where ?Env=(|prg=G,cls=accC,lcl=L|),simplified])
           (simp-all add: wf)

      show ?thesis
      proof (cases the-Bool b)
        case True
        with Loop.hyps obtain
          eval-c: G⊢s1 −c→ s2 and
          eval-while: G⊢abupd (absorb (Cont l)) s2 −l• While(e) c→ s3
          by simp
        from eval-c abr have s2=s1 by auto
        moreover from calculation no-jmp have abupd (absorb (Cont l)) s2=s2
          by (cases s1) (simp add: absorb-def)
        ultimately show ?thesis
          using eval-while abr
          by auto
      next
        case False
        with Loop.hyps show ?thesis by simp
      qed
    qed
    with conf-s1 error-free-s1 wt
    show ?thesis
      by simp
  qed
next
  case (Jmp s j L accC T A)
  note ‹Norm s::≼(G, L)›
  moreover
  from Jmp.prems
  have j=Ret ⟶ Result ∈ dom (locals (store ((Norm s)::state)))
    by (elim da-elim-cases)
  ultimately have (Some (Jump j), s)::≼(G, L) by auto
  then
  show (Some (Jump j), s)::≼(G, L) ∧
         (normal (Some (Jump j), s)
         ⟶ G,L,store (Some (Jump j), s)⊢In1r (Jmp j)≻◇::≼T) ∧
         (error-free (Norm s) = error-free (Some (Jump j), s))
    by simp
next
```

case (*Throw s0 e a s1 L accC T A*)
note ‹*G*⊢*Norm s0* −*e*−≻*a*→ *s1*›
note *hyp* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1l e*) (*In1 a*)›
note *conf-s0* = ‹*Norm s0*::⪯(*G*, *L*)›
note *wt* = ‹⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*In1r* (*Throw e*)::*T*›
then obtain *tn*
  where *wt-e*: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*e*::−*Class tn* **and**
    throwable: *G*⊢*tn*$\preceq_C$ *SXcpt Throwable*
  by (*rule wt-elim-cases*) (*auto*)
from *Throw.prems* **obtain** *E* **where**
  da-e: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
    ⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1l e*» *E*
  by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-e da-e* **obtain**
  s1::⪯(*G*, *L*) **and**
  (*normal s1* ⟶ *G*,*store s1*⊢*a*::⪯*Class tn*) **and**
  error-free-s1: *error-free s1*
  by (*rule hyp* [*elim-format*]) *simp*
with *wf throwable*
have *abupd* (*throw a*) *s1*::⪯(*G*, *L*)
  by (*cases s1*) (*auto dest*: *Throw-lemma*)
with *wt error-free-s1*
show *abupd* (*throw a*) *s1*::⪯(*G*, *L*) ∧
    (*normal* (*abupd* (*throw a*) *s1*) ⟶
    G,*L*,*store* (*abupd* (*throw a*) *s1*)⊢*In1r* (*Throw e*)≻◇::⪯*T*) ∧
    (*error-free* (*Norm s0*) = *error-free* (*abupd* (*throw a*) *s1*))
  by *simp*
next
case (*Try s0 c1 s1 s2 catchC vn c2 s3 L accC T A*)
note *eval-c1* = ‹*G*⊢*Norm s0* −*c1*→ *s1*›
note *sx-alloc* = ‹*G*⊢*s1* −*sxalloc*→ *s2*›
note *hyp-c1* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1r c1*) ◇›
note *conf-s0* = ‹*Norm s0*::⪯(*G*, *L*)›
note *wt* = ‹⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*In1r* (*Try c1 Catch*(*catchC vn*) *c2*)::*T*›
then obtain
  wt-c1: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*c1*::√ **and**
  wt-c2: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*(*VName vn*↦*Class catchC*)⦈⊢*c2*::√ **and**
  fresh-vn: *L*(*VName vn*)=*None*
  by (*rule wt-elim-cases*) *simp*
from *Try.prems* **obtain** *C1 C2* **where**
  da-c1: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
    ⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1r c1*» *C1* **and**
  da-c2:
   ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*(*VName vn*↦*Class catchC*)⦈
   ⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))) ∪ {*VName vn*}) »*In1r c2*» *C2*
  by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-c1 da-c1*
obtain *conf-s1*: *s1*::⪯(*G*, *L*) **and** *error-free-s1*: *error-free s1*
  by (*rule hyp-c1* [*elim-format*]) *simp*
from *conf-s1 sx-alloc wf*
have *conf-s2*: *s2*::⪯(*G*, *L*)
  by (*auto dest*: *sxalloc-type-sound split*: *option.splits abrupt.splits*)
from *sx-alloc error-free-s1*
have *error-free-s2*: *error-free s2*
  by (*rule error-free-sxalloc*)
show *s3*::⪯(*G*, *L*) ∧
   (*normal s3* ⟶ *G*,*L*,*store s3*⊢*In1r* (*Try c1 Catch*(*catchC vn*) *c2*)≻◇::⪯*T*)∧
   (*error-free* (*Norm s0*) = *error-free s3*)
proof (*cases* ∃ *x*. *abrupt s1* = *Some* (*Xcpt x*))

```
    case False
    from sx-alloc wf
    have eq-s2-s1: s2=s1
      by (rule sxalloc-type-sound [elim-format])
         (insert False, auto split: option.splits abrupt.splits )
    with False
    have ¬ G,s2⊢catch catchC
      by (simp add: catch-def)
    with Try
    have s3=s2
      by simp
    with wt conf-s1 error-free-s1 eq-s2-s1
    show ?thesis
      by simp
  next
    case True
    note exception-s1 = this
    show ?thesis
    proof (cases G,s2⊢catch catchC)
      case False
      with Try
      have s3=s2
        by simp
      with wt conf-s2 error-free-s2
      show ?thesis
        by simp
    next
      case True
      with Try have G⊢new-xcpt-var vn s2 −c2→ s3 by simp
      from True Try.hyps
      have ?TypeSafeObj (new-xcpt-var vn s2) s3 (In1r c2) ♢
        by simp
      note hyp-c2 = this [rule-format]
      from exception-s1 sx-alloc wf
      obtain a
        where xcpt-s2: abrupt s2 = Some (Xcpt (Loc a))
        by (auto dest!: sxalloc-type-sound split: option.splits abrupt.splits)
      with True
      have G⊢obj-ty (the (globs (store s2) (Heap a)))≼Class catchC
        by (cases s2) simp
      with xcpt-s2 conf-s2 wf
      have new-xcpt-var vn s2 ::≼(G, L(VName vn↦Class catchC))
        by (auto dest: Try-lemma)
      moreover note wt-c2
      moreover
      obtain C2′ where
        ⦇prg=G,cls=accC,lcl=L(VName vn↦Class catchC)⦈
        ⊢ (dom (locals (store (new-xcpt-var vn s2)))) »In1r c2» C2′
      proof −
        have (dom (locals (store ((Norm s0)::state))) ∪ {VName vn})
                ⊆ dom (locals (store (new-xcpt-var vn s2)))
        proof −
          from ‹G⊢Norm s0 −c1→ s1›
          have dom (locals (store ((Norm s0)::state)))
                  ⊆ dom (locals (store s1))
            by (rule dom-locals-eval-mono-elim)
          also
          from sx-alloc
          have ... ⊆ dom (locals (store s2))
```

```
          by (rule dom-locals-sxalloc-mono)
        also
        have ... ⊆ dom (locals (store (new-xcpt-var vn s2)))
          by (cases s2) (simp add: new-xcpt-var-def, blast)
        also
        have {VName vn} ⊆ ...
          by (cases s2) simp
        ultimately show ?thesis
          by (rule Un-least)
      qed
      with da-c2 show thesis
        by (rule da-weakenE) (rule that)
    qed
    ultimately
    obtain      conf-s3: s3::≼(G, L(VName vn↦Class catchC)) and
          error-free-s3: error-free s3
      by (rule hyp-c2 [elim-format])
         (cases s2, simp add: xcpt-s2 error-free-s2)
    from conf-s3 fresh-vn
    have s3::≼(G,L)
      by (blast intro: conforms-deallocL)
    with wt error-free-s3
    show ?thesis
      by simp
  qed
qed
next
  case (Fin s0 c1 x1 s1 c2 s2 s3 L accC T A)
  note eval-c1 = ‹G⊢Norm s0 −c1→ (x1, s1)›
  note eval-c2 = ‹G⊢Norm s1 −c2→ s2›
  note s3 = ‹s3 = (if ∃ err. x1 = Some (Error err)
                    then (x1, s1)
                    else abupd (abrupt-if (x1 ≠ None) x1) s2)›
  note hyp-c1 = ‹PROP ?TypeSafe (Norm s0) (x1,s1) (In1r c1) ◇›
  note hyp-c2 = ‹PROP ?TypeSafe (Norm s1) s2      (In1r c2) ◇›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1r (c1 Finally c2)::T›
  then obtain
    wt-c1: (|prg=G,cls=accC,lcl=L|)⊢c1::√ and
    wt-c2: (|prg=G,cls=accC,lcl=L|)⊢c2::√
    by (rule wt-elim-cases) blast
  from Fin.prems obtain C1 C2 where
    da-c1: (|prg=G,cls=accC,lcl=L|)
           ⊢ dom (locals (store ((Norm s0)::state))) »In1r c1» C1 and
    da-c2: (|prg=G,cls=accC,lcl=L|)
           ⊢ dom (locals (store ((Norm s0)::state))) »In1r c2» C2
    by (elim da-elim-cases) simp
  from conf-s0 wt-c1 da-c1
  obtain conf-s1: (x1,s1)::≼(G, L) and error-free-s1: error-free (x1,s1)
    by (rule hyp-c1 [elim-format]) simp
  from conf-s1 have Norm s1::≼(G, L)
    by (rule conforms-NormI)
  moreover note wt-c2
  moreover obtain C2'
    where (|prg=G,cls=accC,lcl=L|)
           ⊢ dom (locals (store ((Norm s1)::state))) »In1r c2» C2'
  proof −
    from eval-c1
    have dom (locals (store ((Norm s0)::state)))
```

```
          ⊆ dom (locals (store (x1,s1)))
      by (rule dom-locals-eval-mono-elim)
    hence dom (locals (store ((Norm s0)::state)))
            ⊆ dom (locals (store ((Norm s1)::state)))
      by simp
    with da-c2 show thesis
      by (rule da-weakenE) (rule that)
  qed
  ultimately
  obtain conf-s2: s2::≼(G, L) and error-free-s2: error-free s2
    by (rule hyp-c2 [elim-format]) simp
  from error-free-s1 s3
  have s3': s3=abupd (abrupt-if (x1 ≠ None) x1) s2
    by simp
  show s3::≼(G, L) ∧
       (normal s3 ⟶ G,L,store s3 ⊢In1r (c1 Finally c2)≻◇::≼T) ∧
       (error-free (Norm s0) = error-free s3)
  proof (cases x1)
    case None with conf-s2 s3' wt error-free-s2
    show ?thesis by auto
  next
    case (Some x)
    from eval-c2 have
      dom (locals (store ((Norm s1)::state))) ⊆ dom (locals (store s2))
      by (rule dom-locals-eval-mono-elim)
    with Some eval-c2 wf conf-s1 conf-s2
    have conf: (abrupt-if True (Some x) (abrupt s2), store s2)::≼(G, L)
      by (cases s2) (auto dest: Fin-lemma)
    from Some error-free-s1
    have ¬ (∃ err. x=Error err)
      by (simp add: error-free-def)
    with error-free-s2
    have error-free (abrupt-if True (Some x) (abrupt s2), store s2)
      by (cases s2) simp
    with Some wt conf s3' show ?thesis
      by (cases s2) auto
  qed
next
  case (Init C c s0 s3 s1 s2 L accC T A)
  note cls = ‹the (class G C) = c›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1r (Init C)::T›
  with cls
  have cls-C: class G C = Some c
    by − (erule wt-elim-cases, auto)
  show s3::≼(G, L) ∧ (normal s3 ⟶ G,L,store s3⊢In1r (Init C)≻◇::≼T) ∧
       (error-free (Norm s0) = error-free s3)
  proof (cases inited C (globs s0))
    case True
    with Init.hyps have s3 = Norm s0
      by simp
    with conf-s0 wt show ?thesis
      by simp
  next
    case False
    with Init.hyps obtain
         eval-init-super:
         G⊢Norm ((init-class-obj G C) s0)
           −(if C = Object then Skip else Init (super c))→ s1 and
```

eval-init: G⊢(*set-lvars Map.empty*) *s1* −*init c*→ *s2* **and**
s3: *s3* = (*set-lvars* (*locals* (*store s1*))) *s2*
by *simp*
have *?TypeSafeObj* (*Norm* ((*init-class-obj G C*) *s0*)) *s1*
(*In1r* (*if C* = *Object then Skip else Init* (*super c*))) ♢
using *False Init.hyps* **by** *simp*
note *hyp-init-super* = *this* [*rule-format*]
have *?TypeSafeObj* ((*set-lvars Map.empty*) *s1*) *s2* (*In1r* (*init c*)) ♢
using *False Init.hyps* **by** *simp*
note *hyp-init-c* = *this* [*rule-format*]
from *conf-s0 wf cls-C False*
have (*Norm* ((*init-class-obj G C*) *s0*))::≼(*G*, *L*)
by (*auto dest*: *conforms-init-class-obj*)
moreover from *wf cls-C* **have**
wt-init-super: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈
⊢(*if C* = *Object then Skip else Init* (*super c*))::√
by (*cases C*=*Object*)
(*auto dest*: *wf-prog-cdecl wf-cdecl-supD is-acc-classD*)
moreover
obtain *S* **where**
da-init-super:
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ *dom* (*locals* (*store* ((*Norm* ((*init-class-obj G C*) *s0*))::*state*)))
»*In1r* (*if C* = *Object then Skip else Init* (*super c*))» *S*
proof (*cases C*=*Object*)
case *True*
with *da-Skip* **show** *?thesis*
using *that* **by** (*auto intro*: *assigned.select-convs*)
next
case *False*
with *da-Init* **show** *?thesis*
by − (*rule that*, *auto intro*: *assigned.select-convs*)
qed
ultimately
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and** *error-free-s1*: *error-free s1*
by (*rule hyp-init-super* [*elim-format*]) *simp*
from *eval-init-super wt-init-super wf*
have *s1-no-ret*: ⋀ *j*. *abrupt s1* ≠ *Some* (*Jump j*)
by − (*rule eval-statement-no-jump* [**where** *?Env*=⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈],
auto)
with *conf-s1*
have (*set-lvars Map.empty*) *s1*::≼(*G*, *Map.empty*)
by (*cases s1*) (*auto intro*: *conforms-set-locals*)
moreover
from *error-free-s1*
have *error-free-empty*: *error-free* ((*set-lvars Map.empty*) *s1*)
by *simp*
from *cls-C wf* **have** *wt-init-c*: ⦇*prg*=*G*, *cls*=*C*,*lcl*=*Map.empty*⦈⊢(*init c*)::√
by (*rule wf-prog-cdecl* [*THEN wf-cdecl-wt-init*])
moreover from *cls-C wf* **obtain** *I*
where ⦇*prg*=*G*,*cls*=*C*,*lcl*=*Map.empty*⦈⊢ {} »*In1r* (*init c*)» *I*
by (*rule wf-prog-cdecl* [*THEN wf-cdeclE*,*simplified*]) *blast*

then obtain *I′* **where**
⦇*prg*=*G*,*cls*=*C*,*lcl*=*Map.empty*⦈⊢*dom* (*locals* (*store* ((*set-lvars Map.empty*) *s1*)))
»*In1r* (*init c*)» *I′*
by (*rule da-weakenE*) *simp*
ultimately
obtain *conf-s2*: *s2*::≼(*G*, *Map.empty*) **and** *error-free-s2*: *error-free s2*

by (*rule hyp-init-c* [*elim-format*]) (*simp add*: *error-free-empty*)
have *abrupt s2* $\neq$ *Some* (*Jump Ret*)
proof −
from *s1-no-ret*
have $\bigwedge$ *j. abrupt* ((*set-lvars Map.empty*) *s1*) $\neq$ *Some* (*Jump j*)
by *simp*
moreover
from *cls-C wf* **have** *jumpNestingOkS* {} (*init c*)
by (*rule wf-prog-cdecl* [*THEN wf-cdeclE*])
ultimately
show *?thesis*
using *eval-init wt-init-c wf*
by − (*rule eval-statement-no-jump*
[**where** *?Env*=(|*prg*=*G*,*cls*=*C*,*lcl*=*Map.empty*|)],*simp*+)
qed
with *conf-s2 s3 conf-s1 eval-init*
have *s3*::$\preceq$(*G*, *L*)
by (*cases s2*,*cases s1*) (*force dest*: *conforms-return eval-gext'*)
moreover from *error-free-s2 s3*
have *error-free s3*
by *simp*
moreover note *wt*
ultimately show *?thesis*
by *simp*
qed
next
case (*NewC s0 C s1 a s2 L accC T A*)
note ‹*G*⊢*Norm s0* −*Init C*→ *s1*›
note *halloc* = ‹*G*⊢*s1* −*halloc CInst C*≻*a*→ *s2*›
note *hyp* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1r* (*Init C*)) ◇›
note *conf-s0* = ‹*Norm s0*::$\preceq$(*G*, *L*)›
moreover
note *wt* = ‹(|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)⊢*In1l* (*NewC C*)::*T*›
then obtain *is-cls-C*: *is-class G C* **and**
T: *T*=*Inl* (*Class C*)
by (*rule wt-elim-cases*) (*auto dest*: *is-acc-classD*)
hence (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)⊢*Init C*::$\surd$ **by** *auto*
moreover obtain *I* **where**
(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1r* (*Init C*)» *I*
by (*auto intro*: *da-Init* [*simplified*] *assigned.select-convs*)

ultimately
obtain *conf-s1*: *s1*::$\preceq$(*G*, *L*) **and** *error-free-s1*: *error-free s1*
by (*rule hyp* [*elim-format*]) *simp*
from *conf-s1 halloc wf is-cls-C*
obtain *halloc-type-safe*: *s2*::$\preceq$(*G*, *L*)
(*normal s2* ⟶ *G*,*store s2*⊢*Addr a*::$\preceq$*Class C*)
by (*cases s2*) (*auto dest*!: *halloc-type-sound*)
from *halloc error-free-s1*
have *error-free s2*
by (*rule error-free-halloc*)
with *halloc-type-safe T*
show *s2*::$\preceq$(*G*, *L*) ∧
(*normal s2* ⟶ *G*,*L*,*store s2*⊢*In1l* (*NewC C*)≻*In1* (*Addr a*)::$\preceq$*T*) ∧
(*error-free* (*Norm s0*) = *error-free s2*)
by *auto*
next
case (*NewA s0 elT s1 e i s2 a s3 L accC T A*)

note *eval-init* = ‹G⊢Norm s0 −init-comp-ty elT→ s1›
note *eval-e* = ‹G⊢s1 −e−≻i→ s2›
note *halloc* = ‹G⊢abupd (check-neg i) s2−halloc Arr elT (the-Intg i)≻a→ s3›
note *hyp-init* = ‹PROP ?TypeSafe (Norm s0) s1 (In1r (init-comp-ty elT)) ◇›
note *hyp-size* = ‹PROP ?TypeSafe s1 s2 (In1l e) (In1 i)›
note *conf-s0* = ‹Norm s0::≼(G, L)›
note *wt* = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l (New elT[e])::T›
then obtain
  wt-init: (|prg = G, cls = accC, lcl = L|)⊢init-comp-ty elT::√ **and**
  wt-size: (|prg = G, cls = accC, lcl = L|)⊢e::−PrimT Integer **and**
    elT: is-type G elT **and**
    T: T=Inl (elT.[])
  by (rule wt-elim-cases) (auto intro: wt-init-comp-ty dest: is-acc-typeD)
from *NewA.prems*
have *da-e*:(|prg=G,cls=accC,lcl=L|)
      ⊢ dom (locals (store ((Norm s0)::state))) »In1l e» A
  by (elim da-elim-cases) simp
obtain *conf-s1*: s1::≼(G, L) **and** *error-free-s1*: error-free s1
proof −
  note *conf-s0 wt-init*
  moreover obtain *I* **where**
    (|prg=G,cls=accC,lcl=L|)
    ⊢ dom (locals (store ((Norm s0)::state))) »In1r (init-comp-ty elT)» I
  proof (cases ∃ C. elT = Class C)
    case *True*
    thus *?thesis*
      by − (rule that, (auto intro: da-Init [simplified]
                assigned.select-convs
            simp add: init-comp-ty-def))

  next
    case *False*
    thus *?thesis*
    by − (rule that, (auto intro: da-Skip [simplified]
                assigned.select-convs
            simp add: init-comp-ty-def))

  qed
  ultimately show *thesis*
    by (rule hyp-init [elim-format]) (auto intro: that)
qed
obtain *conf-s2*: s2::≼(G, L) **and** *error-free-s2*: error-free s2
proof −
  from *eval-init*
  have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
    by (rule dom-locals-eval-mono-elim)
  with *da-e*
  obtain *A′* **where**
   (|prg=G,cls=accC,lcl=L|)
      ⊢ dom (locals (store s1)) »In1l e» A′
    by (rule da-weakenE)
  with *conf-s1 wt-size*
  show *?thesis*
    by (rule hyp-size [elim-format]) (simp add: that error-free-s1)
qed
from *conf-s2* **have** abupd (check-neg i) s2::≼(G, L)
  by (cases s2) auto
with *halloc wf elT*
have *halloc-type-safe*:

s3::≼(G, L) ∧ (normal s3 ⟶ G,store s3⊢Addr a::≼elT.[])
by (cases s3) (auto dest!: halloc-type-sound)
from halloc error-free-s2
have error-free s3
by (auto dest: error-free-halloc)
with halloc-type-safe T
show s3::≼(G, L) ∧
(normal s3 ⟶ G,L,store s3⊢In1l (New elT[e])≻In1 (Addr a)::≼T) ∧
(error-free (Norm s0) = error-free s3)
by simp
next
case (Cast s0 e v s1 s2 castT L accC T A)
note ‹G⊢Norm s0 −e−≻v→ s1›
note s2 = ‹s2 = abupd (raise-if (¬ G,store s1⊢v fits castT) ClassCast) s1›
note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›
note conf-s0 = ‹Norm s0::≼(G, L)›
note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l (Cast castT e)::T›
then obtain eT
where wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT **and**
eT: G⊢eT≼? castT **and**
T: T=Inl castT
by (rule wt-elim-cases) auto
from Cast.prems
have (|prg=G,cls=accC,lcl=L|)
⊢ dom (locals (store ((Norm s0)::state))) »In1l e» A
by (elim da-elim-cases) simp
with conf-s0 wt-e
obtain conf-s1: s1::≼(G, L) **and**
v-ok: normal s1 ⟶ G,store s1⊢v::≼eT **and**
error-free-s1: error-free s1
by (rule hyp [elim-format]) simp
from conf-s1 s2
have conf-s2: s2::≼(G, L)
by (cases s1) simp
from error-free-s1 s2
have error-free-s2: error-free s2
by simp
{
assume norm-s2: normal s2
have G,L,store s2⊢In1l (Cast castT e)≻In1 v::≼T
proof −
from s2 norm-s2 **have** normal s1
by (cases s1) simp
with v-ok
have G,store s1⊢v::≼eT
by simp
with eT wf s2 T norm-s2
show ?thesis
by (cases s1) (auto dest: fits-conf)
qed
}
with conf-s2 error-free-s2
show s2::≼(G, L) ∧
(normal s2 ⟶ G,L,store s2⊢In1l (Cast castT e)≻In1 v::≼T) ∧
(error-free (Norm s0) = error-free s2)
by blast
next
case (Inst s0 e v s1 b instT L accC T A)
note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›

```
    note conf-s0 = ‹Norm s0::≼(G, L)›
    from Inst.prems obtain eT
    where wt-e: ⦇prg = G, cls = accC, lcl = L⦈⊢e::−RefT eT  and
              T: T=Inl (PrimT Boolean)
      by (elim wt-elim-cases) simp
    from Inst.prems
    have da-e: ⦇prg=G,cls=accC,lcl=L⦈
                 ⊢ dom (locals (store ((Norm s0)::state))) »In1l e» A
      by (elim da-elim-cases) simp
    from conf-s0 wt-e da-e
    obtain conf-s1: s1::≼(G, L) and
              v-ok: normal s1 ⟶ G,store s1⊢v::≼RefT eT and
      error-free-s1: error-free s1
      by (rule hyp [elim-format]) simp
    with T show ?case
      by simp
  next
    case (Lit s v L accC T A)
    then show ?case
      by (auto elim!: wt-elim-cases
                intro: conf-litval simp add: empty-dt-def)
  next
    case (UnOp s0 e v s1 unop L accC T A)
    note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    note wt = ‹⦇prg = G, cls = accC, lcl = L⦈⊢In1l (UnOp unop e)::T›
    then obtain eT
      where     wt-e: ⦇prg = G, cls = accC, lcl = L⦈⊢e::−eT and
            wt-unop: wt-unop unop eT and
                  T: T=Inl (PrimT (unop-type unop))
      by (auto elim!: wt-elim-cases)
    from UnOp.prems obtain A where
       da-e: ⦇prg=G,cls=accC,lcl=L⦈
                 ⊢ dom (locals (store ((Norm s0)::state))) »In1l e» A
      by (elim da-elim-cases) simp
    from conf-s0 wt-e da-e
    obtain      conf-s1: s1::≼(G, L)  and
                wt-v: normal s1 ⟶ G,store s1⊢v::≼eT and
         error-free-s1: error-free s1
      by (rule hyp [elim-format]) simp
    from wt-v T wt-unop
    have normal s1⟶G,L,snd s1⊢In1l (UnOp unop e)≻In1 (eval-unop unop v)::≼T
      by (cases unop) auto
    with conf-s1 error-free-s1
    show s1::≼(G, L) ∧
     (normal s1 ⟶ G,L,snd s1⊢In1l (UnOp unop e)≻In1 (eval-unop unop v)::≼T) ∧
     error-free (Norm s0) = error-free s1
      by simp
  next
    case (BinOp s0 e1 v1 s1 binop e2 v2 s2 L accC T A)
    note eval-e1 = ‹G⊢Norm s0 −e1−≻v1→ s1›
    note eval-e2 = ‹G⊢s1 −(if need-second-arg binop v1 then In1l e2
                              else In1r Skip)≻→ (In1 v2, s2)›
    note hyp-e1 = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e1) (In1 v1)›
    note hyp-e2 = ‹PROP ?TypeSafe       s1  s2
                  (if need-second-arg binop v1 then In1l e2 else In1r Skip)
                  (In1 v2)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    note wt = ‹⦇prg = G, cls = accC, lcl = L⦈⊢In1l (BinOp binop e1 e2)::T›
```

then obtain *e1T e2T* **where**
wt-e1: $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash e1::-e1T$ **and**
wt-e2: $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash e2::-e2T$ **and**
wt-binop: *wt-binop G binop e1T e2T* **and**
T: $T{=}Inl\ (PrimT\ (binop\text{-}type\ binop))$
by (*elim wt-elim-cases*) *simp*
have *wt-Skip*: $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash Skip::\surd$
by *simp*
obtain *S* **where**
daSkip: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\vdash dom\ (locals\ (store\ s1))\ \gg In1r\ Skip\gg\ S$
by (*auto intro*: *da-Skip* [*simplified*] *assigned.select-convs*)
note $da = \langle(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash\ dom\ (locals\ (store\ ((Norm\ s0::state))))$
$\gg\langle BinOp\ binop\ e1\ e2\rangle_e\gg\ A\rangle$
then obtain *E1* **where**
da-e1: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\vdash dom\ (locals\ (store\ ((Norm\ s0)::state)))\ \gg In1l\ e1\gg\ E1$
by (*elim da-elim-cases*) *simp+*
from *conf-s0 wt-e1 da-e1*
obtain *conf-s1*: $s1::\preceq(G,\ L)$ **and**
wt-v1: $normal\ s1\ \longrightarrow\ G,store\ s1\vdash v1::\preceq e1T$ **and**
error-free-s1: *error-free s1*
by (*rule hyp-e1* [*elim-format*]) *simp*
from *wt-binop T*
have *conf-v*:
$G,L,snd\ s2\vdash In1l\ (BinOp\ binop\ e1\ e2)\succ In1\ (eval\text{-}binop\ binop\ v1\ v2)::\preceq T$
by (*cases binop*) *auto*

— Note that we don't use the information that v1 really is compatible with the expected type e1T and v2 is compatible with e2T, because *eval-binop* will anyway produce an output of the right type. So evaluating the addition of an integer with a string is type safe. This is a little bit annoying since we may regard such a behaviour as not type safe. If we want to avoid this we can redefine *eval-binop* so that it only produces a output of proper type if it is assigned to values of the expected types, and arbitrary if the inputs have unexpected types. The proof can easily be adapted since we have the hypothesis that the values have a proper type. This also applies to unary operations.

from *eval-e1* **have**
s0-s1: $dom\ (locals\ (store\ ((Norm\ s0)::state)))\subseteq dom\ (locals\ (store\ s1))$
by (*rule dom-locals-eval-mono-elim*)
show $s2::\preceq(G,\ L)\ \wedge$
$(normal\ s2\ \longrightarrow$
$G,L,snd\ s2\vdash In1l\ (BinOp\ binop\ e1\ e2)\succ In1\ (eval\text{-}binop\ binop\ v1\ v2)::\preceq T)\ \wedge$
$error\text{-}free\ (Norm\ s0) = error\text{-}free\ s2$
proof (*cases normal s1*)
case *False*
with *eval-e2* **have** *s2=s1* **by** *auto*
with *conf-s1 error-free-s1 False* **show** *?thesis*
by *auto*
next
case *True*
note *normal-s1 = this*
show *?thesis*
proof (*cases need-second-arg binop v1*)
case *False*
with *normal-s1 eval-e2* **have** *s2=s1*
by (*cases s1*) (*simp*, *elim eval-elim-cases*,*simp*)
with *conf-s1 conf-v error-free-s1*
show *?thesis* **by** *simp*
next
case *True*
note *need-second-arg = this*

```
      with hyp-e2
      have hyp-e2': PROP ?TypeSafe s1 s2 (In1l e2) (In1 v2) by simp
      from da wt-e1 wt-e2 wt-binop conf-s0 normal-s1 eval-e1
        wt-v1 [rule-format,OF normal-s1] wf
      obtain E2 where
        ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals (store s1)) »In1l e2» E2
        by (rule da-e2-BinOp [elim-format])
           (auto simp add: need-second-arg )
      with conf-s1 wt-e2
      obtain s2::≼(G, L) and error-free s2
        by (rule hyp-e2' [elim-format]) (simp add: error-free-s1)
      with conf-v show ?thesis by simp
    qed
  qed
next
  case (Super s L accC T A)
  note conf-s = ‹Norm s::≼(G, L)›
  note wt = ‹⦇prg = G, cls = accC, lcl = L⦈⊢In1l Super::T›
  then obtain C c where
          C: L This = Some (Class C) and
     neq-Obj: C≠Object and
       cls-C: class G C = Some c and
          T: T=Inl (Class (super c))
    by (rule wt-elim-cases) auto
  from Super.prems
  obtain This ∈ dom (locals s)
    by (elim da-elim-cases) simp
  with conf-s C  have G,s⊢val-this s::≼Class C
    by (auto dest: conforms-localD [THEN wlconfD])
  with neq-Obj cls-C wf
  have G,s⊢val-this s::≼Class (super c)
    by (auto intro: conf-widen
               dest: subcls-direct[THEN widen.subcls])
  with T conf-s
  show Norm s::≼(G, L) ∧
         (normal (Norm s) ⟶
            G,L,store (Norm s)⊢In1l Super≻In1 (val-this s)::≼T) ∧
         (error-free (Norm s) = error-free (Norm s))
    by simp
next
  case (Acc s0 v w upd s1 L accC T A)
  note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In2 v) (In2 (w,upd))›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  from Acc.prems obtain vT where
    wt-v: ⦇prg = G, cls = accC, lcl = L⦈⊢v::=vT and
       T: T=Inl vT
    by (elim wt-elim-cases) simp
  from Acc.prems obtain V where
    da-v: ⦇prg=G,cls=accC,lcl=L⦈
               ⊢ dom (locals (store ((Norm s0)::state))) »In2 v» V
    by (cases ∃ n. v=LVar n) (insert da.LVar, auto elim!: da-elim-cases)
  {
    fix n assume lvar: v=LVar n
    have locals (store s1) n ≠ None
    proof −
      from Acc.prems lvar have
        n ∈ dom (locals s0)
        by (cases ∃ n. v=LVar n) (auto elim!: da-elim-cases)
      also
```

```
      have dom (locals s0) ⊆ dom (locals (store s1))
      proof -
        from ‹G⊢Norm s0 −v=≻(w, upd)→ s1›
        show ?thesis
          by (rule dom-locals-eval-mono-elim) simp
      qed
      finally show ?thesis
        by blast
    qed
  } note lvar-in-locals = this
  from conf-s0 wt-v da-v
  obtain conf-s1: s1::≼(G, L)
    and  conf-var: (normal s1 ⟶ G,L,store s1⊢In2 v≻In2 (w, upd)::≼Inl vT)
    and  error-free-s1: error-free s1
    by (rule hyp [elim-format]) simp
  from lvar-in-locals conf-var T
  have (normal s1 ⟶ G,L,store s1⊢In1l (Acc v)≻In1 w::≼T)
    by (cases ∃ n. v=LVar n) auto
  with conf-s1 error-free-s1 show ?case
    by simp
next
  case (Ass s0 var w upd s1 e v s2 L accC T A)
  note eval-var = ‹G⊢Norm s0 −var=≻(w, upd)→ s1›
  note eval-e = ‹G⊢s1 −e−≻v→ s2›
  note hyp-var = ‹PROP ?TypeSafe (Norm s0) s1 (In2 var) (In2 (w,upd))›
  note hyp-e = ‹PROP ?TypeSafe s1 s2 (In1l e) (In1 v)›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l (var:=e)::T›
  then obtain varT eT where
       wt-var: (|prg = G, cls = accC, lcl = L|)⊢var::=varT and
         wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT and
        widen: G⊢eT≼varT and
            T: T=Inl eT
    by (rule wt-elim-cases) auto
  show assign upd v s2::≼(G, L) ∧
         (normal (assign upd v s2) ⟶
          G,L,store (assign upd v s2)⊢In1l (var:=e)≻In1 v::≼T) ∧
    (error-free (Norm s0) = error-free (assign upd v s2))
  proof (cases ∃ vn. var=LVar vn)
    case False
    with Ass.prems
    obtain V E where
      da-var: (|prg=G,cls=accC,lcl=L|)
                ⊢ dom (locals (store ((Norm s0)::state))) »In2 var» V and
      da-e:   (|prg=G,cls=accC,lcl=L|) ⊢ nrm V »In1l e» E
      by (elim da-elim-cases) simp+
    from conf-s0 wt-var da-var
    obtain conf-s1: s1::≼(G, L)
      and  conf-var: normal s1
                   ⟶ G,L,store s1⊢In2 var≻In2 (w, upd)::≼Inl varT
      and  error-free-s1: error-free s1
      by (rule hyp-var [elim-format]) simp
    show ?thesis
    proof (cases normal s1)
      case False
      with eval-e have s2=s1 by auto
      with False have assign upd v s2=s1
        by simp
      with conf-s1 error-free-s1 False show ?thesis
```

```
        by auto
    next
      case True
      note normal-s1=this
      obtain A' where (|prg=G,cls=accC,lcl=L|)
                      ⊢ dom (locals (store s1)) »In1l e» A'
      proof −
        from eval-var wt-var da-var wf normal-s1
        have nrm V ⊆  dom (locals (store s1))
          by (cases rule: da-good-approxE') iprover
        with da-e show thesis
          by (rule da-weakenE) (rule that)
      qed
      with conf-s1 wt-e
      obtain conf-s2: s2::≼(G, L) and
        conf-v: normal s2 ⟶ G,store s2⊢v::≼eT and
        error-free-s2: error-free s2
        by (rule hyp-e [elim-format]) (simp add: error-free-s1)
      show ?thesis
      proof (cases normal s2)
        case False
        with conf-s2 error-free-s2
        show ?thesis
          by auto
      next
        case True
        from True conf-v
        have conf-v-eT: G,store s2⊢v::≼eT
          by simp
        with widen wf
        have conf-v-varT: G,store s2⊢v::≼varT
          by (auto intro: conf-widen)
        from normal-s1 conf-var
        have G,L,store s1⊢In2 var≻In2 (w, upd)::≼Inl varT
          by simp
        then
        have conf-assign:  store s1≤|upd≼varT::≼(G, L)
          by (simp add: rconf-def)
        from conf-v-eT conf-v-varT conf-assign normal-s1 True wf eval-var
          eval-e T conf-s2 error-free-s2
        show ?thesis
          by (cases s1, cases s2)
             (auto dest!: Ass-lemma simp add: assign-conforms-def)
      qed
    qed
  next
    case True
    then obtain vn where vn: var=LVar vn
      by blast
    with Ass.prems
    obtain E where
      da-e: (|prg=G,cls=accC,lcl=L|)
              ⊢ dom (locals (store ((Norm s0)::state))) »In1l e» E
      by (elim da-elim-cases) simp+
    from da.LVar vn obtain V where
      da-var: (|prg=G,cls=accC,lcl=L|)
              ⊢ dom (locals (store ((Norm s0)::state))) »In2 var» V
      by auto
    obtain E' where
```

```
    da-e′: (|prg=G,cls=accC,lcl=L|)
              ⊢ dom (locals (store s1)) »In1l e» E′
  proof –
    have dom (locals (store ((Norm s0)::state)))
           ⊆ dom (locals (store (s1)))
      by (rule dom-locals-eval-mono-elim) (rule Ass.hyps)
    with da-e show thesis
      by (rule da-weakenE) (rule that)
  qed
  from conf-s0 wt-var da-var
  obtain conf-s1: s1::≼(G, L)
    and  conf-var: normal s1
                      ⟶ G,L,store s1⊢In2 var≻In2 (w, upd)::≼Inl varT
    and  error-free-s1: error-free s1
    by (rule hyp-var [elim-format]) simp
  show ?thesis
  proof (cases normal s1)
    case False
    with eval-e have s2=s1 by auto
    with False have assign upd v s2=s1
      by simp
    with conf-s1 error-free-s1 False show ?thesis
      by auto
  next
    case True
    note normal-s1 = this
    from conf-s1 wt-e da-e′
    obtain conf-s2: s2::≼(G, L) and
      conf-v: normal s2 ⟶ G,store s2⊢v::≼eT and
      error-free-s2: error-free s2
      by (rule hyp-e [elim-format]) (simp add: error-free-s1)
    show ?thesis
    proof (cases normal s2)
      case False
      with conf-s2 error-free-s2
      show ?thesis
        by auto
    next
      case True
      from True conf-v
      have conf-v-eT: G,store s2⊢v::≼eT
        by simp
      with widen wf
      have conf-v-varT: G,store s2⊢v::≼varT
        by (auto intro: conf-widen)
      from normal-s1 conf-var
      have G,L,store s1⊢In2 var≻In2 (w, upd)::≼Inl varT
        by simp
      then
      have conf-assign:  store s1≤|upd≼varT::≼(G, L)
        by (simp add: rconf-def)
      from conf-v-eT conf-v-varT conf-assign normal-s1 True wf eval-var
        eval-e T conf-s2 error-free-s2
      show ?thesis
        by (cases s1, cases s2)
           (auto dest!: Ass-lemma simp add: assign-conforms-def)
    qed
  qed
qed
```

```
next
  case (Cond s0 e0 b s1 e1 e2 v s2 L accC T A)
  note eval-e0 = ‹G⊢Norm s0 −e0−≻b→ s1›
  note eval-e1-e2 = ‹G⊢s1 −(if the-Bool b then e1 else e2)−≻v→ s2›
  note hyp-e0 = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e0) (In1 b)›
  note hyp-if = ‹PROP ?TypeSafe s1 s2
                    (In1l (if the-Bool b then e1 else e2)) (In1 v)›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l (e0 ? e1 : e2)::T›
  then obtain T1 T2 statT where
    wt-e0: (|prg = G, cls = accC, lcl = L|)⊢e0::−PrimT Boolean and
    wt-e1: (|prg = G, cls = accC, lcl = L|)⊢e1::−T1 and
    wt-e2: (|prg = G, cls = accC, lcl = L|)⊢e2::−T2 and
    statT: G⊢T1≼T2 ∧ statT = T2  ∨  G⊢T2≼T1 ∧ statT =  T1 and
    T    : T=Inl statT
    by (rule wt-elim-cases) auto
  with Cond.prems obtain E0 E1 E2 where
       da-e0: (|prg=G,cls=accC,lcl=L|)
               ⊢ dom (locals (store ((Norm s0)::state)))
                   »In1l e0» E0 and
       da-e1: (|prg=G,cls=accC,lcl=L|)
               ⊢ (dom (locals (store ((Norm s0)::state)))
                    ∪ assigns-if True e0) »In1l e1» E1 and
       da-e2: (|prg=G,cls=accC,lcl=L|)
               ⊢ (dom (locals (store ((Norm s0)::state)))
                   ∪ assigns-if False e0) »In1l e2» E2
     by (elim da-elim-cases) simp+
  from conf-s0 wt-e0 da-e0
  obtain conf-s1: s1::≼(G, L) and error-free-s1: error-free s1
    by (rule hyp-e0 [elim-format]) simp
  show s2::≼(G, L) ∧
         (normal s2 ⟶ G,L,store s2⊢In1l (e0 ? e1 : e2)≻In1 v::≼T) ∧
         (error-free (Norm s0) = error-free s2)
  proof (cases normal s1)
    case False
    with eval-e1-e2 have s2=s1 by auto
    with conf-s1 error-free-s1 False show ?thesis
      by auto
  next
    case True
    have s0-s1: dom (locals (store ((Norm s0)::state)))
                 ∪ assigns-if (the-Bool b) e0 ⊆ dom (locals (store s1))
    proof −
      from eval-e0 have
        dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
        by (rule dom-locals-eval-mono-elim)
      moreover
      from eval-e0 True wt-e0
      have assigns-if (the-Bool b) e0 ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx′)
      ultimately show ?thesis by (rule Un-least)
    qed
    show ?thesis
    proof (cases the-Bool b)
      case True
      with hyp-if have hyp-e1: PROP ?TypeSafe s1 s2 (In1l e1) (In1 v)
        by simp
      from da-e1 s0-s1 True obtain E1′ where
        (|prg=G,cls=accC,lcl=L|)⊢ (dom (locals (store s1)))»In1l e1» E1′
```

by − (*rule da-weakenE*, *auto iff del*: *Un-subset-iff sup.bounded-iff*)
with *conf-s1 wt-e1*
obtain
s2::≼(*G*, *L*)
(*normal s2* ⟶ *G*,*L*,*store s2*⊢*In1l e1*≻*In1 v*::≼*Inl T1*)
error-free s2
by (*rule hyp-e1* [*elim-format*]) (*simp add*: *error-free-s1*)
moreover
from *statT*
have *G*⊢*T1*≼*statT*
by *auto*
ultimately show *?thesis*
using *T wf* **by** *auto*
next
case *False*
with *hyp-if* **have** *hyp-e2*: *PROP ?TypeSafe s1 s2* (*In1l e2*) (*In1 v*)
by *simp*
from *da-e2 s0-s1 False* **obtain** *E2′* **where**
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ (*dom* (*locals* (*store s1*)))»*In1l e2*» *E2′*
by − (*rule da-weakenE*, *auto iff del*: *Un-subset-iff sup.bounded-iff*)
with *conf-s1 wt-e2*
obtain
s2::≼(*G*, *L*)
(*normal s2* ⟶ *G*,*L*,*store s2*⊢*In1l e2*≻*In1 v*::≼*Inl T2*)
error-free s2
by (*rule hyp-e2* [*elim-format*]) (*simp add*: *error-free-s1*)
moreover
from *statT*
have *G*⊢*T2*≼*statT*
by *auto*
ultimately show *?thesis*
using *T wf* **by** *auto*
qed
qed
next
case (*Call s0 e a s1 args vs s2 invDeclC mode statT mn pTs′ s3 s3′ accC′*
v s4 L accC T A)
note *eval-e* = ‹*G*⊢*Norm s0* −*e*−≻*a*→ *s1*›
note *eval-args* = ‹*G*⊢*s1* −*args*≐≻*vs*→ *s2*›
note *invDeclC* = ‹*invDeclC*
= *invocation-declclass G mode* (*store s2*) *a statT*
⦇*name* = *mn*, *parTs* = *pTs′*⦈›
note *init-lvars* =
‹*s3* = *init-lvars G invDeclC* ⦇*name* = *mn*, *parTs* = *pTs′*⦈ *mode a vs s2*›
note *check* = ‹*s3′* =
check-method-access G accC′ statT mode ⦇*name* = *mn*, *parTs* = *pTs′*⦈ *a s3*›
note *eval-methd* =
‹*G*⊢*s3′* −*Methd invDeclC* ⦇*name* = *mn*, *parTs* = *pTs′*⦈−≻*v*→ *s4*›
note *hyp-e* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1l e*) (*In1 a*)›
note *hyp-args* = ‹*PROP ?TypeSafe s1 s2* (*In3 args*) (*In3 vs*)›
note *hyp-methd* = ‹*PROP ?TypeSafe s3′ s4*
(*In1l* (*Methd invDeclC* ⦇*name* = *mn*, *parTs* = *pTs′*⦈)) (*In1 v*)›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹⦇*prg*=*G*, *cls*=*accC*, *lcl*=*L*⦈
⊢*In1l* ({*accC′*,*statT*,*mode*}*e*·*mn*({*pTs′*}*args*))::*T*›
from *wt* **obtain** *pTs statDeclT statM* **where**
wt-e: ⦇*prg*=*G*, *cls*=*accC*, *lcl*=*L*⦈⊢*e*::−*RefT statT* **and**
wt-args: ⦇*prg*=*G*, *cls*=*accC*, *lcl*=*L*⦈⊢*args*::≐*pTs* **and**
statM: *max-spec G accC statT* ⦇*name*=*mn*,*parTs*=*pTs*⦈

= {((*statDeclT*,*statM*),*pTs'*)} **and**
mode: *mode* = *invmode statM e* **and**
T: *T* =*Inl* (*resTy statM*) **and**
eq-accC-accC': *accC*=*accC'*
by (*rule wt-elim-cases*) *fastforce+*
from *Call.prems* **obtain** *E* **where**
da-e: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))»*In1l e*» *E* **and**
da-args: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *nrm E* »*In3 args*» *A*
by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-e da-e*
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and**
conf-a: *normal s1* ⟹ *G*, *store s1*⊢*a*::≼*RefT statT* **and**
error-free-s1: *error-free s1*
by (*rule hyp-e* [*elim-format*]) *simp*
{
assume *abnormal-s2*: ¬ *normal s2*
have *set-lvars* (*locals* (*store s2*)) *s4* = *s2*
proof −
from *abnormal-s2 init-lvars*
obtain *keep-abrupt*: *abrupt s3* = *abrupt s2* **and**
store s3 = *store* (*init-lvars G invDeclC* ⦇*name* = *mn*, *parTs* = *pTs'*⦈
mode a vs s2)
by (*auto simp add*: *init-lvars-def2*)
moreover
from *keep-abrupt abnormal-s2 check*
have *eq-s3'-s3*: *s3'*=*s3*
by (*auto simp add*: *check-method-access-def Let-def*)
moreover
from *eq-s3'-s3 abnormal-s2 keep-abrupt eval-methd*
have *s4*=*s3'*
by *auto*
ultimately show
set-lvars (*locals* (*store s2*)) *s4* = *s2*
by (*cases s2*,*cases s3*) (*simp add*: *init-lvars-def2*)
qed
} note *propagate-abnormal-s2* = *this*
show (*set-lvars* (*locals* (*store s2*))) *s4*::≼(*G*, *L*) ∧
(*normal* ((*set-lvars* (*locals* (*store s2*))) *s4*) ⟶
G,*L*,*store* ((*set-lvars* (*locals* (*store s2*))) *s4*)
⊢*In1l* ({*accC'*,*statT*,*mode*}*e*·*mn*({*pTs'*}*args*))≻*In1 v*::≼*T*) ∧
(*error-free* (*Norm s0*) =
error-free ((*set-lvars* (*locals* (*store s2*))) *s4*))
proof (*cases normal s1*)
case *False*
with *eval-args* **have** *s2*=*s1* **by** *auto*
with *False propagate-abnormal-s2 conf-s1 error-free-s1*
show *?thesis*
by *auto*
next
case *True*
note *normal-s1* = *this*
obtain *A'* **where**
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *dom* (*locals* (*store s1*)) »*In3 args*» *A'*
proof −
from *eval-e wt-e da-e wf normal-s1*
have *nrm E* ⊆ *dom* (*locals* (*store s1*))
by (*cases rule*: *da-good-approxE'*) *iprover*
with *da-args* **show** *thesis*

by (*rule da-weakenE*) (*rule that*)
qed
with *conf-s1 wt-args*
obtain *conf-s2*: *s2*::$\preceq$(*G*, *L*) **and**
conf-args: *normal s2*
$\Longrightarrow$ *list-all2* (*conf G* (*store s2*)) *vs pTs* **and**
error-free-s2: *error-free s2*
by (*rule hyp-args* [*elim-format*]) (*simp add*: *error-free-s1*)
from *error-free-s2 init-lvars*
have *error-free-s3*: *error-free s3*
by (*auto simp add*: *init-lvars-def2*)
from *statM*
obtain
statM': (*statDeclT*,*statM*)$\in$*mheads G accC statT* (|*name*=*mn*,*parTs*=*pTs'*|) **and**
pTs-widen: *G*$\vdash$*pTs*[$\preceq$]*pTs'*
by (*blast dest*: *max-spec2mheads*)
from *check*
have *eq-store-s3'-s3*: *store s3'*=*store s3*
by (*cases s3*) (*simp add*: *check-method-access-def Let-def*)
obtain *invC*
where *invC*: *invC* = *invocation-class mode* (*store s2*) *a statT*
by *simp*
with *init-lvars*
have *invC'*: *invC* = (*invocation-class mode* (*store s3*) *a statT*)
by (*cases s2*,*cases mode*) (*auto simp add*: *init-lvars-def2*)
show *?thesis*
proof (*cases normal s2*)
case *False*
with *propagate-abnormal-s2 conf-s2 error-free-s2*
show *?thesis*
by *auto*
next
case *True*
note *normal-s2* = *True*
with *normal-s1 conf-a eval-args*
have *conf-a-s2*: *G*, *store s2*$\vdash$*a*::$\preceq$*RefT statT*
by (*auto dest*: *eval-gext intro*: *conf-gext*)
show *?thesis*
proof (*cases a*=*Null* $\longrightarrow$ *is-static statM*)
case *False*
then obtain *not-static*: $\neg$ *is-static statM* **and** *Null*: *a*=*Null*
by *blast*
with *normal-s2 init-lvars mode*
obtain *np*: *abrupt s3* = *Some* (*Xcpt* (*Std NullPointer*)) **and**
store s3 = *store* (*init-lvars G invDeclC*
(|*name* = *mn*, *parTs* = *pTs'*|) *mode a vs s2*)
by (*auto simp add*: *init-lvars-def2*)
moreover
from *np check*
have *eq-s3'-s3*: *s3'*=*s3*
by (*auto simp add*: *check-method-access-def Let-def*)
moreover
from *eq-s3'-s3 np eval-methd*
have *s4*=*s3'*
by *auto*
ultimately have
set-lvars (*locals* (*store s2*)) *s4*
= (*Some* (*Xcpt* (*Std NullPointer*)),*store s2*)
by (*cases s2*,*cases s3*) (*simp add*: *init-lvars-def2*)

```
        with conf-s2 error-free-s2
        show ?thesis
          by (cases s2) (auto dest: conforms-NormI)
      next
        case True
        with mode have notNull: mode = IntVir ⟶ a ≠ Null
          by (auto dest!: Null-staticD)
        with conf-s2 conf-a-s2 wf invC
        have dynT-prop: G⊢mode→invC≼statT
          by (cases s2) (auto intro: DynT-propI)
        with wt-e statM' invC mode wf
        obtain dynM where
          dynM: dynlookup G statT invC  (|name=mn,parTs=pTs'|) = Some dynM and
          acc-dynM: G ⊢Methd  (|name=mn,parTs=pTs'|) dynM
                        in invC dyn-accessible-from accC
          by (force dest!: call-access-ok)
        with invC' check eq-accC-accC'
        have eq-s3'-s3: s3'=s3
          by (auto simp add: check-method-access-def Let-def)
        from dynT-prop wf wt-e statM' mode invC invDeclC dynM
        obtain
          wf-dynM: wf-mdecl G invDeclC ((|name=mn,parTs=pTs'|),mthd dynM) and
            dynM': methd G invDeclC (|name=mn,parTs=pTs'|) = Some dynM and
          iscls-invDeclC: is-class G invDeclC and
               invDeclC': invDeclC = declclass dynM and
            invC-widen: G⊢invC≼C invDeclC and
           resTy-widen: G⊢resTy dynM≼resTy statM and
          is-static-eq: is-static dynM = is-static statM and
          involved-classes-prop:
           (if invmode statM e = IntVir
             then ∀ statC. statT = ClassT statC ⟶ G⊢invC≼C statC
             else ((∃ statC. statT = ClassT statC ∧ G⊢statC≼C invDeclC) ∨
                   (∀ statC. statT ≠ ClassT statC ∧ invDeclC = Object)) ∧
                    statDeclT = ClassT invDeclC)
          by (cases rule: DynT-mheadsE) simp
        obtain L' where
         L':L'=(λ k.
               (case k of
                  EName e
                  ⇒ (case e of
                         VNam v
                         ⇒(table-of (lcls (mbody (mthd dynM)))
                            (pars (mthd dynM)[↦]pTs')) v
                       | Res ⇒ Some (resTy dynM))
                | This ⇒ if is-static statM
                            then None else Some (Class invDeclC)))
          by simp
        from wf-dynM [THEN wf-mdeclD1, THEN conjunct1] normal-s2 conf-s2 wt-e
          wf eval-args conf-a mode notNull wf-dynM involved-classes-prop
        have conf-s3: s3::≼(G,L')
          apply -

          apply (drule conforms-init-lvars [of G invDeclC
                  (|name=mn,parTs=pTs'|) dynM store s2 vs pTs abrupt s2
                  L statT invC a (statDeclT,statM) e])
          apply (rule wf)
          apply (rule conf-args,assumption)
          apply (simp add: pTs-widen)
          apply (cases s2,simp)
```

```
  apply (rule dynM')
  apply (force dest: ty-expr-is-type)
  apply (rule invC-widen)
  apply (force intro: conf-gext dest: eval-gext)
  apply simp
  apply simp
  apply (simp add: invC)
  apply (simp add: invDeclC)
  apply (simp add: normal-s2)
  apply (cases s2, simp add: L' init-lvars
                  cong add: lname.case-cong ename.case-cong)
  done
with eq-s3'-s3
have conf-s3': s3'::≼(G,L') by simp
moreover
from is-static-eq wf-dynM L'
obtain mthdT where
  ⦇prg=G,cls=invDeclC,lcl=L'⦈
    ⊢Body invDeclC (stmt (mbody (mthd dynM)))::−mthdT and
  mthdT-widen: G⊢mthdT≼resTy dynM
  by − (drule wf-mdecl-bodyD,
       auto simp add: callee-lcl-def
            cong add: lname.case-cong ename.case-cong)
with dynM' iscls-invDeclC invDeclC'
have
  ⦇prg=G,cls=invDeclC,lcl=L'⦈
    ⊢(Methd invDeclC ⦇name = mn, parTs = pTs'⦈)::−mthdT
  by (auto intro: wt.Methd)
moreover
obtain M where
  ⦇prg=G,cls=invDeclC,lcl=L'⦈
    ⊢ dom (locals (store s3'))
       »In1l (Methd invDeclC ⦇name = mn, parTs = pTs'⦈)» M
proof −
  from wf-dynM
  obtain M' where
    da-body:
    ⦇prg=G, cls=invDeclC
     ,lcl=callee-lcl invDeclC ⦇name = mn, parTs = pTs'⦈ (mthd dynM)
     ⦈ ⊢ parameters (mthd dynM) »⟨stmt (mbody (mthd dynM))⟩» M' and
    res: Result ∈ nrm M'
    by (rule wf-mdeclE) iprover
  from da-body is-static-eq L' have
    ⦇prg=G, cls=invDeclC,lcl=L'⦈
      ⊢ parameters (mthd dynM) »⟨stmt (mbody (mthd dynM))⟩» M'
    by (simp add: callee-lcl-def
         cong add: lname.case-cong ename.case-cong)
  moreover have parameters (mthd dynM) ⊆  dom (locals (store s3'))
  proof −
    from is-static-eq
    have (invmode (mthd dynM) e) = (invmode statM e)
      by (simp add: invmode-def)
    moreover
    have length (pars (mthd dynM)) = length vs
    proof −
      from normal-s2 conf-args
      have length vs = length pTs
        by (simp add: list-all2-iff)
      also from pTs-widen
```

have . . . = *length pTs′*
by (*simp add*: *widens-def list-all2-iff*)
also from *wf-dynM*
have . . . = *length* (*pars* (*mthd dynM*))
by (*simp add*: *wf-mdecl-def wf-mhead-def*)
finally show *?thesis* **..**
qed
moreover note *init-lvars dynM′ is-static-eq normal-s2 mode*
ultimately
have *parameters* (*mthd dynM*) = *dom* (*locals* (*store s3*))
using *dom-locals-init-lvars*
[*of mthd dynM G invDeclC* (|*name*=*mn*,*parTs*=*pTs′*|) *vs e a s2*]
by *simp*
also from *check*
have *dom* (*locals* (*store s3*)) ⊆ *dom* (*locals* (*store s3′*))
by (*simp add*: *eq-s3′-s3*)
finally show *?thesis* **.**
qed
ultimately obtain *M2* **where**
da:
(|*prg*=*G*, *cls*=*invDeclC*,*lcl*=*L′*|)
⊢ *dom* (*locals* (*store s3′*)) »⟨*stmt* (*mbody* (*mthd dynM*))⟩» *M2* **and**
M2: *nrm M′* ⊆ *nrm M2*
by (*rule da-weakenE*)
from *res M2* **have** *Result* ∈ *nrm M2*
by *blast*
moreover from *wf-dynM*
have *jumpNestingOkS* {*Ret*} (*stmt* (*mbody* (*mthd dynM*)))
by (*rule wf-mdeclE*)
ultimately
obtain *M3* **where**
(|*prg*=*G*, *cls*=*invDeclC*,*lcl*=*L′*|) ⊢ *dom* (*locals* (*store s3′*))
»⟨*Body* (*declclass dynM*) (*stmt* (*mbody* (*mthd dynM*)))⟩» *M3*
using *da*
by (*iprover intro*: *da.Body assigned.select-convs*)
from - *this* [*simplified*]
show *?thesis*
by (*rule da.Methd* [*simplified*,*elim-format*]) (*auto intro*: *dynM′ that*)
qed
ultimately obtain
conf-s4: *s4*::≼(*G*, *L′*) **and**
conf-Res: *normal s4* ⟶ *G*,*store s4*⊢*v*::≼*mthdT* **and**
error-free-s4: *error-free s4*
by (*rule hyp-methd* [*elim-format*])
(*simp add*: *error-free-s3 eq-s3′-s3*)
from *init-lvars eval-methd eq-s3′-s3*
have *store s2*≤|*store s4*
by (*cases s2*) (*auto dest*!: *eval-gext simp add*: *init-lvars-def2*)
moreover
have *abrupt s4* ≠ *Some* (*Jump Ret*)
proof −
from *normal-s2 init-lvars*
have *abrupt s3* ≠ *Some* (*Jump Ret*)
by (*cases s2*) (*simp add*: *init-lvars-def2 abrupt-if-def*)
with *check*
have *abrupt s3′* ≠ *Some* (*Jump Ret*)
by (*cases s3*) (*auto simp add*: *check-method-access-def Let-def*)
with *eval-methd*
show *?thesis*

by (*rule Methd-no-jump*)
qed
ultimately
have (*set-lvars* (*locals* (*store s2*))) *s4*::$\preceq$(*G*, *L*)
using *conf-s2 conf-s4*
by (*cases s2*,*cases s4*) (*auto intro*: *conforms-return*)
moreover
from *conf-Res mthdT-widen resTy-widen wf*
have *normal s4*
$\longrightarrow$ *G*,*store s4*$\vdash$*v*::$\preceq$(*resTy statM*)
by (*auto dest*: *widen-trans*)
then
have *normal* ((*set-lvars* (*locals* (*store s2*))) *s4*)
$\longrightarrow$ *G*,*store*((*set-lvars* (*locals* (*store s2*))) *s4*) $\vdash$*v*::$\preceq$(*resTy statM*)
by (*cases s4*) *auto*
moreover note *error-free-s4 T*
ultimately
show *?thesis*
by *simp*
qed
qed
qed
next
case (*Methd s0 D sig v s1 L accC T A*)
note ‹*G*$\vdash$*Norm s0* $-$*body G D sig*$-\succ$*v*$\rightarrow$ *s1*›
note *hyp* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1l* (*body G D sig*)) (*In1 v*)›
note *conf-s0* = ‹*Norm s0*::$\preceq$(*G*, *L*)›
note *wt* = ‹(|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)$\vdash$*In1l* (*Methd D sig*)::*T*›
then obtain *m bodyT* **where**
D: *is-class G D* **and**
m: *methd G D sig* = *Some m* **and**
wt-body: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)
$\vdash$*Body* (*declclass m*) (*stmt* (*mbody* (*mthd m*)))::$-$*bodyT* **and**
T: *T*=*Inl bodyT*
by (*rule wt-elim-cases*) *auto*
moreover
from *Methd.prems m* **have**
da-body: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
$\vdash$ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))
»*In1l* (*Body* (*declclass m*) (*stmt* (*mbody* (*mthd m*))))» *A*
by $-$ (*erule da-elim-cases*,*simp*)
ultimately
show *s1*::$\preceq$(*G*, *L*) $\wedge$
(*normal s1* $\longrightarrow$ *G*,*L*,*snd s1*$\vdash$*In1l* (*Methd D sig*)$\succ$*In1 v*::$\preceq$*T*) $\wedge$
(*error-free* (*Norm s0*) = *error-free s1*)
using *hyp* [*of* - - (*Inl bodyT*)] *conf-s0*
by (*auto simp add*: *Let-def body-def*)
next
case (*Body s0 D s1 c s2 s3 L accC T A*)
note *eval-init* = ‹*G*$\vdash$*Norm s0* $-$*Init D*$\rightarrow$ *s1*›
note *eval-c* = ‹*G*$\vdash$*s1* $-$*c*$\rightarrow$ *s2*›
note *hyp-init* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1r* (*Init D*)) $\diamondsuit$›
note *hyp-c* = ‹*PROP ?TypeSafe s1 s2* (*In1r c*) $\diamondsuit$›
note *conf-s0* = ‹*Norm s0*::$\preceq$(*G*, *L*)›
note *wt* = ‹(|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)$\vdash$*In1l* (*Body D c*)::*T*›
then obtain *bodyT* **where**
iscls-D: *is-class G D* **and**
wt-c: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)$\vdash$*c*::$\surd$ **and**
resultT: *L Result* = *Some bodyT* **and**

```
    isty-bodyT: is-type G bodyT and
          T: T=Inl bodyT
    by (rule wt-elim-cases) auto
  from Body.prems obtain C where
    da-c: (|prg=G,cls=accC,lcl=L|)
             ⊢ (dom (locals (store ((Norm s0)::state))))»In1r c» C and
    jmpOk: jumpNestingOkS {Ret} c and
    res: Result ∈ nrm C
    by (elim da-elim-cases) simp
  note conf-s0
  moreover from iscls-D
  have (|prg=G, cls=accC, lcl=L|)⊢Init D::√ by auto
  moreover obtain I where
    (|prg=G,cls=accC,lcl=L|)
      ⊢ dom (locals (store ((Norm s0)::state))) »In1r (Init D)» I
    by (auto intro: da-Init [simplified] assigned.select-convs)
  ultimately obtain
    conf-s1: s1::≼(G, L) and error-free-s1:  error-free s1
     by (rule hyp-init [elim-format]) simp
  obtain C' where da-C': (|prg=G,cls=accC,lcl=L|)
                          ⊢ (dom (locals (store s1)))»In1r c» C'
          and nrm-C': nrm C ⊆ nrm C'
  proof -
    from eval-init
    have (dom (locals (store ((Norm s0)::state))))
                 ⊆ (dom (locals (store s1)))
      by (rule dom-locals-eval-mono-elim)
    with da-c show thesis by (rule da-weakenE) (rule that)
  qed
  from conf-s1 wt-c da-C'
  obtain conf-s2: s2::≼(G, L) and error-free-s2: error-free s2
    by (rule hyp-c [elim-format]) (simp add: error-free-s1)
  from conf-s2
  have abupd (absorb Ret) s2::≼(G, L)
    by (cases s2) (auto intro: conforms-absorb)
  moreover
  from error-free-s2
  have error-free (abupd (absorb Ret) s2)
    by simp
  moreover have abrupt (abupd (absorb Ret) s3) ≠ Some (Jump Ret)
    by (cases s3) (simp add: absorb-def)
  moreover have s3=s2
  proof -
    from iscls-D
    have wt-init: (|prg=G, cls=accC, lcl=L|)⊢(Init D)::√
      by auto
    from eval-init wf
    have s1-no-jmp: ⋀ j. abrupt s1 ≠ Some (Jump j)
      by - (rule eval-statement-no-jump [OF - - - wt-init],auto)
    from eval-c - wt-c wf
    have ⋀ j. abrupt s2 = Some (Jump j) ⟹ j=Ret
      by (rule jumpNestingOk-evalE) (auto intro: jmpOk simp add: s1-no-jmp)
    moreover
    note ‹s3 =
          (if ∃ l. abrupt s2 = Some (Jump (Break l)) ∨
                  abrupt s2 = Some (Jump (Cont l))
           then abupd (λx. Some (Error CrossMethodJump)) s2 else s2)›
    ultimately show ?thesis
      by force
```

```
  qed
  moreover
  {
    assume normal-upd-s2: normal (abupd (absorb Ret) s2)
    have Result ∈ dom (locals (store s2))
    proof −
      from normal-upd-s2
      have normal s2 ∨ abrupt s2 = Some (Jump Ret)
        by (cases s2) (simp add: absorb-def)
      thus ?thesis
      proof
        assume normal s2
        with eval-c wt-c da-C′ wf res nrm-C′
        show ?thesis
          by (cases rule: da-good-approxE′) blast
      next
        assume abrupt s2 = Some (Jump Ret)
        with conf-s2 show ?thesis
          by (cases s2) (auto dest: conforms-RetD simp add: dom-def)
      qed
    qed
  }
  moreover note T resultT
  ultimately
  show abupd (absorb Ret) s3::≼(G, L) ∧
         (normal (abupd (absorb Ret) s3) −→
           G,L,store (abupd (absorb Ret) s3)
           ⊢In1l (Body D c)≻In1 (the (locals (store s2) Result))::≼T) ∧
        (error-free (Norm s0) = error-free (abupd (absorb Ret) s3))
    by (cases s2) (auto intro: conforms-locals)
next
  case (LVar s vn L accC T)
  note conf-s = ‹Norm s::≼(G, L)› and
    wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In2 (LVar vn)::T›
  then obtain vnT where
    vnT: L vn = Some vnT and
      T: T=Inl vnT
    by (auto elim!: wt-elim-cases)
  from conf-s vnT
  have conf-fst: locals s vn ≠ None −→ G,s⊢fst (lvar vn s)::≼vnT
   by (auto elim: conforms-localD [THEN wlconfD]
            simp add: lvar-def)
  moreover
  from conf-s conf-fst vnT
  have s≤|snd (lvar vn s)≼vnT::≼(G, L)
    by (auto elim: conforms-lupd simp add: assign-conforms-def lvar-def)
  moreover note conf-s T
  ultimately
  show Norm s::≼(G, L) ∧
              (normal (Norm s) −→
                 G,L,store (Norm s)⊢In2 (LVar vn)≻In2 (lvar vn s)::≼T) ∧
              (error-free (Norm s) = error-free (Norm s))
    by (simp add: lvar-def)
next
  case (FVar s0 statDeclC s1 e a s2 v s2′ stat fn s3 accC L accC′ T A)
  note eval-init = ‹G⊢Norm s0 −Init statDeclC→ s1›
  note eval-e = ‹G⊢s1 −e−≻a→ s2›
  note fvar = ‹(v, s2′) = fvar statDeclC stat fn a s2›
  note check = ‹s3 = check-field-access G accC statDeclC fn stat a s2′›
```

note *hyp-init* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1r* (*Init statDeclC*)) ◇›
note *hyp-e* = ‹*PROP ?TypeSafe s1 s2* (*In1l e*) (*In1 a*)›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹(|*prg*=*G*, *cls*=*accC*′, *lcl*=*L*|)⊢*In2* ({*accC*,*statDeclC*,*stat*}*e*..*fn*)::*T*›
then obtain *statC f* **where**
wt-e: (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)⊢*e*::−*Class statC* **and**
accfield: *accfield G accC statC fn* = *Some* (*statDeclC*,*f*) **and**
eq-accC-accC′: *accC*=*accC*′ **and**
stat: *stat*=*is-static f* **and**
T: *T*=(*Inl* (*type f*))
by (*rule wt-elim-cases*) (*auto simp add*: *member-is-static-simp*)
from *FVar.prems eq-accC-accC*′
have *da-e*: (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))»*In1l e*» *A*
by (*elim da-elim-cases*) *simp*
note *conf-s0*
moreover
from *wf wt-e*
have *iscls-statC*: *is-class G statC*
by (*auto dest*: *ty-expr-is-type type-is-class*)
with *wf accfield*
have *iscls-statDeclC*: *is-class G statDeclC*
by (*auto dest*!: *accfield-fields dest*: *fields-declC*)
hence (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)⊢(*Init statDeclC*)::√
by *simp*
moreover obtain *I* **where**
(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1r* (*Init statDeclC*)» *I*
by (*auto intro*: *da-Init* [*simplified*] *assigned.select-convs*)
ultimately
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and** *error-free-s1*: *error-free s1*
by (*rule hyp-init* [*elim-format*]) *simp*
obtain *A*′ **where**
(|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|) ⊢ (*dom* (*locals* (*store s1*)))»*In1l e*» *A*′
proof −
from *eval-init*
have (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))
⊆ (*dom* (*locals* (*store s1*)))
by (*rule dom-locals-eval-mono-elim*)
with *da-e* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *conf-s1 wt-e*
obtain *conf-s2*: *s2*::≼(*G*, *L*) **and**
conf-a: *normal s2* ⟶ *G*,*store s2*⊢*a*::≼*Class statC* **and**
error-free-s2: *error-free s2*
by (*rule hyp-e* [*elim-format*]) (*simp add*: *error-free-s1*)
from *fvar*
have *store-s2*′: *store s2*′=*store s2*
by (*cases s2*) (*simp add*: *fvar-def2*)
with *fvar conf-s2*
have *conf-s2*′: *s2*′::≼(*G*, *L*)
by (*cases s2*,*cases stat*) (*auto simp add*: *fvar-def2*)
from *eval-init*
have *initd-statDeclC-s1*: *initd statDeclC s1*
by (*rule init-yields-initd*)
from *accfield wt-e eval-init eval-e conf-s2 conf-a fvar stat check wf*
have *eq-s3-s2*′: *s3*=*s2*′
by (*auto dest*!: *error-free-field-access*)

```
    have conf-v: normal s2′ ⟹
          G,store s2′⊢fst v::≼type f ∧ store s2′≤|snd v≼type f::≼(G, L)
    proof −
      assume normal: normal s2′
      obtain vv vf x2 store2 store2′
        where  v: v=(vv,vf) and
              s2: s2=(x2,store2) and
         store2′: store s2′ = store2′
        by (cases v,cases s2,cases s2′) blast
      from iscls-statDeclC obtain c
        where c: class G statDeclC = Some c
        by auto
      have G,store2′⊢vv::≼type f ∧ store2′≤|vf≼type f::≼(G, L)
      proof (rule FVar-lemma [of vv vf store2′ statDeclC f fn a x2 store2
                              statC G c L store s1])
        from v normal s2 fvar stat store2′
        show ((vv, vf), Norm store2′) =
               fvar statDeclC (static f) fn a (x2, store2)
          by (auto simp add: member-is-static-simp)
        from accfield iscls-statC wf
        show G⊢statC≼C statDeclC
          by (auto dest!: accfield-fields dest: fields-declC)
        from accfield
        show fld: table-of (DeclConcepts.fields G statC) (fn, statDeclC) = Some f
          by (auto dest!: accfield-fields)
        from wf show wf-prog G .
        from conf-a s2 show x2 = None ⟶ G,store2⊢a::≼Class statC
          by auto
        from fld wf iscls-statC
        show statDeclC ≠ Object
          by (cases statDeclC=Object) (drule fields-declC,simp+)+
        from c show class G statDeclC = Some c .
        from conf-s2 s2 show (x2, store2)::≼(G, L) by simp
        from eval-e s2 show snd s1≤|store2 by (auto dest: eval-gext)
        from initd-statDeclC-s1 show inited statDeclC (globs (snd s1))
          by simp
      qed
      with v s2 store2′
      show ?thesis
        by simp
    qed
    from fvar error-free-s2
    have error-free s2′
      by (cases s2)
         (auto simp add: fvar-def2 intro!: error-free-FVar-lemma)
    with conf-v T conf-s2′ eq-s3-s2′
    show s3::≼(G, L) ∧
          (normal s3
           ⟶ G,L,store s3⊢In2 ({accC,statDeclC,stat}e..fn)≻In2 v::≼T) ∧
          (error-free (Norm s0) = error-free s3)
      by auto
  next
    case (AVar s0 e1 a s1 e2 i s2 v s2′ L accC T A)
    note eval-e1 = ‹G⊢Norm s0 −e1−≻a→ s1›
    note eval-e2 = ‹G⊢s1 −e2−≻i→ s2›
    note hyp-e1 = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e1) (In1 a)›
    note hyp-e2 = ‹PROP ?TypeSafe s1 s2 (In1l e2) (In1 i)›
    note avar = ‹(v, s2′) = avar G i a s2›
    note conf-s0 = ‹Norm s0::≼(G, L)›
```

note *wt* = ‹⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*In2* (*e1*.[*e2*])::*T*›
then obtain *elemT*
  where *wt-e1*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*e1*::−*elemT*.[] **and**
    wt-e2: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*e2*::−*PrimT Integer* **and**
      T: *T*= *Inl elemT*
 by (*rule wt-elim-cases*) *auto*
from *AVar.prems* **obtain** *E1* **where**
 da-e1: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
   ⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))»*In1l e1*» *E1* **and**
 da-e2: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *nrm E1* »*In1l e2*» *A*
 by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-e1 da-e1*
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and**
   conf-a: (*normal s1* ⟶ *G*,*store s1*⊢*a*::≼*elemT*.[]) **and**
   error-free-s1: *error-free s1*
 by (*rule hyp-e1* [*elim-format*]) *simp*
show *s2'*::≼(*G*, *L*) ∧
  (*normal s2'* ⟶ *G*,*L*,*store s2'*⊢*In2* (*e1*.[*e2*])≻*In2 v*::≼*T*) ∧
  (*error-free* (*Norm s0*) = *error-free s2'*)
proof (*cases normal s1*)
 case *False*
 moreover
 from *False eval-e2* **have** *eq-s2-s1*: *s2*=*s1* **by** *auto*
 moreover
 from *eq-s2-s1 False* **have** ¬ *normal s2* **by** *simp*
 then have *snd* (*avar G i a s2*) = *s2*
  by (*cases s2*) (*simp add*: *avar-def2*)
 with *avar* **have** *s2'*=*s2*
  by (*cases* (*avar G i a s2*)) *simp*
 ultimately show *?thesis*
  using *conf-s1 error-free-s1*
  by *auto*
next
 case *True*
 obtain *A'* **where**
  ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *dom* (*locals* (*store s1*)) »*In1l e2*» *A'*
 proof −
  from *eval-e1 wt-e1 da-e1 wf True*
  have *nrm E1* ⊆ *dom* (*locals* (*store s1*))
   by (*cases rule*: *da-good-approxE'*) *iprover*
  with *da-e2* **show** *thesis*
   by (*rule da-weakenE*) (*rule that*)
 qed
 with *conf-s1 wt-e2*
 obtain *conf-s2*: *s2*::≼(*G*, *L*) **and** *error-free-s2*: *error-free s2*
  by (*rule hyp-e2* [*elim-format*]) (*simp add*: *error-free-s1*)
 from *avar*
 have *store s2'*=*store s2*
  by (*cases s2*) (*simp add*: *avar-def2*)
 with *avar conf-s2*
 have *conf-s2'*: *s2'*::≼(*G*, *L*)
  by (*cases s2*) (*auto simp add*: *avar-def2*)
 from *avar error-free-s2*
 have *error-free-s2'*: *error-free s2'*
  by (*cases s2*) (*auto simp add*: *avar-def2*)
 have *normal s2'* ⟹
  G,*store s2'*⊢*fst v*::≼*elemT* ∧ *store s2'*≤|*snd v*≼*elemT*::≼(*G*, *L*)
 proof −
  assume *normal*: *normal s2'*

```
      show ?thesis
      proof –
        obtain vv vf x1 store1 x2 store2 store2'
          where  v: v=(vv,vf) and
                 s1: s1=(x1,store1) and
                 s2: s2=(x2,store2) and
             store2': store2'=store s2'
          by (cases v,cases s1, cases s2, cases s2') blast
        have G,store2'⊢vv::≼elemT ∧ store2'≤|vf≼elemT::≼(G, L)
        proof (rule AVar-lemma [of G x1 store1 e2 i x2 store2 vv vf store2' a,
                            OF wf])
          from s1 s2 eval-e2 show G⊢(x1, store1) –e2–≻i→ (x2, store2)
            by simp
          from v normal s2 store2' avar
          show ((vv, vf), Norm store2') = avar G i a (x2, store2)
            by auto
          from s2 conf-s2 show (x2, store2)::≼(G, L) by simp
          from s1 conf-a show  x1 = None ⟶ G,store1⊢a::≼elemT.[] by simp
          from eval-e2 s1 s2 show store1≤|store2 by (auto dest: eval-gext)
        qed
        with v s1 s2 store2'
        show ?thesis
          by simp
      qed
    qed
    with conf-s2' error-free-s2' T
    show ?thesis
      by auto
  qed
next
  case (Nil s0 L accC T)
  then show ?case
    by (auto elim!: wt-elim-cases)
next
  case (Cons s0 e v s1 es vs s2 L accC T A)
  note eval-e = ‹G⊢Norm s0 –e–≻v→ s1›
  note eval-es = ‹G⊢s1 –es≐≻vs→ s2›
  note hyp-e = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›
  note hyp-es = ‹PROP ?TypeSafe s1 s2 (In3 es) (In3 vs)›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹⦇prg = G, cls = accC, lcl = L⦈⊢In3 (e # es)::T›
  then obtain eT esT where
     wt-e: ⦇prg = G, cls = accC, lcl = L⦈⊢e::–eT and
     wt-es: ⦇prg = G, cls = accC, lcl = L⦈⊢es::≐esT and
     T: T=Inr (eT#esT)
    by (rule wt-elim-cases) blast
  from Cons.prems obtain E where
    da-e: ⦇prg=G,cls=accC,lcl=L⦈
            ⊢ (dom (locals (store ((Norm s0)::state))))»In1l e» E and
    da-es: ⦇prg=G,cls=accC,lcl=L⦈⊢ nrm E »In3 es» A
    by (elim da-elim-cases) simp
  from conf-s0 wt-e da-e
  obtain conf-s1: s1::≼(G, L) and error-free-s1: error-free s1 and
    conf-v: normal s1 ⟶ G,store s1⊢v::≼eT
    by (rule hyp-e [elim-format]) simp
  show
    s2::≼(G, L) ∧
    (normal s2 ⟶ G,L,store s2⊢In3 (e # es)≻In3 (v # vs)::≼T) ∧
    (error-free (Norm s0) = error-free s2)
```

proof (*cases normal s1*)
case *False*
with *eval-es* **have** *s2=s1* **by** *auto*
with *False conf-s1 error-free-s1*
show *?thesis*
by *auto*
next
case *True*
obtain A' **where**
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash$ *dom* (*locals* (*store s1*)) »*In3 es*» A'
proof −
from *eval-e wt-e da-e wf True*
have *nrm E* ⊆ *dom* (*locals* (*store s1*))
by (*cases rule*: *da-good-approxE'*) *iprover*
with *da-es* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *conf-s1 wt-es*
obtain *conf-s2*: *s2*::≼(*G*, *L*) **and**
error-free-s2: *error-free s2* **and**
conf-vs: *normal s2* ⟶ *list-all2* (*conf G* (*store s2*)) *vs esT*
by (*rule hyp-es* [*elim-format*]) (*simp add*: *error-free-s1*)
moreover
from *True eval-es conf-v*
have *conf-v'*: *G*,*store s2*⊢*v*::≼*eT*
apply *clarify*
apply (*rule conf-gext*)
apply (*auto dest*: *eval-gext*)
done
ultimately show *?thesis* **using** *T* **by** *simp*
qed
qed
from *this* **and** *conf-s0 wt da* **show** *?thesis* **.**
qed

corollary *eval-type-soundE* [*consumes 5*]:
assumes *eval*: *G*⊢*s0* −*t*≻→ (*v*, *s1*)
and *conf*: *s0*::≼(*G*, *L*)
and *wt*: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*t*::*T*
and *da*: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢ *dom* (*locals* (*snd s0*)) »*t*» *A*
and *wf*: *wf-prog G*
and *elim*: ⟦*s1*::≼(*G*, *L*); *normal s1* ⟹ *G*,*L*,*snd s1*⊢*t*≻*v*::≼*T*;
error-free s0 = *error-free s1*⟧ ⟹ *P*
shows *P*
using *eval wt da wf conf*
by (*rule eval-type-sound* [*elim-format*]) (*iprover intro*: *elim*)

corollary *eval-ts*:
⟦*G*⊢*s* −*e*−≻*v* → *s'*; *wf-prog G*; *s*::≼(*G*,*L*); (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*e*::−*T*;
(|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*dom* (*locals* (*store s*))»*In1l e*»*A*⟧
⟹ *s'*::≼(*G*,*L*) ∧ (*normal s'* ⟶ *G*,*store s'*⊢*v*::≼*T*) ∧
(*error-free s* = *error-free s'*)
apply (*drule* (*4*) *eval-type-sound*)
apply *clarsimp*
done

corollary *evals-ts*:
⟦*G*⊢*s* −*es*≐≻*vs*→ *s'*; *wf-prog G*; *s*::≼(*G*,*L*); (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*es*::≐*Ts*;

```
  ⦇prg=G,cls=C,lcl=L⦈⊢dom (locals (store s))»In3 es»A⟧
⟹  s'::≼(G,L) ∧ (normal s' ⟶ list-all2 (conf G (store s')) vs Ts) ∧
     (error-free s = error-free s')
apply (drule (4) eval-type-sound)
apply clarsimp
done

corollary evar-ts:
⟦G⊢s −v=≻vf→ s'; wf-prog G; s::≼(G,L); ⦇prg=G,cls=C,lcl=L⦈⊢v::=T;
 ⦇prg=G,cls=C,lcl=L⦈⊢dom (locals (store s))»In2 v»A⟧ ⟹
  s'::≼(G,L) ∧ (normal s' ⟶ G,L,(store s')⊢In2 v≻In2 vf::≼Inl T) ∧
  (error-free s = error-free s')
apply (drule (4) eval-type-sound)
apply clarsimp
done

theorem exec-ts:
⟦G⊢s −c→ s'; wf-prog G; s::≼(G,L); ⦇prg=G,cls=C,lcl=L⦈⊢c::√;
 ⦇prg=G,cls=C,lcl=L⦈⊢dom (locals (store s))»In1r c»A⟧
 ⟹ s'::≼(G,L) ∧ (error-free s ⟶ error-free s')
apply (drule (4) eval-type-sound)
apply clarsimp
done


lemma wf-eval-Fin:
  assumes wf:    wf-prog G
    and   wt-c1: ⦇prg = G, cls = C, lcl = L⦈⊢In1r c1::Inl (PrimT Void)
    and   da-c1: ⦇prg=G,cls=C,lcl=L⦈⊢dom (locals (store (Norm s0)))»In1r c1»A
    and conf-s0: Norm s0::≼(G, L)
    and eval-c1: G⊢Norm s0 −c1→ (x1,s1)
    and eval-c2: G⊢Norm s1 −c2→ s2
    and      s3: s3=abupd (abrupt-if (x1≠None) x1) s2
  shows G⊢Norm s0 −c1 Finally c2→ s3
proof −
  from eval-c1 wt-c1 da-c1 wf conf-s0
  have error-free (x1,s1)
    by (auto dest: eval-type-sound)
  with eval-c1 eval-c2 s3
  show ?thesis
    by − (rule eval.Fin, auto simp add: error-free-def)
qed
```

3 Ideas for the future

In the type soundness proof and the correctness proof of definite assignment we perform induction on the evaluation relation with the further preconditions that the term is welltyped and definitely assigned. During the proofs we have to establish the welltypedness and definite assignment of the subterms to be able to apply the induction hypothesis. So large parts of both proofs are the same work in propagating welltypedness and definite assignment. So we can derive a new induction rule for induction on the evaluation of a wellformed term, were these propagations is already done, once and forever. Then we can do the proofs with this rule and can enjoy the time we have saved. Here is a first and incomplete sketch of such a rule.

```
theorem wellformed-eval-induct [consumes 4, case-names Abrupt Skip Expr Lab
                                Comp If]:
  assumes  eval: G⊢s0 −t≻→ (v,s1)
   and       wt: ⦇prg=G,cls=accC,lcl=L⦈⊢t::T
```

and *da*: ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0))»t»A
and *wf*: wf-prog G
and *abrupt*: ⋀ s t abr L accC T A.
⟦⦇prg=G,cls=accC,lcl=L⦈⊢t::T;
⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store (Some abr,s)))»t»A
⟧ ⟹ P L accC (Some abr, s) t (undefined3 t) (Some abr, s)
and *skip*: ⋀ s L accC. P L accC (Norm s) $\langle Skip\rangle_s$ ◇ (Norm s)
and *expr*: ⋀ e s0 s1 v L accC eT E.
⟦⦇prg=G,cls=accC,lcl=L⦈⊢e::−eT;
⦇prg=G,cls=accC,lcl=L⦈
⊢dom (locals (store ((Norm s0)::state)))»$\langle e\rangle_e$»E;
P L accC (Norm s0) $\langle e\rangle_e$ $\lfloor v\rfloor_e$ s1⟧
⟹ P L accC (Norm s0) $\langle Expr\ e\rangle_s$ ◇ s1
and *lab*: ⋀ c l s0 s1 L accC C.
⟦⦇prg=G,cls=accC,lcl=L⦈⊢c::√;
⦇prg=G,cls=accC, lcl=L⦈
⊢dom (locals (store ((Norm s0)::state)))»$\langle c\rangle_s$»C;
P L accC (Norm s0) $\langle c\rangle_s$ ◇ s1⟧
⟹ P L accC (Norm s0) $\langle l\cdot\ c\rangle_s$ ◇ (abupd (absorb l) s1)
and *comp*: ⋀ c1 c2 s0 s1 s2 L accC C1.
⟦G⊢Norm s0 −c1 → s1;G⊢s1 −c2 → s2;
⦇prg=G,cls=accC,lcl=L⦈⊢c1::√;
⦇prg=G,cls=accC,lcl=L⦈⊢c2::√;
⦇prg=G,cls=accC,lcl=L⦈⊢
dom (locals (store ((Norm s0)::state))) »$\langle c1\rangle_s$» C1;
P L accC (Norm s0) $\langle c1\rangle_s$ ◇ s1;
⋀ Q. ⟦normal s1;
⋀ C2.⟦⦇prg=G,cls=accC,lcl=L⦈
⊢dom (locals (store s1)) »$\langle c2\rangle_s$» C2;
P L accC s1 $\langle c2\rangle_s$ ◇ s2⟧ ⟹ Q
⟧ ⟹ Q
⟧⟹ P L accC (Norm s0) $\langle c1;;\ c2\rangle_s$ ◇ s2
and *if*: ⋀ b c1 c2 e s0 s1 s2 L accC E.
⟦G⊢Norm s0 −e−≻b→ s1;
G⊢s1 −(if the-Bool b then c1 else c2)→ s2;
⦇prg=G,cls=accC,lcl=L⦈⊢e::−PrimT Boolean;
⦇prg=G, cls=accC, lcl=L⦈⊢(if the-Bool b then c1 else c2)::√;
⦇prg=G,cls=accC,lcl=L⦈⊢
dom (locals (store ((Norm s0)::state))) »$\langle e\rangle_e$» E;
P L accC (Norm s0) $\langle e\rangle_e$ $\lfloor b\rfloor_e$ s1;
⋀ Q. ⟦normal s1;
⋀ C. ⟦⦇prg=G,cls=accC,lcl=L⦈⊢ (dom (locals (store s1)))
»$\langle if\ the\text{-}Bool\ b\ then\ c1\ else\ c2\rangle_s$» C;
P L accC s1 $\langle if\ the\text{-}Bool\ b\ then\ c1\ else\ c2\rangle_s$ ◇ s2
⟧ ⟹ Q
⟧ ⟹ Q
⟧ ⟹ P L accC (Norm s0) $\langle If(e)\ c1\ Else\ c2\rangle_s$ ◇ s2
shows *P L accC s0 t v s1*
proof −
note *inj-term-simps* [*simp*]
from *eval*
have ⋀L accC T A. ⟦⦇prg=G,cls=accC,lcl=L⦈⊢t::T;
⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0))»t»A⟧
⟹ P L accC s0 t v s1 (**is** *PROP ?Hyp s0 t v s1*)
proof (*induct*)
case *Abrupt* **with** *abrupt* **show** *?case* **.**
next
case *Skip* **from** *skip* **show** *?case* **by** *simp*
next

case ($Expr\ s0\ e\ v\ s1\ L\ accC\ T\ A$)
from $Expr.prems$ **obtain** eT **where**
$(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash e{::}{-}eT$
by ($elim\ wt\text{-}elim\text{-}cases$)
moreover
from $Expr.prems$ **obtain** E **where**
$(\!|prg{=}G,cls{=}accC,\ lcl{=}L|\!)\vdash dom\ (locals\ (store\ ((Norm\ s0){::}state)))$»$\langle e\rangle_e$»$E$
by ($elim\ da\text{-}elim\text{-}cases$) $simp$
moreover from $calculation$
have $P\ L\ accC\ (Norm\ s0)\ \langle e\rangle_e\ \lfloor v\rfloor_e\ s1$
by ($rule\ Expr.hyps$)
ultimately show $?case$
by ($rule\ expr$)
next
case ($Lab\ s0\ c\ s1\ l\ L\ accC\ T\ A$)
from $Lab.prems$
have $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash c{::}\surd$
by ($elim\ wt\text{-}elim\text{-}cases$)
moreover
from $Lab.prems$ **obtain** C **where**
$(\!|prg{=}G,cls{=}accC,\ lcl{=}L|\!)\vdash dom\ (locals\ (store\ ((Norm\ s0){::}state)))$»$\langle c\rangle_s$»$C$
by ($elim\ da\text{-}elim\text{-}cases$) $simp$
moreover from $calculation$
have $P\ L\ accC\ (Norm\ s0)\ \langle c\rangle_s\ \diamondsuit\ s1$
by ($rule\ \ Lab.hyps$)
ultimately show $?case$
by ($rule\ lab$)
next
case ($Comp\ s0\ c1\ s1\ c2\ s2\ L\ accC\ T\ A$)
note $eval\text{-}c1 = \langle G\vdash Norm\ s0\ {-}c1{\rightarrow}\ s1\rangle$
note $eval\text{-}c2 = \langle G\vdash s1\ {-}c2{\rightarrow}\ s2\rangle$
from $Comp.prems$ **obtain**
$wt\text{-}c1$: $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash c1{::}\surd$ **and**
$wt\text{-}c2$: $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash c2{::}\surd$
by ($elim\ wt\text{-}elim\text{-}cases$)
from $Comp.prems$
obtain $C1\ C2$
where $da\text{-}c1$: $(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash$
$dom\ (locals\ (store\ ((Norm\ s0){::}state)))$ »$\langle c1\rangle_s$» $C1$ **and**
$da\text{-}c2$: $(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash\ \ nrm\ C1$ »$\langle c2\rangle_s$» $C2$
by ($elim\ da\text{-}elim\text{-}cases$) $simp$
from $wt\text{-}c1\ da\text{-}c1$
have $P\text{-}c1$: $P\ L\ accC\ (Norm\ s0)\ \langle c1\rangle_s\ \diamondsuit\ s1$
by ($rule\ Comp.hyps$)
{
fix Q
assume $normal\text{-}s1$: $normal\ s1$
assume $elim$: $\bigwedge\ C2'.$
$[\![(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash dom\ (locals\ (store\ s1))$»$\langle c2\rangle_s$»$C2'$;
$P\ L\ accC\ s1\ \langle c2\rangle_s\ \diamondsuit\ s2]\!]\Longrightarrow Q$
have Q
proof −
obtain $C2'$ **where**
da: $(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash\ dom\ (locals\ (store\ s1))$ »$\langle c2\rangle_s$» $C2'$
proof −
from $eval\text{-}c1\ wt\text{-}c1\ da\text{-}c1\ wf\ normal\text{-}s1$
have $nrm\ C1 \subseteq dom\ (locals\ (store\ s1))$
by ($cases\ rule$: $da\text{-}good\text{-}approxE'$) $iprover$
with $da\text{-}c2$ **show** $thesis$

```
        by (rule da-weakenE) (rule that)
      qed
      with wt-c2 have P L accC s1 ⟨c2⟩ₛ ◇ s2
        by (rule Comp.hyps)
      with da show ?thesis
        using elim by iprover
    qed
  }
  with eval-c1 eval-c2 wt-c1 wt-c2 da-c1 P-c1
  show ?case
    by (rule comp) iprover+
next
  case (If s0 e b s1 c1 c2 s2 L accC T A)
  note eval-e = ‹G⊢Norm s0 −e−≻b→ s1›
  note eval-then-else = ‹G⊢s1 −(if the-Bool b then c1 else c2)→ s2›
  from If.prems
  obtain
          wt-e: (|prg=G, cls=accC, lcl=L|)⊢e::−PrimT Boolean and
    wt-then-else: (|prg=G, cls=accC, lcl=L|)⊢(if the-Bool b then c1 else c2)::√
    by (elim wt-elim-cases) auto
  from If.prems obtain E C where
    da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store ((Norm s0)::state)))
                                  »⟨e⟩ₑ» E and
    da-then-else:
    (|prg=G,cls=accC,lcl=L|)⊢
       (dom (locals (store ((Norm s0)::state))) ∪ assigns-if (the-Bool b) e)
        »⟨if the-Bool b then c1 else c2⟩ₛ» C
    by (elim da-elim-cases) (cases the-Bool b,auto)
  from wt-e da-e
  have P-e: P L accC (Norm s0) ⟨e⟩ₑ ⌊b⌋ₑ s1
    by (rule If.hyps)
  {
    fix Q
    assume normal-s1: normal s1
    assume elim: ⋀ C. ⟦(|prg=G,cls=accC,lcl=L|)⊢ (dom (locals (store s1)))
                          »⟨if the-Bool b then c1 else c2⟩ₛ» C;
                       P L accC s1 ⟨if the-Bool b then c1 else c2⟩ₛ ◇ s2
                       ⟧ ⟹ Q
    have Q
    proof −
      obtain C' where
        da: (|prg=G,cls=accC,lcl=L|)⊢
              (dom (locals (store s1)))»⟨if the-Bool b then c1 else c2⟩ₛ » C'
      proof −
        from eval-e have
          dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
          by (rule dom-locals-eval-mono-elim)
        moreover
        from eval-e normal-s1 wt-e
        have assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
          by (rule assigns-if-good-approx')
        ultimately
        have dom (locals (store ((Norm s0)::state)))
          ∪ assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
          by (rule Un-least)
        with da-then-else show thesis
          by (rule da-weakenE) (rule that)
      qed
      with wt-then-else
```

have *P L accC s1* $\langle$*if the-Bool b then c1 else c2*$\rangle_s$ $\diamondsuit$ *s2*
by (*rule If.hyps*)
with *da* **show** *?thesis* **using** *elim* **by** *iprover*
qed
}
with *eval-e eval-then-else wt-e wt-then-else da-e P-e*
show *?case*
by (*rule if*) *iprover+*
next
oops

end

Chapter 20

Evaln

1 Operational evaluation (big-step) semantics of Java expressions and statements

theory *Evaln* **imports** *TypeSafe* **begin**

Variant of *eval* relation with counter for bounded recursive depth. In principal *evaln* could replace *eval*.

Validity of the axiomatic semantics builds on *evaln*. For recursive method calls the axiomatic semantics rule assumes the method ok to derive a proof for the body. To prove the method rule sound we need to perform induction on the recursion depth. For the completeness proof of the axiomatic semantics the notion of the most general formula is used. The most general formula right now builds on the ordinary evaluation relation *eval*. So sometimes we have to switch between *evaln* and *eval* and vice versa. To make this switch easy *evaln* also does all the technical accessibility tests *check-field-access* and *check-method-access* like *eval*. If it would omit them *evaln* and *eval* would only be equivalent for welltyped, and definitely assigned terms.

inductive
  evaln :: [*prog*, *state*, *term*, *nat*, *vals*, *state*] ⇒ *bool*
    (-⊢- −-≻−-→ ′(-, -′) [*61*,*61*,*80*,*61*,*0*,*0*] *60*)
  and *evarn* :: [*prog*, *state*, *var*, *vvar*, *nat*, *state*] ⇒ *bool*
    (-⊢- −-=≻-−-→ - [*61*,*61*,*90*,*61*,*61*,*61*] *60*)
  and *eval-n*:: [*prog*, *state*, *expr*, *val*, *nat*, *state*] ⇒ *bool*
    (-⊢- −-−≻-−-→ - [*61*,*61*,*80*,*61*,*61*,*61*] *60*)
  and *evalsn* :: [*prog*, *state*, *expr list*, *val list*, *nat*, *state*] ⇒ *bool*
    (-⊢- −-≐≻-−-→ - [*61*,*61*,*61*,*61*,*61*,*61*] *60*)
  and *execn* :: [*prog*, *state*, *stmt*, *nat*, *state*] ⇒ *bool*
    (-⊢- −-−-→ - [*61*,*61*,*65*, *61*,*61*] *60*)
  for *G* :: *prog*
where

  $G \vdash s\ -c\ \ -n\rightarrow\ \ s' \equiv G \vdash s\ -In1r\ \ c \succ -n\rightarrow (\Diamond\ ,\ s')$
| $G \vdash s\ -e-\succ v\ \ -n\rightarrow\ \ s' \equiv G \vdash s\ -In1l\ e \succ -n\rightarrow (In1\ v\ ,\ s')$
| $G \vdash s\ -e=\succ vf\ -n\rightarrow\ \ s' \equiv G \vdash s\ -In2\ \ e \succ -n\rightarrow (In2\ vf,\ s')$
| $G \vdash s\ -e\doteq\succ v\ \ -n\rightarrow\ \ s' \equiv G \vdash s\ -In3\ \ e \succ -n\rightarrow (In3\ v\ ,\ s')$

— propagation of abrupt completion

| *Abrupt*: $G \vdash (Some\ xc,s)\ -t \succ -n\rightarrow (undefined3\ t,(Some\ xc,s))$

— evaluation of variables

| *LVar*: $G \vdash Norm\ s\ -LVar\ vn=\succ lvar\ vn\ s-n\rightarrow\ Norm\ s$

$$\begin{array}{l}
\mid FVar\colon \llbracket G\vdash Norm\ s0\ -Init\ statDeclC-n\rightarrow\ s1;\ G\vdash s1\ -e-\!\succ a-n\rightarrow\ s2;\\
\qquad (v,s2') = fvar\ statDeclC\ stat\ fn\ a\ s2;\\
\qquad s3 = check\text{-}field\text{-}access\ G\ accC\ statDeclC\ fn\ stat\ a\ s2'\rrbracket \Longrightarrow\\
\qquad G\vdash Norm\ s0\ -\{accC,statDeclC,stat\}e..fn{=}\!\succ v-n\rightarrow\ s3
\end{array}$$

$$\begin{array}{l}
\mid AVar\colon \llbracket G\vdash\ Norm\ s0\ -e1-\!\succ a-n\rightarrow\ s1\ ;\ G\vdash s1\ -e2-\!\succ i-n\rightarrow\ s2;\\
\qquad (v,s2') = avar\ G\ i\ a\ s2\rrbracket \Longrightarrow\\
\qquad\qquad G\vdash Norm\ s0\ -e1.[e2]{=}\!\succ v-n\rightarrow\ s2'
\end{array}$$

— evaluation of expressions

$$\begin{array}{l}
\mid NewC\colon \llbracket G\vdash Norm\ s0\ -Init\ C-n\rightarrow\ s1;\\
\qquad G\vdash\quad s1\ -halloc\ (CInst\ C)\!\succ a\rightarrow\ s2\rrbracket \Longrightarrow\\
\qquad\qquad G\vdash Norm\ s0\ -NewC\ C-\!\succ Addr\ a-n\rightarrow\ s2
\end{array}$$

$$\begin{array}{l}
\mid NewA\colon \llbracket G\vdash Norm\ s0\ -init\text{-}comp\text{-}ty\ T-n\rightarrow\ s1;\ G\vdash s1\ -e-\!\succ i'-n\rightarrow\ s2;\\
\qquad G\vdash abupd\ (check\text{-}neg\ i')\ s2\ -halloc\ (Arr\ T\ (the\text{-}Intg\ i'))\!\succ a\rightarrow\ s3\rrbracket \Longrightarrow\\
\qquad\qquad G\vdash Norm\ s0\ -New\ T[e]-\!\succ Addr\ a-n\rightarrow\ s3
\end{array}$$

$$\begin{array}{l}
\mid Cast\colon \llbracket G\vdash Norm\ s0\ -e-\!\succ v-n\rightarrow\ s1;\\
\qquad s2 = abupd\ (raise\text{-}if\ (\neg G,snd\ s1\vdash v\ fits\ T)\ ClassCast)\ s1\rrbracket \Longrightarrow\\
\qquad\qquad G\vdash Norm\ s0\ -Cast\ T\ e-\!\succ v-n\rightarrow\ s2
\end{array}$$

$$\begin{array}{l}
\mid Inst\colon \llbracket G\vdash Norm\ s0\ -e-\!\succ v-n\rightarrow\ s1;\\
\qquad b = (v\neq Null\ \wedge\ G,store\ s1\vdash v\ fits\ RefT\ T)\rrbracket \Longrightarrow\\
\qquad\qquad G\vdash Norm\ s0\ -e\ InstOf\ T-\!\succ Bool\ b-n\rightarrow\ s1
\end{array}$$

$$\mid Lit\colon \qquad G\vdash Norm\ s\ -Lit\ v-\!\succ v-n\rightarrow\ Norm\ s$$

$$\begin{array}{l}
\mid UnOp\colon \llbracket G\vdash Norm\ s0\ -e-\!\succ v-n\rightarrow\ s1\rrbracket\\
\qquad \Longrightarrow G\vdash Norm\ s0\ -UnOp\ unop\ e-\!\succ(eval\text{-}unop\ unop\ v)-n\rightarrow\ s1
\end{array}$$

$$\begin{array}{l}
\mid BinOp\colon \llbracket G\vdash Norm\ s0\ -e1-\!\succ v1-n\rightarrow\ s1;\\
\qquad G\vdash s1\ -(if\ need\text{-}second\text{-}arg\ binop\ v1\ then\ (In1l\ e2)\ else\ (In1r\ Skip))\\
\qquad\ \succ-n\rightarrow\ (In1\ v2,s2)\rrbracket\\
\qquad \Longrightarrow G\vdash Norm\ s0\ -BinOp\ binop\ e1\ e2-\!\succ(eval\text{-}binop\ binop\ v1\ v2)-n\rightarrow\ s2
\end{array}$$

$$\mid Super\colon \qquad G\vdash Norm\ s\ -Super-\!\succ val\text{-}this\ s-n\rightarrow\ Norm\ s$$

$$\begin{array}{l}
\mid Acc\colon\ \llbracket G\vdash Norm\ s0\ -va{=}\!\succ(v,f)-n\rightarrow\ s1\rrbracket \Longrightarrow\\
\qquad\qquad G\vdash Norm\ s0\ -Acc\ va-\!\succ v-n\rightarrow\ s1
\end{array}$$

$$\begin{array}{l}
\mid Ass\colon\ \llbracket G\vdash Norm\ s0\ -va{=}\!\succ(w,f)-n\rightarrow\ s1;\\
\qquad G\vdash\quad s1\ -e-\!\succ v\quad -n\rightarrow\ s2\rrbracket \Longrightarrow\\
\qquad\qquad G\vdash Norm\ s0\ -va{:=}e-\!\succ v-n\rightarrow\ assign\ f\ v\ s2
\end{array}$$

$$\begin{array}{l}
\mid Cond\colon \llbracket G\vdash Norm\ s0\ -e0-\!\succ b-n\rightarrow\ s1;\\
\qquad G\vdash\quad s1\ -(if\ the\text{-}Bool\ b\ then\ e1\ else\ e2)-\!\succ v-n\rightarrow\ s2\rrbracket \Longrightarrow\\
\qquad\qquad G\vdash Norm\ s0\ -e0\ ?\ e1\ :\ e2-\!\succ v-n\rightarrow\ s2
\end{array}$$

$$\begin{array}{l}
\mid Call\colon\\
\ \llbracket G\vdash Norm\ s0\ -e-\!\succ a'-n\rightarrow\ s1;\ G\vdash s1\ -args\doteq\!\succ vs-n\rightarrow\ s2;\\
\quad D = invocation\text{-}declclass\ G\ mode\ (store\ s2)\ a'\ statT\ (\!|name{=}mn,parTs{=}pTs|\!);\\
\quad s3{=}init\text{-}lvars\ G\ D\ (\!|name{=}mn,parTs{=}pTs|\!)\ mode\ a'\ vs\ s2;\\
\quad s3' = check\text{-}method\text{-}access\ G\ accC\ statT\ mode\ (\!|name{=}mn,parTs{=}pTs|\!)\ a'\ s3;
\end{array}$$

$G\vdash s3'-Methd\ D\ (|name=mn,parTs=pTs|)-\succ v-n\rightarrow\ s4$
$]\!]$
$\Longrightarrow$
$G\vdash Norm\ s0\ -\{accC,statT,mode\}e\cdot mn(\{pTs\}args)-\succ v-n\rightarrow\ (restore\text{-}lvars\ s2\ s4)$

$|\ Methd{:}[\![G\vdash Norm\ s0\ -body\ G\ D\ sig-\succ v-n\rightarrow\ s1]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -Methd\ D\ sig-\succ v-Suc\ n\rightarrow\ s1$

$|\ Body{:}\ [\![G\vdash Norm\ s0-Init\ D-n\rightarrow\ s1;\ G\vdash s1\ -c-n\rightarrow\ s2;$
$s3 = (if\ (\exists\ l.\ abrupt\ s2 = Some\ (Jump\ (Break\ l))\ \vee$
$abrupt\ s2 = Some\ (Jump\ (Cont\ l)))$
$then\ abupd\ (\lambda\ x.\ Some\ (Error\ CrossMethodJump))\ s2$
$else\ s2)]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -Body\ D\ c$
$-\succ the\ (locals\ (store\ s2)\ Result)-n\rightarrow abupd\ (absorb\ Ret)\ s3$

— evaluation of expression lists

$|\ Nil{:}$
$G\vdash Norm\ s0\ -[]\dot{=}\succ[]-n\rightarrow\ Norm\ s0$

$|\ Cons{:}\ [\![G\vdash Norm\ s0\ -e\ -\succ\ v\ -n\rightarrow\ s1;$
$G\vdash\quad s1\ -es\dot{=}\succ vs-n\rightarrow\ s2]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -e\#es\dot{=}\succ v\#vs-n\rightarrow\ s2$

— execution of statements

$|\ Skip{:}\qquad G\vdash Norm\ s\ -Skip-n\rightarrow\ Norm\ s$

$|\ Expr{:}\ [\![G\vdash Norm\ s0\ -e-\succ v-n\rightarrow\ s1]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -Expr\ e-n\rightarrow\ s1$

$|\ Lab{:}\ [\![G\vdash Norm\ s0\ -c\ -n\rightarrow\ s1]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -l\bullet\ c-n\rightarrow\ abupd\ (absorb\ l)\ s1$

$|\ Comp{:}\ [\![G\vdash Norm\ s0\ -c1\ -n\rightarrow\ s1;$
$G\vdash\quad s1\ -c2\ -n\rightarrow\ s2]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -c1;;\ c2-n\rightarrow\ s2$

$|\ If{:}\ [\![G\vdash Norm\ s0\ -e-\succ b-n\rightarrow\ s1;$
$G\vdash\quad s1-(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)-n\rightarrow\ s2]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -If(e)\ c1\ Else\ c2\ -n\rightarrow\ s2$

$|\ Loop{:}\ [\![G\vdash Norm\ s0\ -e-\succ b-n\rightarrow\ s1;$
$if\ the\text{-}Bool\ b$
$then\ (G\vdash s1\ -c-n\rightarrow\ s2\ \wedge$
$G\vdash(abupd\ (absorb\ (Cont\ l))\ s2)\ -l\bullet\ While(e)\ c-n\rightarrow\ s3)$
$else\ s3 = s1]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -l\bullet\ While(e)\ c-n\rightarrow\ s3$

$|\ Jmp{:}\ G\vdash Norm\ s\ -Jmp\ j-n\rightarrow\ (Some\ (Jump\ j),\ s)$

$|\ Throw{:}[\![G\vdash Norm\ s0\ -e-\succ a'-n\rightarrow\ s1]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -Throw\ e-n\rightarrow\ abupd\ (throw\ a')\ s1$

$|\ Try{:}\ [\![G\vdash Norm\ s0\ -c1-n\rightarrow\ s1;\ G\vdash s1\ -sxalloc\rightarrow\ s2;$
$if\ G,s2\vdash catch\ tn\ then\ G\vdash new\text{-}xcpt\text{-}var\ vn\ s2\ -c2-n\rightarrow\ s3\ else\ s3 = s2]\!]$
$\Longrightarrow$

```
            G⊢Norm s0 −Try c1 Catch(tn vn) c2−n→ s3

| Fin: ⟦G⊢Norm s0 −c1−n→ (x1,s1);
        G⊢Norm s1 −c2−n→ s2;
        s3=(if (∃ err. x1=Some (Error err))
             then (x1,s1)
             else abupd (abrupt-if (x1≠None) x1) s2)⟧ ⟹
             G⊢Norm s0 −c1 Finally c2−n→ s3

| Init: ⟦the (class G C) = c;
        if inited C (globs s0) then s3 = Norm s0
        else (G⊢Norm (init-class-obj G C s0)
               −(if C = Object then Skip else Init (super c))−n→ s1 ∧
              G⊢set-lvars Map.empty s1 −init c−n→ s2 ∧
              s3 = restore-lvars s1 s2)⟧
        ⟹
            G⊢Norm s0 −Init C−n→ s3
monos
  if-bool-eq-conj


declare if-split     [split del] if-split-asm     [split del]
        option.split [split del] option.split-asm [split del]
        not-None-eq [simp del]
        split-paired-All [simp del] split-paired-Ex [simp del]
setup ‹map-theory-simpset (fn ctxt => ctxt delloop split-all-tac)›

inductive-cases evaln-cases: G⊢s −t≻−n→ (v, s′)

inductive-cases evaln-elim-cases:
        G⊢(Some xc, s) −t                          ≻−n→ (v, s′)
        G⊢Norm s −In1r Skip                        ≻−n→ (x, s′)
        G⊢Norm s −In1r (Jmp j)                     ≻−n→ (x, s′)
        G⊢Norm s −In1r (l• c)                      ≻−n→ (x, s′)
        G⊢Norm s −In3 ([])                         ≻−n→ (v, s′)
        G⊢Norm s −In3 (e#es)                       ≻−n→ (v, s′)
        G⊢Norm s −In1l (Lit w)                     ≻−n→ (v, s′)
        G⊢Norm s −In1l (UnOp unop e)               ≻−n→ (v, s′)
        G⊢Norm s −In1l (BinOp binop e1 e2)         ≻−n→ (v, s′)
        G⊢Norm s −In2 (LVar vn)                    ≻−n→ (v, s′)
        G⊢Norm s −In1l (Cast T e)                  ≻−n→ (v, s′)
        G⊢Norm s −In1l (e InstOf T)                ≻−n→ (v, s′)
        G⊢Norm s −In1l (Super)                     ≻−n→ (v, s′)
        G⊢Norm s −In1l (Acc va)                    ≻−n→ (v, s′)
        G⊢Norm s −In1r (Expr e)                    ≻−n→ (x, s′)
        G⊢Norm s −In1r (c1;; c2)                   ≻−n→ (x, s′)
        G⊢Norm s −In1l (Methd C sig)               ≻−n→ (x, s′)
        G⊢Norm s −In1l (Body D c)                  ≻−n→ (x, s′)
        G⊢Norm s −In1l (e0 ? e1 : e2)              ≻−n→ (v, s′)
        G⊢Norm s −In1r (If(e) c1 Else c2)          ≻−n→ (x, s′)
        G⊢Norm s −In1r (l• While(e) c)             ≻−n→ (x, s′)
        G⊢Norm s −In1r (c1 Finally c2)             ≻−n→ (x, s′)
        G⊢Norm s −In1r (Throw e)                   ≻−n→ (x, s′)
        G⊢Norm s −In1l (NewC C)                    ≻−n→ (v, s′)
        G⊢Norm s −In1l (New T[e])                  ≻−n→ (v, s′)
        G⊢Norm s −In1l (Ass va e)                  ≻−n→ (v, s′)
        G⊢Norm s −In1r (Try c1 Catch(tn vn) c2)    ≻−n→ (x, s′)
        G⊢Norm s −In2 ({accC,statDeclC,stat}e..fn) ≻−n→ (v, s′)
        G⊢Norm s −In2 (e1.[e2])                    ≻−n→ (v, s′)
```

G⊢Norm s −In1l ({accC,statT,mode}e·mn({pT}p)) ≻−n→ (v, s′)
G⊢Norm s −In1r (Init C) ≻−n→ (x, s′)

declare *if-split* [*split*] *if-split-asm* [*split*]
option.split [*split*] *option.split-asm* [*split*]
not-None-eq [*simp*]
split-paired-All [*simp*] *split-paired-Ex* [*simp*]
declaration ‹*K* (*Simplifier.map-ss* (*fn ss => ss addloop* (*split-all-tac*, *split-all-tac*)))›

lemma *evaln-Inj-elim*: *G⊢s −t≻−n→ (w,s′) ⟹ case t of In1 ec ⇒*
(*case ec of Inl e ⇒* (∃ *v. w = In1 v*) | *Inr c ⇒ w = ♢*)
| *In2 e ⇒* (∃ *v. w = In2 v*) | *In3 e ⇒* (∃ *v. w = In3 v*)
apply (*erule evaln-cases* , *auto*)
apply (*induct-tac t*)
apply (*rename-tac a*, *induct-tac a*)
apply *auto*
done

The following simplification procedures set up the proper injections of terms and their corresponding values in the evaluation relation: E.g. an expression (injection *In1l* into terms) always evaluates to ordinary values (injection *In1* into generalised values *vals*).

lemma *evaln-expr-eq*: *G⊢s −In1l t≻−n→ (w, s′) = (∃ v. w=In1 v ∧ G⊢s −t−≻v −n→ s′)*
by (*auto*, *frule evaln-Inj-elim*, *auto*)

lemma *evaln-var-eq*: *G⊢s −In2 t≻−n→ (w, s′) = (∃ vf. w=In2 vf ∧ G⊢s −t=≻vf−n→ s′)*
by (*auto*, *frule evaln-Inj-elim*, *auto*)

lemma *evaln-exprs-eq*: *G⊢s −In3 t≻−n→ (w, s′) = (∃ vs. w=In3 vs ∧ G⊢s −t≐≻vs−n→ s′)*
by (*auto*, *frule evaln-Inj-elim*, *auto*)

lemma *evaln-stmt-eq*: *G⊢s −In1r t≻−n→ (w, s′) = (w=♢ ∧ G⊢s −t −n→ s′)*
by (*auto*, *frule evaln-Inj-elim*, *auto*, *frule evaln-Inj-elim*, *auto*)

simproc-setup *evaln-expr* (*G⊢s −In1l t≻−n→ (w, s′)*) = ‹
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- $ - $ - $ - $ - $ (*Const* - $ -) $ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm evaln-expr-eq*}))›

simproc-setup *evaln-var* (*G⊢s −In2 t≻−n→ (w, s′)*) = ‹
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- $ - $ - $ - $ - $ (*Const* - $ -) $ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm evaln-var-eq*}))›

simproc-setup *evaln-exprs* (*G⊢s −In3 t≻−n→ (w, s′)*) = ‹
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- $ - $ - $ - $ - $ (*Const* - $ -) $ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm evaln-exprs-eq*}))›

simproc-setup *evaln-stmt* (*G⊢s −In1r t≻−n→ (w, s′)*) = ‹
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*

```
    (- $ - $ - $ - $ - $ (Const - $ -) $ -) => NONE
  | - => SOME (mk-meta-eq @{thm evaln-stmt-eq}))›
```

ML ‹*ML-Thms.bind-thms* (*evaln-AbruptIs*, *sum3-instantiate* ***context*** @{*thm evaln.Abrupt*})›
declare *evaln-AbruptIs* [*intro*!]

lemma *evaln-Callee*: $G \vdash Norm\ s -In1l\ (Callee\ l\ e) \succ -n \rightarrow (v,s') = False$
proof −
  { **fix** *s t v s'*
    assume *eval*: $G \vdash s\ -t \succ -n \rightarrow (v,s')$ **and**
        normal: *normal s* **and**
        callee: $t=In1l\ (Callee\ l\ e)$
    then have *False* **by** *induct auto*
  }
  then show *?thesis*
    by (*cases s'*) *fastforce*
qed

lemma *evaln-InsInitE*: $G \vdash Norm\ s -In1l\ (InsInitE\ c\ e) \succ -n \rightarrow (v,s') = False$
proof −
  { **fix** *s t v s'*
    assume *eval*: $G \vdash s\ -t \succ -n \rightarrow (v,s')$ **and**
        normal: *normal s* **and**
        callee: $t=In1l\ (InsInitE\ c\ e)$
    then have *False* **by** *induct auto*
  }
  then show *?thesis*
    by (*cases s'*) *fastforce*
qed

lemma *evaln-InsInitV*: $G \vdash Norm\ s -In2\ (InsInitV\ c\ w) \succ -n \rightarrow (v,s') = False$
proof −
  { **fix** *s t v s'*
    assume *eval*: $G \vdash s\ -t \succ -n \rightarrow (v,s')$ **and**
        normal: *normal s* **and**
        callee: $t=In2\ (InsInitV\ c\ w)$
    then have *False* **by** *induct auto*
  }
  then show *?thesis*
    by (*cases s'*) *fastforce*
qed

lemma *evaln-FinA*: $G \vdash Norm\ s -In1r\ (FinA\ a\ c) \succ -n \rightarrow (v,s') = False$
proof −
  { **fix** *s t v s'*
    assume *eval*: $G \vdash s\ -t \succ -n \rightarrow (v,s')$ **and**
        normal: *normal s* **and**
        callee: $t=In1r\ (FinA\ a\ c)$
    then have *False* **by** *induct auto*
  }
  then show *?thesis*
    by (*cases s'*) *fastforce*
qed

lemma *evaln-abrupt-lemma*: $G\vdash s\ -e\succ-n\to\ (v,s') \Longrightarrow$
$fst\ s = Some\ xc \longrightarrow s' = s \wedge v = undefined3\ e$
apply (*erule evaln-cases* , *auto*)
done

lemma *evaln-abrupt*:
$\bigwedge s'.\ G\vdash(Some\ xc,s)\ -e\succ-n\to\ (w,s') = (s' = (Some\ xc,s) \wedge$
$w{=}undefined3\ e \wedge G\vdash(Some\ xc,s)\ -e\succ-n\to\ (undefined3\ e,(Some\ xc,s)))$
apply *auto*
apply (*frule evaln-abrupt-lemma*, *auto*)+
done

simproc-setup *evaln-abrupt* ($G\vdash(Some\ xc,s)\ -e\succ-n\to\ (w,s')$) = ‹
fn - => *fn* - => *fn ct* =>
(*case Thm.term-of ct of*
(- \$ - \$ - \$ - \$ - \$ - \$ (*Const* (***const-name*** ‹*Pair*›, -) \$ (*Const* (***const-name*** ‹*Some*›,-) \$ -)\$ -))
=> *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm evaln-abrupt*}))
›

lemma *evaln-LitI*: $G\vdash s\ -Lit\ v-\succ(if\ normal\ s\ then\ v\ else\ undefined)-n\to\ s$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *evaln.Lit*)

lemma *CondI*:
$\bigwedge s1.\ [\![G\vdash s\ -e-\succ b-n\to\ s1;\ G\vdash s1\ -(if\ the\text{-}Bool\ b\ then\ e1\ else\ e2)-\succ v-n\to\ s2]\!] \Longrightarrow$
$G\vdash s\ -e\ ?\ e1 : e2-\succ(if\ normal\ s1\ then\ v\ else\ undefined)-n\to\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *evaln.Cond*)

lemma *evaln-SkipI* [*intro*!]: $G\vdash s\ -Skip-n\to\ s$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *evaln.Skip*)

lemma *evaln-ExprI*: $G\vdash s\ -e-\succ v-n\to\ s' \Longrightarrow G\vdash s\ -Expr\ e-n\to\ s'$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *evaln.Expr*)

lemma *evaln-CompI*: $[\![G\vdash s\ -c1-n\to\ s1;\ G\vdash s1\ -c2-n\to\ s2]\!] \Longrightarrow G\vdash s\ -c1;;\ c2-n\to\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *evaln.Comp*)

lemma *evaln-IfI*:
$[\![G\vdash s\ -e-\succ v-n\to\ s1;\ G\vdash s1\ -(if\ the\text{-}Bool\ v\ then\ c1\ else\ c2)-n\to\ s2]\!] \Longrightarrow$
$G\vdash s\ -If(e)\ c1\ Else\ c2-n\to\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *evaln.If*)

lemma *evaln-SkipD* [*dest*!]: $G\vdash s\ -Skip-n\to\ s' \Longrightarrow s' = s$
by (*erule evaln-cases*, *auto*)

lemma *evaln-Skip-eq* [*simp*]: $G \vdash s \ {-}Skip{-}n{\to}\ s' = (s = s')$
apply *auto*
done

evaln implies eval

lemma *evaln-eval*:
  assumes *evaln*: $G \vdash s0\ {-}t{\succ}{-}n{\to}\ (v,s1)$
  shows $G \vdash s0\ {-}t{\succ}{\to}\ (v,s1)$
using *evaln*
proof (*induct*)
  case (*Loop s0 e b n s1 c s2 l s3*)
  note ‹$G \vdash Norm\ s0\ {-}e{-}{\succ}b{\to}\ s1$›
  moreover
  have *if the-Bool b*
    then $(G \vdash s1\ {-}c{\to}\ s2) \wedge$
      $G \vdash abupd\ (absorb\ (Cont\ l))\ s2\ {-}l{\bullet}\ While(e)\ c{\to}\ s3$
    else $s3 = s1$
   using *Loop.hyps* **by** *simp*
  ultimately show *?case* **by** (*rule eval.Loop*)
next
  case (*Try s0 c1 n s1 s2 C vn c2 s3*)
  note ‹$G \vdash Norm\ s0\ {-}c1{\to}\ s1$›
  moreover
  note ‹$G \vdash s1\ {-}sxalloc{\to}\ s2$›
  moreover
  have *if* $G,s2 \vdash catch\ C$ *then* $G \vdash new\text{-}xcpt\text{-}var\ vn\ s2\ {-}c2{\to}\ s3$ *else* $s3 = s2$
   using *Try.hyps* **by** *simp*
  ultimately show *?case* **by** (*rule eval.Try*)
next
  case (*Init C c s0 s3 n s1 s2*)
  note ‹*the* (*class G C*) = *c*›
  moreover
  have *if inited C* (*globs s0*)
     then $s3 = Norm\ s0$
     else $G \vdash Norm\ ((init\text{-}class\text{-}obj\ G\ C)\ s0)$
       ${-}(if\ C = Object\ then\ Skip\ else\ Init\ (super\ c)){\to}\ s1 \wedge$
      $G \vdash (set\text{-}lvars\ Map.empty)\ s1\ {-}init\ c{\to}\ s2 \wedge$
      $s3 = (set\text{-}lvars\ (locals\ (store\ s1)))\ s2$
   using *Init.hyps* **by** *simp*
  ultimately show *?case* **by** (*rule eval.Init*)
qed (*rule eval.intros*,(*assumption+* | *assumption?*))+

lemma *Suc-le-D-lemma*: $[\![Suc\ n <= m';\ (\bigwedge m.\ n <= m \Longrightarrow P\ (Suc\ m))\ ]\!] \Longrightarrow P\ m'$
apply (*frule Suc-le-D*)
apply *fast*
done

lemma *evaln-nonstrict* [*rule-format* (*no-asm*), *elim*]:
  $G \vdash s\ {-}t{\succ}{-}n{\to}\ (w,\ s') \Longrightarrow \forall m.\ n \le m \longrightarrow G \vdash s\ {-}t{\succ}{-}m{\to}\ (w,\ s')$
apply (*erule evaln.induct*)
apply (*tactic* ‹*ALLGOALS* (*EVERY'* [*strip-tac* ***context***,
  TRY o eresolve-tac ***context*** @{*thms Suc-le-D-lemma*},
  REPEAT o smp-tac ***context*** *1*,
  resolve-tac ***context*** @{*thms evaln.intros*} *THEN-ALL-NEW TRY o assume-tac* ***context***])›)

apply (*auto split del*: *if-split*)
done

lemmas *evaln-nonstrict-Suc* = *evaln-nonstrict* [*OF* - *le-refl* [*THEN le-SucI*]]

lemma *evaln-max2*: $\llbracket G\vdash s1\ -t1\succ-n1\rightarrow (w1,\ s1');\ G\vdash s2\ -t2\succ-n2\rightarrow (w2,\ s2')\rrbracket \Longrightarrow$
$G\vdash s1\ -t1\succ-max\ n1\ n2\rightarrow (w1,\ s1') \wedge G\vdash s2\ -t2\succ-max\ n1\ n2\rightarrow (w2,\ s2')$
by (*fast intro*: *max.cobounded1 max.cobounded2*)

corollary *evaln-max2E* [*consumes 2*]:
$\llbracket G\vdash s1\ -t1\succ-n1\rightarrow (w1,\ s1');\ G\vdash s2\ -t2\succ-n2\rightarrow (w2,\ s2');$
$\llbracket G\vdash s1\ -t1\succ-max\ n1\ n2\rightarrow (w1,\ s1');G\vdash s2\ -t2\succ-max\ n1\ n2\rightarrow (w2,\ s2')\ \rrbracket \Longrightarrow P\ \rrbracket \Longrightarrow P$
by (*drule* (*1*) *evaln-max2*) *simp*

lemma *evaln-max3*:
$\llbracket G\vdash s1\ -t1\succ-n1\rightarrow (w1,\ s1');\ G\vdash s2\ -t2\succ-n2\rightarrow (w2,\ s2');\ G\vdash s3\ -t3\succ-n3\rightarrow (w3,\ s3')\rrbracket \Longrightarrow$
$G\vdash s1\ -t1\succ-max\ (max\ n1\ n2)\ n3\rightarrow (w1,\ s1') \wedge$
$G\vdash s2\ -t2\succ-max\ (max\ n1\ n2)\ n3\rightarrow (w2,\ s2') \wedge$
$G\vdash s3\ -t3\succ-max\ (max\ n1\ n2)\ n3\rightarrow (w3,\ s3')$
apply (*drule* (*1*) *evaln-max2*, *erule thin-rl*)
apply (*fast intro*!: *max.cobounded1 max.cobounded2*)
done

corollary *evaln-max3E*:
$\llbracket G\vdash s1\ -t1\succ-n1\rightarrow (w1,\ s1');\ G\vdash s2\ -t2\succ-n2\rightarrow (w2,\ s2');\ G\vdash s3\ -t3\succ-n3\rightarrow (w3,\ s3');$
$\llbracket G\vdash s1\ -t1\succ-max\ (max\ n1\ n2)\ n3\rightarrow (w1,\ s1');$
$G\vdash s2\ -t2\succ-max\ (max\ n1\ n2)\ n3\rightarrow (w2,\ s2');$
$G\vdash s3\ -t3\succ-max\ (max\ n1\ n2)\ n3\rightarrow (w3,\ s3')$
$\rrbracket \Longrightarrow P$
$\rrbracket \Longrightarrow P$
by (*drule* (*2*) *evaln-max3*) *simp*

lemma *le-max3I1*: $(n2::nat) \leq max\ n1\ (max\ n2\ n3)$
proof −
have $n2 \leq max\ n2\ n3$
by (*rule max.cobounded1*)
also
have $max\ n2\ n3 \leq max\ n1\ (max\ n2\ n3)$
by (*rule max.cobounded2*)
finally
show *?thesis* **.**
qed

lemma *le-max3I2*: $(n3::nat) \leq max\ n1\ (max\ n2\ n3)$
proof −
have $n3 \leq max\ n2\ n3$
by (*rule max.cobounded2*)
also
have $max\ n2\ n3 \leq max\ n1\ (max\ n2\ n3)$
by (*rule max.cobounded2*)
finally
show *?thesis* **.**
qed

declare [[*simproc del*: *wt-expr wt-var wt-exprs wt-stmt*]]

eval implies evaln

lemma *eval-evaln*:
  assumes *eval*: $G\vdash s0\ -t\succ\rightarrow\ (v,s1)$
  shows $\exists n.\ G\vdash s0\ -t\succ-n\rightarrow\ (v,s1)$
using *eval*
proof (*induct*)
  case (*Abrupt xc s t*)
  obtain *n* **where**
    $G\vdash(Some\ xc,\ s)\ -t\succ-n\rightarrow\ (undefined3\ t,\ (Some\ xc,\ s))$
    by (*iprover intro*: *evaln.Abrupt*)
  then show *?case* **..**
next
  case *Skip*
  show *?case* **by** (*blast intro*: *evaln.Skip*)
next
  case (*Expr s0 e v s1*)
  then obtain *n* **where**
    $G\vdash Norm\ s0\ -e-\succ v-n\rightarrow\ s1$
    by (*iprover*)
  then have $G\vdash Norm\ s0\ -Expr\ e-n\rightarrow\ s1$
    by (*rule evaln.Expr*)
  then show *?case* **..**
next
  case (*Lab s0 c s1 l*)
  then obtain *n* **where**
    $G\vdash Norm\ s0\ -c-n\rightarrow\ s1$
    by (*iprover*)
  then have $G\vdash Norm\ s0\ -l\bullet\ c-n\rightarrow\ abupd\ (absorb\ l)\ s1$
    by (*rule evaln.Lab*)
  then show *?case* **..**
next
  case (*Comp s0 c1 s1 c2 s2*)
  then obtain *n1 n2* **where**
    $G\vdash Norm\ s0\ -c1-n1\rightarrow\ s1$
    $G\vdash s1\ -c2-n2\rightarrow\ s2$
    by (*iprover*)
  then have $G\vdash Norm\ s0\ -c1;;\ c2-max\ n1\ n2\rightarrow\ s2$
    by (*blast intro*: *evaln.Comp dest*: *evaln-max2*)
  then show *?case* **..**
next
  case (*If s0 e b s1 c1 c2 s2*)
  then obtain *n1 n2* **where**
    $G\vdash Norm\ s0\ -e-\succ b-n1\rightarrow\ s1$
    $G\vdash s1\ -(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)-n2\rightarrow\ s2$
    by (*iprover*)
  then have $G\vdash Norm\ s0\ -If(e)\ c1\ Else\ c2-max\ n1\ n2\rightarrow\ s2$
    by (*blast intro*: *evaln.If dest*: *evaln-max2*)
  then show *?case* **..**
next
  case (*Loop s0 e b s1 c s2 l s3*)
  from *Loop.hyps* **obtain** *n1* **where**
    $G\vdash Norm\ s0\ -e-\succ b-n1\rightarrow\ s1$
    by (*iprover*)
  moreover from *Loop.hyps* **obtain** *n2* **where**
    if the-Bool b

```
        then (G⊢s1 −c−n2→ s2 ∧
              G⊢(abupd (absorb (Cont l)) s2)−l• While(e) c−n2→ s3)
        else s3 = s1
    by simp (iprover intro: evaln-nonstrict max.cobounded1 max.cobounded2)
  ultimately
  have G⊢Norm s0 −l• While(e) c−max n1 n2→ s3
    apply −
    apply (rule evaln.Loop)
    apply  (iprover intro: evaln-nonstrict intro: max.cobounded1)
    apply  (auto intro: evaln-nonstrict intro: max.cobounded2)
    done
  then show ?case ..
next
  case (Jmp s j)
  fix n have G⊢Norm s −Jmp j−n→ (Some (Jump j), s)
    by (rule evaln.Jmp)
  then show ?case ..
next
  case (Throw s0 e a s1)
  then obtain n where
    G⊢Norm s0 −e−≻a−n→ s1
    by (iprover)
  then have G⊢Norm s0 −Throw e−n→ abupd (throw a) s1
    by (rule evaln.Throw)
  then show ?case ..
next
  case (Try s0 c1 s1 s2 catchC vn c2 s3)
  from Try.hyps obtain n1 where
    G⊢Norm s0 −c1−n1→ s1
    by (iprover)
  moreover
  note sxalloc = ‹G⊢s1 −sxalloc→ s2›
  moreover
  from Try.hyps obtain n2 where
    if G,s2⊢catch catchC then G⊢new-xcpt-var vn s2 −c2−n2→ s3 else s3 = s2
    by fastforce
  ultimately
  have G⊢Norm s0 −Try c1 Catch(catchC vn) c2−max n1 n2→ s3
    by (auto intro!: evaln.Try max.cobounded1 max.cobounded2)
  then show ?case ..
next
  case (Fin s0 c1 x1 s1 c2 s2 s3)
  from Fin obtain n1 n2 where
    G⊢Norm s0 −c1−n1→ (x1, s1)
    G⊢Norm s1 −c2−n2→ s2
    by iprover
  moreover
  note s3 = ‹s3 = (if ∃ err. x1 = Some (Error err)
                  then (x1, s1)
                  else abupd (abrupt-if (x1 ≠ None) x1) s2)›
  ultimately
  have
    G⊢Norm s0 −c1 Finally c2−max n1 n2→ s3
    by (blast intro: evaln.Fin dest: evaln-max2)
  then show ?case ..
next
  case (Init C c s0 s3 s1 s2)
  note cls = ‹the (class G C) = c›
  moreover from Init.hyps obtain n where
```

```
     if inited C (globs s0) then s3 = Norm s0
      else (G⊢Norm (init-class-obj G C s0)
             −(if C = Object then Skip else Init (super c))−n→ s1 ∧
                  G⊢set-lvars Map.empty s1 −init c−n→ s2 ∧
                  s3 = restore-lvars s1 s2)
    by (auto intro: evaln-nonstrict max.cobounded1 max.cobounded2)
  ultimately have G⊢Norm s0 −Init C−n→ s3
    by (rule evaln.Init)
  then show ?case ..
next
  case (NewC s0 C s1 a s2)
  then obtain n where
    G⊢Norm s0 −Init C−n→ s1
    by (iprover)
  with NewC
  have G⊢Norm s0 −NewC C−≻Addr a−n→ s2
    by (iprover intro: evaln.NewC)
  then show ?case ..
next
  case (NewA s0 T s1 e i s2 a s3)
  then obtain n1 n2 where
    G⊢Norm s0 −init-comp-ty T−n1→ s1
    G⊢s1 −e−≻i−n2→ s2
    by (iprover)
  moreover
  note ‹G⊢abupd (check-neg i) s2 −halloc Arr T (the-Intg i)≻a→ s3›
  ultimately
  have G⊢Norm s0 −New T[e]−≻Addr a−max n1 n2→ s3
    by (blast intro: evaln.NewA dest: evaln-max2)
  then show ?case ..
next
  case (Cast s0 e v s1 s2 castT)
  then obtain n where
    G⊢Norm s0 −e−≻v−n→ s1
    by (iprover)
  moreover
  note ‹s2 = abupd (raise-if (¬ G,snd s1⊢v fits castT) ClassCast) s1›
  ultimately
  have G⊢Norm s0 −Cast castT e−≻v−n→ s2
    by (rule evaln.Cast)
  then show ?case ..
next
  case (Inst s0 e v s1 b T)
  then obtain n where
    G⊢Norm s0 −e−≻v−n→ s1
    by (iprover)
  moreover
  note ‹b = (v ≠ Null ∧ G,snd s1⊢v fits RefT T)›
  ultimately
  have G⊢Norm s0 −e InstOf T−≻Bool b−n→ s1
    by (rule evaln.Inst)
  then show ?case ..
next
  case (Lit s v)
  fix n have G⊢Norm s −Lit v−≻v−n→ Norm s
    by (rule evaln.Lit)
  then show ?case ..
next
  case (UnOp s0 e v s1 unop)
```

```
  then obtain n where
    G⊢Norm s0 −e−≻v−n→ s1
    by (iprover)
  hence G⊢Norm s0 −UnOp unop e−≻eval-unop unop v−n→ s1
    by (rule evaln.UnOp)
  then show ?case ..
next
  case (BinOp s0 e1 v1 s1 binop e2 v2 s2)
  then obtain n1 n2 where
    G⊢Norm s0 −e1−≻v1−n1→ s1
    G⊢s1 −(if need-second-arg binop v1 then In1l e2
               else In1r Skip)≻−n2→ (In1 v2, s2)
    by (iprover)
  hence G⊢Norm s0 −BinOp binop e1 e2−≻(eval-binop binop v1 v2)−max n1 n2
          → s2
    by (blast intro!: evaln.BinOp dest: evaln-max2)
  then show ?case ..
next
  case (Super s )
  fix n have G⊢Norm s −Super−≻val-this s−n→ Norm s
    by (rule evaln.Super)
  then show ?case ..
next
  case (Acc s0 va v f s1)
  then obtain n where
    G⊢Norm s0 −va=≻(v, f)−n→ s1
    by (iprover)
  then
  have G⊢Norm s0 −Acc va−≻v−n→ s1
    by (rule evaln.Acc)
  then show ?case ..
next
  case (Ass s0 var w f s1 e v s2)
  then obtain n1 n2 where
    G⊢Norm s0 −var=≻(w, f)−n1→ s1
    G⊢s1 −e−≻v−n2→ s2
    by (iprover)
  then
  have G⊢Norm s0 −var:=e−≻v−max n1 n2→ assign f v s2
    by (blast intro: evaln.Ass dest: evaln-max2)
  then show ?case ..
next
  case (Cond s0 e0 b s1 e1 e2 v s2)
  then obtain n1 n2 where
    G⊢Norm s0 −e0−≻b−n1→ s1
    G⊢s1 −(if the-Bool b then e1 else e2)−≻v−n2→ s2
    by (iprover)
  then
  have G⊢Norm s0 −e0 ? e1 : e2−≻v−max n1 n2→ s2
    by (blast intro: evaln.Cond dest: evaln-max2)
  then show ?case ..
next
  case (Call s0 e a' s1 args vs s2 invDeclC mode statT mn pTs' s3 s3' accC' v s4)
  then obtain n1 n2 where
    G⊢Norm s0 −e−≻a'−n1→ s1
    G⊢s1 −args≐≻vs−n2→ s2
    by iprover
  moreover
  note ‹invDeclC = invocation-declclass G mode (store s2) a' statT
```

(|*name*=*mn*,*parTs*=*pTs*′|)›
moreover
note ‹*s3* = *init-lvars G invDeclC* (|*name*=*mn*,*parTs*=*pTs*′|) *mode a*′ *vs s2*›
moreover
note ‹*s3*′=*check-method-access G accC*′ *statT mode* (|*name*=*mn*,*parTs*=*pTs*′|) *a*′ *s3*›
moreover
from *Call.hyps*
obtain *m* **where**
G⊢*s3*′ −*Methd invDeclC* (|*name*=*mn*, *parTs*=*pTs*′|)−≻*v*−*m*→ *s4*
by *iprover*
ultimately
have *G*⊢*Norm s0* −{*accC*′,*statT*,*mode*}*e*·*mn*({*pTs*′}*args*)−≻*v*−*max n1* (*max n2 m*)→
(*set-lvars* (*locals* (*store s2*))) *s4*
by (*auto intro*!: *evaln.Call max.cobounded1 le-max3I1 le-max3I2*)
thus *?case* **..**
next
case (*Methd s0 D sig v s1*)
then obtain *n* **where**
G⊢*Norm s0* −*body G D sig*−≻*v*−*n*→ *s1*
by *iprover*
then have *G*⊢*Norm s0* −*Methd D sig*−≻*v*−*Suc n*→ *s1*
by (*rule evaln.Methd*)
then show *?case* **..**
next
case (*Body s0 D s1 c s2 s3*)
from *Body.hyps* **obtain** *n1 n2* **where**
evaln-init: *G*⊢*Norm s0* −*Init D*−*n1*→ *s1* **and**
evaln-c: *G*⊢*s1* −*c*−*n2*→ *s2*
by (*iprover*)
moreover
note ‹*s3* = (*if* ∃ *l*. *fst s2* = *Some* (*Jump* (*Break l*)) ∨
fst s2 = *Some* (*Jump* (*Cont l*))
then abupd (λ*x*. *Some* (*Error CrossMethodJump*)) *s2*
else s2)›
ultimately
have
G⊢*Norm s0* −*Body D c*−≻*the* (*locals* (*store s2*) *Result*)−*max n1 n2*
→ *abupd* (*absorb Ret*) *s3*
by (*iprover intro*: *evaln.Body dest*: *evaln-max2*)
then show *?case* **..**
next
case (*LVar s vn*)
obtain *n* **where**
G⊢*Norm s* −*LVar vn*=≻*lvar vn s*−*n*→ *Norm s*
by (*iprover intro*: *evaln.LVar*)
then show *?case* **..**
next
case (*FVar s0 statDeclC s1 e a s2 v s2*′ *stat fn s3 accC*)
then obtain *n1 n2* **where**
G⊢*Norm s0* −*Init statDeclC*−*n1*→ *s1*
G⊢*s1* −*e*−≻*a*−*n2*→ *s2*
by *iprover*
moreover
note ‹*s3* = *check-field-access G accC statDeclC fn stat a s2*′›
and ‹(*v*, *s2*′) = *fvar statDeclC stat fn a s2*›
ultimately
have *G*⊢*Norm s0* −{*accC*,*statDeclC*,*stat*}*e*..*fn*=≻*v*−*max n1 n2*→ *s3*
by (*iprover intro*: *evaln.FVar dest*: *evaln-max2*)
then show *?case* **..**

```
next
  case (AVar s0 e1 a s1 e2 i s2 v s2')
  then obtain n1 n2 where
    G⊢Norm s0 −e1−≻a−n1→ s1
    G⊢s1 −e2−≻i−n2→ s2
    by iprover
  moreover
  note ‹(v, s2') = avar G i a s2›
  ultimately
  have G⊢Norm s0 −e1.[e2]=≻v−max n1 n2→ s2'
    by (blast intro!: evaln.AVar dest: evaln-max2)
  then show ?case ..
next
  case (Nil s0)
  show ?case by (iprover intro: evaln.Nil)
next
  case (Cons s0 e v s1 es vs s2)
  then obtain n1 n2 where
    G⊢Norm s0 −e−≻v−n1→ s1
    G⊢s1 −es≐≻vs−n2→ s2
    by iprover
  then
  have G⊢Norm s0 −e # es≐≻v # vs−max n1 n2→ s2
    by (blast intro!: evaln.Cons dest: evaln-max2)
  then show ?case ..
qed

end
```

Chapter 21

Trans

theory *Trans* **imports** *Evaln* **begin**

definition
  groundVar :: *var* ⇒ *bool* **where**
  groundVar v ⟷ (*case v of*
        LVar ln ⇒ *True*
      | {*accC*,*statDeclC*,*stat*}*e*..*fn* ⇒ ∃ *a*. *e*=*Lit a*
      | *e1*.[*e2*] ⇒ ∃ *a i*. *e1*= *Lit a* ∧ *e2* = *Lit i*
      | *InsInitV c v* ⇒ *False*)

lemma *groundVar-cases*:
  assumes *ground*: *groundVar v*
  obtains (*LVar*) *ln* **where** *v*=*LVar ln*
    | (*FVar*) *accC statDeclC stat a fn* **where** *v*={*accC*,*statDeclC*,*stat*}(*Lit a*)..*fn*
    | (*AVar*) *a i* **where** *v*=(*Lit a*).[*Lit i*]
  using *ground LVar FVar AVar*
  by (*cases v*) (*auto simp add*: *groundVar-def*)

definition
  groundExprs :: *expr list* ⇒ *bool*
  where *groundExprs es* ⟷ (∀ *e* ∈ *set es*. ∃ *v*. *e* = *Lit v*)

primrec *the-val*:: *expr* ⇒ *val*
  where *the-val* (*Lit v*) = *v*

primrec *the-var*:: *prog* ⇒ *state* ⇒ *var* ⇒ (*vvar* × *state*) **where**
  the-var G s (*LVar ln*) = (*lvar ln* (*store s*),*s*)
| *the-var-FVar-def*: *the-var G s* ({*accC*,*statDeclC*,*stat*}*a*..*fn*) =*fvar statDeclC stat fn* (*the-val a*) *s*
| *the-var-AVar-def*: *the-var G s*(*a*.[*i*])   =*avar G* (*the-val i*) (*the-val a*) *s*

lemma *the-var-FVar-simp*[*simp*]:
the-var G s ({*accC*,*statDeclC*,*stat*}(*Lit a*)..*fn*) = *fvar statDeclC stat fn a s*
by (*simp*)
declare *the-var-FVar-def* [*simp del*]

lemma *the-var-AVar-simp*:
the-var G s ((*Lit a*).[*Lit i*]) = *avar G i a s*
by (*simp*)
declare *the-var-AVar-def* [*simp del*]

abbreviation
  Ref :: *loc* ⇒ *expr*
  where *Ref a* == *Lit* (*Addr a*)

abbreviation
  SKIP :: *expr*
  where *SKIP* == *Lit Unit*

inductive
  step :: [*prog*,*term* × *state*,*term* × *state*] ⇒ *bool* (-⊢- ↦*1* -[*61*,*82*,*82*] *81*)
  for *G* :: *prog*
where

Abrupt: ⟦∀ *v*. *t* ≠ ⟨*Lit v*⟩;
  ∀ *t*. *t* ≠ ⟨*l*• *Skip*⟩;
  ∀ *C vn c*. *t* ≠ ⟨*Try Skip Catch*(*C vn*) *c*⟩;
  ∀ *x c*. *t* ≠ ⟨*Skip Finally c*⟩ ∧ *xc* ≠ *Xcpt x*;
  ∀ *a c*. *t* ≠ ⟨*FinA a c*⟩⟧
  ⟹
  G⊢(*t*,*Some xc*,*s*) ↦*1* (⟨*Lit undefined*⟩,*Some xc*,*s*)

| *InsInitE*: ⟦*G*⊢(⟨*c*⟩,*Norm s*) ↦*1* (⟨*c*′⟩, *s*′)⟧
  ⟹
  G⊢(⟨*InsInitE c e*⟩,*Norm s*) ↦*1* (⟨*InsInitE c*′ *e*⟩, *s*′)

| *NewC*: *G*⊢(⟨*NewC C*⟩,*Norm s*) ↦*1* (⟨*InsInitE* (*Init C*) (*NewC C*)⟩, *Norm s*)
| *NewCInited*: ⟦*G*⊢ *Norm s* −*halloc* (*CInst C*)≻*a*→ *s*′⟧
  ⟹
  G⊢(⟨*InsInitE Skip* (*NewC C*)⟩,*Norm s*) ↦*1* (⟨*Ref a*⟩, *s*′)

| *NewA*:
  G⊢(⟨*New T*[*e*]⟩,*Norm s*) ↦*1* (⟨*InsInitE* (*init-comp-ty T*) (*New T*[*e*])⟩,*Norm s*)
| *InsInitNewAIdx*:
  ⟦*G*⊢(⟨*e*⟩,*Norm s*) ↦*1* (⟨*e*′⟩, *s*′)⟧
  ⟹
  G⊢(⟨*InsInitE Skip* (*New T*[*e*])⟩,*Norm s*) ↦*1* (⟨*InsInitE Skip* (*New T*[*e*′])⟩,*s*′)
| *InsInitNewA*:
  ⟦*G*⊢*abupd* (*check-neg i*) (*Norm s*) −*halloc* (*Arr T* (*the-Intg i*))≻*a*→ *s*′ ⟧
  ⟹
  G⊢(⟨*InsInitE Skip* (*New T*[*Lit i*])⟩,*Norm s*) ↦*1* (⟨*Ref a*⟩,*s*′)

| *CastE*:
  ⟦*G*⊢(⟨*e*⟩,*Norm s*) ↦*1* (⟨*e*′⟩,*s*′)⟧
  ⟹
  G⊢(⟨*Cast T e*⟩,*None*,*s*) ↦*1* (⟨*Cast T e*′⟩,*s*′)
| *Cast*:
  ⟦*s*′ = *abupd* (*raise-if* (¬*G*,*s*⊢*v fits T*) *ClassCast*) (*Norm s*)⟧

$\Longrightarrow$
$G \vdash (\langle Cast\ T\ (Lit\ v)\rangle, Norm\ s) \mapsto 1\ (\langle Lit\ v\rangle, s')$

| $InstE$: $[\![G \vdash (\langle e\rangle, Norm\ s) \mapsto 1\ (\langle e'::expr\rangle, s')]\!]$
$\Longrightarrow$
$G \vdash (\langle e\ InstOf\ T\rangle, Norm\ s) \mapsto 1\ (\langle e'\rangle, s')$
| $Inst$: $[\![b = (v \neq Null \wedge G, s \vdash v\ fits\ RefT\ T)]\!]$
$\Longrightarrow$
$G \vdash (\langle (Lit\ v)\ InstOf\ T\rangle, Norm\ s) \mapsto 1\ (\langle Lit\ (Bool\ b)\rangle, s')$

| $UnOpE$: $[\![G \vdash (\langle e\rangle, Norm\ s) \mapsto 1\ (\langle e'\rangle, s')\ ]\!]$
$\Longrightarrow$
$G \vdash (\langle UnOp\ unop\ e\rangle, Norm\ s) \mapsto 1\ (\langle UnOp\ unop\ e'\rangle, s')$
| $UnOp$: $G \vdash (\langle UnOp\ unop\ (Lit\ v)\rangle, Norm\ s) \mapsto 1\ (\langle Lit\ (eval\text{-}unop\ unop\ v)\rangle, Norm\ s)$

| $BinOpE1$: $[\![G \vdash (\langle e1\rangle, Norm\ s) \mapsto 1\ (\langle e1'\rangle, s')\ ]\!]$
$\Longrightarrow$
$G \vdash (\langle BinOp\ binop\ e1\ e2\rangle, Norm\ s) \mapsto 1\ (\langle BinOp\ binop\ e1'\ e2\rangle, s')$
| $BinOpE2$: $[\![need\text{-}second\text{-}arg\ binop\ v1;\ G \vdash (\langle e2\rangle, Norm\ s) \mapsto 1\ (\langle e2'\rangle, s')\ ]\!]$
$\Longrightarrow$
$G \vdash (\langle BinOp\ binop\ (Lit\ v1)\ e2\rangle, Norm\ s)$
$\mapsto 1\ (\langle BinOp\ binop\ (Lit\ v1)\ e2'\rangle, s')$
| $BinOpTerm$: $[\![\neg\ need\text{-}second\text{-}arg\ binop\ v1]\!]$
$\Longrightarrow$
$G \vdash (\langle BinOp\ binop\ (Lit\ v1)\ e2\rangle, Norm\ s)$
$\mapsto 1\ (\langle Lit\ v1\rangle, Norm\ s)$
| $BinOp$: $G \vdash (\langle BinOp\ binop\ (Lit\ v1)\ (Lit\ v2)\rangle, Norm\ s)$
$\mapsto 1\ (\langle Lit\ (eval\text{-}binop\ binop\ v1\ v2)\rangle, Norm\ s)$

| $Super$: $G \vdash (\langle Super\rangle, Norm\ s) \mapsto 1\ (\langle Lit\ (val\text{-}this\ s)\rangle, Norm\ s)$

| $AccVA$: $[\![G \vdash (\langle va\rangle, Norm\ s) \mapsto 1\ (\langle va'\rangle, s')\ ]\!]$
$\Longrightarrow$
$G \vdash (\langle Acc\ va\rangle, Norm\ s) \mapsto 1\ (\langle Acc\ va'\rangle, s')$
| Acc: $[\![groundVar\ va;\ ((v, vf), s') = the\text{-}var\ G\ (Norm\ s)\ va]\!]$
$\Longrightarrow$
$G \vdash (\langle Acc\ va\rangle, Norm\ s) \mapsto 1\ (\langle Lit\ v\rangle, s')$

| $AssVA$: $[\![G \vdash (\langle va\rangle, Norm\ s) \mapsto 1\ (\langle va'\rangle, s')]\!]$
$\Longrightarrow$
$G \vdash (\langle va{:=}e\rangle, Norm\ s) \mapsto 1\ (\langle va'{:=}e\rangle, s')$
| $AssE$: $[\![groundVar\ va;\ G \vdash (\langle e\rangle, Norm\ s) \mapsto 1\ (\langle e'\rangle, s')]\!]$
$\Longrightarrow$
$G \vdash (\langle va{:=}e\rangle, Norm\ s) \mapsto 1\ (\langle va{:=}e'\rangle, s')$
| Ass: $[\![groundVar\ va;\ ((w, f), s') = the\text{-}var\ G\ (Norm\ s)\ va]\!]$
$\Longrightarrow$
$G \vdash (\langle va{:=}(Lit\ v)\rangle, Norm\ s) \mapsto 1\ (\langle Lit\ v\rangle, assign\ f\ v\ s')$

| $CondC$: $[\![G \vdash (\langle e0\rangle, Norm\ s) \mapsto 1\ (\langle e0'\rangle, s')]\!]$
$\Longrightarrow$
$G \vdash (\langle e0?\ e1{:}e2\rangle, Norm\ s) \mapsto 1\ (\langle e0'?\ e1{:}e2\rangle, s')$
| $Cond$: $G \vdash (\langle Lit\ b?\ e1{:}e2\rangle, Norm\ s) \mapsto 1\ (\langle if\ the\text{-}Bool\ b\ then\ e1\ else\ e2\rangle, Norm\ s)$

$$\begin{array}{l}
\mid \mathit{CallTarget}\colon [\![G\vdash(\langle e\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle e'\rangle,s')]\!]\\
\qquad\Longrightarrow\\
\qquad G\vdash(\langle\{\mathit{accC},\mathit{statT},\mathit{mode}\}e{\cdot}\mathit{mn}(\{\mathit{pTs}\}\mathit{args})\rangle,\mathit{Norm}\ s)\\
\qquad\ \mapsto\mathit{1}\ (\langle\{\mathit{accC},\mathit{statT},\mathit{mode}\}e'{\cdot}\mathit{mn}(\{\mathit{pTs}\}\mathit{args})\rangle,s')\\
\mid \mathit{CallArgs}\colon\ [\![G\vdash(\langle \mathit{args}\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle \mathit{args}'\rangle,s')]\!]\\
\qquad\Longrightarrow\\
\qquad G\vdash(\langle\{\mathit{accC},\mathit{statT},\mathit{mode}\}\mathit{Lit}\ a{\cdot}\mathit{mn}(\{\mathit{pTs}\}\mathit{args})\rangle,\mathit{Norm}\ s)\\
\qquad\ \mapsto\mathit{1}\ (\langle\{\mathit{accC},\mathit{statT},\mathit{mode}\}\mathit{Lit}\ a{\cdot}\mathit{mn}(\{\mathit{pTs}\}\mathit{args}')\rangle,s')\\
\mid \mathit{Call}\colon\quad [\![\mathit{groundExprs}\ \mathit{args};\ \mathit{vs} = \mathit{map}\ \mathit{the\text{-}val}\ \mathit{args};\\
\qquad D = \mathit{invocation\text{-}declclass}\ G\ \mathit{mode}\ s\ a\ \mathit{statT}\ (\!|\mathit{name}{=}\mathit{mn},\mathit{parTs}{=}\mathit{pTs}|\!);\\
\qquad s'{=}\mathit{init\text{-}lvars}\ G\ D\ (\!|\mathit{name}{=}\mathit{mn},\mathit{parTs}{=}\mathit{pTs}|\!)\ \mathit{mode}\ a'\ \mathit{vs}\ (\mathit{Norm}\ s)]\!]\\
\qquad\Longrightarrow\\
\qquad G\vdash(\langle\{\mathit{accC},\mathit{statT},\mathit{mode}\}\mathit{Lit}\ a{\cdot}\mathit{mn}(\{\mathit{pTs}\}\mathit{args})\rangle,\mathit{Norm}\ s)\\
\qquad\ \mapsto\mathit{1}\ (\langle\mathit{Callee}\ (\mathit{locals}\ s)\ (\mathit{Methd}\ D\ (\!|\mathit{name}{=}\mathit{mn},\mathit{parTs}{=}\mathit{pTs}|\!))\rangle,s')
\end{array}$$

$$\begin{array}{l}
\mid \mathit{Callee}\colon\quad [\![G\vdash(\langle e\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle e'{::}\mathit{expr}\rangle,s')]\!]\\
\qquad\Longrightarrow\\
\qquad G\vdash(\langle\mathit{Callee}\ \mathit{lcls\text{-}caller}\ e\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle e'\rangle,s')
\end{array}$$

$$\begin{array}{l}
\mid \mathit{CalleeRet}\colon\quad G\vdash(\langle\mathit{Callee}\ \mathit{lcls\text{-}caller}\ (\mathit{Lit}\ v)\rangle,\mathit{Norm}\ s)\\
\qquad\ \mapsto\mathit{1}\ (\langle\mathit{Lit}\ v\rangle,(\mathit{set\text{-}lvars}\ \mathit{lcls\text{-}caller}\ (\mathit{Norm}\ s)))
\end{array}$$

$$\mid \mathit{Methd}\colon G\vdash(\langle\mathit{Methd}\ D\ \mathit{sig}\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle\mathit{body}\ G\ D\ \mathit{sig}\rangle,\mathit{Norm}\ s)$$

$$\mid \mathit{Body}\colon G\vdash(\langle\mathit{Body}\ D\ c\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle\mathit{InsInitE}\ (\mathit{Init}\ D)\ (\mathit{Body}\ D\ c)\rangle,\mathit{Norm}\ s)$$

$$\begin{array}{l}
\mid \mathit{InsInitBody}\colon\\
\quad [\![G\vdash(\langle c\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle c'\rangle,s')]\!]\\
\quad\Longrightarrow\\
\quad G\vdash(\langle\mathit{InsInitE}\ \mathit{Skip}\ (\mathit{Body}\ D\ c)\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}(\langle\mathit{InsInitE}\ \mathit{Skip}\ (\mathit{Body}\ D\ c')\rangle,s')\\
\mid \mathit{InsInitBodyRet}\colon\\
\quad G\vdash(\langle\mathit{InsInitE}\ \mathit{Skip}\ (\mathit{Body}\ D\ \mathit{Skip})\rangle,\mathit{Norm}\ s)\\
\quad\ \mapsto\mathit{1}\ (\langle\mathit{Lit}\ (\mathit{the}\ ((\mathit{locals}\ s)\ \mathit{Result}))\rangle,\mathit{abupd}\ (\mathit{absorb}\ \mathit{Ret})\ (\mathit{Norm}\ s))
\end{array}$$

$$\begin{array}{l}
\mid \mathit{FVar}\colon [\![\neg\ \mathit{inited}\ \mathit{statDeclC}\ (\mathit{globs}\ s)]\!]\\
\qquad\Longrightarrow\\
\qquad G\vdash(\langle\{\mathit{accC},\mathit{statDeclC},\mathit{stat}\}e..\mathit{fn}\rangle,\mathit{Norm}\ s)\\
\qquad\ \mapsto\mathit{1}\ (\langle\mathit{InsInitV}\ (\mathit{Init}\ \mathit{statDeclC})\ (\{\mathit{accC},\mathit{statDeclC},\mathit{stat}\}e..\mathit{fn})\rangle,\mathit{Norm}\ s)\\
\mid \mathit{InsInitFVarE}\colon\\
\quad [\![G\vdash(\langle e\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle e'\rangle,s')]\!]\\
\quad\Longrightarrow\\
\quad G\vdash(\langle\mathit{InsInitV}\ \mathit{Skip}\ (\{\mathit{accC},\mathit{statDeclC},\mathit{stat}\}e..\mathit{fn})\rangle,\mathit{Norm}\ s)\\
\quad\ \mapsto\mathit{1}\ (\langle\mathit{InsInitV}\ \mathit{Skip}\ (\{\mathit{accC},\mathit{statDeclC},\mathit{stat}\}e'..\mathit{fn})\rangle,s')\\
\mid \mathit{InsInitFVar}\colon\\
\quad G\vdash(\langle\mathit{InsInitV}\ \mathit{Skip}\ (\{\mathit{accC},\mathit{statDeclC},\mathit{stat}\}\mathit{Lit}\ a..\mathit{fn})\rangle,\mathit{Norm}\ s)\\
\quad\ \mapsto\mathit{1}\ (\langle\{\mathit{accC},\mathit{statDeclC},\mathit{stat}\}\mathit{Lit}\ a..\mathit{fn}\rangle,\mathit{Norm}\ s)
\end{array}$$

— Notice, that we do not have literal values for *vars*. The rules for accessing variables (*Acc*) and assigning to variables (*Ass*), test this with the predicate *groundVar*. After initialisation is done and the *FVar* is evaluated, we can't just throw away the *InsInitFVar* term and return a literal value, as in the cases of *New* or *NewC*. Instead we just return the evaluated *FVar* and test for initialisation in the rule *FVar*.

$$\begin{array}{l}
\mid \mathit{AVarE1}\colon [\![G\vdash(\langle e1\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle e1'\rangle,s')]\!]\\
\qquad\Longrightarrow\\
\qquad G\vdash(\langle e1.[e2]\rangle,\mathit{Norm}\ s) \mapsto\mathit{1}\ (\langle e1'.[e2]\rangle,s')
\end{array}$$

| *AVarE2*: $G\vdash(\langle e2\rangle,Norm\ s) \mapsto 1\ (\langle e2'\rangle,s')$
$\Longrightarrow$
$G\vdash(\langle Lit\ a.[e2]\rangle,Norm\ s) \mapsto 1\ (\langle Lit\ a.[e2']\rangle,s')$

— *Nil* is fully evaluated

| *ConsHd*: $[\![G\vdash(\langle e{::}expr\rangle,Norm\ s) \mapsto 1\ (\langle e'{::}expr\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle e\#es\rangle,Norm\ s) \mapsto 1\ (\langle e'\#es\rangle,s')$

| *ConsTl*: $[\![G\vdash(\langle es\rangle,Norm\ s) \mapsto 1\ (\langle es'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle (Lit\ v)\#es\rangle,Norm\ s) \mapsto 1\ (\langle (Lit\ v)\#es'\rangle,s')$

| *Skip*: $G\vdash(\langle Skip\rangle,Norm\ s) \mapsto 1\ (\langle SKIP\rangle,Norm\ s)$

| *ExprE*: $[\![G\vdash(\langle e\rangle,Norm\ s) \mapsto 1\ (\langle e'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle Expr\ e\rangle,Norm\ s) \mapsto 1\ (\langle Expr\ e'\rangle,s')$
| *Expr*: $G\vdash(\langle Expr\ (Lit\ v)\rangle,Norm\ s) \mapsto 1\ (\langle Skip\rangle,Norm\ s)$

| *LabC*: $[\![G\vdash(\langle c\rangle,Norm\ s) \mapsto 1\ (\langle c'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle l\bullet\ c\rangle,Norm\ s) \mapsto 1\ (\langle l\bullet\ c'\rangle,s')$
| *Lab*: $G\vdash(\langle l\bullet\ Skip\rangle,s) \mapsto 1\ (\langle Skip\rangle,\ abupd\ (absorb\ l)\ s)$

| *CompC1*: $[\![G\vdash(\langle c1\rangle,Norm\ s) \mapsto 1\ (\langle c1'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle c1;;\ c2\rangle,Norm\ s) \mapsto 1\ (\langle c1';;\ c2\rangle,s')$

| *Comp*: $G\vdash(\langle Skip;;\ c2\rangle,Norm\ s) \mapsto 1\ (\langle c2\rangle,Norm\ s)$

| *IfE*: $[\![G\vdash(\langle e\rangle\ ,Norm\ s) \mapsto 1\ (\langle e'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle If(e)\ s1\ Else\ s2\rangle,Norm\ s) \mapsto 1\ (\langle If(e')\ s1\ Else\ s2\rangle,s')$
| *If*: $G\vdash(\langle If(Lit\ v)\ s1\ Else\ s2\rangle,Norm\ s)$
$\mapsto 1\ (\langle if\ the\text{-}Bool\ v\ then\ s1\ else\ s2\rangle,Norm\ s)$

| *Loop*: $G\vdash(\langle l\bullet\ While(e)\ c\rangle,Norm\ s)$
$\mapsto 1\ (\langle If(e)\ (Cont\ l\bullet c;;\ l\bullet\ While(e)\ c)\ Else\ Skip\rangle,Norm\ s)$

| *Jmp*: $G\vdash(\langle Jmp\ j\rangle,Norm\ s) \mapsto 1\ (\langle Skip\rangle,(Some\ (Jump\ j),\ s))$

| *ThrowE*: $[\![G\vdash(\langle e\rangle,Norm\ s) \mapsto 1\ (\langle e'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle Throw\ e\rangle,Norm\ s) \mapsto 1\ (\langle Throw\ e'\rangle,s')$
| *Throw*: $G\vdash(\langle Throw\ (Lit\ a)\rangle,Norm\ s) \mapsto 1\ (\langle Skip\rangle,abupd\ (throw\ a)\ (Norm\ s))$

$|$ *TryC1*: $[\![G\vdash(\langle c1\rangle,Norm\ s) \mapsto 1\ (\langle c1'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle Try\ c1\ Catch(C\ vn)\ c2\rangle,\ Norm\ s) \mapsto 1\ (\langle Try\ c1'\ Catch(C\ vn)\ c2\rangle,s')$
$|$ *Try*: $[\![G\vdash s\ -sxalloc\rightarrow\ s']\!]$
$\Longrightarrow$
$G\vdash(\langle Try\ Skip\ Catch(C\ vn)\ c2\rangle,\ s)$
$\mapsto 1\ (if\ G,s'\vdash catch\ C\ then\ (\langle c2\rangle,new\text{-}xcpt\text{-}var\ vn\ s')$
$else\ (\langle Skip\rangle,s'))$

$|$ *FinC1*: $[\![G\vdash(\langle c1\rangle,Norm\ s) \mapsto 1\ (\langle c1'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle c1\ Finally\ c2\rangle,Norm\ s) \mapsto 1\ (\langle c1'\ Finally\ c2\rangle,s')$

$|$ *Fin*: $G\vdash(\langle Skip\ Finally\ c2\rangle,(a,s)) \mapsto 1\ (\langle FinA\ a\ c2\rangle,Norm\ s)$

$|$ *FinAC*: $[\![G\vdash(\langle c\rangle,s) \mapsto 1\ (\langle c'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle FinA\ a\ c\rangle,s) \mapsto 1\ (\langle FinA\ a\ c'\rangle,s')$
$|$ *FinA*: $G\vdash(\langle FinA\ a\ Skip\rangle,s) \mapsto 1\ (\langle Skip\rangle,abupd\ (abrupt\text{-}if\ (a\neq None)\ a)\ s)$

$|$ *Init1*: $[\![inited\ C\ (globs\ s)]\!]$
$\Longrightarrow$
$G\vdash(\langle Init\ C\rangle,Norm\ s) \mapsto 1\ (\langle Skip\rangle,Norm\ s)$
$|$ *Init*: $[\![the\ (class\ G\ C)=c;\ \neg\ inited\ C\ (globs\ s)]\!]$
$\Longrightarrow$
$G\vdash(\langle Init\ C\rangle,Norm\ s)$
$\mapsto 1\ (\langle(if\ C = Object\ then\ Skip\ else\ (Init\ (super\ c)));;$
$Expr\ (Callee\ (locals\ s)\ (InsInitE\ (init\ c)\ SKIP))\rangle$
$,Norm\ (init\text{-}class\text{-}obj\ G\ C\ s))$
— *InsInitE* is just used as trick to embed the statement *init c* into an expression
$|$ *InsInitESKIP*:
$G\vdash(\langle InsInitE\ Skip\ SKIP\rangle,Norm\ s) \mapsto 1\ (\langle SKIP\rangle,Norm\ s)$

abbreviation
stepn:: $[prog,\ term \times state,nat,term \times state] \Rightarrow bool$ (-⊢- ↦- -[*61*,*82*,*82*] *81*)
where $G\vdash p \mapsto n\ p' \equiv (p,p') \in \{(x,\ y).\ step\ G\ x\ y\}\hat{}\hat{}n$

abbreviation
steptr:: $[prog,term \times state,term \times state] \Rightarrow bool$ (-⊢- ↦∗ -[*61*,*82*,*82*] *81*)
where $G\vdash p \mapsto * \ p' \equiv (p,p') \in \{(x,\ y).\ step\ G\ x\ y\}^*$

end

Chapter 22

AxSem

1 Axiomatic semantics of Java expressions and statements (see also Eval.thy)

theory *AxSem* **imports** *Evaln TypeSafe* **begin**

design issues:

- a strong version of validity for triples with premises, namely one that takes the recursive depth needed to complete execution, enables correctness proof
- auxiliary variables are handled first-class (-> Thomas Kleymann)
- expressions not flattened to elementary assignments (as usual for axiomatic semantics) but treated first-class => explicit result value handling
- intermediate values not on triple, but on assertion level (with result entry)
- multiple results with semantical substitution mechnism not requiring a stack
- because of dynamic method binding, terms need to be dependent on state. this is also useful for conditional expressions and statements
- result values in triples exactly as in eval relation (also for xcpt states)
- validity: additional assumption of state conformance and well-typedness, which is required for soundness and thus rule hazard required of completeness

restrictions:

- all triples in a derivation are of the same type (due to weak polymorphism)

type-synonym *res* = *vals* — result entry

abbreviation (*input*)
  Val **where** *Val x* == *In1 x*

abbreviation (*input*)
  Var **where** *Var x* == *In2 x*

abbreviation (*input*)
  Vals **where** *Vals x* == *In3 x*

syntax
  -Val :: [*pttrn*] => *pttrn* (*Val*:- [*951*] *950*)
  -Var :: [*pttrn*] => *pttrn* (*Var*:- [*951*] *950*)

  -Vals :: [*pttrn*] => *pttrn* (*Vals*:- [*951*] *950*)

translations
  λ*Val*:*v* . *b* == ($\lambda v.\ b$) $\circ$ *CONST the-In1*
  λ*Var*:*v* . *b* == ($\lambda v.\ b$) $\circ$ *CONST the-In2*
  λ*Vals*:*v*. *b* == ($\lambda v.\ b$) $\circ$ *CONST the-In3*

— relation on result values, state and auxiliary variables
type-synonym *'a assn* = *res* $\Rightarrow$ *state* $\Rightarrow$ *'a* $\Rightarrow$ *bool*
translations
  (*type*) *'a assn* <= (*type*) *vals* $\Rightarrow$ *state* $\Rightarrow$ *'a* $\Rightarrow$ *bool*

definition
  assn-imp :: *'a assn* $\Rightarrow$ *'a assn* $\Rightarrow$ *bool* (**infixr** $\Rightarrow$ *25*)
  where $(P \Rightarrow Q) = (\forall\, Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow Q\ Y\ s\ Z)$

lemma *assn-imp-def2* [*iff*]: $(P \Rightarrow Q) = (\forall\, Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow Q\ Y\ s\ Z)$
apply (*unfold assn-imp-def*)
apply (*rule HOL.refl*)
done

assertion transformers

2 peek-and

definition
  peek-and :: *'a assn* $\Rightarrow$ (*state* $\Rightarrow$ *bool*) $\Rightarrow$ *'a assn* (**infixl** $\wedge$. *13*)
  where $(P \wedge .\ p) = (\lambda Y\ s\ Z.\ P\ Y\ s\ Z \wedge p\ s)$

lemma *peek-and-def2* [*simp*]: *peek-and P p Y s* = $(\lambda Z.\ (P\ Y\ s\ Z \wedge p\ s))$
apply (*unfold peek-and-def*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-Not* [*simp*]: $(P \wedge .\ (\lambda s.\ \neg\ f\ s)) = (P \wedge .\ Not \circ f)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-and* [*simp*]: *peek-and* (*peek-and P p*) *p* = *peek-and P p*
apply (*unfold peek-and-def*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-commut*: $(P \wedge .\ p \wedge .\ q) = (P \wedge .\ q \wedge .\ p)$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply *auto*
done

abbreviation
  Normal :: *'a assn* $\Rightarrow$ *'a assn*

where *Normal* $P == P \wedge.\ normal$

lemma *peek-and-Normal* [*simp*]: *peek-and* (*Normal* P) p = *Normal* (*peek-and* P p)
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply *auto*
done

3 assn-supd

definition
assn-supd :: $'a$ *assn* $\Rightarrow$ (*state* $\Rightarrow$ *state*) $\Rightarrow$ $'a$ *assn* (**infixl** ;. *13*)
where $(P\ ;.\ f) = (\lambda Y\ s'\ Z.\ \exists s.\ P\ Y\ s\ Z \wedge s' = f\ s)$

lemma *assn-supd-def2* [*simp*]: *assn-supd* $P\ f\ Y\ s'\ Z = (\exists s.\ P\ Y\ s\ Z \wedge s' = f\ s)$
apply (*unfold assn-supd-def*)
apply (*simp* (*no-asm*))
done

4 supd-assn

definition
supd-assn :: (*state* $\Rightarrow$ *state*) $\Rightarrow$ $'a$ *assn* $\Rightarrow$ $'a$ *assn* (**infixr** .; *13*)
where $(f\ .;\ P) = (\lambda Y\ s.\ P\ Y\ (f\ s))$

lemma *supd-assn-def2* [*simp*]: $(f\ .;\ P)\ Y\ s = P\ Y\ (f\ s)$
apply (*unfold supd-assn-def*)
apply (*simp* (*no-asm*))
done

lemma *supd-assn-supdD* [*elim*]: $((f\ .;\ Q)\ ;.\ f)\ Y\ s\ Z \Longrightarrow Q\ Y\ s\ Z$
apply *auto*
done

lemma *supd-assn-supdI* [*elim*]: $Q\ Y\ s\ Z \Longrightarrow (f\ .;\ (Q\ ;.\ f))\ Y\ s\ Z$
apply (*auto simp del*: *split-paired-Ex*)
done

5 subst-res

definition
subst-res :: $'a$ *assn* $\Rightarrow$ *res* $\Rightarrow$ $'a$ *assn* (-←- [*60*,*61*] *60*)
where $P{\leftarrow}w = (\lambda Y.\ P\ w)$

lemma *subst-res-def2* [*simp*]: $(P{\leftarrow}w)\ Y = P\ w$
apply (*unfold subst-res-def*)
apply (*simp* (*no-asm*))
done

lemma *subst-subst-res* [*simp*]: $P{\leftarrow}w{\leftarrow}v = P{\leftarrow}w$

apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-subst-res* [*simp*]: $(P \wedge. \ p) \leftarrow w = (P \leftarrow w \wedge. \ p)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

6 subst-Bool

definition
subst-Bool :: $'a$ *assn* $\Rightarrow$ *bool* $\Rightarrow$ $'a$ *assn* (-$\leftarrow$=- [*60*,*61*] *60*)
where $P \leftarrow = b = (\lambda Y\ s\ Z.\ \exists\, v.\ P\ (Val\ v)\ s\ Z \wedge (normal\ s \longrightarrow the\text{-}Bool\ v{=}b))$

lemma *subst-Bool-def2* [*simp*]:
$(P \leftarrow = b)\ Y\ s\ Z = (\exists\, v.\ P\ (Val\ v)\ s\ Z \wedge (normal\ s \longrightarrow the\text{-}Bool\ v{=}b))$
apply (*unfold subst-Bool-def*)
apply (*simp* (*no-asm*))
done

lemma *subst-Bool-the-BoolI*: $P\ (Val\ b)\ s\ Z \Longrightarrow (P \leftarrow = the\text{-}Bool\ b)\ Y\ s\ Z$
apply *auto*
done

7 peek-res

definition
peek-res :: (*res* $\Rightarrow$ $'a$ *assn*) $\Rightarrow$ $'a$ *assn*
where *peek-res Pf* = $(\lambda Y.\ Pf\ Y\ Y)$

syntax
-peek-res :: *pttrn* $\Rightarrow$ $'a$ *assn* $\Rightarrow$ $'a$ *assn* (λ-:. - [*0*,*3*] *3*)
translations
$\lambda w{:}.\ P$ == *CONST peek-res* $(\lambda w.\ P)$

lemma *peek-res-def2* [*simp*]: *peek-res* $P\ Y = P\ Y\ Y$
apply (*unfold peek-res-def*)
apply (*simp* (*no-asm*))
done

lemma *peek-res-subst-res* [*simp*]: *peek-res* $P \leftarrow w = P\ w \leftarrow w$
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-subst-res-allI*:
$(\bigwedge a.\ T\ a\ (P\ (f\ a) \leftarrow f\ a)) \Longrightarrow \forall\, a.\ T\ a\ (peek\text{-}res\ P \leftarrow f\ a)$
apply (*rule allI*)
apply (*simp* (*no-asm*))
apply *fast*

done

8 ign-res

definition
ign-res :: *′a assn* ⇒ *′a assn* (-↓ [*1000*] *1000*)
where $P{\downarrow} = (\lambda Y\ s\ Z.\ \exists\ Y.\ P\ Y\ s\ Z)$

lemma *ign-res-def2* [*simp*]: $P{\downarrow}\ Y\ s\ Z = (\exists\ Y.\ P\ Y\ s\ Z)$
apply (*unfold ign-res-def*)
apply (*simp* (*no-asm*))
done

lemma *ign-ign-res* [*simp*]: $P{\downarrow}{\downarrow} = P{\downarrow}$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *ign-subst-res* [*simp*]: $P{\downarrow}{\leftarrow}w = P{\downarrow}$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-ign-res* [*simp*]: $(P \wedge.\ p){\downarrow} = (P{\downarrow} \wedge.\ p)$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

9 peek-st

definition
peek-st :: (*st* ⇒ *′a assn*) ⇒ *′a assn*
where *peek-st* $P = (\lambda Y\ s.\ P\ (store\ s)\ Y\ s)$

syntax
-peek-st :: *pttrn* ⇒ *′a assn* ⇒ *′a assn* (λ-.. - [*0*,*3*] *3*)
translations
λ*s*.. *P* == *CONST peek-st* (λ*s*. *P*)

lemma *peek-st-def2* [*simp*]: (λ*s*.. *Pf s*) *Y s* = *Pf* (*store s*) *Y s*
apply (*unfold peek-st-def*)
apply (*simp* (*no-asm*))
done

lemma *peek-st-triv* [*simp*]: (λ*s*.. *P*) = *P*
apply (*rule ext*)
apply (*rule ext*)

apply (*simp* (*no-asm*))
done

lemma *peek-st-st* [*simp*]: $(\lambda s.. \lambda s'.. P\ s\ s') = (\lambda s.. P\ s\ s)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-st-split* [*simp*]: $(\lambda s.. \lambda Y\ s'.\ P\ s\ Y\ s') = (\lambda Y\ s.\ P\ (store\ s)\ Y\ s)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-st-subst-res* [*simp*]: $(\lambda s.. P\ s){\leftarrow}w = (\lambda s.. P\ s{\leftarrow}w)$
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-st-Normal* [*simp*]: $(\lambda s..(Normal\ (P\ s))) = Normal\ (\lambda s.. P\ s)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

10 ign-res-eq

definition
  $ign\text{-}res\text{-}eq :: 'a\ assn \Rightarrow res \Rightarrow 'a\ assn$ (-↓=- [*60*,*61*] *60*)
  where $P{\downarrow}{=}w \equiv (\lambda Y{:}.\ P{\downarrow} \wedge.\ (\lambda s.\ Y{=}w))$

lemma *ign-res-eq-def2* [*simp*]: $(P{\downarrow}{=}w)\ Y\ s\ Z = ((\exists\ Y.\ P\ Y\ s\ Z) \wedge Y{=}w)$
apply (*unfold ign-res-eq-def*)
apply *auto*
done

lemma *ign-ign-res-eq* [*simp*]: $(P{\downarrow}{=}w){\downarrow} = P{\downarrow}$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *ign-res-eq-subst-res*: $P{\downarrow}{=}w{\leftarrow}w = P{\downarrow}$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *subst-Bool-ign-res-eq*: $((P\leftarrow=b)\downarrow=x)\ Y\ s\ Z = ((P\leftarrow=b)\ Y\ s\ Z\ \wedge\ Y=x)$
apply (*simp* (*no-asm*))
done

11 RefVar

definition
RefVar :: $(state \Rightarrow vvar \times state) \Rightarrow {}'a\ assn \Rightarrow {}'a\ assn$ (**infixr** ..; *13*)
where $(vf\ ..;\ P) = (\lambda Y\ s.\ let\ (v,s') = vf\ s\ in\ P\ (Var\ v)\ s')$

lemma *RefVar-def2* [*simp*]: $(vf\ ..;\ P)\ Y\ s =$
$P\ (Var\ (fst\ (vf\ s)))\ (snd\ (vf\ s))$
apply (*unfold RefVar-def Let-def*)
apply (*simp* (*no-asm*) *add*: *split-beta*)
done

12 allocation

definition
Alloc :: $prog \Rightarrow obj\text{-}tag \Rightarrow {}'a\ assn \Rightarrow {}'a\ assn$
where $Alloc\ G\ otag\ P = (\lambda Y\ s\ Z.\ \forall s'\ a.\ G\vdash s\ -halloc\ otag\succ a\rightarrow s' \longrightarrow P\ (Val\ (Addr\ a))\ s'\ Z)$

definition
SXAlloc :: $prog \Rightarrow {}'a\ assn \Rightarrow {}'a\ assn$
where $SXAlloc\ G\ P = (\lambda Y\ s\ Z.\ \forall s'.\ G\vdash s\ -sxalloc\rightarrow s' \longrightarrow P\ Y\ s'\ Z)$

lemma *Alloc-def2* [*simp*]: $Alloc\ G\ otag\ P\ Y\ s\ Z =$
$(\forall s'\ a.\ G\vdash s\ -halloc\ otag\succ a\rightarrow s' \longrightarrow P\ (Val\ (Addr\ a))\ s'\ Z)$
apply (*unfold Alloc-def*)
apply (*simp* (*no-asm*))
done

lemma *SXAlloc-def2* [*simp*]:
$SXAlloc\ G\ P\ Y\ s\ Z = (\forall s'.\ G\vdash s\ -sxalloc\rightarrow s' \longrightarrow P\ Y\ s'\ Z)$
apply (*unfold SXAlloc-def*)
apply (*simp* (*no-asm*))
done

validity

definition
type-ok :: $prog \Rightarrow term \Rightarrow state \Rightarrow bool$ **where**
$type\text{-}ok\ G\ t\ s =$
$(\exists L\ T\ C\ A.\ (normal\ s \longrightarrow (\!|prg=G,cls=C,lcl=L|\!)\vdash t::T\ \wedge$
$(\!|prg=G,cls=C,lcl=L|\!)\vdash dom\ (locals\ (store\ s))\gg t\gg A\ )$
$\wedge\ s::\preceq(G,L))$

datatype $'a\ triple = triple\ ('a\ assn)\ term\ ('a\ assn)$
$(\{(1\text{-})\}/\ \text{-}\succ/\ \{(1\text{-})\}\quad [3,65,3]\ 75)$
type-synonym $'a\ triples = {}'a\ triple\ set$

abbreviation
var-triple :: $['a\ assn,\ var\qquad ,'a\ assn] \Rightarrow {}'a\ triple$

({(1-)}/ -=≻/ {(1-)} [3,80,3] 75)
where {P} e=≻ {Q} == {P} In2 e≻ {Q}

abbreviation
expr-triple :: ['a assn, expr ,'a assn] ⇒ 'a triple
({(1-)}/ -−≻/ {(1-)} [3,80,3] 75)
where {P} e−≻ {Q} == {P} In1l e≻ {Q}

abbreviation
exprs-triple :: ['a assn, expr list ,'a assn] ⇒ 'a triple
({(1-)}/ -≐≻/ {(1-)} [3,65,3] 75)
where {P} e≐≻ {Q} == {P} In3 e≻ {Q}

abbreviation
stmt-triple :: ['a assn, stmt, 'a assn] ⇒ 'a triple
({(1-)}/ .-./ {(1-)} [3,65,3] 75)
where {P} .c. {Q} == {P} In1r c≻ {Q}

notation (*ASCII*)
triple ({(1-)}/ ->/ {(1-)} [3,65,3]75) **and**
var-triple ({(1-)}/ -=>/ {(1-)} [3,80,3] 75) **and**
expr-triple ({(1-)}/ -->/ {(1-)} [3,80,3] 75) **and**
exprs-triple ({(1-)}/ -#>/ {(1-)} [3,65,3] 75)

lemma *inj-triple*: *inj* (λ(P,t,Q). {P} t≻ {Q})
apply (*rule inj-onI*)
apply *auto*
done

lemma *triple-inj-eq*: ({P} t≻ {Q} = {P'} t'≻ {Q'}) = (P=P' ∧ t=t' ∧ Q=Q')
apply *auto*
done

definition *mtriples* :: ('c ⇒ 'sig ⇒ 'a assn) ⇒ ('c ⇒ 'sig ⇒ expr) ⇒
('c ⇒ 'sig ⇒ 'a assn) ⇒ ('c × 'sig) set ⇒ 'a triples ({{(1-)}/ -−≻/ {(1-)} | -}[3,65,3,65]75)
where
{{P} tf−≻ {Q} | ms} = (λ(C,sig). {Normal(P C sig)} tf C sig−≻ {Q C sig})‘ms

definition
triple-valid :: *prog* ⇒ *nat* ⇒ 'a triple ⇒ *bool* (-|=-:- [61,0, 58] 57)
where
G|=n:t =
(*case* t *of* {P} t≻ {Q} ⇒
∀ Y s Z. P Y s Z ⟶ *type-ok* G t s ⟶
(∀ Y' s'. G⊢s −t≻−n→ (Y',s') ⟶ Q Y' s' Z))

abbreviation
triples-valid:: *prog* ⇒ *nat* ⇒ 'a triples ⇒ *bool* (-||=-:- [61,0, 58] 57)
where G||=n:ts == *Ball ts* (*triple-valid* G n)

notation (*ASCII*)
triples-valid (-||=-:- [61,0, 58] 57)

definition
ax-valids :: *prog* ⇒ 'b triples ⇒ 'a triples ⇒ *bool* (-,-||=- [61,58,58] 57)
where (G,A||=ts) = (∀ n. G||=n:A ⟶ G||=n:ts)

abbreviation
ax-valid :: *prog* ⇒ *'b triples* ⇒ *'a triple* ⇒ *bool* (-,-⊨- [*61*,*58*,*58*] *57*)
where *G*,*A* ⊨*t* == *G*,*A*|⊨{*t*}

notation (*ASCII*)
ax-valid (-,-|=- [*61*,*58*,*58*] *57*)

lemma *triple-valid-def2*: *G*⊨*n*:{*P*} *t*≻ {*Q*} =
(∀ *Y s Z*. *P Y s Z*
⟶ (∃ *L*. (*normal s* ⟶ (∃ *C T A*. ⦇*prg*=*G*,*cls*=*C*,*lcl*=*L*⦈⊢*t*::*T* ∧
⦇*prg*=*G*,*cls*=*C*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s*))»*t*»*A*)) ∧
s::≼(*G*,*L*))
⟶ (∀ *Y′ s′*. *G*⊢*s* −*t*≻−*n*→ (*Y′*,*s′*)⟶ *Q Y′ s′ Z*))
apply (*unfold triple-valid-def type-ok-def*)
apply (*simp* (*no-asm*))
done

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declare *if-split* [*split del*] *if-split-asm* [*split del*]
option.split [*split del*] *option.split-asm* [*split del*]
setup ‹*map-theory-simpset* (*fn ctxt* => *ctxt delloop split-all-tac*)›
setup ‹*map-theory-claset* (*fn ctxt* => *ctxt delSWrapper split-all-tac*)›

inductive
ax-derivs :: *prog* ⇒ *'a triples* ⇒ *'a triples* ⇒ *bool* (-,-|⊢- [*61*,*58*,*58*] *57*)
and *ax-deriv* :: *prog* ⇒ *'a triples* ⇒ *'a triple* ⇒ *bool* (-,-⊢- [*61*,*58*,*58*] *57*)
for *G* :: *prog*
where

G,*A* ⊢*t* ≡ *G*,*A*|⊢{*t*}

| *empty*: *G*,*A*|⊢{}
| *insert*:⟦*G*,*A*⊢*t*; *G*,*A*|⊢*ts*⟧ ⟹
G,*A*|⊢*insert t ts*

| *asm*: *ts*⊆*A* ⟹ *G*,*A*|⊢*ts*

| *weaken*:⟦*G*,*A*|⊢*ts′*; *ts* ⊆ *ts′*⟧ ⟹ *G*,*A*|⊢*ts*

| *conseq*:∀ *Y s Z* . *P Y s Z* ⟶ (∃ *P′ Q′*. *G*,*A*⊢{*P′*} *t*≻ {*Q′*} ∧ (∀ *Y′ s′*.
(∀ *Y Z′*. *P′ Y s Z′* ⟶ *Q′ Y′ s′ Z′*) ⟶
Q Y′ s′ Z))
⟹ *G*,*A*⊢{*P* } *t*≻ {*Q* }

| *hazard*:*G*,*A*⊢{*P* ∧. *Not* ∘ *type-ok G t*} *t*≻ {*Q*}

| *Abrupt*: *G*,*A*⊢{*P*←(*undefined3 t*) ∧. *Not* ∘ *normal*} *t*≻ {*P*}

— variables
| *LVar*: *G*,*A*⊢{*Normal* (λ*s*.. *P*←*Var* (*lvar vn s*))} *LVar vn*=≻ {*P*}

| *FVar*: ⟦*G*,*A*⊢{*Normal P*} .*Init C*. {*Q*};
G,*A*⊢{*Q*} *e*−≻ {λ*Val*:*a*:. *fvar C stat fn a* ..; *R*}⟧ ⟹
G,*A*⊢{*Normal P*} {*accC*,*C*,*stat*}*e*..*fn*=≻ {*R*}

| $AVar$: $\llbracket G,A\vdash\{Normal\ P\}\ e1{-}\!\succ\ \{Q\};$
$\forall a.\ G,A\vdash\{Q\leftarrow Val\ a\}\ e2{-}\!\succ\ \{\lambda Val{:}i{:}.\ avar\ G\ i\ a\ ..;\ R\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ e1.[e2]{=}\!\succ\ \{R\}$

— expressions

| $NewC$: $\llbracket G,A\vdash\{Normal\ P\}\ .Init\ C.\ \{Alloc\ G\ (CInst\ C)\ Q\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ NewC\ C{-}\!\succ\ \{Q\}$

| $NewA$: $\llbracket G,A\vdash\{Normal\ P\}\ .init\text{-}comp\text{-}ty\ T.\ \{Q\};\ \ G,A\vdash\{Q\}\ e{-}\!\succ$
$\{\lambda Val{:}i{:}.\ abupd\ (check\text{-}neg\ i)\ .;\ Alloc\ G\ (Arr\ T\ (the\text{-}Intg\ i))\ R\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ New\ T[e]{-}\!\succ\ \{R\}$

| $Cast$: $\llbracket G,A\vdash\{Normal\ P\}\ e{-}\!\succ\ \{\lambda Val{:}v{:}.\ \lambda s..$
$abupd\ (raise\text{-}if\ (\neg G,s\vdash v\ fits\ T)\ ClassCast)\ .;\ Q\leftarrow Val\ v\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ Cast\ T\ e{-}\!\succ\ \{Q\}$

| $Inst$: $\llbracket G,A\vdash\{Normal\ P\}\ e{-}\!\succ\ \{\lambda Val{:}v{:}.\ \lambda s..$
$Q\leftarrow Val\ (Bool\ (v\neq Null\ \wedge\ G,s\vdash v\ fits\ RefT\ T))\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ e\ InstOf\ T{-}\!\succ\ \{Q\}$

| Lit: $G,A\vdash\{Normal\ (P\leftarrow Val\ v)\}\ Lit\ v{-}\!\succ\ \{P\}$

| $UnOp$: $\llbracket G,A\vdash\{Normal\ P\}\ e{-}\!\succ\ \{\lambda Val{:}v{:}.\ Q\leftarrow Val\ (eval\text{-}unop\ unop\ v)\}\rrbracket$
$\Longrightarrow$
$G,A\vdash\{Normal\ P\}\ UnOp\ unop\ e{-}\!\succ\ \{Q\}$

| $BinOp$:
$\llbracket G,A\vdash\{Normal\ P\}\ e1{-}\!\succ\ \{Q\};$
$\forall v1.\ G,A\vdash\{Q\leftarrow Val\ v1\}$
$(if\ need\text{-}second\text{-}arg\ binop\ v1\ then\ (In1l\ e2)\ else\ (In1r\ Skip)){\succ}$
$\{\lambda Val{:}v2{:}.\ R\leftarrow Val\ (eval\text{-}binop\ binop\ v1\ v2)\}\rrbracket$
$\Longrightarrow$
$G,A\vdash\{Normal\ P\}\ BinOp\ binop\ e1\ e2{-}\!\succ\ \{R\}$

| $Super$: $G,A\vdash\{Normal\ (\lambda s..\ P\leftarrow Val\ (val\text{-}this\ s))\}\ Super{-}\!\succ\ \{P\}$

| Acc: $\llbracket G,A\vdash\{Normal\ P\}\ va{=}\!\succ\ \{\lambda Var{:}(v,f){:}.\ Q\leftarrow Val\ v\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ Acc\ va{-}\!\succ\ \{Q\}$

| Ass: $\llbracket G,A\vdash\{Normal\ P\}\ va{=}\!\succ\ \{Q\};$
$\forall vf.\ G,A\vdash\{Q\leftarrow Var\ vf\}\ e{-}\!\succ\ \{\lambda Val{:}v{:}.\ assign\ (snd\ vf)\ v\ .;\ R\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ va{:=}e{-}\!\succ\ \{R\}$

| $Cond$: $\llbracket G,A\ \vdash\{Normal\ P\}\ e0{-}\!\succ\ \{P'\};$
$\forall b.\ G,A\vdash\{P'\leftarrow{=}b\}\ (if\ b\ then\ e1\ else\ e2){-}\!\succ\ \{Q\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ e0\ ?\ e1\ :\ e2{-}\!\succ\ \{Q\}$

| $Call$:
$\llbracket G,A\vdash\{Normal\ P\}\ e{-}\!\succ\ \{Q\};\ \forall a.\ G,A\vdash\{Q\leftarrow Val\ a\}\ args{\doteq}\!\succ\ \{R\ a\};$
$\forall a\ vs\ invC\ declC\ l.\ G,A\vdash\{(R\ a\leftarrow Vals\ vs\ \wedge.$
$(\lambda s.\ declC{=}invocation\text{-}declclass\ G\ mode\ (store\ s)\ a\ statT\ (\!|name{=}mn,parTs{=}pTs|\!)\ \wedge$
$invC\ =\ invocation\text{-}class\ mode\ (store\ s)\ a\ statT\ \wedge$
$l\ =\ locals\ (store\ s))\ ;.$
$init\text{-}lvars\ G\ declC\ (\!|name{=}mn,parTs{=}pTs|\!)\ mode\ a\ vs)\ \wedge.$
$(\lambda s.\ normal\ s\ \longrightarrow\ G\vdash mode\rightarrow invC\preceq statT)\}$
$Methd\ declC\ (\!|name{=}mn,parTs{=}pTs|\!){-}\!\succ\ \{set\text{-}lvars\ l\ .;\ S\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ \{accC,statT,mode\}e{\cdot}mn(\{pTs\}args){-}\!\succ\ \{S\}$

| *Methd*:⟦*G*,*A*∪ {{*P*} *Methd*−≻ {*Q*} | *ms*} |⊢ {{*P*} *body G*−≻ {*Q*} | *ms*}⟧ ⟹
G,*A*|⊢{{*P*} *Methd*−≻ {*Q*} | *ms*}

| *Body*: ⟦*G*,*A*⊢{*Normal P*} *.Init D.* {*Q*};
G,*A*⊢{*Q*} *.c.* {λ*s*.. *abupd* (*absorb Ret*) .; *R*←(*In1* (*the* (*locals s Result*)))}⟧
⟹
G,*A*⊢{*Normal P*} *Body D c*−≻ {*R*}

— expression lists

| *Nil*: *G*,*A*⊢{*Normal* (*P*← *Vals* [])} []≐≻ {*P*}

| *Cons*: ⟦*G*,*A*⊢{*Normal P*} *e*−≻ {*Q*};
∀ *v*. *G*,*A*⊢{*Q*← *Val v*} *es*≐≻ {λ *Vals*:*vs*:. *R*← *Vals* (*v*#*vs*)}⟧ ⟹
G,*A*⊢{*Normal P*} *e*#*es*≐≻ {*R*}

— statements

| *Skip*: *G*,*A*⊢{*Normal* (*P*←◇)} *.Skip.* {*P*}

| *Expr*: ⟦*G*,*A*⊢{*Normal P*} *e*−≻ {*Q*←◇}⟧ ⟹
G,*A*⊢{*Normal P*} *.Expr e.* {*Q*}

| *Lab*: ⟦*G*,*A*⊢{*Normal P*} *.c.* {*abupd* (*absorb l*) .; *Q*}⟧ ⟹
G,*A*⊢{*Normal P*} *.l*• *c.* {*Q*}

| *Comp*: ⟦*G*,*A*⊢{*Normal P*} *.c1.* {*Q*};
G,*A*⊢{*Q*} *.c2.* {*R*}⟧ ⟹
G,*A*⊢{*Normal P*} *.c1*;;*c2.* {*R*}

| *If*: ⟦*G*,*A* ⊢{*Normal P*} *e*−≻ {*P′*};
∀ *b*. *G*,*A*⊢{*P′*←=*b*} .(*if b then c1 else c2*). {*Q*}⟧ ⟹
G,*A*⊢{*Normal P*} *.If*(*e*) *c1 Else c2.* {*Q*}

| *Loop*: ⟦*G*,*A*⊢{*P*} *e*−≻ {*P′*};
G,*A*⊢{*Normal* (*P′*←=*True*)} *.c.* {*abupd* (*absorb* (*Cont l*)) .; *P*}⟧ ⟹
G,*A*⊢{*P*} *.l*• *While*(*e*) *c.* {(*P′*←=*False*)↓=◇}

| *Jmp*: *G*,*A*⊢{*Normal* (*abupd* (λ*a*. (*Some* (*Jump j*))) .; *P*←◇)} *.Jmp j.* {*P*}

| *Throw*:⟦*G*,*A*⊢{*Normal P*} *e*−≻ {λ *Val*:*a*:. *abupd* (*throw a*) .; *Q*←◇}⟧ ⟹
G,*A*⊢{*Normal P*} *.Throw e.* {*Q*}

| *Try*: ⟦*G*,*A*⊢{*Normal P*} *.c1.* {*SXAlloc G Q*};
G,*A*⊢{*Q* ∧. (λ*s*. *G*,*s*⊢*catch C*) ;. *new-xcpt-var vn*} *.c2.* {*R*};
(*Q* ∧. (λ*s*. ¬*G*,*s*⊢*catch C*)) ⇒ *R*⟧ ⟹
G,*A*⊢{*Normal P*} *.Try c1 Catch*(*C vn*) *c2.* {*R*}

| *Fin*: ⟦*G*,*A*⊢{*Normal P*} *.c1.* {*Q*};
∀ *x*. *G*,*A*⊢{*Q* ∧. (λ*s*. *x* = *fst s*) ;. *abupd* (λ*x*. *None*)}
.c2. {*abupd* (*abrupt-if* (*x*≠*None*) *x*) .; *R*}⟧ ⟹
G,*A*⊢{*Normal P*} *.c1 Finally c2.* {*R*}

| *Done*: *G*,*A*⊢{*Normal* (*P*←◇ ∧. *initd C*)} *.Init C.* {*P*}

| *Init*: ⟦*the* (*class G C*) = *c*;
G,*A*⊢{*Normal* ((*P* ∧. *Not* ∘ *initd C*) ;. *supd* (*init-class-obj G C*))}
.(*if C* = *Object then Skip else Init* (*super c*)). {*Q*};
∀ *l*. *G*,*A*⊢{*Q* ∧. (λ*s*. *l* = *locals* (*store s*)) ;. *set-lvars Map.empty*}

$.init\ c.\ \{set\text{-}lvars\ l\ .;\ R\}\rrbracket \Longrightarrow$
$G,A\vdash\{Normal\ (P\ \wedge.\ Not\ \circ\ initd\ C)\}\ .Init\ C.\ \{R\}$

— Some dummy rules for the intermediate terms *Callee*, *InsInitE*, *InsInitV*, *FinA* only used by the smallstep semantics.
| *InsInitV*: $G,A\vdash\{Normal\ P\}\ InsInitV\ c\ v{=}\succ\ \{Q\}$
| *InsInitE*: $G,A\vdash\{Normal\ P\}\ InsInitE\ c\ e{-}\succ\ \{Q\}$
| *Callee*: $G,A\vdash\{Normal\ P\}\ Callee\ l\ e{-}\succ\ \{Q\}$
| *FinA*: $G,A\vdash\{Normal\ P\}\ .FinA\ a\ c.\ \{Q\}$

definition
adapt-pre :: $'a\ assn \Rightarrow\ 'a\ assn \Rightarrow\ 'a\ assn \Rightarrow\ 'a\ assn$
where $adapt\text{-}pre\ P\ Q\ Q' = (\lambda Y\ s\ Z.\ \forall\,Y'\ s'.\ \exists\,Z'.\ P\ Y\ s\ Z' \wedge (Q\ Y'\ s'\ Z' \longrightarrow Q'\ Y'\ s'\ Z))$

rules derived by induction

lemma *cut-valid*: $\llbracket G,A'|\models ts;\ G,A|\models A'\rrbracket \Longrightarrow G,A|\models ts$
apply (*unfold ax-valids-def*)
apply *fast*
done

lemma *ax-thin* [*rule-format* (*no-asm*)]:
$G,(A'::'a\ triple\ set)|\vdash(ts::'a\ triple\ set) \Longrightarrow \forall\,A.\ A' \subseteq A \longrightarrow G,A|\vdash ts$
apply (*erule ax-derivs.induct*)
apply (*tactic ALLGOALS* (*EVERY'*[*clarify-tac* ***context***, *REPEAT o smp-tac* ***context*** *1*]))
apply (*rule ax-derivs.empty*)
apply (*erule* (*1*) *ax-derivs.insert*)
apply (*fast intro*: *ax-derivs.asm*)

apply (*fast intro*: *ax-derivs.weaken*)
apply (*rule ax-derivs.conseq*, *intro strip*, *tactic smp-tac* ***context*** *3 1*,*clarify*,
tactic smp-tac ***context*** *1 1*,*rule exI*, *rule exI*, *erule* (*1*) *conjI*)

prefer *18*
apply (*rule ax-derivs.Methd*, *drule spec*, *erule mp*, *fast*)
apply (*tactic* ‹*TRYALL* (*resolve-tac* ***context*** ((*funpow 5 tl*) @{*thms ax-derivs.intros*}))›)
apply *auto*
done

lemma *ax-thin-insert*: $G,(A::'a\ triple\ set)\vdash(t::'a\ triple) \Longrightarrow G,insert\ x\ A\vdash t$
apply (*erule ax-thin*)
apply *fast*
done

lemma *subset-mtriples-iff*:
$ts \subseteq \{\{P\}\ mb{-}\succ\ \{Q\}\ |\ ms\} = (\exists\,ms'.\ ms' {\subseteq} ms \wedge\ \ ts = \{\{P\}\ mb{-}\succ\ \{Q\}\ |\ ms'\})$
apply (*unfold mtriples-def*)
apply (*rule subset-image-iff*)
done

lemma *weaken*:
$G,(A::'a\ triple\ set)|\vdash(ts'::'a\ triple\ set) \Longrightarrow \forall\,ts.\ ts \subseteq ts' \longrightarrow G,A|\vdash ts$
apply (*erule ax-derivs.induct*)

apply (*tactic ALLGOALS* (*strip-tac* ***context***))
apply (*tactic* ‹*ALLGOALS*(*REPEAT o* (*EVERY'*[*dresolve-tac* ***context*** @{*thms subset-singletonD*},
eresolve-tac ***context*** [*disjE*],
fast-tac (***context*** *addSIs* @{*thms ax-derivs.empty*})]))›)
apply (*tactic TRYALL* (*hyp-subst-tac* ***context***))
apply (*simp*, *rule ax-derivs.empty*)
apply (*drule subset-insertD*)
apply (*blast intro*: *ax-derivs.insert*)
apply (*fast intro*: *ax-derivs.asm*)

apply (*fast intro*: *ax-derivs.weaken*)
apply (*rule ax-derivs.conseq*, *clarify*, *tactic smp-tac* ***context*** *3 1*, *blast*)

apply (*tactic* ‹*TRYALL* (*resolve-tac* ***context*** ((*funpow 5 tl*) @{*thms ax-derivs.intros*})
THEN-ALL-NEW fast-tac ***context***)›)

apply (*clarsimp simp add*: *subset-mtriples-iff*)
apply (*rule ax-derivs.Methd*)
apply (*drule spec*)
apply (*erule impE*)
apply (*rule exI*)
apply (*erule conjI*)
apply (*rule HOL.refl*)
oops

rules derived from conseq

In the following rules we often have to give some type annotations like: $G,A\vdash\{P\}\ t\succ\ \{Q\}$. Given only the term above without annotations, Isabelle would infer a more general type were we could have different types of auxiliary variables in the assumption set (A) and in the triple itself (P and Q). But *ax-derivs.Methd* enforces the same type in the inductive definition of the derivation. So we have to restrict the types to be able to apply the rules.

lemma *conseq12*: $[\![G,(A::'a\ triple\ set)\vdash\{P'::'a\ assn\}\ t\succ\ \{Q'\};$
$\forall Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow (\forall Y'\ s'.\ (\forall Y\ Z'.\ P'\ Y\ s\ Z' \longrightarrow Q'\ Y'\ s'\ Z') \longrightarrow$
$Q\ Y'\ s'\ Z)]\!]$
$\Longrightarrow\ G,A\vdash\{P\ ::'a\ assn\}\ t\succ\ \{Q\ \}$
apply (*rule ax-derivs.conseq*)
apply *clarsimp*
apply *blast*
done

— Nice variant, since it is so symmetric we might be able to memorise it.

lemma *conseq12'*: $[\![G,(A::'a\ triple\ set)\vdash\{P'::'a\ assn\}\ t\succ\ \{Q'\};\ \forall s\ Y'\ s'.$
$(\forall Y\ Z.\ P'\ Y\ s\ Z \longrightarrow Q'\ Y'\ s'\ Z) \longrightarrow$
$(\forall Y\ Z.\ P\ \ Y\ s\ Z \longrightarrow Q\ \ Y'\ s'\ Z)]\!]$
$\Longrightarrow\ G,A\vdash\{P::'a\ assn\ \}\ t\succ\ \{Q\ \}$
apply (*erule conseq12*)
apply *fast*
done

lemma *conseq12-from-conseq12'*: $[\![G,(A::'a\ triple\ set)\vdash\{P'::'a\ assn\}\ t\succ\ \{Q'\};$
$\forall Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow (\forall Y'\ s'.\ (\forall Y\ Z'.\ P'\ Y\ s\ Z' \longrightarrow Q'\ Y'\ s'\ Z') \longrightarrow$
$Q\ Y'\ s'\ Z)]\!]$
$\Longrightarrow\ G,A\vdash\{P::'a\ assn\}\ t\succ\ \{Q\ \}$
apply (*erule conseq12'*)

apply *blast*
done

lemma *conseq1*: $[\![G,(A::'a\ triple\ set)\vdash\{P'::'a\ assn\}\ t\succ\ \{Q\};\ P\Rightarrow P']\!]$
$\Longrightarrow G,A\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}$
apply (*erule conseq12*)
apply *blast*
done

lemma *conseq2*: $[\![G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ\ \{Q'\};\ Q'\Rightarrow Q]\!]$
$\Longrightarrow G,A\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}$
apply (*erule conseq12*)
apply *blast*
done

lemma *ax-escape*:
$[\![\forall\ Y\ s\ Z.\ P\ Y\ s\ Z$
$\longrightarrow G,(A::'a\ triple\ set)\vdash\{\lambda Y'\ s'\ (Z'::'a).\ (Y',s')=(Y,s)\}$
$t\succ$
$\{\lambda Y\ s\ Z'.\ Q\ Y\ s\ Z\}$
$]\!]\Longrightarrow\ G,A\vdash\{P::'a\ assn\}\ t\succ\ \{Q::'a\ assn\}$
apply (*rule ax-derivs.conseq*)
apply *force*
done

lemma *ax-constant*: $[\![\ C\Longrightarrow G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}]\!]$
$\Longrightarrow G,A\vdash\{\lambda Y\ s\ Z.\ C\wedge P\ Y\ s\ Z\}\ t\succ\ \{Q\}$
apply (*rule ax-escape*)
apply *clarify*
apply (*rule conseq12*)
apply *fast*
apply *auto*
done

lemma *ax-impossible* [*intro*]:
$G,(A::'a\ triple\ set)\vdash\{\lambda Y\ s\ Z.\ False\}\ t\succ\ \{Q::'a\ assn\}$
apply (*rule ax-escape*)
apply *clarify*
done

lemma *ax-nochange-lemma*: $[\![P\ Y\ s;\ All\ ((=)\ w)]\!]\Longrightarrow P\ w\ s$
apply *auto*
done

lemma *ax-nochange*:
$G,(A::(res\times state)\ triple\ set)\vdash\{\lambda Y\ s\ Z.\ (Y,s)=Z\}\ t\succ\ \{\lambda Y\ s\ Z.\ (Y,s)=Z\}$
$\Longrightarrow G,A\vdash\{P::(res\times state)\ assn\}\ t\succ\ \{P\}$
apply (*erule conseq12*)

apply *auto*
apply (*erule* (*1*) *ax-nochange-lemma*)
done

lemma *ax-trivial*: $G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ \ t\succ\ \{\lambda Y\ s\ Z.\ True\}$
apply (*rule ax-derivs.conseq*)
apply *auto*
done

lemma *ax-disj*:
$[\![G,(A::'a\ triple\ set)\vdash\{P1::'a\ assn\}\ t\succ\ \{Q1\};\ G,A\vdash\{P2::'a\ assn\}\ t\succ\ \{Q2\}]\!]$
$\Longrightarrow\ G,A\vdash\{\lambda Y\ s\ Z.\ P1\ Y\ s\ Z \vee P2\ Y\ s\ Z\}\ t\succ\ \{\lambda Y\ s\ Z.\ Q1\ Y\ s\ Z \vee Q2\ Y\ s\ Z\}$
apply (*rule ax-escape*)
apply *safe*
apply (*erule conseq12*, *fast*)+
done

lemma *ax-supd-shuffle*:
$(\exists\, Q.\ G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ .c1.\ \{Q\} \wedge G,A\vdash\{Q\ ;.\ f\}\ .c2.\ \{R\}) =$
$(\exists\, Q'.\ G,A\vdash\{P\}\ .c1.\ \{f\ .;\ Q'\} \wedge G,A\vdash\{Q'\}\ .c2.\ \{R\})$
apply (*best elim*!: *conseq1 conseq2*)
done

lemma *ax-cases*:
$[\![G,(A::'a\ triple\ set)\vdash\{P\ \wedge.\qquad C\}\ t\succ\ \{Q::'a\ assn\};$
$G,A\vdash\{P\ \wedge.\ Not \circ C\}\ t\succ\ \{Q\}]\!] \Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Q\}$
apply (*unfold peek-and-def*)
apply (*rule ax-escape*)
apply *clarify*
apply (*case-tac C s*)
apply (*erule conseq12*, *force*)+
done

lemma *ax-adapt*: $G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}$
$\Longrightarrow G,A\vdash\{adapt\text{-}pre\ P\ Q\ Q'\}\ t\succ\ \{Q'\}$
apply (*unfold adapt-pre-def*)
apply (*erule conseq12*)
apply *fast*
done

lemma *adapt-pre-adapts*: $G,(A::'a\ triple\ set)\models\{P::'a\ assn\}\ t\succ\ \{Q\}$
$\longrightarrow G,A\models\{adapt\text{-}pre\ P\ Q\ Q'\}\ t\succ\ \{Q'\}$
apply (*unfold adapt-pre-def*)
apply (*simp add*: *ax-valids-def triple-valid-def2*)
apply *fast*
done

lemma *adapt-pre-weakest*:
$\forall G\ (A{::}'a\ triple\ set)\ t.\ G{,}A{\models}\{P\}\ t{\succ}\ \{Q\} \longrightarrow G{,}A{\models}\{P'\}\ t{\succ}\ \{Q'\} \Longrightarrow$
$P' \Rightarrow adapt\text{-}pre\ P\ Q\ (Q'{::}'a\ assn)$
apply (*unfold adapt-pre-def*)
apply (*drule spec*)
apply (*drule-tac* $x = \{\}$ **in** *spec*)
apply (*drule-tac* $x = In1r\ Skip$ **in** *spec*)
apply (*simp add*: *ax-valids-def triple-valid-def2*)
oops

lemma *peek-and-forget1-Normal*:
$G{,}(A{::}'a\ triple\ set){\vdash}\{Normal\ P\}\ t{\succ}\ \{Q{::}'a\ assn\}$
$\Longrightarrow G{,}A{\vdash}\{Normal\ (P\ \wedge.\ p)\}\ t{\succ}\ \{Q\}$
apply (*erule conseq1*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-forget1*:
$G{,}(A{::}'a\ triple\ set){\vdash}\{P{::}'a\ assn\}\ t{\succ}\ \{Q\}$
$\Longrightarrow G{,}A{\vdash}\{P\ \wedge.\ p\}\ t{\succ}\ \{Q\}$
apply (*erule conseq1*)
apply (*simp* (*no-asm*))
done

lemmas *ax-NormalD* = *peek-and-forget1* [*of* - - - - - *normal*]

lemma *peek-and-forget2*:
$G{,}(A{::}'a\ triple\ set){\vdash}\{P{::}'a\ assn\}\ t{\succ}\ \{Q\ \wedge.\ p\}$
$\Longrightarrow G{,}A{\vdash}\{P\}\ t{\succ}\ \{Q\}$
apply (*erule conseq2*)
apply (*simp* (*no-asm*))
done

lemma *ax-subst-Val-allI*:
$\forall v.\ G{,}(A{::}'a\ triple\ set){\vdash}\{(P'\qquad v\ ){\leftarrow}Val\ v\}\ t{\succ}\ \{(Q\ v){::}'a\ assn\}$
$\Longrightarrow\ \forall v.\ G{,}A{\vdash}\{(\lambda w{:}.\ P'\ (the\text{-}In1\ w)){\leftarrow}Val\ v\}\ t{\succ}\ \{Q\ v\}$
apply (*force elim*!: *conseq1*)
done

lemma *ax-subst-Var-allI*:
$\forall v.\ G{,}(A{::}'a\ triple\ set){\vdash}\{(P'\qquad v\ ){\leftarrow}Var\ v\}\ t{\succ}\ \{(Q\ v){::}'a\ assn\}$
$\Longrightarrow\ \forall v.\ G{,}A{\vdash}\{(\lambda w{:}.\ P'\ (the\text{-}In2\ w)){\leftarrow}Var\ v\}\ t{\succ}\ \{Q\ v\}$
apply (*force elim*!: *conseq1*)
done

lemma *ax-subst-Vals-allI*:
$(\forall v.\ G{,}(A{::}'a\ triple\ set){\vdash}\{(\qquad P'\qquad v\ ){\leftarrow}Vals\ v\}\ t{\succ}\ \{(Q\ v){::}'a\ assn\})$
$\Longrightarrow\ \forall v.\ G{,}A{\vdash}\{(\lambda w{:}.\ P'\ (the\text{-}In3\ w)){\leftarrow}Vals\ v\}\ t{\succ}\ \{Q\ v\}$
apply (*force elim*!: *conseq1*)
done

alternative axioms

lemma *ax-Lit2*:
$G,(A::'a\ triple\ set)\vdash\{Normal\ P::'a\ assn\}\ Lit\ v{-}\succ\ \{Normal\ (P{\downarrow}{=}Val\ v)\}$
apply (*rule ax-derivs.Lit* [*THEN conseq1*])
apply *force*
done

lemma *ax-Lit2-test-complete*:
$G,(A::'a\ triple\ set)\vdash\{Normal\ (P{\leftarrow}Val\ v)::'a\ assn\}\ Lit\ v{-}\succ\ \{P\}$
apply (*rule ax-Lit2* [*THEN conseq2*])
apply *force*
done

lemma *ax-LVar2*: $G,(A::'a\ triple\ set)\vdash\{Normal\ P::'a\ assn\}\ LVar\ vn{=}\succ\ \{Normal\ (\lambda s..\ P{\downarrow}{=}Var\ (lvar\ vn\ s))\}$
apply (*rule ax-derivs.LVar* [*THEN conseq1*])
apply *force*
done

lemma *ax-Super2*: $G,(A::'a\ triple\ set)\vdash$
$\{Normal\ P::'a\ assn\}\ Super{-}\succ\ \{Normal\ (\lambda s..\ P{\downarrow}{=}Val\ (val\text{-}this\ s))\}$
apply (*rule ax-derivs.Super* [*THEN conseq1*])
apply *force*
done

lemma *ax-Nil2*:
$G,(A::'a\ triple\ set)\vdash\{Normal\ P::'a\ assn\}\ []\doteq\succ\ \{Normal\ (P{\downarrow}{=}Vals\ [])\}$
apply (*rule ax-derivs.Nil* [*THEN conseq1*])
apply *force*
done

misc derived structural rules

lemma *ax-finite-mtriples-lemma*: $[\![F \subseteq ms;\ finite\ ms;\ \forall\,(C,sig){\in}ms.$
$G,(A::'a\ triple\ set)\vdash\{Normal\ (P\ C\ sig)::'a\ assn\}\ mb\ C\ sig{-}\succ\ \{Q\ C\ sig\}]\!] \Longrightarrow$
$G,A|\vdash\{\{P\}\ mb{-}\succ\ \{Q\}\ |\ F\}$
apply (*frule* (*1*) *finite-subset*)
apply (*erule rev-mp*)
apply (*erule thin-rl*)
apply (*erule finite-induct*)
apply (*unfold mtriples-def*)
apply (*clarsimp intro*!: *ax-derivs.empty ax-derivs.insert*)+
apply *force*
done
lemmas *ax-finite-mtriples* = *ax-finite-mtriples-lemma* [*OF subset-refl*]

lemma *ax-derivs-insertD*:
$G,(A::'a\ triple\ set)|\vdash insert\ (t::'a\ triple)\ ts \Longrightarrow G,A\vdash t \wedge G,A|\vdash ts$
apply (*fast intro*: *ax-derivs.weaken*)
done

lemma *ax-methods-spec*:
$[\![G,(A::'a\ triple\ set)|\vdash case\text{-}prod\ f\ `\ ms;\ (C,sig) \in ms]\!]\Longrightarrow G,A\vdash((f\ C\ sig)::'a\ triple)$
apply (*erule ax-derivs.weaken*)

apply (*force del*: *image-eqI intro*: *rev-image-eqI*)
done

lemma *ax-finite-pointwise-lemma* [*rule-format*]: $\llbracket F \subseteq ms;\ finite\ ms\rrbracket \Longrightarrow$
$((\forall (C,sig)\in F.\ G,(A::'a\ triple\ set)\vdash(f\ C\ sig::'a\ triple)) \longrightarrow (\forall (C,sig)\in ms.\ G,A\vdash(g\ C\ sig::'a\ triple))) \longrightarrow$
$G,A|\vdash case\text{-}prod\ f\ `\ F \longrightarrow G,A|\vdash case\text{-}prod\ g\ `\ F$
apply (*frule* (*1*) *finite-subset*)
apply (*erule rev-mp*)
apply (*erule thin-rl*)
apply (*erule finite-induct*)
apply *clarsimp+*
apply (*drule ax-derivs-insertD*)
apply (*rule ax-derivs.insert*)
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*auto elim*: *ax-methods-spec*)
done
lemmas *ax-finite-pointwise* = *ax-finite-pointwise-lemma* [*OF subset-refl*]

lemma *ax-no-hazard*:
$G,(A::'a\ triple\ set)\vdash\{P \wedge.\ type\text{-}ok\ G\ t\}\ t\succ\ \{Q::'a\ assn\} \Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Q\}$
apply (*erule ax-cases*)
apply (*rule ax-derivs.hazard* [*THEN conseq1*])
apply *force*
done

lemma *ax-free-wt*:
$(\exists\ T\ L\ C.\ (|prg=G,cls=C,lcl=L|)\vdash t::T)$
$\longrightarrow G,(A::'a\ triple\ set)\vdash\{Normal\ P\}\ t\succ\ \{Q::'a\ assn\} \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ t\succ\ \{Q\}$
apply (*rule ax-no-hazard*)
apply (*rule ax-escape*)
apply *clarify*
apply (*erule mp* [*THEN conseq12*])
apply (*auto simp add*: *type-ok-def*)
done

ML ‹*ML-Thms.bind-thms* (*ax-Abrupts*, *sum3-instantiate* ***context*** @{*thm ax-derivs.Abrupt*})›
declare *ax-Abrupts* [*intro*!]

lemmas *ax-Normal-cases* = *ax-cases* [*of - - - normal*]

lemma *ax-Skip* [*intro*!]: $G,(A::'a\ triple\ set)\vdash\{P\leftarrow\diamondsuit\}\ .Skip.\ \{P::'a\ assn\}$
apply (*rule ax-Normal-cases*)
apply (*rule ax-derivs.Skip*)
apply *fast*
done
lemmas *ax-SkipI* = *ax-Skip* [*THEN conseq1*]

derived rules for methd call

lemma *ax-Call-known-DynT*:
$\llbracket G\vdash IntVir\rightarrow C\preceq statT;$
$\forall\ a\ vs\ l.\ G,A\vdash\{(R\ a\leftarrow Vals\ vs \wedge.\ (\lambda s.\ l = locals\ (store\ s))\ ;.$
$init\text{-}lvars\ G\ C\ (|name=mn,parTs=pTs|)\ IntVir\ a\ vs)\}$

Methd C (|*name=mn,parTs=pTs*|)−≻ {*set-lvars l .; S*};
∀ *a. G,A*⊢{*Q*← *Val a*} *args*≐≻
{*R a* ∧. (λ*s. C* = *obj-class* (*the* (*heap* (*store s*) (*the-Addr a*))) ∧
C = *invocation-declclass*
G IntVir (*store s*) *a statT* (|*name=mn,parTs=pTs*|))};
G,(*A*::′*a triple set*)⊢{*Normal P*} *e*−≻ {*Q*::′*a assn*}⟧
⟹ *G,A*⊢{*Normal P*} {*accC,statT,IntVir*}*e*·*mn*({*pTs*}*args*)−≻ {*S*}
apply (*erule ax-derivs.Call*)
apply *safe*
apply (*erule spec*)
apply (*rule ax-escape*, *clarsimp*)
apply (*drule spec*, *drule spec*, *drule spec*,*erule conseq12*)
apply *force*
done

lemma *ax-Call-Static*:
⟦∀ *a vs l. G,A*⊢{*R a*← *Vals vs* ∧. (λ*s. l* = *locals* (*store s*)) ;.
init-lvars G C (|*name=mn,parTs=pTs*|) *Static any-Addr vs*}
Methd C (|*name=mn,parTs=pTs*|)−≻ {*set-lvars l .; S*};
G,A⊢{*Normal P*} *e*−≻ {*Q*};
∀ *a. G*,(*A*::′*a triple set*)⊢{*Q*← *Val a*} *args*≐≻ {(*R*::*val* ⇒ ′*a assn*) *a*
∧. (λ *s. C*=*invocation-declclass*
G Static (*store s*) *a statT* (|*name=mn,parTs=pTs*|))}
⟧ ⟹ *G,A*⊢{*Normal P*} {*accC,statT,Static*}*e*·*mn*({*pTs*}*args*)−≻ {*S*}
apply (*erule ax-derivs.Call*)
apply *safe*
apply (*erule spec*)
apply (*rule ax-escape*, *clarsimp*)
apply (*erule-tac V* = *P* ⟶ *Q* **for** *P Q* **in** *thin-rl*)
apply (*drule spec*,*drule spec*,*drule spec*, *erule conseq12*)
apply (*force simp add*: *init-lvars-def Let-def*)
done

lemma *ax-Methd1*:
⟦*G,A*∪{{*P*} *Methd*−≻ {*Q*} | *ms*}|⊢ {{*P*} *body G*−≻ {*Q*} | *ms*}; (*C,sig*)∈ *ms*⟧ ⟹
G,A⊢{*Normal* (*P C sig*)} *Methd C sig*−≻ {*Q C sig*}
apply (*drule ax-derivs.Methd*)
apply (*unfold mtriples-def*)
apply (*erule* (*1*) *ax-methods-spec*)
done

lemma *ax-MethdN*:
G,insert({*Normal P*} *Methd C sig*−≻ {*Q*}) *A*⊢
{*Normal P*} *body G C sig*−≻ {*Q*} ⟹
G,A⊢{*Normal P*} *Methd C sig*−≻ {*Q*}
apply (*rule ax-Methd1*)
apply (*rule-tac* [*2*] *singletonI*)
apply (*unfold mtriples-def*)
apply *clarsimp*
done

lemma *ax-StatRef*:
G,(*A*::′*a triple set*)⊢{*Normal* (*P*←*Val Null*)} *StatRef rt*−≻ {*P*::′*a assn*}
apply (*rule ax-derivs.Cast*)

apply (*rule ax-Lit2* [*THEN conseq2*])
apply *clarsimp*
done

rules derived from Init and Done

lemma *ax-InitS*: $[\![$*the* (*class G C*) = *c*; *C* $\neq$ *Object*;
$\forall$ *l*. *G*,*A*$\vdash${*Q* $\wedge$. (λ*s*. *l* = *locals* (*store s*)) ;. *set-lvars Map.empty*}
.*init c*. {*set-lvars l* .; *R*};
G,*A*$\vdash${*Normal* ((*P* $\wedge$. *Not* $\circ$ *initd C*) ;. *supd* (*init-class-obj G C*))}
.*Init* (*super c*). {*Q*}$]\!]$ $\Longrightarrow$
G,(*A*::'*a triple set*)$\vdash${*Normal* (*P* $\wedge$. *Not* $\circ$ *initd C*)} .*Init C*. {*R*::'*a assn*}
apply (*erule ax-derivs.Init*)
apply (*simp* (*no-asm-simp*))
apply *assumption*
done

lemma *ax-Init-Skip-lemma*:
$\forall$ *l*. *G*,(*A*::'*a triple set*)$\vdash${*P*$\leftarrow\diamondsuit$ $\wedge$. (λ*s*. *l* = *locals* (*store s*)) ;. *set-lvars l'*}
.*Skip*. {(*set-lvars l* .; *P*)::'*a assn*}
apply (*rule allI*)
apply (*rule ax-SkipI*)
apply *clarsimp*
done

lemma *ax-triv-InitS*: $[\![$*the* (*class G C*) = *c*;*init c* = *Skip*; *C* $\neq$ *Object*;
P$\leftarrow\diamondsuit$ $\Rightarrow$ (*supd* (*init-class-obj G C*) .; *P*);
G,*A*$\vdash${*Normal* (*P* $\wedge$. *initd C*)} .*Init* (*super c*). {(*P* $\wedge$. *initd C*)$\leftarrow\diamondsuit$}$]\!]$ $\Longrightarrow$
G,(*A*::'*a triple set*)$\vdash${*Normal P*$\leftarrow\diamondsuit$} .*Init C*. {(*P* $\wedge$. *initd C*)::'*a assn*}
apply (*rule-tac C* = *initd C* **in** *ax-cases*)
apply (*rule conseq1*, *rule ax-derivs.Done*, *clarsimp*)
apply (*simp* (*no-asm*))
apply (*erule* (*1*) *ax-InitS*)
apply *simp*
apply (*rule ax-Init-Skip-lemma*)
apply (*erule conseq1*)
apply *force*
done

lemma *ax-Init-Object*: *wf-prog G* $\Longrightarrow$ *G*,(*A*::'*a triple set*)$\vdash$
{*Normal* ((*supd* (*init-class-obj G Object*) .; *P*$\leftarrow\diamondsuit$) $\wedge$. *Not* $\circ$ *initd Object*)}
.*Init Object*. {(*P* $\wedge$. *initd Object*)::'*a assn*}
apply (*rule ax-derivs.Init*)
apply (*drule class-Object*, *force*)
apply (*simp-all* (*no-asm*))
apply (*rule-tac* [*2*] *ax-Init-Skip-lemma*)
apply (*rule ax-SkipI*, *force*)
done

lemma *ax-triv-Init-Object*: $[\![$*wf-prog G*;
(*P*::'*a assn*) $\Rightarrow$ (*supd* (*init-class-obj G Object*) .; *P*)$]\!]$ $\Longrightarrow$
G,(*A*::'*a triple set*)$\vdash${*Normal P*$\leftarrow\diamondsuit$} .*Init Object*. {*P* $\wedge$. *initd Object*}
apply (*rule-tac C* = *initd Object* **in** *ax-cases*)
apply (*rule conseq1*, *rule ax-derivs.Done*, *clarsimp*)

apply (*erule ax-Init-Object* [*THEN conseq1*])
apply *force*
done

introduction rules for Alloc and SXAlloc

lemma *ax-SXAlloc-Normal*:
$G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ .c.\ \{Normal\ Q\}$
$\Longrightarrow G,A\vdash\{P\}\ .c.\ \{SXAlloc\ G\ Q\}$
apply (*erule conseq2*)
apply (*clarsimp elim*!: *sxalloc-elim-cases simp add*: *split-tupled-all*)
done

lemma *ax-Alloc*:
$G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ$
$\{Normal\ (\lambda Y\ (x,s)\ Z.\ (\forall a.\ new\text{-}Addr\ (heap\ s) = Some\ a \longrightarrow$
$Q\ (Val\ (Addr\ a))\ (Norm(init\text{-}obj\ G\ (CInst\ C)\ (Heap\ a)\ s))\ Z)) \wedge.$
$heap\text{-}free\ (Suc\ (Suc\ 0))\}$
$\Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Alloc\ G\ (CInst\ C)\ Q\}$
apply (*erule conseq2*)
apply (*auto elim*!: *halloc-elim-cases*)
done

lemma *ax-Alloc-Arr*:
$G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ$
$\{\lambda Val{:}i{:}.\ Normal\ (\lambda Y\ (x,s)\ Z.\ \neg the\text{-}Intg\ i{<}0\ \wedge$
$(\forall a.\ new\text{-}Addr\ (heap\ s) = Some\ a \longrightarrow$
$Q\ (Val\ (Addr\ a))\ (Norm\ (init\text{-}obj\ G\ (Arr\ T\ (the\text{-}Intg\ i))\ (Heap\ a)\ s))\ Z)) \wedge.$
$heap\text{-}free\ (Suc\ (Suc\ 0))\}$
$\Longrightarrow$
$G,A\vdash\{P\}\ t\succ\ \{\lambda Val{:}i{:}.\ abupd\ (check\text{-}neg\ i)\ .;\ Alloc\ G\ (Arr\ T(the\text{-}Intg\ i))\ Q\}$
apply (*erule conseq2*)
apply (*auto elim*!: *halloc-elim-cases*)
done

lemma *ax-SXAlloc-catch-SXcpt*:
$[\![G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ$
$\{(\lambda Y\ (x,s)\ Z.\ x{=}Some\ (Xcpt\ (Std\ xn)) \wedge$
$(\forall a.\ new\text{-}Addr\ (heap\ s) = Some\ a \longrightarrow$
$Q\ Y\ (Some\ (Xcpt\ (Loc\ a)),init\text{-}obj\ G\ (CInst\ (SXcpt\ xn))\ (Heap\ a)\ s)\ Z))$
$\wedge.\ heap\text{-}free\ (Suc\ (Suc\ 0))\}]\!]$
$\Longrightarrow$
$G,A\vdash\{P\}\ t\succ\ \{SXAlloc\ G\ (\lambda Y\ s\ Z.\ Q\ Y\ s\ Z \wedge G,s\vdash catch\ SXcpt\ xn)\}$
apply (*erule conseq2*)
apply (*auto elim*!: *sxalloc-elim-cases halloc-elim-cases*)
done

end

Chapter 23

AxSound

1 Soundness proof for Axiomatic semantics of Java expressions and statements

theory *AxSound* **imports** *AxSem* **begin**

validity

definition
  triple-valid2 :: *prog* ⇒ *nat* ⇒ *'a triple* ⇒ *bool* (-⊨-::-[*61*,*0*, *58*] *57*)
  where
    G⊨n::t =
     (*case t of* {P} t≻ {Q} ⇒
       ∀ Y s Z. P Y s Z ⟶ (∀ L. s::≼(G,L)
         ⟶ (∀ T C A. (*normal s* ⟶ ((|*prg*=G,*cls*=C,*lcl*=L|)⊢t::T ∧
          (|*prg*=G,*cls*=C,*lcl*=L|)⊢*dom* (*locals* (*store s*))»t»A)) ⟶
           (∀ Y' s'. G⊢s −t≻−n→ (Y',s') ⟶ Q Y' s' Z ∧ s'::≼(G,L)))))

This definition differs from the ordinary *triple-valid-def* manly in the conclusion: We also ensures conformance of the result state. So we don't have to apply the type soundness lemma all the time during induction. This definition is only introduced for the soundness proof of the axiomatic semantics, in the end we will conclude to the ordinary definition.

definition
  ax-valids2 :: *prog* ⇒ *'a triples* ⇒ *'a triples* ⇒ *bool* (-,-|⊨::- [*61*,*58*,*58*] *57*)
  where G,A|⊨::ts = (∀ n. (∀ t∈A. G⊨n::t) ⟶ (∀ t∈ts. G⊨n::t))

lemma *triple-valid2-def2*: G⊨n::{P} t≻ {Q} =
 (∀ Y s Z. P Y s Z ⟶ (∀ Y' s'. G⊢s −t≻−n→ (Y',s')⟶
  (∀ L. s::≼(G,L) ⟶ (∀ T C A. (*normal s* ⟶ ((|*prg*=G,*cls*=C,*lcl*=L|)⊢t::T ∧
                (|*prg*=G,*cls*=C,*lcl*=L|)⊢*dom* (*locals* (*store s*))»t»A)) ⟶
  Q Y' s' Z ∧ s'::≼(G,L)))))
apply (*unfold triple-valid2-def*)
apply (*simp* (*no-asm*) *add*: *split-paired-All*)
apply *blast*
done

lemma *triple-valid2-eq* [*rule-format* (*no-asm*)]:
  wf-prog G ==> *triple-valid2 G* = *triple-valid G*
apply (*rule ext*)
apply (*rule ext*)
apply (*rule triple.induct*)
apply (*simp* (*no-asm*) *add*: *triple-valid-def2 triple-valid2-def2*)
apply (*rule iffI*)
apply *fast*

apply *clarify*
apply (*tactic smp-tac* ***context*** *3 1*)
apply (*case-tac normal s*)
apply *clarsimp*
apply (*elim conjE impE*)
apply *blast*

apply (*tactic smp-tac* ***context*** *2 1*)
apply (*drule evaln-eval*)
apply (*drule* (*1*) *eval-type-sound* [*THEN conjunct1*],*simp*, *assumption*+)
apply *simp*

apply *clarsimp*
done

lemma *ax-valids2-eq*: *wf-prog G* $\Longrightarrow$ *G*,*A*$|\models$::*ts* = *G*,*A*$|\models$*ts*
apply (*unfold ax-valids-def ax-valids2-def*)
apply (*force simp add*: *triple-valid2-eq*)
done

lemma *triple-valid2-Suc* [*rule-format* (*no-asm*)]: *G*$\models$*Suc n*::*t* $\longrightarrow$ *G*$\models$*n*::*t*
apply (*induct-tac t*)
apply (*subst triple-valid2-def2*)
apply (*subst triple-valid2-def2*)
apply (*fast intro*: *evaln-nonstrict-Suc*)
done

lemma *Methd-triple-valid2-0*: *G*$\models$*0*::{*Normal P*} *Methd C sig*$-\succ$ {*Q*}
by (*auto elim*!: *evaln-elim-cases simp add*: *triple-valid2-def2*)

lemma *Methd-triple-valid2-SucI*:
$[\![$*G*$\models$*n*::{*Normal P*} *body G C sig*$-\succ${*Q*}$]\!]$
$\Longrightarrow$ *G*$\models$*Suc n*::{*Normal P*} *Methd C sig*$-\succ$ {*Q*}
apply (*simp* (*no-asm-use*) *add*: *triple-valid2-def2*)
apply (*intro strip*, *tactic smp-tac* ***context*** *3 1*, *clarify*)
apply (*erule wt-elim-cases*, *erule da-elim-cases*, *erule evaln-elim-cases*)
apply (*unfold body-def Let-def*)
apply (*clarsimp simp add*: *inj-term-simps*)
apply *blast*
done

lemma *triples-valid2-Suc*:
Ball ts (*triple-valid2 G* (*Suc n*)) $\Longrightarrow$ *Ball ts* (*triple-valid2 G n*)
apply (*fast intro*: *triple-valid2-Suc*)
done

lemma *G*$|\models$*n*:*insert t A* = (*G*$\models$*n*:*t* $\wedge$ *G*$|\models$*n*:*A*)
oops

soundness

lemma *Methd-sound*:

assumes *recursive*: *G*,*A*∪ {{*P*} *Methd*−≻ {*Q*} | *ms*}|⊨::{{*P*} *body G*−≻ {*Q*} | *ms*}
shows *G*,*A*|⊨::{{*P*} *Methd*−≻ {*Q*} | *ms*}
proof −
{
fix *n*
assume *recursive*: ⋀ *n*. ∀ *t*∈(*A* ∪ {{*P*} *Methd*−≻ {*Q*} | *ms*}). *G*⊨*n*::*t*
⟹ ∀ *t*∈{{*P*} *body G*−≻ {*Q*} | *ms*}. *G*⊨*n*::*t*
have ∀ *t*∈*A*. *G*⊨*n*::*t* ⟹ ∀ *t*∈{{*P*} *Methd*−≻ {*Q*} | *ms*}. *G*⊨*n*::*t*
proof (*induct n*)
case *0*
show ∀ *t*∈{{*P*} *Methd*−≻ {*Q*} | *ms*}. *G*⊨*0*::*t*
proof −
{
fix *C sig*
assume (*C*,*sig*) ∈ *ms*
have *G*⊨*0*::{*Normal* (*P C sig*)} *Methd C sig*−≻ {*Q C sig*}
by (*rule Methd-triple-valid2-0*)
}
thus *?thesis*
by (*simp add*: *mtriples-def split-def*)
qed
next
case (*Suc m*)
note *hyp* = ‹∀ *t*∈*A*. *G*⊨*m*::*t* ⟹ ∀ *t*∈{{*P*} *Methd*−≻ {*Q*} | *ms*}. *G*⊨*m*::*t*›
note *prem* = ‹∀ *t*∈*A*. *G*⊨*Suc m*::*t*›
show ∀ *t*∈{{*P*} *Methd*−≻ {*Q*} | *ms*}. *G*⊨*Suc m*::*t*
proof −
{
fix *C sig*
assume *m*: (*C*,*sig*) ∈ *ms*
have *G*⊨*Suc m*::{*Normal* (*P C sig*)} *Methd C sig*−≻ {*Q C sig*}
proof −
from *prem* **have** *prem-m*: ∀ *t*∈*A*. *G*⊨*m*::*t*
by (*rule triples-valid2-Suc*)
hence ∀ *t*∈{{*P*} *Methd*−≻ {*Q*} | *ms*}. *G*⊨*m*::*t*
by (*rule hyp*)
with *prem-m*
have ∀ *t*∈(*A* ∪ {{*P*} *Methd*−≻ {*Q*} | *ms*}). *G*⊨*m*::*t*
by (*simp add*: *ball-Un*)
hence ∀ *t*∈{{*P*} *body G*−≻ {*Q*} | *ms*}. *G*⊨*m*::*t*
by (*rule recursive*)
with *m* **have** *G*⊨*m*::{*Normal* (*P C sig*)} *body G C sig*−≻ {*Q C sig*}
by (*auto simp add*: *mtriples-def split-def*)
thus *?thesis*
by (*rule Methd-triple-valid2-SucI*)
qed
}
thus *?thesis*
by (*simp add*: *mtriples-def split-def*)
qed
qed
}
with *recursive* **show** *?thesis*
by (*unfold ax-valids2-def*) *blast*
qed

lemma *valids2-inductI*: ∀ *s t n Y′ s′*. *G*⊢*s*−*t*≻−*n*→ (*Y′*,*s′*) ⟶ *t* = *c* ⟶

$\mathit{Ball}\ A\ (\mathit{triple\text{-}valid2}\ G\ n) \longrightarrow (\forall\ Y\ Z.\ P\ Y\ s\ Z \longrightarrow$
$(\forall\ L.\ s{::}\preceq(G,L) \longrightarrow$
$(\forall\ T\ C\ A.\ (\mathit{normal}\ s \longrightarrow ((\!|\mathit{prg}{=}G,\mathit{cls}{=}C,\mathit{lcl}{=}L|\!)\vdash t{::}T) \wedge$
$(\!|\mathit{prg}{=}G,\mathit{cls}{=}C,\mathit{lcl}{=}L|\!)\vdash \mathit{dom}\ (\mathit{locals}\ (\mathit{store}\ s))\gg t\gg A) \longrightarrow$
$Q\ Y'\ s'\ Z \wedge s'{::}\preceq(G,\ L)))) \Longrightarrow$
$G,A|\!\models{::}\{\ \{P\}\ c\succ\ \{Q\}\}$
apply (*simp* (*no-asm*) *add*: *ax-valids2-def triple-valid2-def2*)
apply *clarsimp*
done

lemma *da-good-approx-evalnE* [*consumes 4*]:
assumes *evaln*: $G\vdash s0\ -t\succ-n\rightarrow\ (v,\ s1)$
and *wt*: $(\!|\mathit{prg}{=}G,\mathit{cls}{=}C,\mathit{lcl}{=}L|\!)\vdash t{::}T$
and *da*: $(\!|\mathit{prg}{=}G,\mathit{cls}{=}C,\mathit{lcl}{=}L|\!)\vdash\ \mathit{dom}\ (\mathit{locals}\ (\mathit{store}\ s0))\ \gg t\gg\ A$
and *wf*: *wf-prog G*
and *elim*: $[\![\mathit{normal}\ s1 \Longrightarrow \mathit{nrm}\ A \subseteq \mathit{dom}\ (\mathit{locals}\ (\mathit{store}\ s1));$
$\bigwedge\ l.\ [\![\mathit{abrupt}\ s1\ =\ \mathit{Some}\ (\mathit{Jump}\ (\mathit{Break}\ l));\ \mathit{normal}\ s0]\!]$
$\Longrightarrow\ \mathit{brk}\ A\ l \subseteq \mathit{dom}\ (\mathit{locals}\ (\mathit{store}\ s1));$
$[\![\mathit{abrupt}\ s1\ =\ \mathit{Some}\ (\mathit{Jump}\ \mathit{Ret});\mathit{normal}\ s0]\!]$
$\Longrightarrow \mathit{Result} \in \mathit{dom}\ (\mathit{locals}\ (\mathit{store}\ s1))$
$]\!] \Longrightarrow P$
shows P
proof −
from *evaln* **have** $G\vdash s0\ -t\succ\rightarrow\ (v,\ s1)$
by (*rule evaln-eval*)
from *this wt da wf elim* **show** P
by (*rule da-good-approxE'*) *iprover+*
qed

lemma *validI*:
assumes I: $\bigwedge\ n\ s0\ L\ \mathit{accC}\ T\ C\ v\ s1\ Y\ Z.$
$[\![\forall\ t{\in}A.\ G|\!\!=n{::}t;\ s0{::}\preceq(G,L);$
$\mathit{normal}\ s0 \Longrightarrow (\!|\mathit{prg}{=}G,\mathit{cls}{=}\mathit{accC},\mathit{lcl}{=}L|\!)\vdash t{::}T;$
$\mathit{normal}\ s0 \Longrightarrow (\!|\mathit{prg}{=}G,\mathit{cls}{=}\mathit{accC},\mathit{lcl}{=}L|\!)\vdash \mathit{dom}\ (\mathit{locals}\ (\mathit{store}\ s0))\gg t\gg C;$
$G\vdash s0\ -t\succ-n\rightarrow\ (v,s1);\ P\ Y\ s0\ Z]\!] \Longrightarrow Q\ v\ s1\ Z \wedge s1{::}\preceq(G,L)$
shows $G,A|\!\models{::}\{\ \{P\}\ t\succ\ \{Q\}\ \}$
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)
apply (*intro allI impI*)
apply (*case-tac normal s*)
apply *clarsimp*
apply (*rule I*,(*assumption*|*simp*)+)

apply (*rule I*,*auto*)
done

declare [[*simproc add*: *wt-expr wt-var wt-exprs wt-stmt*]]

lemma *valid-stmtI*:
assumes I: $\bigwedge\ n\ s0\ L\ \mathit{accC}\ C\ s1\ Y\ Z.$
$[\![\forall\ t{\in}A.\ G|\!\!=n{::}t;\ s0{::}\preceq(G,L);$
$\mathit{normal}\ s0 \Longrightarrow (\!|\mathit{prg}{=}G,\mathit{cls}{=}\mathit{accC},\mathit{lcl}{=}L|\!)\vdash c{::}\surd;$
$\mathit{normal}\ s0 \Longrightarrow (\!|\mathit{prg}{=}G,\mathit{cls}{=}\mathit{accC},\mathit{lcl}{=}L|\!)\vdash \mathit{dom}\ (\mathit{locals}\ (\mathit{store}\ s0))\gg\langle c\rangle_s\gg C;$
$G\vdash s0\ -c-n\rightarrow\ s1;\ P\ Y\ s0\ Z]\!] \Longrightarrow Q\ \Diamond\ s1\ Z \wedge s1{::}\preceq(G,L)$
shows $G,A|\!\models{::}\{\ \{P\}\ \langle c\rangle_s\succ\ \{Q\}\ \}$
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)

apply (*intro allI impI*)
apply (*case-tac normal s*)
apply *clarsimp*
apply (*rule I*,(*assumption*|*simp*)+)

apply (*rule I*,*auto*)
done

lemma *valid-stmt-NormalI*:
assumes *I*: $\bigwedge$ *n s0 L accC C s1 Y Z*.
⟦∀ *t*∈*A*. *G*⊨*n*::*t*; *s0*::⪯(*G*,*L*); *normal s0*; ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*c*::√;
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»⟨*c*⟩$_s$»*C*;
G⊢*s0* −*c*−*n*→ *s1*; (*Normal P*) *Y s0 Z*⟧ ⟹ *Q* ◇ *s1 Z* ∧ *s1*::⪯(*G*,*L*)
shows *G*,*A*|⊨::{ {*Normal P*} ⟨*c*⟩$_s$≻ {*Q*} }
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)
apply (*intro allI impI*)
apply (*elim exE conjE*)
apply (*rule I*)
by *auto*

lemma *valid-var-NormalI*:
assumes *I*: $\bigwedge$ *n s0 L accC T C vf s1 Y Z*.
⟦∀ *t*∈*A*. *G*⊨*n*::*t*; *s0*::⪯(*G*,*L*); *normal s0*;
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*t*::=*T*;
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»⟨*t*⟩$_v$»*C*;
G⊢*s0* −*t*=≻*vf*−*n*→ *s1*; (*Normal P*) *Y s0 Z*⟧
⟹ *Q* (*In2 vf*) *s1 Z* ∧ *s1*::⪯(*G*,*L*)
shows *G*,*A*|⊨::{ {*Normal P*} ⟨*t*⟩$_v$≻ {*Q*} }
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)
apply (*intro allI impI*)
apply (*elim exE conjE*)
apply *simp*
apply (*rule I*)
by *auto*

lemma *valid-expr-NormalI*:
assumes *I*: $\bigwedge$ *n s0 L accC T C v s1 Y Z*.
⟦∀ *t*∈*A*. *G*⊨*n*::*t*; *s0*::⪯(*G*,*L*); *normal s0*;
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*t*::−*T*;
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»⟨*t*⟩$_e$»*C*;
G⊢*s0* −*t*−≻*v*−*n*→ *s1*; (*Normal P*) *Y s0 Z*⟧
⟹ *Q* (*In1 v*) *s1 Z* ∧ *s1*::⪯(*G*,*L*)
shows *G*,*A*|⊨::{ {*Normal P*} ⟨*t*⟩$_e$≻ {*Q*} }
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)
apply (*intro allI impI*)
apply (*elim exE conjE*)
apply *simp*
apply (*rule I*)
by *auto*

lemma *valid-expr-list-NormalI*:
assumes *I*: $\bigwedge$ *n s0 L accC T C vs s1 Y Z*.
⟦∀ *t*∈*A*. *G*⊨*n*::*t*; *s0*::⪯(*G*,*L*); *normal s0*;
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*t*::≐*T*;
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»⟨*t*⟩$_l$»*C*;

```
              G⊢s0 −t≐≻vs−n→ s1; (Normal P) Y s0 Z⟧
              ⟹ Q (In3 vs) s1 Z ∧ s1::≼(G,L)
  shows G,A|⊨::{ {Normal P} ⟨t⟩l≻ {Q} }
apply (simp add: ax-valids2-def triple-valid2-def2)
apply (intro allI impI)
apply (elim exE conjE)
apply simp
apply (rule I)
by auto
```

```
lemma validE [consumes 5]:
 assumes valid: G,A|⊨::{ {P} t≻ {Q} }
  and    P: P Y s0 Z
  and    valid-A: ∀ t∈A. G⊨n::t
  and    conf: s0::≼(G,L)
  and    eval: G⊢s0 −t≻−n→ (v,s1)
  and    wt: normal s0 ⟹ ⦇prg=G,cls=accC,lcl=L⦈⊢t::T
  and    da: normal s0 ⟹ ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0))»t»C
  and    elim: ⟦Q v s1 Z; s1::≼(G,L)⟧ ⟹ concl
 shows concl
using assms
by (simp add: ax-valids2-def triple-valid2-def2) fast
```

```
lemma all-empty: (∀ x. P) = P
by simp
```

```
corollary evaln-type-sound:
 assumes evaln: G⊢s0 −t≻−n→ (v,s1) and
           wt: ⦇prg=G,cls=accC,lcl=L⦈⊢t::T and
           da: ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0)) »t» A and
      conf-s0: s0::≼(G,L) and
           wf: wf-prog G
 shows s1::≼(G,L) ∧ (normal s1 ⟶ G,L,store s1⊢t≻v::≼T) ∧
       (error-free s0 = error-free s1)
proof −
 from evaln have G⊢s0 −t≻→ (v,s1)
   by (rule evaln-eval)
 from this wt da wf conf-s0 show ?thesis
   by (rule eval-type-sound)
qed
```

```
corollary dom-locals-evaln-mono-elim [consumes 1]:
 assumes
 evaln: G⊢ s0 −t≻−n→ (v,s1) and
   hyps: ⟦dom (locals (store s0)) ⊆ dom (locals (store s1));
         ⋀ vv s val. ⟦v=In2 vv; normal s1⟧
                     ⟹ dom (locals (store s))
                         ⊆ dom (locals (store ((snd vv) val s)))⟧ ⟹ P
 shows P
proof −
 from evaln have G⊢ s0 −t≻→ (v,s1) by (rule evaln-eval)
 from this hyps show ?thesis
   by (rule dom-locals-eval-mono-elim) iprover+
qed
```

```
lemma evaln-no-abrupt:
  ⋀s s'. ⟦G⊢s −t≻−n→ (w,s'); normal s'⟧ ⟹ normal s
by (erule evaln-cases,auto)

declare inj-term-simps [simp]

lemma ax-sound2:
  assumes   wf: wf-prog G
    and   deriv: G,A|⊢ts
  shows G,A|⊨::ts
using deriv
proof (induct)
  case (empty A)
  show ?case
    by (simp add: ax-valids2-def triple-valid2-def2)
next
  case (insert A t ts)
  note valid-t = ‹G,A|⊨::{t}›
  moreover
  note valid-ts = ‹G,A|⊨::ts›
  {
    fix n assume valid-A: ∀t∈A. G⊨n::t
    have G⊨n::t and ∀t∈ts. G⊨n::t
    proof −
      from valid-A valid-t show G⊨n::t
        by (simp add: ax-valids2-def)
    next
      from valid-A valid-ts show ∀t∈ts. G⊨n::t
        by (unfold ax-valids2-def) blast
    qed
    hence ∀t'∈insert t ts. G⊨n::t'
      by simp
  }
  thus ?case
    by (unfold ax-valids2-def) blast
next
  case (asm ts A)
  from ‹ts ⊆ A›
  show G,A|⊨::ts
    by (auto simp add: ax-valids2-def triple-valid2-def)
next
  case (weaken A ts' ts)
  note ‹G,A|⊨::ts'›
  moreover note ‹ts ⊆ ts'›
  ultimately show G,A|⊨::ts
    by (unfold ax-valids2-def triple-valid2-def) blast
next
  case (conseq P A t Q)
  note con = ‹∀Y s Z. P Y s Z ⟶
             (∃P' Q'.
                (G,A⊢{P'} t≻ {Q'} ∧ G,A|⊨::{ {P'} t≻ {Q'} }) ∧
                (∀Y' s'. (∀Y Z'. P' Y s Z' ⟶ Q' Y' s' Z') ⟶ Q Y' s' Z))›
  show G,A|⊨::{ {P} t≻ {Q} }
  proof (rule validI)
    fix n s0 L accC T C v s1 Y Z
    assume valid-A: ∀t∈A. G⊨n::t
    assume conf: s0::≼(G,L)
```

```
    assume wt: normal s0 ⟹ (|prg=G,cls=accC,lcl=L|)⊢t::T
    assume da: normal s0
               ⟹ (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0)) »t» C
    assume eval: G⊢s0 −t≻−n→ (v, s1)
    assume P: P Y s0 Z
    show Q v s1 Z ∧ s1::≼(G, L)
    proof −
      from valid-A conf wt da eval P con
      have Q v s1 Z
        apply (simp add: ax-valids2-def triple-valid2-def2)
        apply (tactic smp-tac context 3 1)
        apply clarify
        apply (tactic smp-tac context 1 1)
        apply (erule allE,erule allE, erule mp)
        apply (intro strip)
        apply (tactic smp-tac context 3 1)
        apply (tactic smp-tac context 2 1)
        apply (tactic smp-tac context 1 1)
        by blast
      moreover have s1::≼(G, L)
      proof (cases normal s0)
        case True
        from eval wt [OF True] da [OF True] conf wf
        show ?thesis
          by (rule evaln-type-sound [elim-format]) simp
      next
        case False
        with eval have s1=s0
          by auto
        with conf show ?thesis by simp
      qed
      ultimately show ?thesis ..
    qed
  qed
next
  case (hazard A P t Q)
  show G,A|⊨::{ {P ∧. Not ∘ type-ok G t} t≻ {Q} }
    by (simp add: ax-valids2-def triple-valid2-def2 type-ok-def) fast
next
  case (Abrupt A P t)
  show G,A|⊨::{ {P←undefined3 t ∧. Not ∘ normal} t≻ {P} }
  proof (rule validI)
    fix n s0 L accC T C v s1 Y Z
    assume conf-s0: s0::≼(G, L)
    assume eval: G⊢s0 −t≻−n→ (v, s1)
    assume (P←undefined3 t ∧. Not ∘ normal) Y s0 Z
    then obtain P: P (undefined3 t) s0 Z and abrupt-s0: ¬ normal s0
      by simp
    from eval abrupt-s0 obtain s1=s0 and v=undefined3 t
      by auto
    with P conf-s0
    show P v s1 Z ∧ s1::≼(G, L)
      by simp
  qed
next
  case (LVar A P vn)
  show G,A|⊨::{ {Normal (λs.. P←In2 (lvar vn s))} LVar vn=≻ {P} }
  proof (rule valid-var-NormalI)
    fix n s0 L accC T C vf s1 Y Z
```

```
      assume conf-s0: s0::≼(G, L)
      assume normal-s0: normal s0
      assume wt: ⦇prg = G, cls = accC, lcl = L⦈⊢LVar vn::=T
      assume da: ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals (store s0)) »⟨LVar vn⟩v» C
      assume eval: G⊢s0 −LVar vn=≻vf−n→ s1
      assume P: (Normal (λs.. P←In2 (lvar vn s))) Y s0 Z
      show P (In2 vf) s1 Z ∧ s1::≼(G, L)
      proof
        from eval normal-s0 obtain s1=s0 vf=lvar vn (store s0)
          by (fastforce elim: evaln-elim-cases)
        with P show P (In2 vf) s1 Z
          by simp
      next
        from eval wt da conf-s0 wf
        show s1::≼(G, L)
          by (rule evaln-type-sound [elim-format]) simp
      qed
    qed
  next
    case (FVar A P statDeclC Q e stat fn R accC)
    note valid-init = ‹G,A|⊨::{ {Normal P} .Init statDeclC. {Q} }›
    note valid-e = ‹G,A|⊨::{ {Q} e−≻ {λVal:a:. fvar statDeclC stat fn a ..; R} }›
    show G,A|⊨::{ {Normal P} {accC,statDeclC,stat}e..fn=≻ {R} }
    proof (rule valid-var-NormalI)
      fix n s0 L accC' T V vf s3 Y Z
      assume valid-A: ∀ t∈A. G⊨n::t
      assume conf-s0:  s0::≼(G,L)
      assume normal-s0: normal s0
      assume wt: ⦇prg=G,cls=accC',lcl=L⦈⊢{accC,statDeclC,stat}e..fn::=T
      assume da: ⦇prg=G,cls=accC',lcl=L⦈
                   ⊢ dom (locals (store s0)) »⟨{accC,statDeclC,stat}e..fn⟩v» V
      assume eval: G⊢s0 −{accC,statDeclC,stat}e..fn=≻vf−n→ s3
      assume P: (Normal P) Y s0 Z
      show R ⌊vf⌋v s3 Z ∧ s3::≼(G, L)
      proof −
        from wt obtain statC f where
          wt-e: ⦇prg=G, cls=accC, lcl=L⦈⊢e::−Class statC and
          accfield: accfield G accC statC fn = Some (statDeclC,f) and
          eq-accC: accC=accC' and
          stat: stat=is-static f and
          T: T=(type f)
          by (cases) (auto simp add: member-is-static-simp)
        from da eq-accC
        have da-e: ⦇prg=G, cls=accC, lcl=L⦈⊢dom (locals (store s0))»⟨e⟩e» V
          by cases simp
        from eval obtain a s1 s2 s2' where
          eval-init: G⊢s0 −Init statDeclC−n→ s1 and
          eval-e: G⊢s1 −e−≻a−n→ s2 and
          fvar: (vf,s2')=fvar statDeclC stat fn a s2 and
          s3: s3 = check-field-access G accC statDeclC fn stat a s2'
          using normal-s0 by (fastforce elim: evaln-elim-cases)
        have wt-init: ⦇prg=G, cls=accC, lcl=L⦈⊢(Init statDeclC)::√
        proof −
          from wf wt-e
          have iscls-statC: is-class G statC
            by (auto dest: ty-expr-is-type type-is-class)
          with wf accfield
          have iscls-statDeclC: is-class G statDeclC
            by (auto dest!: accfield-fields dest: fields-declC)
```

thus *?thesis* **by** *simp*
qed
obtain *I* **where**
da-init: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\vdash$ *dom* (*locals* (*store s0*)) »⟨*Init statDeclC*⟩$_s$» *I*
by (*auto intro*: *da-Init* [*simplified*] *assigned.select-convs*)
from *valid-init P valid-A conf-s0 eval-init wt-init da-init*
obtain *Q*: *Q* $\diamondsuit$ *s1 Z* **and** *conf-s1*: *s1*::$\preceq$(*G*, *L*)
by (*rule validE*)
obtain
R: *R* $\lfloor vf\rfloor_v$ *s2′ Z* **and**
conf-s2: *s2*::$\preceq$(*G*, *L*) **and**
conf-a: *normal s2* $\longrightarrow$ *G*,*store s2*$\vdash$*a*::$\preceq$*Class statC*
proof (*cases normal s1*)
case *True*
obtain *V′* **where**
da-e′:
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$ $\vdash$*dom* (*locals* (*store s1*))»⟨*e*⟩$_e$» *V′*
proof −
from *eval-init*
have (*dom* (*locals* (*store s0*))) $\subseteq$ (*dom* (*locals* (*store s1*)))
by (*rule dom-locals-evaln-mono-elim*)
with *da-e* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *valid-e Q valid-A conf-s1 eval-e wt-e*
obtain *R* $\lfloor vf\rfloor_v$ *s2′ Z* **and** *s2*::$\preceq$(*G*, *L*)
by (*rule validE*) (*simp add*: *fvar* [*symmetric*])
moreover
from *eval-e wt-e da-e′ conf-s1 wf*
have *normal s2* $\longrightarrow$ *G*,*store s2*$\vdash$*a*::$\preceq$*Class statC*
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
next
case *False*
with *valid-e Q valid-A conf-s1 eval-e*
obtain *R* $\lfloor vf\rfloor_v$ *s2′ Z* **and** *s2*::$\preceq$(*G*, *L*)
by (*cases rule*: *validE*) (*simp add*: *fvar* [*symmetric*])+
moreover from *False eval-e* **have** ¬ *normal s2*
by *auto*
hence *normal s2* $\longrightarrow$ *G*,*store s2*$\vdash$*a*::$\preceq$*Class statC*
by *auto*
ultimately show *?thesis* **..**
qed
from *accfield wt-e eval-init eval-e conf-s2 conf-a fvar stat s3 wf*
have *eq-s3-s2′*: *s3*=*s2′*
using *normal-s0* **by** (*auto dest*!: *error-free-field-access evaln-eval*)
moreover
from *eval wt da conf-s0 wf*
have *s3*::$\preceq$(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **using** *Q R* **by** *simp*
qed
qed
next
case (*AVar A P e1 Q e2 R*)
note *valid-e1* = ‹*G*,*A*|$\models$::{ {*Normal P*} *e1*$-\succ$ {*Q*} }›
have *valid-e2*: $\bigwedge$ *a*. *G*,*A*|$\models$::{ {*Q*$\leftarrow$*In1 a*} *e2*$-\succ$ {λ*Val*:*i*:. *avar G i a* ..; *R*} }
using *AVar.hyps* **by** *simp*

```
show G,A|⊨::{ {Normal P} e1.[e2]=≻ {R} }
proof (rule valid-var-NormalI)
  fix n s0 L accC T V vf s2′ Y Z
  assume valid-A: ∀ t∈A. G⊨n::t
  assume conf-s0: s0::≼(G,L)
  assume normal-s0: normal s0
  assume wt: (|prg=G,cls=accC,lcl=L|)⊢e1.[e2]::=T
  assume da: (|prg=G,cls=accC,lcl=L|)
              ⊢ dom (locals (store s0)) »⟨e1.[e2]⟩ᵥ» V
  assume eval: G⊢s0 −e1.[e2]=≻vf−n→ s2′
  assume P: (Normal P) Y s0 Z
  show R ⌊vf⌋ᵥ s2′ Z ∧ s2′::≼(G, L)
  proof −
    from wt obtain
      wt-e1: (|prg=G,cls=accC,lcl=L|)⊢e1::−T.[] and
      wt-e2: (|prg=G,cls=accC,lcl=L|)⊢e2::−PrimT Integer
      by (rule wt-elim-cases) simp
    from da obtain E1 where
      da-e1: (|prg=G,cls=accC,lcl=L|) ⊢dom (locals (store s0))»⟨e1⟩ₑ» E1 and
      da-e2: (|prg=G,cls=accC,lcl=L|)⊢ nrm E1 »⟨e2⟩ₑ» V
      by (rule da-elim-cases) simp
    from eval obtain s1 a i s2 where
      eval-e1: G⊢s0 −e1−≻a−n→ s1 and
      eval-e2: G⊢s1 −e2−≻i−n→ s2 and
      avar: avar G i a s2 =(vf, s2′)
      using normal-s0 by (fastforce elim: evaln-elim-cases)
    from valid-e1 P valid-A conf-s0 eval-e1 wt-e1 da-e1
    obtain Q: Q ⌊a⌋ₑ s1 Z and conf-s1: s1::≼(G, L)
      by (rule validE)
    from Q have Q′: ⋀ v. (Q←In1 a) v s1 Z
      by simp
    have R ⌊vf⌋ᵥ s2′ Z
    proof (cases normal s1)
      case True
      obtain V′ where
        (|prg=G,cls=accC,lcl=L|) ⊢dom (locals (store s1))»⟨e2⟩ₑ» V′
      proof −
        from eval-e1 wt-e1 da-e1 wf True
        have nrm E1 ⊆ dom (locals (store s1))
          by (cases rule: da-good-approx-evalnE) iprover
        with da-e2 show thesis
          by (rule da-weakenE) (rule that)
      qed
      with valid-e2 Q′ valid-A conf-s1 eval-e2 wt-e2
      show ?thesis
        by (rule validE) (simp add: avar)
    next
      case False
      with valid-e2 Q′ valid-A conf-s1 eval-e2
      show ?thesis
        by (cases rule: validE) (simp add: avar)+
    qed
    moreover
    from eval wt da conf-s0 wf
    have s2′::≼(G, L)
      by (rule evaln-type-sound [elim-format]) simp
    ultimately show ?thesis ..
  qed
qed
```

next
case (*NewC A P C Q*)
note *valid-init* = ‹*G*,*A*|⊨::{ {*Normal P*} .*Init C*. {*Alloc G* (*CInst C*) *Q*} }›
show *G*,*A*|⊨::{ {*Normal P*} *NewC C*−≻ {*Q*} }
proof (*rule valid-expr-NormalI*)
fix *n s0 L accC T E v s2 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*NewC C*::−*T*
assume *da*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ *dom* (*locals* (*store s0*)) »$\langle$*NewC C*$\rangle_e$» *E*
assume *eval*: *G*⊢*s0* −*NewC C*−≻*v*−*n*→ *s2*
assume *P*: (*Normal P*) *Y s0 Z*
show *Q* $\lfloor v\rfloor_e$ *s2 Z* ∧ *s2*::≼(*G*, *L*)
proof −
from *wt* **obtain** *is-cls-C*: *is-class G C*
by (*rule wt-elim-cases*) (*auto dest*: *is-acc-classD*)
hence *wt-init*: ⦇*prg*=*G*, *cls*=*accC*, *lcl*=*L*⦈⊢*Init C*::√
by *auto*
obtain *I* **where**
da-init: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *dom* (*locals* (*store s0*)) »$\langle$*Init C*$\rangle_s$» *I*
by (*auto intro*: *da-Init* [*simplified*] *assigned.select-convs*)
from *eval* **obtain** *s1 a* **where**
eval-init: *G*⊢*s0* −*Init C*−*n*→ *s1* **and**
alloc: *G*⊢*s1* −*halloc CInst C*≻*a*→ *s2* **and**
v: *v*=*Addr a*
using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)
from *valid-init P valid-A conf-s0 eval-init wt-init da-init*
obtain (*Alloc G* (*CInst C*) *Q*) ♢ *s1 Z*
by (*rule validE*)
with *alloc v* **have** *Q* $\lfloor v\rfloor_e$ *s2 Z*
by *simp*
moreover
from *eval wt da conf-s0 wf*
have *s2*::≼(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*NewA A P T Q e R*)
note *valid-init* = ‹*G*,*A*|⊨::{ {*Normal P*} .*init-comp-ty T*. {*Q*} }›
note *valid-e* = ‹*G*,*A*|⊨::{ {*Q*} *e*−≻ {λ*Val*:*i*:. *abupd* (*check-neg i*) .;
Alloc G (*Arr T* (*the-Intg i*)) *R*}}›
show *G*,*A*|⊨::{ {*Normal P*} *New T*[*e*]−≻ {*R*} }
proof (*rule valid-expr-NormalI*)
fix *n s0 L accC arrT E v s3 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*New T*[*e*]::−*arrT*
assume *da*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*)) »$\langle$*New T*[*e*]$\rangle_e$» *E*
assume *eval*: *G*⊢*s0* −*New T*[*e*]−≻*v*−*n*→ *s3*
assume *P*: (*Normal P*) *Y s0 Z*
show *R* $\lfloor v\rfloor_e$ *s3 Z* ∧ *s3*::≼(*G*, *L*)
proof −
from *wt* **obtain**
wt-init: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*init-comp-ty T*::√ **and**

```
        wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−PrimT Integer
        by (rule wt-elim-cases) (auto intro: wt-init-comp-ty )
      from da obtain
        da-e:(|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨e⟩e» E
        by cases simp
      from eval obtain s1 i s2 a where
        eval-init: G⊢s0 −init-comp-ty T−n→ s1 and
        eval-e: G⊢s1 −e−≻i−n→ s2 and
        alloc: G⊢abupd (check-neg i) s2 −halloc Arr T (the-Intg i)≻a→ s3 and
        v: v=Addr a
        using normal-s0 by (fastforce elim: evaln-elim-cases)
      obtain I where
        da-init:
        (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0)) »⟨init-comp-ty T⟩s» I
      proof (cases ∃ C. T = Class C)
        case True
        thus ?thesis
          by − (rule that, (auto intro: da-Init [simplified]
                                  assigned.select-convs
                         simp add: init-comp-ty-def))

      next
        case False
        thus ?thesis
          by − (rule that, (auto intro: da-Skip [simplified]
                                 assigned.select-convs
                       simp add: init-comp-ty-def))

      qed
      with valid-init P valid-A conf-s0 eval-init wt-init
      obtain Q: Q ♢ s1 Z and conf-s1: s1::≼(G, L)
        by (rule validE)
      obtain E' where
       (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨e⟩e» E'
      proof −
        from eval-init
        have dom (locals (store s0)) ⊆ dom (locals (store s1))
          by (rule dom-locals-evaln-mono-elim)
        with da-e show thesis
          by (rule da-weakenE) (rule that)
      qed
      with valid-e Q valid-A conf-s1 eval-e wt-e
      have (λVal:i:. abupd (check-neg i) .;
                   Alloc G (Arr T (the-Intg i)) R) ⌊i⌋e s2 Z
        by (rule validE)
      with alloc v have R ⌊v⌋e s3 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s3::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Cast A P e T Q)
  note valid-e = ‹G,A|⊨::{ {Normal P} e−≻
               {λVal:v:. λs.. abupd (raise-if (¬ G,s⊢v fits T) ClassCast) .;
                Q←In1 v} }›
```

```
  show G,A|⊨::{ {Normal P} Cast T e−≻ {Q} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC castT E v s2 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Cast T e::−castT
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0)) »⟨Cast T e⟩e» E
    assume eval: G⊢s0 −Cast T e−≻v−n→ s2
    assume P: (Normal P) Y s0 Z
    show Q ⌊v⌋e s2 Z ∧ s2::≼(G, L)
    proof −
      from wt obtain eT where
        wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT
        by cases simp
      from da obtain
        da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨e⟩e» E
        by cases simp
      from eval obtain s1 where
        eval-e: G⊢s0 −e−≻v−n→ s1 and
        s2: s2 = abupd (raise-if (¬ G,snd s1⊢v fits T) ClassCast) s1
        using normal-s0 by (fastforce elim: evaln-elim-cases)
      from valid-e P valid-A conf-s0 eval-e wt-e da-e
      have (λVal:v:. λs.. abupd (raise-if (¬ G,s⊢v fits T) ClassCast) .;
                  Q←In1 v) ⌊v⌋e s1 Z
        by (rule validE)
      with s2 have Q ⌊v⌋e s2 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s2::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Inst A P e Q T)
  assume valid-e: G,A|⊨::{ {Normal P} e−≻
             {λVal:v:. λs.. Q←In1 (Bool (v ≠ Null ∧ G,s⊢v fits RefT T))} }
  show G,A|⊨::{ {Normal P} e InstOf T−≻ {Q} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC instT E v s1 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢e InstOf T::−instT
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨e InstOf T⟩e» E
    assume eval: G⊢s0 −e InstOf T−≻v−n→ s1
    assume P: (Normal P) Y s0 Z
    show Q ⌊v⌋e s1 Z ∧ s1::≼(G, L)
    proof −
      from wt obtain eT where
        wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT
        by cases simp
      from da obtain
        da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨e⟩e» E
        by cases simp
      from eval obtain a where
        eval-e: G⊢s0 −e−≻a−n→ s1 and
```

```
      v: v = Bool (a ≠ Null ∧ G,store s1⊢a fits RefT T)
      using normal-s0 by (fastforce elim: evaln-elim-cases)
    from valid-e P valid-A conf-s0 eval-e wt-e da-e
    have (λVal:v:. λs.. Q←In1 (Bool (v ≠ Null ∧ G,s⊢v fits RefT T)))
           ⌊a⌋_e s1 Z
      by (rule validE)
    with v have Q ⌊v⌋_e s1 Z
      by simp
    moreover
    from eval wt da conf-s0 wf
    have s1::≼(G, L)
      by (rule evaln-type-sound [elim-format]) simp
    ultimately show ?thesis ..
  qed
 qed
next
 case (Lit A P v)
 show G,A|⊨::{ {Normal (P←In1 v)} Lit v−≻ {P} }
 proof (rule valid-expr-NormalI)
  fix n L s0 s1 v′ Y Z
  assume conf-s0: s0::≼(G, L)
  assume normal-s0: normal s0
  assume eval: G⊢s0 −Lit v−≻v′−n→ s1
  assume P: (Normal (P←In1 v)) Y s0 Z
  show P ⌊v′⌋_e s1 Z ∧ s1::≼(G, L)
  proof −
    from eval have s1=s0 and v′=v
      using normal-s0 by (auto elim: evaln-elim-cases)
    with P conf-s0 show ?thesis by simp
  qed
 qed
next
 case (UnOp A P e Q unop)
 assume valid-e: G,A|⊨::{ {Normal P}e−≻{λVal:v:. Q←In1 (eval-unop unop v)} }
 show G,A|⊨::{ {Normal P} UnOp unop e−≻ {Q} }
 proof (rule valid-expr-NormalI)
  fix n s0 L accC T E v s1 Y Z
  assume valid-A: ∀t∈A. G⊨n::t
  assume conf-s0: s0::≼(G,L)
  assume normal-s0: normal s0
  assume wt: ⦇prg=G,cls=accC,lcl=L⦈⊢UnOp unop e::−T
  assume da: ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0))»⟨UnOp unop e⟩_e»E
  assume eval: G⊢s0 −UnOp unop e−≻v−n→ s1
  assume P: (Normal P) Y s0 Z
  show Q ⌊v⌋_e s1 Z ∧ s1::≼(G, L)
  proof −
    from wt obtain eT where
      wt-e: ⦇prg = G, cls = accC, lcl = L⦈⊢e::−eT
      by cases simp
    from da obtain
      da-e: ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals (store s0)) »⟨e⟩_e» E
      by cases simp
    from eval obtain ve where
      eval-e: G⊢s0 −e−≻ve−n→ s1 and
      v: v = eval-unop unop ve
      using normal-s0 by (fastforce elim: evaln-elim-cases)
    from valid-e P valid-A conf-s0 eval-e wt-e da-e
    have (λVal:v:. Q←In1 (eval-unop unop v)) ⌊ve⌋_e s1 Z
      by (rule validE)
```

```
      with v have Q ⌊v⌋e s1 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s1::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (BinOp A P e1 Q binop e2 R)
  assume valid-e1: G,A|⊨::{ {Normal P} e1−≻ {Q} }
  have valid-e2: ⋀ v1.  G,A|⊨::{ {Q←In1 v1}
              (if need-second-arg binop v1 then In1l e2 else In1r Skip)≻
              {λVal:v2:. R←In1 (eval-binop binop v1 v2)} }
    using BinOp.hyps by simp
  show G,A|⊨::{ {Normal P} BinOp binop e1 e2−≻ {R} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s2 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢BinOp binop e1 e2::−T
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0)) »⟨BinOp binop e1 e2⟩e» E
    assume eval: G⊢s0 −BinOp binop e1 e2−≻v−n→ s2
    assume P: (Normal P) Y s0 Z
    show R ⌊v⌋e s2 Z ∧ s2::≼(G, L)
    proof −
      from wt obtain e1T e2T where
        wt-e1: (|prg=G,cls=accC,lcl=L|)⊢e1::−e1T and
        wt-e2: (|prg=G,cls=accC,lcl=L|)⊢e2::−e2T and
        wt-binop: wt-binop G binop e1T e2T
        by cases simp
      have wt-Skip: (|prg = G, cls = accC, lcl = L|)⊢Skip::√
        by simp

      from da obtain E1 where
        da-e1: (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨e1⟩e» E1
        by cases simp+
      from eval obtain v1 s1 v2 where
        eval-e1: G⊢s0 −e1−≻v1−n→ s1 and
        eval-e2: G⊢s1 −(if need-second-arg binop v1 then ⟨e2⟩e else ⟨Skip⟩s)
                      ≻−n→ (⌊v2⌋e, s2) and
        v: v=eval-binop binop v1 v2
        using normal-s0 by (fastforce elim: evaln-elim-cases)
      from valid-e1 P valid-A conf-s0 eval-e1 wt-e1 da-e1
      obtain Q: Q ⌊v1⌋e s1 Z and conf-s1: s1::≼(G,L)
        by (rule validE)
      from Q have Q': ⋀ v. (Q←In1 v1) v s1 Z
        by simp
      have (λVal:v2:. R←In1 (eval-binop binop v1 v2)) ⌊v2⌋e s2 Z
      proof (cases normal s1)
        case True
        from eval-e1 wt-e1 da-e1 conf-s0 wf
        have conf-v1: G,store s1⊢v1::≼e1T
          by (rule evaln-type-sound [elim-format]) (insert True,simp)
        from eval-e1
        have G⊢s0 −e1−≻v1→ s1
```

by (*rule evaln-eval*)
from *da wt-e1 wt-e2 wt-binop conf-s0 True this conf-v1 wf*
obtain *E2* **where**
da-e2: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢ *dom* (*locals* (*store s1*))
»(*if need-second-arg binop v1 then* ⟨*e2*⟩$_e$ *else* ⟨*Skip*⟩$_s$)» *E2*
by (*rule da-e2-BinOp* [*elim-format*]) *iprover*
from *wt-e2 wt-Skip* **obtain** *T2*
where (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢(*if need-second-arg binop v1 then* ⟨*e2*⟩$_e$ *else* ⟨*Skip*⟩$_s$)::*T2*
by (*cases need-second-arg binop v1*) *auto*
note *ve*=*validE* [*OF valid-e2*,*OF Q′ valid-A conf-s1 eval-e2 this da-e2*]

thus *?thesis*
by (*rule ve*)
next
case *False*
note *ve*=*validE* [*OF valid-e2*,*OF Q′ valid-A conf-s1 eval-e2*]
with *False* **show** *?thesis*
by *iprover*
qed
with *v* **have** *R* ⌊*v*⌋$_e$ *s2 Z*
by *simp*
moreover
from *eval wt da conf-s0 wf*
have *s2*::⪯(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*Super A P*)
show *G*,*A*|⊨::{ {*Normal* (λ*s*.. *P*←*In1* (*val-this s*))} *Super*−≻ {*P*} }
proof (*rule valid-expr-NormalI*)
fix *n L s0 s1 v Y Z*
assume *conf-s0*: *s0*::⪯(*G*, *L*)
assume *normal-s0*: *normal s0*
assume *eval*: *G*⊢*s0* −*Super*−≻*v*−*n*→ *s1*
assume *P*: (*Normal* (λ*s*.. *P*←*In1* (*val-this s*))) *Y s0 Z*
show *P* ⌊*v*⌋$_e$ *s1 Z* ∧ *s1*::⪯(*G*, *L*)
proof −
from *eval* **have** *s1*=*s0* **and** *v*=*val-this* (*store s0*)
using *normal-s0* **by** (*auto elim*: *evaln-elim-cases*)
with *P conf-s0* **show** *?thesis* **by** *simp*
qed
qed
next
case (*Acc A P var Q*)
note *valid-var* = ‹*G*,*A*|⊨::{ {*Normal P*} *var*=≻ {λ*Var*:(*v*, *f*):. *Q*←*In1 v*} }›
show *G*,*A*|⊨::{ {*Normal P*} *Acc var*−≻ {*Q*} }
proof (*rule valid-expr-NormalI*)
fix *n s0 L accC T E v s1 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*|⊨*n*::*t*
assume *conf-s0*: *s0*::⪯(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*Acc var*::−*T*
assume *da*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*dom* (*locals* (*store s0*))»⟨*Acc var*⟩$_e$»*E*
assume *eval*: *G*⊢*s0* −*Acc var*−≻*v*−*n*→ *s1*
assume *P*: (*Normal P*) *Y s0 Z*
show *Q* ⌊*v*⌋$_e$ *s1 Z* ∧ *s1*::⪯(*G*, *L*)

```
    proof -
      from wt obtain
        wt-var: (|prg=G,cls=accC,lcl=L|)⊢var::=T
        by cases simp
      from da obtain V where
        da-var: (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨var⟩_v» V
        by (cases ∃ n. var=LVar n) (insert da.LVar,auto elim!: da-elim-cases)
      from eval obtain upd where
        eval-var: G⊢s0 -var=≻(v, upd)-n→ s1
        using normal-s0 by (fastforce elim: evaln-elim-cases)
      from valid-var P valid-A conf-s0 eval-var wt-var da-var
      have (λVar:(v, f):. Q←In1 v) ⌊(v, upd)⌋_v s1 Z
        by (rule validE)
      then have Q ⌊v⌋_e s1 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s1::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Ass A P var Q e R)
  note valid-var = ‹G,A|⊨::{ {Normal P} var=≻ {Q} }›
  have valid-e: ⋀ vf.
                  G,A|⊨::{ {Q←In2 vf} e-≻ {λVal:v:. assign (snd vf) v .; R} }
    using Ass.hyps by simp
  show G,A|⊨::{ {Normal P} var:=e-≻ {R} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s3 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢var:=e::-T
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨var:=e⟩_e»E
    assume eval: G⊢s0 -var:=e-≻v-n→ s3
    assume P: (Normal P) Y s0 Z
    show R ⌊v⌋_e s3 Z ∧ s3::≼(G, L)
    proof -
      from wt obtain varT  where
        wt-var: (|prg=G,cls=accC,lcl=L|)⊢var::=varT and
        wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::-T
        by cases simp
      from eval obtain w upd s1 s2 where
        eval-var: G⊢s0 -var=≻(w, upd)-n→ s1 and
        eval-e: G⊢s1 -e-≻v-n→ s2 and
        s3: s3=assign upd v s2
        using normal-s0 by (auto elim: evaln-elim-cases)
      have R ⌊v⌋_e s3 Z
      proof (cases ∃ vn. var = LVar vn)
        case False
        with da obtain V where
          da-var: (|prg=G,cls=accC,lcl=L|)
                    ⊢ dom (locals (store s0)) »⟨var⟩_v» V and
          da-e:   (|prg=G,cls=accC,lcl=L|) ⊢ nrm V »⟨e⟩_e» E
          by cases simp+
        from valid-var P valid-A conf-s0 eval-var wt-var da-var
        obtain Q: Q ⌊(w,upd)⌋_v s1 Z and conf-s1: s1::≼(G,L)
```

```
      by (rule validE)
    hence Q': ⋀ v. (Q←In2 (w,upd)) v s1 Z
      by simp
    have (λVal:v:. assign (snd (w,upd)) v .; R) ⌊v⌋_e s2 Z
    proof (cases normal s1)
      case True
      obtain E' where
        da-e': ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals (store s1)) »⟨e⟩_e» E'
      proof −
        from eval-var wt-var da-var wf True
        have nrm V ⊆  dom (locals (store s1))
          by (cases rule: da-good-approx-evalnE) iprover
        with da-e show thesis
          by (rule da-weakenE) (rule that)
      qed
      note ve=validE [OF valid-e,OF Q' valid-A conf-s1 eval-e wt-e da-e']
      show ?thesis
        by (rule ve)
    next
      case False
      note ve=validE [OF valid-e,OF Q' valid-A conf-s1 eval-e]
      with False show ?thesis
        by iprover
    qed
    with s3 show R ⌊v⌋_e s3 Z
      by simp
  next
    case True
    then obtain vn where
      vn: var = LVar vn
      by auto
    with da obtain E where
        da-e:   ⦇prg=G,cls=accC,lcl=L⦈ ⊢ dom (locals (store s0)) »⟨e⟩_e» E
      by cases simp+
    from da.LVar vn obtain  V where
      da-var: ⦇prg=G,cls=accC,lcl=L⦈
              ⊢ dom (locals (store s0)) »⟨var⟩_v» V
      by auto
    from valid-var P valid-A conf-s0 eval-var wt-var da-var
    obtain Q: Q ⌊(w,upd)⌋_v s1 Z and conf-s1: s1::≼(G,L)
      by (rule validE)
    hence Q': ⋀ v. (Q←In2 (w,upd)) v s1 Z
      by simp
    have (λVal:v:. assign (snd (w,upd)) v .; R) ⌊v⌋_e s2 Z
    proof (cases normal s1)
      case True
      obtain E' where
        da-e': ⦇prg=G,cls=accC,lcl=L⦈
               ⊢ dom (locals (store s1)) »⟨e⟩_e» E'
      proof −
        from eval-var
        have dom (locals (store s0)) ⊆ dom (locals (store (s1)))
          by (rule dom-locals-evaln-mono-elim)
        with da-e show thesis
          by (rule da-weakenE) (rule that)
      qed
      note ve=validE [OF valid-e,OF Q' valid-A conf-s1 eval-e wt-e da-e']
      show ?thesis
        by (rule ve)
```

next
case *False*
note *ve=validE* [*OF valid-e*,*OF Q′ valid-A conf-s1 eval-e*]
with *False* **show** *?thesis*
by *iprover*
qed
with *s3* **show** $R \lfloor v \rfloor_e$ *s3 Z*
by *simp*
qed
moreover
from *eval wt da conf-s0 wf*
have *s3*::≼(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*Cond A P e0 P′ e1 e2 Q*)
note *valid-e0* = ‹*G*,*A*|⊨::{ {*Normal P*} *e0*−≻ {*P′*} }›
have *valid-then-else*:⋀ *b*. *G*,*A*|⊨::{ {*P′*←=*b*} (*if b then e1 else e2*)−≻ {*Q*} }
using *Cond.hyps* **by** *simp*
show *G*,*A*|⊨::{ {*Normal P*} *e0 ? e1* : *e2*−≻ {*Q*} }
proof (*rule valid-expr-NormalI*)
fix *n s0 L accC T E v s2 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*e0 ? e1* : *e2*::−*T*
assume *da*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*dom* (*locals* (*store s0*))»$\langle$*e0 ? e1*:*e2*$\rangle_e$»*E*
assume *eval*: *G*⊢*s0* −*e0 ? e1* : *e2*−≻*v*−*n*→ *s2*
assume *P*: (*Normal P*) *Y s0 Z*
show *Q* $\lfloor v \rfloor_e$ *s2 Z* ∧ *s2*::≼(*G*, *L*)
proof −
from *wt* **obtain** *T1 T2* **where**
wt-e0: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*e0*::−*PrimT Boolean* **and**
wt-e1: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*e1*::−*T1* **and**
wt-e2: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*e2*::−*T2*
by *cases simp*
from *da* **obtain** *E0 E1 E2* **where**
da-e0: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢ *dom* (*locals* (*store s0*)) »$\langle$*e0*$\rangle_e$» *E0* **and**
da-e1: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢(*dom* (*locals* (*store s0*)) ∪ *assigns-if True e0*)»$\langle$*e1*$\rangle_e$» *E1* **and**
da-e2: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢(*dom* (*locals* (*store s0*)) ∪ *assigns-if False e0*)»$\langle$*e2*$\rangle_e$» *E2*
by *cases simp+*
from *eval* **obtain** *b s1* **where**
eval-e0: *G*⊢*s0* −*e0*−≻*b*−*n*→ *s1* **and**
eval-then-else: *G*⊢*s1* −(*if the-Bool b then e1 else e2*)−≻*v*−*n*→ *s2*
using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)
from *valid-e0 P valid-A conf-s0 eval-e0 wt-e0 da-e0*
obtain *P′* $\lfloor b \rfloor_e$ *s1 Z* **and** *conf-s1*: *s1*::≼(*G*,*L*)
by (*rule validE*)
hence *P′*: ⋀ *v*. (*P′*←=(*the-Bool b*)) *v s1 Z*
by (*cases normal s1*) *auto*
have *Q* $\lfloor v \rfloor_e$ *s2 Z*
proof (*cases normal s1*)
case *True*
note *normal-s1*=*this*
from *wt-e1 wt-e2* **obtain** *T′* **where**

wt-then-else:
(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢(*if the-Bool b then e1 else e2*)::−*T′*
by (*cases the-Bool b*) *simp*+
have *s0-s1*: *dom* (*locals* (*store s0*))
∪ *assigns-if* (*the-Bool b*) *e0* ⊆ *dom* (*locals* (*store s1*))
proof −
from *eval-e0*
have *eval-e0′*: *G*⊢*s0* −*e0*−≻*b*→ *s1*
by (*rule evaln-eval*)
hence
dom (*locals* (*store s0*)) ⊆ *dom* (*locals* (*store s1*))
by (*rule dom-locals-eval-mono-elim*)
moreover
from *eval-e0′ True wt-e0*
have *assigns-if* (*the-Bool b*) *e0* ⊆ *dom* (*locals* (*store s1*))
by (*rule assigns-if-good-approx′*)
ultimately show *?thesis* **by** (*rule Un-least*)
qed
obtain *E′* **where**
da-then-else:
(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢*dom* (*locals* (*store s1*))»⟨*if the-Bool b then e1 else e2*⟩$_e$» *E′*
proof (*cases the-Bool b*)
case *True*
with *that da-e1 s0-s1* **show** *?thesis*
by *simp* (*erule da-weakenE*,*auto*)
next
case *False*
with *that da-e2 s0-s1* **show** *?thesis*
by *simp* (*erule da-weakenE*,*auto*)
qed
with *valid-then-else P′ valid-A conf-s1 eval-then-else wt-then-else*
show *?thesis*
by (*rule validE*)
next
case *False*
with *valid-then-else P′ valid-A conf-s1 eval-then-else*
show *?thesis*
by (*cases rule*: *validE*) *iprover*+
qed
moreover
from *eval wt da conf-s0 wf*
have *s2*::≼(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*Call A P e Q args R mode statT mn pTs′ S accC′*)
note *valid-e* = ‹*G*,*A*|⊨::{ {*Normal P*} *e*−≻ {*Q*} }›
have *valid-args*: ⋀ *a*. *G*,*A*|⊨::{ {*Q*←*In1 a*} *args*≐≻ {*R a*} }
using *Call.hyps* **by** *simp*
have *valid-methd*: ⋀ *a vs invC declC l*.
G,*A*|⊨::{ {*R a*←*In3 vs* ∧.
(λ*s*. *declC* =
invocation-declclass G mode (*store s*) *a statT*
(|*name* = *mn*, *parTs* = *pTs′*|) ∧
invC = *invocation-class mode* (*store s*) *a statT* ∧
l = *locals* (*store s*)) ;.

```
            init-lvars G declC (|name = mn, parTs = pTs'|) mode a vs ∧.
            (λs. normal s ⟶ G⊢mode→invC≼statT)}
        Methd declC (|name=mn,parTs=pTs'|)−≻ {set-lvars l .; S} }
  using Call.hyps by simp
show G,A|⊨::{ {Normal P} {accC',statT,mode}e·mn( {pTs'}args)−≻ {S} }
proof (rule valid-expr-NormalI)
  fix n s0 L accC T E v s5 Y Z
  assume valid-A: ∀ t∈A. G⊨n::t
  assume conf-s0: s0::≼(G,L)
  assume normal-s0: normal s0
  assume wt: (|prg=G,cls=accC,lcl=L|)⊢{accC',statT,mode}e·mn( {pTs'}args)::−T
  assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))
              »⟨{accC',statT,mode}e·mn( {pTs'}args)⟩ₑ» E
  assume eval: G⊢s0 −{accC',statT,mode}e·mn( {pTs'}args)−≻v−n→ s5
  assume P: (Normal P) Y s0 Z
  show S ⌊v⌋ₑ s5 Z ∧ s5::≼(G, L)
  proof −
    from wt obtain pTs statDeclT statM where
             wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−RefT statT and
          wt-args: (|prg=G,cls=accC,lcl=L|)⊢args::≐pTs and
            statM: max-spec G accC statT (|name=mn,parTs=pTs|)
                     = {((statDeclT,statM),pTs')} and
             mode: mode = invmode statM e and
                T: T =(resTy statM) and
      eq-accC-accC': accC=accC'
      by cases fastforce+
    from da obtain C where
      da-e: (|prg=G,cls=accC,lcl=L|)⊢ (dom (locals (store s0)))»⟨e⟩ₑ» C and
      da-args: (|prg=G,cls=accC,lcl=L|)⊢ nrm C »⟨args⟩ₗ» E
      by cases simp
    from eval eq-accC-accC' obtain a s1 vs s2 s3 s3' s4 invDeclC where
      evaln-e: G⊢s0 −e−≻a−n→ s1 and
      evaln-args: G⊢s1 −args≐≻vs−n→ s2 and
      invDeclC: invDeclC = invocation-declclass
             G mode (store s2) a statT (|name=mn,parTs=pTs'|) and
      s3: s3 = init-lvars G invDeclC (|name=mn,parTs=pTs'|) mode a vs s2 and
      check: s3' = check-method-access G
                        accC' statT mode (|name = mn, parTs = pTs'|) a s3 and
      evaln-methd:
        G⊢s3' −Methd invDeclC  (|name=mn,parTs=pTs'|)−≻v−n→ s4 and
      s5: s5=(set-lvars (locals (store s2))) s4
      using normal-s0 by (auto elim: evaln-elim-cases)

    from evaln-e
    have eval-e: G⊢s0 −e−≻a→ s1
      by (rule evaln-eval)

    from eval-e - wt-e wf
    have s1-no-return: abrupt s1 ≠ Some (Jump Ret)
      by (rule eval-expression-no-jump
               [where ?Env=(|prg=G,cls=accC,lcl=L|),simplified])
         (insert normal-s0,auto)

    from valid-e P valid-A conf-s0 evaln-e wt-e da-e
    obtain Q ⌊a⌋ₑ s1 Z and conf-s1: s1::≼(G,L)
      by (rule validE)
    hence Q: ⋀ v. (Q←In1 a) v s1 Z
      by simp
    obtain
```

```
    R: (R a) ⌊vs⌋l s2 Z and
    conf-s2: s2::≼(G,L) and
    s2-no-return: abrupt s2 ≠ Some (Jump Ret)
  proof (cases normal s1)
    case True
    obtain E' where
      da-args':
      (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨args⟩l» E'
    proof −
      from evaln-e wt-e da-e wf True
      have nrm C ⊆  dom (locals (store s1))
        by (cases rule: da-good-approx-evalnE) iprover
      with da-args show thesis
        by (rule da-weakenE) (rule that)
    qed
    with valid-args Q valid-A conf-s1 evaln-args wt-args
    obtain (R a) ⌊vs⌋l s2 Z s2::≼(G,L)
      by (rule validE)
    moreover
    from evaln-args
    have e: G⊢s1 −args≐≻vs→ s2
      by (rule evaln-eval)
    from this s1-no-return wt-args wf
    have abrupt s2 ≠ Some (Jump Ret)
      by (rule eval-expression-list-no-jump
             [where ?Env=(|prg=G,cls=accC,lcl=L|),simplified])
    ultimately show ?thesis ..
  next
    case False
    with valid-args Q valid-A conf-s1 evaln-args
    obtain (R a) ⌊vs⌋l s2 Z s2::≼(G,L)
      by (cases rule: validE) iprover+
    moreover
    from False evaln-args have s2=s1
      by auto
    with s1-no-return have abrupt s2 ≠ Some (Jump Ret)
      by simp
    ultimately show ?thesis ..
  qed

  obtain invC where
    invC: invC = invocation-class mode (store s2) a statT
    by simp
  with s3
  have invC': invC = (invocation-class mode (store s3) a statT)
    by (cases s2,cases mode) (auto simp add: init-lvars-def2 )
  obtain l where
    l: l = locals (store s2)
    by simp

  from eval wt da conf-s0 wf
  have conf-s5: s5::≼(G, L)
    by (rule evaln-type-sound [elim-format]) simp
  let PROP ?R = ⋀ v.
       (R a←In3 vs ∧.
           (λs. invDeclC = invocation-declclass G mode (store s) a statT
                            (|name = mn, parTs = pTs'|) ∧
                 invC = invocation-class mode (store s) a statT ∧
                    l = locals (store s)) ;.
```

```
          init-lvars G invDeclC (|name = mn, parTs = pTs'|) mode a vs ∧.
          (λs. normal s ⟶ G⊢mode→invC≼statT)
        ) v s3' Z
  {
    assume abrupt-s3: ¬ normal s3
    have S ⌊v⌋e s5 Z
    proof –
      from abrupt-s3 check have eq-s3'-s3: s3'=s3
        by (auto simp add: check-method-access-def Let-def)
      with R s3 invDeclC invC l abrupt-s3
      have R': PROP ?R
        by auto
      have conf-s3': s3'::≼(G, Map.empty)

      proof –
        from s2-no-return s3
        have abrupt s3 ≠ Some (Jump Ret)
          by (cases s2) (auto simp add: init-lvars-def2 split: if-split-asm)
        moreover
        obtain abr2 str2 where s2: s2=(abr2,str2)
          by (cases s2)
        from s3 s2 conf-s2 have (abrupt s3,str2)::≼(G, L)
          by (auto simp add: init-lvars-def2 split: if-split-asm)
        ultimately show ?thesis
          using s3 s2 eq-s3'-s3
          apply (simp add: init-lvars-def2)
          apply (rule conforms-set-locals [OF - wlconf-empty])
          by auto
      qed
      from valid-methd R' valid-A conf-s3' evaln-methd abrupt-s3 eq-s3'-s3
      have (set-lvars l .; S) ⌊v⌋e s4 Z
        by (cases rule: validE) simp+
      with s5 l show ?thesis
        by simp
    qed
  } note abrupt-s3-lemma = this

  have S ⌊v⌋e s5 Z
  proof (cases normal s2)
    case False
    with s3 have abrupt-s3: ¬ normal s3
      by (cases s2) (simp add: init-lvars-def2)
    thus ?thesis
      by (rule abrupt-s3-lemma)
  next
    case True
    note normal-s2 = this
    with evaln-args
    have normal-s1: normal s1
      by (rule evaln-no-abrupt)
    obtain E' where
      da-args':
      (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨args⟩l» E'
    proof –
      from evaln-e wt-e da-e wf normal-s1
      have nrm C ⊆ dom (locals (store s1))
        by (cases rule: da-good-approx-evalnE) iprover
      with da-args show thesis
        by (rule da-weakenE) (rule that)
```

qed
from *evaln-args*
have *eval-args*: *G*⊢*s1* −*args*≐≻*vs*→ *s2*
  by (*rule evaln-eval*)
from *evaln-e wt-e da-e conf-s0 wf*
have *conf-a*: *G*, *store s1*⊢*a*::⪯*RefT statT*
  by (*rule evaln-type-sound* [*elim-format*]) (*insert normal-s1*,*simp*)
with *normal-s1 normal-s2 eval-args*
have *conf-a-s2*: *G*, *store s2*⊢*a*::⪯*RefT statT*
  by (*auto dest*: *eval-gext*)
from *evaln-args wt-args da-args′ conf-s1 wf*
have *conf-args*: *list-all2* (*conf G* (*store s2*)) *vs pTs*
  by (*rule evaln-type-sound* [*elim-format*]) (*insert normal-s2*,*simp*)
from *statM*
obtain
  statM′: (*statDeclT*,*statM*)∈*mheads G accC statT* (|*name*=*mn*,*parTs*=*pTs′*|)
  and
  pTs-widen: *G*⊢*pTs*[⪯]*pTs′*
  by (*blast dest*: *max-spec2mheads*)
show *?thesis*
proof (*cases normal s3*)
  case *False*
  thus *?thesis*
    by (*rule abrupt-s3-lemma*)
next
  case *True*
  note *normal-s3* = *this*
  with *s3* **have** *notNull*: *mode* = *IntVir* ⟶ *a* ≠ *Null*
    by (*cases s2*) (*auto simp add*: *init-lvars-def2*)
  from *conf-s2 conf-a-s2 wf notNull invC*
  have *dynT-prop*: *G*⊢*mode*→*invC*⪯*statT*
    by (*cases s2*) (*auto intro*: *DynT-propI*)

  with *wt-e statM′ invC mode wf*
  obtain *dynM* **where**
    dynM: *dynlookup G statT invC* (|*name*=*mn*,*parTs*=*pTs′*|) = *Some dynM* **and**
    acc-dynM: *G* ⊢*Methd* (|*name*=*mn*,*parTs*=*pTs′*|) *dynM*
                in invC dyn-accessible-from accC
    by (*force dest*!: *call-access-ok*)
  with *invC′ check eq-accC-accC′*
  have *eq-s3′-s3*: *s3′*=*s3*
    by (*auto simp add*: *check-method-access-def Let-def*)

  with *dynT-prop R s3 invDeclC invC l*
  have *R′*: *PROP ?R*
    by *auto*

  from *dynT-prop wf wt-e statM′ mode invC invDeclC dynM*
  obtain
    dynM: *dynlookup G statT invC* (|*name*=*mn*,*parTs*=*pTs′*|) = *Some dynM* **and**
    wf-dynM: *wf-mdecl G invDeclC* ((|*name*=*mn*,*parTs*=*pTs′*|),*mthd dynM*) **and**
      dynM′: *methd G invDeclC* (|*name*=*mn*,*parTs*=*pTs′*|) = *Some dynM* **and**
    iscls-invDeclC: *is-class G invDeclC* **and**
        invDeclC′: *invDeclC* = *declclass dynM* **and**
      invC-widen: *G*⊢*invC*$\preceq_C$ *invDeclC* **and**
     resTy-widen: *G*⊢*resTy dynM*⪯*resTy statM* **and**
    is-static-eq: *is-static dynM* = *is-static statM* **and**
    involved-classes-prop:
     (*if invmode statM e* = *IntVir*

```
    then ∀ statC. statT = ClassT statC ⟶ G⊢invC≼C statC
    else ((∃ statC. statT = ClassT statC ∧ G⊢statC≼C invDeclC) ∨
          (∀ statC. statT ≠ ClassT statC ∧ invDeclC = Object)) ∧
           statDeclT = ClassT invDeclC)
  by (cases rule: DynT-mheadsE) simp
obtain L' where
  L':L'=(λ k.
        (case k of
           EName e
           ⇒ (case e of
                 VNam v
                 ⇒(table-of (lcls (mbody (mthd dynM)))
                    (pars (mthd dynM)[↦]pTs')) v
               | Res ⇒ Some (resTy dynM))
         | This ⇒ if is-static statM
                   then None else Some (Class invDeclC)))
  by simp
from wf-dynM [THEN wf-mdeclD1, THEN conjunct1] normal-s2 conf-s2 wt-e
  wf eval-args conf-a mode notNull wf-dynM involved-classes-prop
have conf-s3: s3::≼(G,L')
  apply −

  apply (drule conforms-init-lvars [of G invDeclC
        (|name=mn,parTs=pTs'|) dynM store s2 vs pTs abrupt s2
        L statT invC a (statDeclT,statM) e])
  apply (rule wf)
  apply (rule conf-args)
  apply (simp add: pTs-widen)
  apply (cases s2,simp)
  apply (rule dynM')
  apply (force dest: ty-expr-is-type)
  apply (rule invC-widen)
  apply (force dest: eval-gext)
  apply simp
  apply simp
  apply (simp add: invC)
  apply (simp add: invDeclC)
  apply (simp add: normal-s2)
  apply (cases s2, simp add: L' init-lvars-def2 s3
               cong add: lname.case-cong ename.case-cong)
  done
with eq-s3'-s3 have conf-s3': s3'::≼(G,L') by simp
from is-static-eq wf-dynM L'
obtain mthdT where
  (|prg=G,cls=invDeclC,lcl=L'|)
    ⊢Body invDeclC (stmt (mbody (mthd dynM)))::−mthdT and
  mthdT-widen: G⊢mthdT≼resTy dynM
  by − (drule wf-mdecl-bodyD,
       auto simp add: callee-lcl-def
            cong add: lname.case-cong ename.case-cong)
with dynM' iscls-invDeclC invDeclC'
have
  wt-methd:
  (|prg=G,cls=invDeclC,lcl=L'|)
    ⊢(Methd invDeclC (|name = mn, parTs = pTs'|))::−mthdT
  by (auto intro: wt.Methd)
obtain M where
  da-methd:
  (|prg=G,cls=invDeclC,lcl=L'|)
```

$\vdash$ *dom* (*locals* (*store s3'*))
»⟨*Methd invDeclC* (|*name=mn,parTs=pTs'*|)⟩$_e$» *M*
proof –
from *wf-dynM*
obtain *M'* **where**
da-body:
(|*prg=G*, *cls=invDeclC*
,*lcl=callee-lcl invDeclC* (|*name* = *mn*, *parTs* = *pTs'*|) (*mthd dynM*)
|) $\vdash$ *parameters* (*mthd dynM*) »⟨*stmt* (*mbody* (*mthd dynM*))⟩» *M'* **and**
res: *Result* ∈ *nrm M'*
by (*rule wf-mdeclE*) *iprover*
from *da-body is-static-eq L'* **have**
(|*prg=G*, *cls=invDeclC*,*lcl=L'*|)
$\vdash$ *parameters* (*mthd dynM*) »⟨*stmt* (*mbody* (*mthd dynM*))⟩» *M'*
by (*simp add*: *callee-lcl-def*
cong add: *lname.case-cong ename.case-cong*)
moreover have *parameters* (*mthd dynM*) ⊆ *dom* (*locals* (*store s3'*))
proof –
from *is-static-eq*
have (*invmode* (*mthd dynM*) *e*) = (*invmode statM e*)
by (*simp add*: *invmode-def*)
moreover
have *length* (*pars* (*mthd dynM*)) = *length vs*
proof –
from *normal-s2 conf-args*
have *length vs* = *length pTs*
by (*simp add*: *list-all2-iff*)
also from *pTs-widen*
have ... = *length pTs'*
by (*simp add*: *widens-def list-all2-iff*)
also from *wf-dynM*
have ... = *length* (*pars* (*mthd dynM*))
by (*simp add*: *wf-mdecl-def wf-mhead-def*)
finally show *?thesis* **..**
qed
moreover note *s3 dynM' is-static-eq normal-s2 mode*
ultimately
have *parameters* (*mthd dynM*) = *dom* (*locals* (*store s3*))
using *dom-locals-init-lvars*
[*of mthd dynM G invDeclC* (|*name=mn,parTs=pTs'*|) *vs e a s2*]
by *simp*
thus *?thesis* **using** *eq-s3'-s3* **by** *simp*
qed
ultimately obtain *M2* **where**
da:
(|*prg=G*, *cls=invDeclC*,*lcl=L'*|)
$\vdash$ *dom* (*locals* (*store s3'*)) »⟨*stmt* (*mbody* (*mthd dynM*))⟩» *M2* **and**
M2: *nrm M'* ⊆ *nrm M2*
by (*rule da-weakenE*)
from *res M2* **have** *Result* ∈ *nrm M2*
by *blast*
moreover from *wf-dynM*
have *jumpNestingOkS* {*Ret*} (*stmt* (*mbody* (*mthd dynM*)))
by (*rule wf-mdeclE*)
ultimately
obtain *M3* **where**
(|*prg=G*, *cls=invDeclC*,*lcl=L'*|) $\vdash$ *dom* (*locals* (*store s3'*))
»⟨*Body* (*declclass dynM*) (*stmt* (*mbody* (*mthd dynM*)))⟩» *M3*
using *da*

by (*iprover intro*: *da.Body assigned.select-convs*)
from - *this* [*simplified*]
show *thesis*
by (*rule da.Methd* [*simplified*,*elim-format*])
(*auto intro*: *dynM′ that*)
qed
from *valid-methd R′ valid-A conf-s3′ evaln-methd wt-methd da-methd*
have (*set-lvars l* .; *S*) $\lfloor v \rfloor_e$ *s4 Z*
by (*cases rule*: *validE*) *iprover*+
with *s5 l* **show** *?thesis*
by *simp*
qed
qed
with *conf-s5* **show** *?thesis* **by** *iprover*
qed
qed
next
case (*Methd A P Q ms*)
note *valid-body* = ‹*G*,*A* ∪ {{*P*} *Methd*−≻ {*Q*} | *ms*}|⊨::{{*P*} *body G*−≻ {*Q*} | *ms*}›
show *G*,*A*|⊨::{{*P*} *Methd*−≻ {*Q*} | *ms*}
by (*rule Methd-sound*) (*rule Methd.hyps*)
next
case (*Body A P D Q c R*)
note *valid-init* = ‹*G*,*A*|⊨::{ {*Normal P*} .*Init D*. {*Q*} }›
note *valid-c* = ‹*G*,*A*|⊨::{ {*Q*} .*c*.
{λ*s*.. *abupd* (*absorb Ret*) .; *R*←*In1* (*the* (*locals s Result*))} }›
show *G*,*A*|⊨::{ {*Normal P*} *Body D c*−≻ {*R*} }
proof (*rule valid-expr-NormalI*)
fix *n s0 L accC T E v s4 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*Body D c*::−*T*
assume *da*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»$\langle$*Body D c*$\rangle_e$»*E*
assume *eval*: *G*⊢*s0* −*Body D c*−≻*v*−*n*→ *s4*
assume *P*: (*Normal P*) *Y s0 Z*
show *R* $\lfloor v \rfloor_e$ *s4 Z* ∧ *s4*::≼(*G*, *L*)
proof −
from *wt* **obtain**
iscls-D: *is-class G D* **and**
wt-init: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*Init D*::√ **and**
wt-c: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*c*::√
by *cases auto*
obtain *I* **where**
da-init:⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈ ⊢ *dom* (*locals* (*store s0*)) »$\langle$*Init D*$\rangle_s$» *I*
by (*auto intro*: *da-Init* [*simplified*] *assigned.select-convs*)
from *da* **obtain** *C* **where**
da-c: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ (*dom* (*locals* (*store s0*)))»$\langle c \rangle_s$» *C* **and**
jmpOk: *jumpNestingOkS* {*Ret*} *c*
by *cases simp*
from *eval* **obtain** *s1 s2 s3* **where**
eval-init: *G*⊢*s0* −*Init D*−*n*→ *s1* **and**
eval-c: *G*⊢*s1* −*c*−*n*→ *s2* **and**
v: *v* = *the* (*locals* (*store s2*) *Result*) **and**
s3: *s3* =(*if* ∃ *l*. *abrupt s2* = *Some* (*Jump* (*Break l*)) ∨
abrupt s2 = *Some* (*Jump* (*Cont l*))
then abupd (λ*x*. *Some* (*Error CrossMethodJump*)) *s2 else s2*)**and**
s4: *s4* = *abupd* (*absorb Ret*) *s3*
using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)

obtain C' **where**
da-c': $(\!|prg=G,cls=accC,lcl=L|\!)\vdash$ (*dom* (*locals* (*store s1*)))»$\langle c\rangle_s$» C'
proof −
from *eval-init*
have (*dom* (*locals* (*store s0*))) $\subseteq$ (*dom* (*locals* (*store s1*)))
by (*rule dom-locals-evaln-mono-elim*)
with *da-c* **show** *thesis* **by** (*rule da-weakenE*) (*rule that*)
qed
from *valid-init P valid-A conf-s0 eval-init wt-init da-init*
obtain *Q*: *Q* $\diamondsuit$ *s1 Z* **and** *conf-s1*: *s1*::$\preceq$(*G*,*L*)
by (*rule validE*)
from *valid-c Q valid-A conf-s1 eval-c wt-c da-c'*
have *R*: (λ*s*.. *abupd* (*absorb Ret*) .; *R*$\leftarrow$*In1* (*the* (*locals s Result*)))
$\diamondsuit$ *s2 Z*
by (*rule validE*)
have *s3*=*s2*
proof −
from *eval-init* [*THEN evaln-eval*] *wf*
have *s1-no-jmp*: $\bigwedge$ *j*. *abrupt s1* $\neq$ *Some* (*Jump j*)
by − (*rule eval-statement-no-jump* [*OF - - - wt-init*],
insert normal-s0,*auto*)
from *eval-c* [*THEN evaln-eval*] - *wt-c wf*
have $\bigwedge$ *j*. *abrupt s2* = *Some* (*Jump j*) $\Longrightarrow$ *j*=*Ret*
by (*rule jumpNestingOk-evalE*) (*auto intro*: *jmpOk simp add*: *s1-no-jmp*)
moreover note *s3*
ultimately show *?thesis*
by (*force split*: *if-split*)
qed
with *R v s4*
have *R* $\lfloor v\rfloor_e$ *s4 Z*
by *simp*
moreover
from *eval wt da conf-s0 wf*
have *s4*::$\preceq$(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*Nil A P*)
show *G*,*A*$|\!\models$::{ {*Normal* (*P*$\leftarrow\lfloor$[]$\rfloor_l$)} []$\doteq\succ$ {*P*} }
proof (*rule valid-expr-list-NormalI*)
fix *s0 s1 vs n L Y Z*
assume *conf-s0*: *s0*::$\preceq$(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *eval*: *G*$\vdash$*s0* −[]$\doteq\succ$*vs*−*n*$\rightarrow$ *s1*
assume *P*: (*Normal* (*P*$\leftarrow\lfloor$[]$\rfloor_l$)) *Y s0 Z*
show *P* $\lfloor vs\rfloor_l$ *s1 Z* $\wedge$ *s1*::$\preceq$(*G*, *L*)
proof −
from *eval* **obtain** *vs*=[] *s1*=*s0*
using *normal-s0* **by** (*auto elim*: *evaln-elim-cases*)
with *P conf-s0* **show** *?thesis*
by *simp*
qed
qed
next
case (*Cons A P e Q es R*)
note *valid-e* = ‹*G*,*A*$|\!\models$::{ {*Normal P*} *e*−$\succ$ {*Q*} }›
have *valid-es*: $\bigwedge$ *v*. *G*,*A*$|\!\models$::{ {*Q*$\leftarrow\lfloor v\rfloor_e$} *es*$\doteq\succ$ {λ*Vals*:*vs*:. *R*$\leftarrow\lfloor$(*v* # *vs*)$\rfloor_l$} }

```
    using Cons.hyps by simp
  show G,A|⊨::{ {Normal P} e # es≐≻ {R} }
  proof (rule valid-expr-list-NormalI)
    fix n s0 L accC T E v s2 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢e # es::≐T
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0)) »⟨e # es⟩ₗ» E
    assume eval: G⊢s0 −e # es≐≻v−n→ s2
    assume P: (Normal P) Y s0 Z
    show R ⌊v⌋ₗ s2 Z ∧ s2::≼(G, L)
    proof −
      from wt obtain eT esT where
        wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−eT and
        wt-es: (|prg=G,cls=accC,lcl=L|)⊢es::≐esT
        by cases simp
      from da obtain E1 where
        da-e: (|prg=G,cls=accC,lcl=L|)⊢ (dom (locals (store s0)))»⟨e⟩ₑ» E1 and
        da-es: (|prg=G,cls=accC,lcl=L|)⊢ nrm E1 »⟨es⟩ₗ» E
        by cases simp
      from eval obtain s1 ve vs where
        eval-e: G⊢s0 −e−≻ve−n→ s1 and
        eval-es: G⊢s1 −es≐≻vs−n→ s2 and
        v: v=ve#vs
        using normal-s0 by (fastforce elim: evaln-elim-cases)
      from valid-e P valid-A conf-s0 eval-e wt-e da-e
      obtain Q: Q ⌊ve⌋ₑ s1 Z and conf-s1: s1::≼(G,L)
        by (rule validE)
      from Q have Q′: ⋀ v. (Q←⌊ve⌋ₑ) v s1 Z
        by simp
      have (λVals:vs:. R←⌊(ve # vs)⌋ₗ) ⌊vs⌋ₗ s2 Z
      proof (cases normal s1)
        case True
        obtain E′ where
          da-es′: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨es⟩ₗ» E′
        proof −
          from eval-e wt-e da-e wf True
          have nrm E1 ⊆ dom (locals (store s1))
            by (cases rule: da-good-approx-evalnE) iprover
          with da-es show thesis
            by (rule da-weakenE) (rule that)
        qed
        from valid-es Q′ valid-A conf-s1 eval-es wt-es da-es′
        show ?thesis
          by (rule validE)
      next
        case False
        with valid-es Q′ valid-A conf-s1 eval-es
        show ?thesis
          by (cases rule: validE) iprover+
      qed
      with v have R ⌊v⌋ₗ s2 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s2::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
```

qed
qed
next
case (*Skip A P*)
show *G,A*|⊨::{ {*Normal* (*P*←♢)} *.Skip.* {*P*} }
proof (*rule valid-stmt-NormalI*)
fix *s0 s1 n L Y Z*
assume *conf-s0*: *s0*::⪯(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *eval*: *G*⊢*s0* −*Skip*−*n*→ *s1*
assume *P*: (*Normal* (*P*←♢)) *Y s0 Z*
show *P* ♢ *s1 Z* ∧ *s1*::⪯(*G*, *L*)
proof −
from *eval* **obtain** *s1*=*s0*
using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)
with *P conf-s0* **show** *?thesis*
by *simp*
qed
qed
next
case (*Expr A P e Q*)
note *valid-e* = ‹*G*,*A*|⊨::{ {*Normal P*} *e*−≻ {*Q*←♢} }›
show *G*,*A*|⊨::{ {*Normal P*} *.Expr e.* {*Q*} }
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s1 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*|⊨*n*::*t*
assume *conf-s0*: *s0*::⪯(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*Expr e*::√
assume *da*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*)) »⟨*Expr e*⟩$_s$» *C*
assume *eval*: *G*⊢*s0* −*Expr e*−*n*→ *s1*
assume *P*: (*Normal P*) *Y s0 Z*
show *Q* ♢ *s1 Z* ∧ *s1*::⪯(*G*, *L*)
proof −
from *wt* **obtain** *eT* **where**
wt-e: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*e*::−*eT*
by *cases simp*
from *da* **obtain** *E* **where**
da-e: ⦇*prg*=*G*,*cls*=*accC*, *lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»⟨*e*⟩$_e$»*E*
by *cases simp*
from *eval* **obtain** *v* **where**
eval-e: *G*⊢*s0* −*e*−≻*v*−*n*→ *s1*
using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)
from *valid-e P valid-A conf-s0 eval-e wt-e da-e*
obtain *Q*: (*Q*←♢) ⌊*v*⌋$_e$ *s1 Z* **and** *s1*::⪯(*G*,*L*)
by (*rule validE*)
thus *?thesis* **by** *simp*
qed
qed
next
case (*Lab A P c l Q*)
note *valid-c* = ‹*G*,*A*|⊨::{ {*Normal P*} *.c.* {*abupd* (*absorb l*) .; *Q*} }›
show *G*,*A*|⊨::{ {*Normal P*} *.l*• *c.* {*Q*} }
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s2 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*|⊨*n*::*t*
assume *conf-s0*: *s0*::⪯(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*l*• *c*::√

    assume *da*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*dom* (*locals* (*store s0*)) »⟨*l*• *c*⟩*s*» *C*
    assume *eval*: *G*⊢*s0* −*l*• *c*−*n*→ *s2*
    assume *P*: (*Normal P*) *Y s0 Z*
    show *Q* ◇ *s2 Z* ∧ *s2*::≼(*G*, *L*)
    proof −
      from *wt* **obtain**
        wt-c: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*c*::√
        by *cases simp*
      from *da* **obtain** *E* **where**
        da-c: (|*prg*=*G*,*cls*=*accC*, *lcl*=*L*|)⊢*dom* (*locals* (*store s0*))»⟨*c*⟩*s*»*E*
        by *cases simp*
      from *eval* **obtain** *s1* **where**
        eval-c: *G*⊢*s0* −*c*−*n*→ *s1* **and**
        s2: *s2* = *abupd* (*absorb l*) *s1*
        using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)
      from *valid-c P valid-A conf-s0 eval-c wt-c da-c*
      obtain *Q*: (*abupd* (*absorb l*) .; *Q*) ◇ *s1 Z*
        by (*rule validE*)
      with *s2* **have** *Q* ◇ *s2 Z*
        by *simp*
      moreover
      from *eval wt da conf-s0 wf*
      have *s2*::≼(*G*, *L*)
        by (*rule evaln-type-sound* [*elim-format*]) *simp*
      ultimately show *?thesis* **..**
    qed
  qed
next
  case (*Comp A P c1 Q c2 R*)
  note *valid-c1* = ‹*G*,*A*|⊨::{ {*Normal P*} .*c1*. {*Q*} }›
  note *valid-c2* = ‹*G*,*A*|⊨::{ {*Q*} .*c2*. {*R*} }›
  show *G*,*A*|⊨::{ {*Normal P*} .*c1*;; *c2*. {*R*} }
  proof (*rule valid-stmt-NormalI*)
    fix *n s0 L accC C s2 Y Z*
    assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
    assume *conf-s0*: *s0*::≼(*G*,*L*)
    assume *normal-s0*: *normal s0*
    assume *wt*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢(*c1*;; *c2*)::√
    assume *da*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*dom* (*locals* (*store s0*))»⟨*c1*;;*c2*⟩*s*»*C*
    assume *eval*: *G*⊢*s0* −*c1*;; *c2*−*n*→ *s2*
    assume *P*: (*Normal P*) *Y s0 Z*
    show *R* ◇ *s2 Z* ∧ *s2*::≼(*G*,*L*)
    proof −
      from *eval* **obtain** *s1* **where**
        eval-c1: *G*⊢*s0* −*c1* −*n*→ *s1* **and**
        eval-c2: *G*⊢*s1* −*c2* −*n*→ *s2*
        using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)
      from *wt* **obtain**
        wt-c1: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*c1*::√ **and**
        wt-c2: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*c2*::√
        by *cases simp*
      from *da* **obtain** *C1 C2* **where**
        da-c1: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢ *dom* (*locals* (*store s0*)) »⟨*c1*⟩*s*» *C1* **and**
        da-c2: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*nrm C1* »⟨*c2*⟩*s*» *C2*
        by *cases simp*
      from *valid-c1 P valid-A conf-s0 eval-c1 wt-c1 da-c1*
      obtain *Q*: *Q* ◇ *s1 Z* **and** *conf-s1*: *s1*::≼(*G*,*L*)
        by (*rule validE*)
      have *R* ◇ *s2 Z*

proof (*cases normal s1*)
case *True*
obtain *C2'* **where**
$(\!|prg=G,cls=accC,lcl=L|\!)\vdash$ *dom* (*locals* (*store s1*)) »⟨*c2*⟩$_s$» *C2'*
proof −
from *eval-c1 wt-c1 da-c1 wf True*
have *nrm C1* ⊆ *dom* (*locals* (*store s1*))
by (*cases rule*: *da-good-approx-evalnE*) *iprover*
with *da-c2* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *valid-c2 Q valid-A conf-s1 eval-c2 wt-c2*
show *?thesis*
by (*rule validE*)
next
case *False*
from *valid-c2 Q valid-A conf-s1 eval-c2 False*
show *?thesis*
by (*cases rule*: *validE*) *iprover+*
qed
moreover
from *eval wt da conf-s0 wf*
have *s2*::≼(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*If A P e P' c1 c2 Q*)
note *valid-e* = ‹*G*,*A*|⊨::{ {*Normal P*} *e*−≻ {*P'*} }›
have *valid-then-else*: ⋀ *b*. *G*,*A*|⊨::{ {*P'*←=*b*} .(*if b then c1 else c2*). {*Q*} }
using *If.hyps* **by** *simp*
show *G*,*A*|⊨::{ {*Normal P*} .*If*(*e*) *c1 Else c2*. {*Q*} }
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s2 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: $(\!|prg=G,cls=accC,lcl=L|\!)\vdash$*If*(*e*) *c1 Else c2*::√
assume *da*: $(\!|prg=G,cls=accC,lcl=L|\!)$
⊢*dom* (*locals* (*store s0*))»⟨*If*(*e*) *c1 Else c2*⟩$_s$»*C*
assume *eval*: *G*⊢*s0* −*If*(*e*) *c1 Else c2*−*n*→ *s2*
assume *P*: (*Normal P*) *Y s0 Z*
show *Q* ◇ *s2 Z* ∧ *s2*::≼(*G*,*L*)
proof −
from *eval* **obtain** *b s1* **where**
eval-e: *G*⊢*s0* −*e*−≻*b*−*n*→ *s1* **and**
eval-then-else: *G*⊢*s1* −(*if the-Bool b then c1 else c2*)−*n*→ *s2*
using *normal-s0* **by** (*auto elim*: *evaln-elim-cases*)
from *wt* **obtain**
wt-e: $(\!|prg=G,\ cls=accC,\ lcl=L|\!)\vdash$*e*::−*PrimT Boolean* **and**
wt-then-else: $(\!|prg=G,cls=accC,lcl=L|\!)\vdash$(*if the-Bool b then c1 else c2*)::√
by *cases* (*simp split*: *if-split*)
from *da* **obtain** *E S* **where**
da-e: $(\!|prg=G,cls=accC,lcl=L|\!)\vdash$ *dom* (*locals* (*store s0*)) »⟨*e*⟩$_e$» *E* **and**
da-then-else:
$(\!|prg=G,cls=accC,lcl=L|\!)\vdash$
(*dom* (*locals* (*store s0*)) ∪ *assigns-if* (*the-Bool b*) *e*)
»⟨*if the-Bool b then c1 else c2*⟩$_s$» *S*

```
      by cases (cases the-Bool b,auto)
    from valid-e P valid-A conf-s0 eval-e wt-e da-e
    obtain P′ ⌊b⌋e s1 Z and conf-s1: s1::≼(G,L)
      by (rule validE)
    hence P′: ⋀v. (P′←=the-Bool b) v s1 Z
      by (cases normal s1) auto
    have Q ◇ s2 Z
    proof (cases normal s1)
      case True
      have s0-s1: dom (locals (store s0))
                    ∪ assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
      proof −
        from eval-e
        have eval-e′: G⊢s0 −e−≻b→ s1
          by (rule evaln-eval)
        hence
          dom (locals (store s0)) ⊆ dom (locals (store s1))
          by (rule dom-locals-eval-mono-elim)
        moreover
        from eval-e′ True wt-e
        have assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
          by (rule assigns-if-good-approx′)
        ultimately show ?thesis by (rule Un-least)
      qed
      with da-then-else
      obtain S′ where
        (|prg=G,cls=accC,lcl=L|)
           ⊢dom (locals (store s1))»⟨if the-Bool b then c1 else c2⟩s» S′
        by (rule da-weakenE)
      with valid-then-else P′ valid-A conf-s1 eval-then-else wt-then-else
      show ?thesis
        by (rule validE)
    next
      case False
      with valid-then-else P′ valid-A conf-s1 eval-then-else
      show ?thesis
        by (cases rule: validE) iprover+
    qed
    moreover
    from eval wt da conf-s0 wf
    have s2::≼(G, L)
      by (rule evaln-type-sound [elim-format]) simp
    ultimately show ?thesis ..
  qed
 qed
next
 case (Loop A P e P′ c l)
 note valid-e = ‹G,A|⊨::{ {P} e−≻ {P′} }›
 note valid-c = ‹G,A|⊨::{ {Normal (P′←=True)}
                         .c.
                         {abupd (absorb (Cont l)) .; P} }›
 show G,A|⊨::{ {P} .l• While(e) c. {P′←=False↓=◇} }
 proof (rule valid-stmtI)
  fix n s0 L accC C s3 Y Z
  assume valid-A: ∀ t∈A. G⊨n::t
  assume conf-s0:  s0::≼(G,L)
  assume wt: normal s0 ⟹ (|prg=G,cls=accC,lcl=L|)⊢l• While(e) c::√
  assume da: normal s0 ⟹ (|prg=G,cls=accC,lcl=L|)
               ⊢ dom (locals (store s0)) »⟨l• While(e) c⟩s» C
```

assume *eval*: $G \vdash s0 \ -l\cdot\ While(e)\ c{-}n{\to}\ s3$
assume *P*: $P\ Y\ s0\ Z$
show $(P'{\leftarrow}{=}False{\downarrow}{=}\diamondsuit)\ \diamondsuit\ s3\ Z \wedge s3{::}{\preceq}(G,L)$
proof –

— From the given hypothesises *valid-e* and *valid-c* we can only reach the state after unfolding the loop once, i.e. $P\ \diamondsuit\ s2\ Z$, where *s2* is the state after executing *c*. To gain validity of the further execution of while, to finally get $(P'{\leftarrow}{=}False{\downarrow}{=}\diamondsuit)\ \diamondsuit\ s3\ Z$ we have to get a hypothesis about the subsequent unfoldings (the whole loop again), too. We can achieve this, by performing induction on the evaluation relation, with all the necessary preconditions to apply *valid-e* and *valid-c* in the goal.

{
fix $t\ s\ s'\ v$
assume $G \vdash s\ -t{\succ}{-}n{\to}\ (v,\ s')$
hence $\bigwedge\ Y'\ T\ E.$
$[\![t = \ \langle l\cdot\ While(e)\ c\rangle_s;\ \forall t{\in}A.\ G{\models}n{::}t;\ P\ Y'\ s\ Z;\ s{::}{\preceq}(G,\ L);$
$normal\ s \Longrightarrow (\!|prg{=}G,cls{=}accC,lcl{=}L|\!){\vdash}t{::}T;$
$normal\ s \Longrightarrow (\!|prg{=}G,cls{=}accC,lcl{=}L|\!){\vdash}dom\ (locals\ (store\ s)){\gg}t{\gg}E$
$]\!]{\Longrightarrow}\ (P'{\leftarrow}{=}False{\downarrow}{=}\diamondsuit)\ v\ s'\ Z$
(**is** *PROP ?Hyp n t s v s'*)
proof (*induct*)
case (*Loop s0' e' b n' s1' c' s2' l' s3' Y' T E*)
note *while* = $\langle(\langle l'\cdot\ While(e')\ c'\rangle_s{::}term) = \langle l\cdot\ While(e)\ c\rangle_s\rangle$
hence *eqs*: $l'{=}l\ e'{=}e\ c'{=}c$ **by** *simp-all*
note *valid-A* = $\langle\forall t{\in}A.\ G{\models}n'{::}t\rangle$
note *P* = $\langle P\ Y'\ (Norm\ s0')\ Z\rangle$
note *conf-s0'* = $\langle Norm\ s0'{::}{\preceq}(G,\ L)\rangle$
have *wt*: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!){\vdash}\langle l\cdot\ While(e)\ c\rangle_s{::}T$
using *Loop.prems eqs* **by** *simp*
have *da*: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!){\vdash}$
$dom\ (locals\ (store\ ((Norm\ s0'){::}state))){\gg}\langle l\cdot\ While(e)\ c\rangle_s{\gg}E$
using *Loop.prems eqs* **by** *simp*
have *evaln-e*: $G{\vdash}Norm\ s0'\ -e{-}{\succ}b{-}n'{\to}\ s1'$
using *Loop.hyps eqs* **by** *simp*
show $(P'{\leftarrow}{=}False{\downarrow}{=}\diamondsuit)\ \diamondsuit\ s3'\ Z$
proof –
from *wt* **obtain**
wt-e: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!){\vdash}e{::}{-}PrimT\ Boolean$ **and**
wt-c: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!){\vdash}c{::}\surd$
by *cases* (*simp add*: *eqs*)
from *da* **obtain** *E S* **where**
da-e: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\vdash dom\ (locals\ (store\ ((Norm\ s0'){::}state)))\ {\gg}\langle e\rangle_e{\gg}\ E$ **and**
da-c: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\vdash (dom\ (locals\ (store\ ((Norm\ s0'){::}state)))$
$\cup\ assigns\text{-}if\ True\ e)\ {\gg}\langle c\rangle_s{\gg}\ S$
by *cases* (*simp add*: *eqs*)
from *evaln-e*
have *eval-e*: $G{\vdash}Norm\ s0'\ -e{-}{\succ}b{\to}\ s1'$
by (*rule evaln-eval*)
from *valid-e P valid-A conf-s0' evaln-e wt-e da-e*
obtain *P'*: $P'\ \lfloor b\rfloor_e\ s1'\ Z$ **and** *conf-s1'*: $s1'{::}{\preceq}(G,L)$
by (*rule validE*)
show $(P'{\leftarrow}{=}False{\downarrow}{=}\diamondsuit)\ \diamondsuit\ s3'\ Z$
proof (*cases normal s1'*)
case *True*
note *normal-s1'=this*
show *?thesis*
proof (*cases the-Bool b*)
case *True*
with *P' normal-s1'* **have** *P''*: $(Normal\ (P'{\leftarrow}{=}True))\ \lfloor b\rfloor_e\ s1'\ Z$

```
  by auto
from True Loop.hyps obtain
  eval-c: G⊢s1′ −c−n′→ s2′ and
  eval-while:
     G⊢abupd (absorb (Cont l)) s2′ −l· While(e) c−n′→ s3′
  by (simp add: eqs)
from True Loop.hyps have
  hyp: PROP ?Hyp n′ ⟨l· While(e) c⟩s
         (abupd (absorb (Cont l′)) s2′) ♢ s3′
  apply (simp only: True if-True eqs)
  apply (elim conjE)
  apply (tactic smp-tac context 3 1)
  apply fast
  done
from eval-e
have s0′-s1′: dom (locals (store ((Norm s0′)::state)))
                 ⊆ dom (locals (store s1′))
  by (rule dom-locals-eval-mono-elim)
obtain S′ where
  da-c′:
   (|prg=G,cls=accC,lcl=L|)⊢(dom (locals (store s1′)))»⟨c⟩s» S′
proof −
  note s0′-s1′
  moreover
  from eval-e normal-s1′ wt-e
  have assigns-if True e ⊆ dom (locals (store s1′))
    by (rule assigns-if-good-approx′ [elim-format])
       (simp add: True)
  ultimately
  have dom (locals (store ((Norm s0′)::state)))
          ∪ assigns-if True e ⊆ dom (locals (store s1′))
    by (rule Un-least)
  with da-c show thesis
    by (rule da-weakenE) (rule that)
qed
with valid-c P′′ valid-A conf-s1′ eval-c wt-c
obtain (abupd (absorb (Cont l)) .; P) ♢ s2′ Z and
  conf-s2′: s2′::⪯(G,L)
  by (rule validE)
hence P-s2′: P ♢ (abupd (absorb (Cont l)) s2′) Z
  by simp
from conf-s2′
have conf-absorb: abupd (absorb (Cont l)) s2′ ::⪯(G, L)
  by (cases s2′) (auto intro: conforms-absorb)
moreover
obtain E′ where
  da-while′:
   (|prg=G,cls=accC,lcl=L|)⊢
     dom (locals(store (abupd (absorb (Cont l)) s2′)))
      »⟨l· While(e) c⟩s» E′
proof −
  note s0′-s1′
  also
  from eval-c
  have G⊢s1′ −c→ s2′
    by (rule evaln-eval)
  hence dom (locals (store s1′)) ⊆ dom (locals (store s2′))
    by (rule dom-locals-eval-mono-elim)
  also
```

```
          have …⊆dom (locals (store (abupd (absorb (Cont l)) s2')))
            by simp
          finally
          have dom (locals (store ((Norm s0')::state))) ⊆ … .
          with da show thesis
            by (rule da-weakenE) (rule that)
        qed
        from valid-A P-s2' conf-absorb wt da-while'
        show (P'←=False↓=◇) ◇ s3' Z
          using hyp by (simp add: eqs)
      next
        case False
        with Loop.hyps obtain s3'=s1'
          by simp
        with P' False show ?thesis
          by auto
      qed
    next
      case False
      note abnormal-s1'=this
      have s3'=s1'
      proof −
        from False obtain abr where abr: abrupt s1' = Some abr
          by (cases s1') auto
        from eval-e - wt-e wf
        have no-jmp: ⋀ j. abrupt s1' ≠ Some (Jump j)
          by (rule eval-expression-no-jump
               [where ?Env=⦇prg=G,cls=accC,lcl=L⦈,simplified])
             simp
        show ?thesis
        proof (cases the-Bool b)
          case True
          with Loop.hyps obtain
            eval-c: G⊢s1' −c−n'→ s2' and
            eval-while:
             G⊢abupd (absorb (Cont l)) s2' −l• While(e) c−n'→ s3'
            by (simp add: eqs)
          from eval-c abr have s2'=s1' by auto
          moreover from calculation no-jmp
          have abupd (absorb (Cont l)) s2'=s2'
            by (cases s1') (simp add: absorb-def)
          ultimately show ?thesis
            using eval-while abr
            by auto
        next
          case False
          with Loop.hyps show ?thesis by simp
        qed
      qed
      with P' False show ?thesis
        by auto
    qed
  qed
next
  case (Abrupt abr s t' n' Y' T E)
  note t' = ‹t' = ⟨l• While(e) c⟩ₛ›
  note conf = ‹(Some abr, s)::≼(G, L)›
  note P = ‹P Y' (Some abr, s) Z›
  note valid-A = ‹∀ t∈A. G⊨n'::t›
```

```
        show (P′←=False↓=◇) (undefined3 t′) (Some abr, s) Z
        proof -
          have eval-e:
            G⊢(Some abr,s) −⟨e⟩e≻−n′→ (undefined3 ⟨e⟩e,(Some abr,s))
            by auto
          from valid-e P valid-A conf eval-e
          have P′ (undefined3 ⟨e⟩e) (Some abr,s) Z
            by (cases rule: validE [where ?P=P]) simp+
          with t′ show ?thesis
            by auto
        qed
      qed simp-all
    } note generalized=this
    from eval - valid-A P conf-s0 wt da
    have (P′←=False↓=◇) ◇ s3 Z
      by (rule generalized) simp-all
    moreover
    have s3::≼(G, L)
    proof (cases normal s0)
      case True
      from eval wt [OF True] da [OF True] conf-s0 wf
      show ?thesis
        by (rule evaln-type-sound [elim-format]) simp
    next
      case False
      with eval have s3=s0
        by auto
      with conf-s0 show ?thesis
        by simp
    qed
    ultimately show ?thesis ..
  qed
qed
next
  case (Jmp A j P)
  show G,A|⊨::{ {Normal (abupd (λa. Some (Jump j)) .; P←◇)} .Jmp j. {P} }
  proof (rule valid-stmt-NormalI)
    fix n s0 L accC C s1 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0:  s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Jmp j::√
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0))»⟨Jmp j⟩s»C
    assume eval: G⊢s0 −Jmp j−n→ s1
    assume P: (Normal (abupd (λa. Some (Jump j)) .; P←◇)) Y s0 Z
    show P ◇ s1 Z ∧ s1::≼(G,L)
    proof -
      from eval obtain s where
        s: s0=Norm s s1=(Some (Jump j), s)
        using normal-s0 by (auto elim: evaln-elim-cases)
      with P have P ◇ s1 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s1::≼(G,L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
```

```
  qed
next
  case (Throw A P e Q)
  note valid-e = ‹G,A|⊨::{ {Normal P} e−≻ {λVal:a:. abupd (throw a) .; Q←♢} }›
  show G,A|⊨::{ {Normal P} .Throw e. {Q} }
  proof (rule valid-stmt-NormalI)
    fix n s0 L accC C s2 Y Z
    assume valid-A: ∀t∈A. G⊨n::t
    assume conf-s0:  s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Throw e::√
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0))»⟨Throw e⟩s»C
    assume eval: G⊢s0 −Throw e−n→ s2
    assume P: (Normal P) Y s0 Z
    show Q ♢ s2 Z ∧ s2::≼(G,L)
    proof −
      from eval obtain s1 a where
        eval-e: G⊢s0 −e−≻a−n→ s1 and
        s2: s2 = abupd (throw a) s1
        using normal-s0 by (auto elim: evaln-elim-cases)
      from wt obtain T where
        wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−T
        by cases simp
      from da obtain E where
        da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨e⟩e» E
        by cases simp
      from valid-e P valid-A conf-s0 eval-e wt-e da-e
      obtain (λVal:a:. abupd (throw a) .; Q←♢) ⌊a⌋e s1 Z
        by (rule validE)
      with s2 have Q ♢ s2 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s2::≼(G,L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Try A P c1 Q C vn c2 R)
  note valid-c1 = ‹G,A|⊨::{ {Normal P} .c1. {SXAlloc G Q} }›
  note valid-c2 = ‹G,A|⊨::{ {Q ∧. (λs. G,s⊢catch C) ;. new-xcpt-var vn}
                            .c2.
                           {R} }›
  note Q-R = ‹(Q ∧. (λs. ¬ G,s⊢catch C)) ⇒ R›
  show G,A|⊨::{ {Normal P} .Try c1 Catch(C vn) c2. {R} }
  proof (rule valid-stmt-NormalI)
    fix n s0 L accC E s3 Y Z
    assume valid-A: ∀t∈A. G⊨n::t
    assume conf-s0:  s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Try c1 Catch(C vn) c2::√
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0)) »⟨Try c1 Catch(C vn) c2⟩s» E
    assume eval: G⊢s0 −Try c1 Catch(C vn) c2−n→ s3
    assume P: (Normal P) Y s0 Z
    show R ♢ s3 Z ∧ s3::≼(G,L)
    proof −
```

from *eval* **obtain** *s1 s2* **where**
  eval-c1: $G\vdash s0\ -c1-n\to\ s1$ **and**
  sxalloc: $G\vdash s1\ -sxalloc\to\ s2$ **and**
  s3: *if* $G,s2\vdash catch\ C$
    then $G\vdash new\text{-}xcpt\text{-}var\ vn\ s2\ -c2-n\to\ s3$
    else $s3 = s2$
  using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)
from *wt* **obtain**
  wt-c1: $(|prg=G,cls=accC,lcl=L|)\vdash c1::\surd$ **and**
  wt-c2: $(|prg=G,cls=accC,lcl=L(VName\ vn\mapsto Class\ C)|)\vdash c2::\surd$
  by *cases simp*
from *da* **obtain** *C1 C2* **where**
  da-c1: $(|prg=G,cls=accC,lcl=L|)\vdash dom\ (locals\ (store\ s0))\ »\langle c1\rangle_s»\ C1$ **and**
  da-c2: $(|prg=G,cls=accC,lcl=L(VName\ vn\mapsto Class\ C)|)$
    $\vdash (dom\ (locals\ (store\ s0)) \cup \{VName\ vn\})\ »\langle c2\rangle_s»\ C2$
  by *cases simp*
from *valid-c1 P valid-A conf-s0 eval-c1 wt-c1 da-c1*
obtain *sxQ*: $(SXAlloc\ G\ Q)\ \Diamond\ s1\ Z$ **and** *conf-s1*: $s1::\preceq(G,L)$
  by (*rule validE*)
from *sxalloc sxQ*
have *Q*: $Q\ \Diamond\ s2\ Z$
  by *auto*
have $R\ \Diamond\ s3\ Z$
proof (*cases* $\exists\ x.\ abrupt\ s1 = Some\ (Xcpt\ x)$)
  case *False*
  from *sxalloc wf*
  have $s2=s1$
    by (*rule sxalloc-type-sound* [*elim-format*])
      (*insert False*, *auto split*: *option.splits abrupt.splits*)
  with *False*
  have *no-catch*: $\neg\ \ G,s2\vdash catch\ C$
    by (*simp add*: *catch-def*)
  moreover
  from *no-catch s3*
  have $s3=s2$
    by *simp*
  ultimately show *?thesis*
    using *Q Q-R* **by** *simp*
next
  case *True*
  note *exception-s1* = *this*
  show *?thesis*
  proof (*cases* $G,s2\vdash catch\ C$)
    case *False*
    with *s3*
    have $s3=s2$
      by *simp*
    with *False Q Q-R* **show** *?thesis*
      by *simp*
  next
    case *True*
    with *s3* **have** *eval-c2*: $G\vdash new\text{-}xcpt\text{-}var\ vn\ s2\ -c2-n\to\ s3$
      by *simp*
    from *conf-s1 sxalloc wf*
    have *conf-s2*: $s2::\preceq(G,\ L)$
      by (*auto dest*: *sxalloc-type-sound*
        split: *option.splits abrupt.splits*)
    from *exception-s1 sxalloc wf*
    obtain *a*

```
      where xcpt-s2: abrupt s2 = Some (Xcpt (Loc a))
      by (auto dest!: sxalloc-type-sound
                  split: option.splits abrupt.splits)
    with True
    have G⊢obj-ty (the (globs (store s2) (Heap a)))≼Class C
      by (cases s2) simp
    with xcpt-s2 conf-s2 wf
    have conf-new-xcpt: new-xcpt-var vn s2 ::≼(G, L(VName vn↦Class C))
      by (auto dest: Try-lemma)
    obtain C2' where
      da-c2':
      ⦇prg=G,cls=accC,lcl=L(VName vn↦Class C)⦈
       ⊢ (dom (locals (store (new-xcpt-var vn s2)))) »⟨c2⟩ₛ» C2'
    proof −
      have (dom (locals (store s0)) ∪ {VName vn})
              ⊆ dom (locals (store (new-xcpt-var vn s2)))
      proof −
        from eval-c1
        have dom (locals (store s0))
                ⊆ dom (locals (store s1))
          by (rule dom-locals-evaln-mono-elim)
        also
        from sxalloc
        have . . . ⊆ dom (locals (store s2))
          by (rule dom-locals-sxalloc-mono)
        also
        have . . . ⊆ dom (locals (store (new-xcpt-var vn s2)))
          by (cases s2) (simp add: new-xcpt-var-def, blast)
        also
        have {VName vn} ⊆ . . .
          by (cases s2) simp
        ultimately show ?thesis
          by (rule Un-least)
      qed
      with da-c2 show thesis
        by (rule da-weakenE) (rule that)
    qed
    from Q eval-c2 True
    have (Q ∧. (λs. G,s⊢catch C) ;. new-xcpt-var vn)
            ♢ (new-xcpt-var vn s2) Z
      by auto
    from valid-c2 this valid-A conf-new-xcpt eval-c2 wt-c2 da-c2'
    show R ♢ s3 Z
      by (rule validE)
  qed
 qed
 moreover
 from eval wt da conf-s0 wf
 have s3::≼(G,L)
  by (rule evaln-type-sound [elim-format]) simp
 ultimately show ?thesis ..
 qed
qed
next
 case (Fin A P c1 Q c2 R)
 note valid-c1 = ‹G,A|⊨::{ {Normal P} .c1. {Q} }›
 have valid-c2: ⋀ abr. G,A|⊨::{ {Q ∧. (λs. abr = fst s) ;. abupd (λx. None)}
                              .c2.
                              {abupd (abrupt-if (abr ≠ None) abr) .; R} }
```

using *Fin.hyps* **by** *simp*
show *G*,*A*|⊨::{ {*Normal P*} .*c1 Finally c2*. {*R*} }
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC E s3 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*c1 Finally c2*::√
assume *da*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢*dom* (*locals* (*store s0*)) »⟨*c1 Finally c2*⟩$_s$» *E*
assume *eval*: *G*⊢*s0* −*c1 Finally c2*−*n*→ *s3*
assume *P*: (*Normal P*) *Y s0 Z*
show *R* ◇ *s3 Z* ∧ *s3*::≼(*G*,*L*)
proof −
from *eval* **obtain** *s1 abr1 s2* **where**
eval-c1: *G*⊢*s0* −*c1*−*n*→ (*abr1*, *s1*) **and**
eval-c2: *G*⊢*Norm s1* −*c2*−*n*→ *s2* **and**
s3: *s3* = (*if* ∃ *err*. *abr1* = *Some* (*Error err*)
then (*abr1*, *s1*)
else abupd (*abrupt-if* (*abr1* ≠ *None*) *abr1*) *s2*)
using *normal-s0* **by** (*fastforce elim*: *evaln-elim-cases*)
from *wt* **obtain**
wt-c1: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*c1*::√ **and**
wt-c2: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*c2*::√
by *cases simp*
from *da* **obtain** *C1 C2* **where**
da-c1: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|) ⊢ *dom* (*locals* (*store s0*)) »⟨*c1*⟩$_s$» *C1* **and**
da-c2: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|) ⊢ *dom* (*locals* (*store s0*)) »⟨*c2*⟩$_s$» *C2*
by *cases simp*
from *valid-c1 P valid-A conf-s0 eval-c1 wt-c1 da-c1*
obtain *Q*: *Q* ◇ (*abr1*,*s1*) *Z* **and** *conf-s1*: (*abr1*,*s1*)::≼(*G*,*L*)
by (*rule validE*)
from *Q*
have *Q'*: (*Q* ∧. (λ*s*. *abr1* = *fst s*) ;. *abupd* (λ*x*. *None*)) ◇ (*Norm s1*) *Z*
by *auto*
from *eval-c1 wt-c1 da-c1 conf-s0 wf*
have *error-free* (*abr1*,*s1*)
by (*rule evaln-type-sound* [*elim-format*]) (*insert normal-s0*,*simp*)
with *s3* **have** *s3'*: *s3* = *abupd* (*abrupt-if* (*abr1* ≠ *None*) *abr1*) *s2*
by (*simp add*: *error-free-def*)
from *conf-s1*
have *conf-Norm-s1*: *Norm s1*::≼(*G*,*L*)
by (*rule conforms-NormI*)
obtain *C2'* **where**
da-c2': (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢ *dom* (*locals* (*store* ((*Norm s1*)::*state*))) »⟨*c2*⟩$_s$» *C2'*
proof −
from *eval-c1*
have *dom* (*locals* (*store s0*)) ⊆ *dom* (*locals* (*store* (*abr1*,*s1*)))
by (*rule dom-locals-evaln-mono-elim*)
hence *dom* (*locals* (*store s0*))
⊆ *dom* (*locals* (*store* ((*Norm s1*)::*state*)))
by *simp*
with *da-c2* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
from *valid-c2 Q' valid-A conf-Norm-s1 eval-c2 wt-c2 da-c2'*
have (*abupd* (*abrupt-if* (*abr1* ≠ *None*) *abr1*) .; *R*) ◇ *s2 Z*
by (*rule validE*)

```
      with s3' have R ◇ s3 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s3::≼(G,L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Done A P C)
  show G,A|⊨::{ {Normal (P←◇ ∧. initd C)} .Init C. {P} }
  proof (rule valid-stmt-NormalI)
    fix n s0 L accC E s3 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0:  s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Init C::√
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0)) »⟨Init C⟩s» E
    assume eval: G⊢s0 −Init C−n→ s3
    assume P: (Normal (P←◇ ∧. initd C)) Y s0 Z
    show P ◇ s3 Z ∧ s3::≼(G,L)
    proof −
      from P have inited: inited C (globs (store s0))
        by simp
      with eval have s3=s0
        using normal-s0 by (auto elim: evaln-elim-cases)
      with P conf-s0 show ?thesis
        by simp
    qed
  qed
next
  case (Init C c A P Q R)
  note c = ‹the (class G C) = c›
  note valid-super =
        ‹G,A|⊨::{ {Normal (P ∧. Not ∘ initd C ;. supd (init-class-obj G C))}
                .(if C = Object then Skip else Init (super c)).
                {Q} }›
  have valid-init:
        ⋀ l.  G,A|⊨::{ {Q ∧. (λs. l = locals (snd s)) ;. set-lvars Map.empty}
                        .init c.
                        {set-lvars l .; R} }
    using Init.hyps by simp
  show G,A|⊨::{ {Normal (P ∧. Not ∘ initd C)} .Init C. {R} }
  proof (rule valid-stmt-NormalI)
    fix n s0 L accC E s3 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0:  s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Init C::√
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0)) »⟨Init C⟩s» E
    assume eval: G⊢s0 −Init C−n→ s3
    assume P: (Normal (P ∧. Not ∘ initd C)) Y s0 Z
    show R ◇ s3 Z ∧ s3::≼(G,L)
    proof −
      from P have not-inited: ¬ inited C (globs (store s0)) by simp
      with eval c obtain s1 s2 where
```

eval-super:
G⊢*Norm* ((*init-class-obj G C*) (*store s0*))
−(*if C* = *Object then Skip else Init* (*super c*))−*n*→ *s1* **and**
eval-init: *G*⊢(*set-lvars Map.empty*) *s1* −*init c*−*n*→ *s2* **and**
s3: *s3* = (*set-lvars* (*locals* (*store s1*))) *s2*
using *normal-s0* **by** (*auto elim*!: *evaln-elim-cases*)
from *wt c* **have**
cls-C: *class G C* = *Some c*
by *cases auto*
from *wf cls-C* **have**
wt-super: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢(*if C* = *Object then Skip else Init* (*super c*))::√
by (*cases C*=*Object*)
(*auto dest*: *wf-prog-cdecl wf-cdecl-supD is-acc-classD*)
obtain *S* **where**
da-super:
(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢ *dom* (*locals* (*store* ((*Norm*
((*init-class-obj G C*) (*store s0*)))::*state*)))
»⟨*if C* = *Object then Skip else Init* (*super c*)⟩$_s$» *S*
proof (*cases C*=*Object*)
case *True*
with *da-Skip* **show** *?thesis*
using *that* **by** (*auto intro*: *assigned.select-convs*)
next
case *False*
with *da-Init* **show** *?thesis*
by − (*rule that*, *auto intro*: *assigned.select-convs*)
qed
from *normal-s0 conf-s0 wf cls-C not-inited*
have *conf-init-cls*: (*Norm* ((*init-class-obj G C*) (*store s0*)))::≼(*G*, *L*)
by (*auto intro*: *conforms-init-class-obj*)
from *P*
have *P′*: (*Normal* (*P* ∧. *Not* ∘ *initd C* ;. *supd* (*init-class-obj G C*)))
Y (*Norm* ((*init-class-obj G C*) (*store s0*))) *Z*
by *auto*

from *valid-super P′ valid-A conf-init-cls eval-super wt-super da-super*
obtain *Q*: *Q* ◇ *s1 Z* **and** *conf-s1*: *s1*::≼(*G*,*L*)
by (*rule validE*)

from *cls-C wf* **have** *wt-init*: (|*prg*=*G*, *cls*=*C*,*lcl*=*Map.empty*|)⊢(*init c*)::√
by (*rule wf-prog-cdecl* [*THEN wf-cdecl-wt-init*])
from *cls-C wf* **obtain** *I* **where**
(|*prg*=*G*,*cls*=*C*,*lcl*=*Map.empty*|)⊢ {} »⟨*init c*⟩$_s$» *I*
by (*rule wf-prog-cdecl* [*THEN wf-cdeclE*,*simplified*]) *blast*

then obtain *I′* **where**
da-init:
(|*prg*=*G*,*cls*=*C*,*lcl*=*Map.empty*|)⊢*dom* (*locals* (*store* ((*set-lvars Map.empty*) *s1*)))
»⟨*init c*⟩$_s$» *I′*
by (*rule da-weakenE*) *simp*
have *conf-s1-empty*: (*set-lvars Map.empty*) *s1*::≼(*G*, *Map.empty*)
proof −
from *eval-super* **have**
G⊢*Norm* ((*init-class-obj G C*) (*store s0*))
−(*if C* = *Object then Skip else Init* (*super c*))→ *s1*
by (*rule evaln-eval*)
from *this wt-super wf*

```
        have s1-no-ret: ⋀ j. abrupt s1 ≠ Some (Jump j)
          by − (rule eval-statement-no-jump
                  [where ?Env=(|prg=G,cls=accC,lcl=L|)], auto split: if-split)
        with conf-s1
        show ?thesis
          by (cases s1) (auto intro: conforms-set-locals)
      qed

      obtain l where l: l = locals (store s1)
        by simp
      with Q
      have Q': (Q ∧. (λs. l = locals (snd s)) ;. set-lvars Map.empty)
                  ♢ ((set-lvars Map.empty) s1) Z
        by auto
      from valid-init Q' valid-A conf-s1-empty eval-init wt-init da-init
      have (set-lvars l .; R) ♢ s2 Z
        by (rule validE)
      with s3 l have R ♢ s3 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s3::≼(G,L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (InsInitV A P c v Q)
  show G,A|⊨::{ {Normal P} InsInitV c v=≻ {Q} }
  proof (rule valid-var-NormalI)
    fix s0 vf n s1 L Z
    assume normal s0
    moreover
    assume G⊢s0 −InsInitV c v=≻vf−n→ s1
    ultimately have False
      by (cases s0) (simp add: evaln-InsInitV)
    thus Q ⌊vf⌋v s1 Z ∧ s1::≼(G, L)..
  qed
next
  case (InsInitE A P c e Q)
  show G,A|⊨::{ {Normal P} InsInitE c e−≻ {Q} }
  proof (rule valid-expr-NormalI)
    fix s0 v n s1 L Z
    assume normal s0
    moreover
    assume G⊢s0 −InsInitE c e−≻v−n→ s1
    ultimately have False
      by (cases s0) (simp add: evaln-InsInitE)
    thus Q ⌊v⌋e s1 Z ∧ s1::≼(G, L)..
  qed
next
  case (Callee A P l e Q)
  show G,A|⊨::{ {Normal P} Callee l e−≻ {Q} }
  proof (rule valid-expr-NormalI)
    fix s0 v n s1 L Z
    assume normal s0
    moreover
    assume G⊢s0 −Callee l e−≻v−n→ s1
    ultimately have False
```

by ($cases\ s0$) ($simp\ add$: $evaln\text{-}Callee$)
thus $Q\ \lfloor v\rfloor_e\ s1\ Z \wedge s1{::}\preceq(G,\ L)$..
qed
next
case ($FinA\ A\ P\ a\ c\ Q$)
show $G,A|\models{::}\{\ \{Normal\ P\}\ .FinA\ a\ c.\ \{Q\}\ \}$
proof ($rule\ valid\text{-}stmt\text{-}NormalI$)
fix $s0\ v\ n\ s1\ L\ Z$
assume $normal\ s0$
moreover
assume $G\vdash s0\ -FinA\ a\ c-n\rightarrow\ s1$
ultimately have $False$
by ($cases\ s0$) ($simp\ add$: $evaln\text{-}FinA$)
thus $Q\ \diamondsuit\ s1\ Z \wedge s1{::}\preceq(G,\ L)$..
qed
qed
declare $inj\text{-}term\text{-}simps$ [$simp\ del$]

theorem $ax\text{-}sound$:
$wf\text{-}prog\ G \Longrightarrow G,(A{::}'a\ triple\ set)|\vdash(ts{::}'a\ triple\ set) \Longrightarrow G,A|\models ts$
apply ($subst\ ax\text{-}valids2\text{-}eq$ [$symmetric$])
apply $assumption$
apply ($erule$ (1) $ax\text{-}sound2$)
done

lemma $sound\text{-}valid2\text{-}lemma$:
$[\![\forall v\ n.\ Ball\ A\ (triple\text{-}valid2\ G\ n) \longrightarrow P\ v\ n;\ Ball\ A\ (triple\text{-}valid2\ G\ n)]\!]$
$\Longrightarrow P\ v\ n$
by $blast$

end

Chapter 24

AxCompl

1 Completeness proof for Axiomatic semantics of Java expressions and statements

theory *AxCompl* **imports** *AxSem* **begin**

design issues:

- proof structured by Most General Formulas (-> Thomas Kleymann)

set of not yet initialzed classes

definition
  nyinitcls :: *prog* ⇒ *state* ⇒ *qtname set*
  where *nyinitcls G s* = {*C*. *is-class G C* ∧ ¬ *initd C s*}

lemma *nyinitcls-subset-class*: *nyinitcls G s* ⊆ {*C*. *is-class G C*}
apply (*unfold nyinitcls-def*)
apply *fast*
done

lemmas *finite-nyinitcls* [*simp*] =
  finite-is-class [*THEN nyinitcls-subset-class* [*THEN finite-subset*]]

lemma *card-nyinitcls-bound*: *card* (*nyinitcls G s*) ≤ *card* {*C*. *is-class G C*}
apply (*rule nyinitcls-subset-class* [*THEN finite-is-class* [*THEN card-mono*]])
done

lemma *nyinitcls-set-locals-cong* [*simp*]:
  nyinitcls G (*x*,*set-locals l s*) = *nyinitcls G* (*x*,*s*)
  by (*simp add*: *nyinitcls-def*)

lemma *nyinitcls-abrupt-cong* [*simp*]: *nyinitcls G* (*f x*, *y*) = *nyinitcls G* (*x*, *y*)
  by (*simp add*: *nyinitcls-def*)

lemma *nyinitcls-abupd-cong* [*simp*]: *nyinitcls G* (*abupd f s*) = *nyinitcls G s*
  by (*simp add*: *nyinitcls-def*)

lemma *card-nyinitcls-abrupt-congE* [*elim*!]:
  card (*nyinitcls G* (*x*, *s*)) ≤ *n* ⟹ *card* (*nyinitcls G* (*y*, *s*)) ≤ *n*
  unfolding *nyinitcls-def* **by** *auto*

lemma *nyinitcls-new-xcpt-var* [*simp*]:
  nyinitcls G (*new-xcpt-var vn s*) = *nyinitcls G s*
  by (*induct s*) (*simp-all add*: *nyinitcls-def*)

lemma *nyinitcls-init-lvars* [*simp*]:
  nyinitcls G ((*init-lvars G C sig mode a′ pvs*) *s*) = *nyinitcls G s*
  by (*induct s*) (*simp add*: *init-lvars-def2 split*: *if-split*)

lemma *nyinitcls-emptyD*: ⟦*nyinitcls G s* = {}; *is-class G C*⟧ ⟹ *initd C s*
  unfolding *nyinitcls-def* **by** *fast*

lemma *card-Suc-lemma*:
  ⟦*card* (*insert a A*) ≤ *Suc n*; *a*∉*A*; *finite A*⟧ ⟹ *card A* ≤ *n*
  by *auto*

lemma *nyinitcls-le-SucD*:
⟦*card* (*nyinitcls G* (*x*,*s*)) ≤ *Suc n*; ¬*inited C* (*globs s*); *class G C*=*Some y*⟧ ⟹
  card (*nyinitcls G* (*x*,*init-class-obj G C s*)) ≤ *n*
apply (*subgoal-tac*
        nyinitcls G (*x*,*s*) = *insert C* (*nyinitcls G* (*x*,*init-class-obj G C s*)))
apply *clarsimp*
apply (*erule-tac V*=*nyinitcls G* (*x*, *s*) = *rhs* **for** *rhs* **in** *thin-rl*)
apply (*rule card-Suc-lemma* [*OF - - finite-nyinitcls*])
apply (*auto dest*!: *not-initedD elim*!:
          simp add: *nyinitcls-def inited-def split*: *if-split-asm*)
done

lemma *inited-gext′*: ⟦*s*≤|*s′*;*inited C* (*globs s*)⟧ ⟹ *inited C* (*globs s′*)
  by (*rule inited-gext*)

lemma *nyinitcls-gext*: *snd s*≤|*snd s′* ⟹ *nyinitcls G s′* ⊆ *nyinitcls G s*
  unfolding *nyinitcls-def* **by** (*force dest*!: *inited-gext′*)

lemma *card-nyinitcls-gext*:
  ⟦*snd s*≤|*snd s′*; *card* (*nyinitcls G s*) ≤ *n*⟧⟹ *card* (*nyinitcls G s′*) ≤ *n*
apply (*rule le-trans*)
apply (*rule card-mono*)
apply (*rule finite-nyinitcls*)
apply (*erule nyinitcls-gext*)
apply *assumption*
done

init-le

definition
  init-le :: *prog* ⇒ *nat* ⇒ *state* ⇒ *bool* (-⊢*init*≤- [*51*,*51*] *50*)
  where *G*⊢*init*≤*n* = (λ*s*. *card* (*nyinitcls G s*) ≤ *n*)

lemma *init-le-def2* [*simp*]: $(G\vdash init\leq n)\ s = (card\ (nyinitcls\ G\ s)\leq n)$
apply (*unfold init-le-def*)
apply *auto*
done

lemma *All-init-leD*:
$\forall n{::}nat.\ G,(A{::}'a\ triple\ set)\vdash\{P \wedge.\ G\vdash init\leq n\}\ t\succ\ \{Q{::}'a\ assn\}$
$\Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Q\}$
apply (*drule spec*)
apply (*erule conseq1*)
apply *clarsimp*
apply (*rule card-nyinitcls-bound*)
done

Most General Triples and Formulas

definition
remember-init-state :: *state assn* ($\doteq$)
where $\doteq\ \equiv \lambda Y\ s\ Z.\ s = Z$

lemma *remember-init-state-def2* [*simp*]: $\doteq\ Y = (=)$
apply (*unfold remember-init-state-def*)
apply (*simp* (*no-asm*))
done

definition
MGF ::[*state assn*, *term*, *prog*] $\Rightarrow$ *state triple* ({-} -$\succ$ {-$\rightarrow$}[*3*,*65*,*3*]*62*)
where $\{P\}\ t\succ\ \{G\rightarrow\} = \{P\}\ t\succ\ \{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\}$

definition
MGFn :: [*nat*, *term*, *prog*] $\Rightarrow$ *state triple* ({=:-} -$\succ$ {-$\rightarrow$}[*3*,*65*,*3*]*62*)
where $\{=:n\}\ t\succ\ \{G\rightarrow\} = \{\doteq\ \wedge.\ G\vdash init\leq n\}\ t\succ\ \{G\rightarrow\}$

lemma *MGF-valid*: *wf-prog G* $\Longrightarrow G,\{\}\models\{\doteq\}\ t\succ\ \{G\rightarrow\}$
apply (*unfold MGF-def*)
apply (*simp add*: *ax-valids-def triple-valid-def2*)
apply (*auto elim*: *evaln-eval*)
done

lemma *MGF-res-eq-lemma* [*simp*]:
$(\forall Y'\ Y\ s.\ Y = Y' \wedge P\ s \longrightarrow Q\ s) = (\forall s.\ P\ s \longrightarrow Q\ s)$
by *auto*

lemma *MGFn-def2*:
$G,A\vdash\{=:n\}\ t\succ\ \{G\rightarrow\} = G,A\vdash\{\doteq\ \wedge.\ G\vdash init\leq n\}$
$t\succ\ \{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\}$
unfolding *MGFn-def MGF-def* **by** *fast*

lemma *MGF-MGFn-iff*:

$G,(A::\mathit{state\ triple\ set})\vdash\{\doteq\}\ t\succ\ \{G\rightarrow\} = (\forall\, n.\ G,A\vdash\{=:n\}\ t\succ\ \{G\rightarrow\})$
apply (*simp add*: *MGFn-def2 MGF-def*)
apply *safe*
apply (*erule-tac* [*2*] *All-init-leD*)
apply (*erule conseq1*)
apply *clarsimp*
done

lemma *MGFnD*:
$G,(A::\mathit{state\ triple\ set})\vdash\{=:n\}\ t\succ\ \{G\rightarrow\} \Longrightarrow$
$G,A\vdash\{(\lambda Y'\ s'\ s.\ s' = s \quad\quad \wedge\ P\ s)\ \wedge.\ G\vdash init\le n\}$
$t\succ\ \{(\lambda Y'\ s'\ s.\ G\vdash s{-}t\succ\rightarrow(Y',s')\ \wedge\ P\ s)\ \wedge.\ G\vdash init\le n\}$
apply (*unfold init-le-def*)
apply (*simp* (*no-asm-use*) *add*: *MGFn-def2*)
apply (*erule conseq12*)
apply *clarsimp*
apply (*erule* (*1*) *eval-gext* [*THEN card-nyinitcls-gext*])
done
lemmas $MGFnD' = MGFnD$ [*of - - - - λx. True*]

To derive the most general formula, we can always assume a normal state in the precondition, since abrupt cases can be handled uniformally by the abrupt rule.

lemma *MGFNormalI*: $G,A\vdash\{Normal\ \doteq\}\ t\succ\ \{G\rightarrow\} \Longrightarrow$
$G,(A::\mathit{state\ triple\ set})\vdash\{\doteq::\mathit{state\ assn}\}\ t\succ\ \{G\rightarrow\}$
apply (*unfold MGF-def*)
apply (*rule ax-Normal-cases*)
apply (*erule conseq1*)
apply *clarsimp*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*])
apply (*clarsimp simp add*: *Let-def*)
done

lemma *MGFNormalD*:
$G,(A::\mathit{state\ triple\ set})\vdash\{\doteq\}\ t\succ\ \{G\rightarrow\} \Longrightarrow G,A\vdash\{Normal\ \doteq\}\ t\succ\ \{G\rightarrow\}$
apply (*unfold MGF-def*)
apply (*erule conseq1*)
apply *clarsimp*
done

Additionally to *MGFNormalI*, we also expand the definition of the most general formula here

lemma *MGFn-NormalI*:
$G,(A::\mathit{state\ triple\ set})\vdash\{Normal((\lambda Y'\ s'\ s.\ s'{=}s\ \wedge\ normal\ s)\ \wedge.\ G\vdash init\le n)\}t\succ$
$\{\lambda Y\ s'\ s.\ G\vdash s\ {-}t\succ\rightarrow\ (Y,s')\} \Longrightarrow G,A\vdash\{=:n\}t\succ\{G\rightarrow\}$
apply (*simp* (*no-asm-use*) *add*: *MGFn-def2*)
apply (*rule ax-Normal-cases*)
apply (*erule conseq1*)
apply *clarsimp*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*])
apply (*clarsimp simp add*: *Let-def*)
done

To derive the most general formula, we can restrict ourselves to welltyped terms, since all others can be uniformally handled by the hazard rule.

lemma *MGFn-free-wt*:
$(\exists\ T\ L\ C.\ (\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash t::T)$
$\longrightarrow G,(A::\mathit{state\ triple\ set})\vdash\{=:n\}\ t\succ\ \{G\rightarrow\}$

$\Longrightarrow G,A\vdash\{=:n\}\ t\succ\ \{G\rightarrow\}$
apply (*rule MGFn-NormalI*)
apply (*rule ax-free-wt*)
apply (*auto elim*: *conseq12 simp add*: *MGFn-def MGF-def*)
done

To derive the most general formula, we can restrict ourselves to welltyped terms and assume that the state in the precondition conforms to the environment. All type violations can be uniformally handled by the hazard rule.

lemma *MGFn-free-wt-NormalConformI*:
$(\forall\ T\ L\ C\ .\ (\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash t{::}\ T$
$\longrightarrow G,(A{::}state\ triple\ set)$
$\vdash\{Normal((\lambda Y'\ s'\ s.\ s'{=}s \wedge normal\ s) \wedge.\ G\vdash init{\le}n) \wedge.\ (\lambda\ s.\ s{::}{\preceq}(G,\ L))\}$
$t\succ$
$\{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\})$
$\Longrightarrow G,A\vdash\{=:n\}t\succ\{G\rightarrow\}$
apply (*rule MGFn-NormalI*)
apply (*rule ax-no-hazard*)
apply (*rule ax-escape*)
apply (*intro strip*)
apply (*simp only*: *type-ok-def peek-and-def*)
apply (*erule conjE*)+
apply (*erule exE*,*erule exE*, *erule exE*, *erule exE*,*erule conjE*,*drule* (*1*) *mp*,
erule conjE)
apply (*drule spec*,*drule spec*, *drule spec*, *drule* (*1*) *mp*)
apply (*erule conseq12*)
apply *blast*
done

To derive the most general formula, we can restrict ourselves to welltyped terms and assume that the state in the precondition conforms to the environment and that the term is definetly assigned with respect to this state. All type violations can be uniformally handled by the hazard rule.

lemma *MGFn-free-wt-da-NormalConformI*:
$(\forall\ T\ L\ C\ B.\ (\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash t{::}\ T$
$\longrightarrow G,(A{::}state\ triple\ set)$
$\vdash\{Normal((\lambda Y'\ s'\ s.\ s'{=}s \wedge normal\ s) \wedge.\ G\vdash init{\le}n) \wedge.\ (\lambda\ s.\ s{::}{\preceq}(G,\ L))$
$\wedge.\ (\lambda\ s.\ (\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash dom\ (locals\ (store\ s))\gg t\gg B)\}$
$t\succ$
$\{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\})$
$\Longrightarrow G,A\vdash\{=:n\}t\succ\{G\rightarrow\}$
apply (*rule MGFn-NormalI*)
apply (*rule ax-no-hazard*)
apply (*rule ax-escape*)
apply (*intro strip*)
apply (*simp only*: *type-ok-def peek-and-def*)
apply (*erule conjE*)+
apply (*erule exE*,*erule exE*, *erule exE*, *erule exE*,*erule conjE*,*drule* (*1*) *mp*,
erule conjE)
apply (*drule spec*,*drule spec*, *drule spec*,*drule spec*, *drule* (*1*) *mp*)
apply (*erule conseq12*)
apply *blast*
done

main lemmas

lemma *MGFn-Init*:
assumes *mgf-hyp*: $\forall m.\ Suc\ m{\le}n \longrightarrow (\forall t.\ G,A\vdash\{=:m\}\ t\succ\ \{G\rightarrow\})$
shows $G,(A{::}state\ triple\ set)\vdash\{=:n\}\ \langle Init\ C\rangle_s\succ\ \{G\rightarrow\}$

```
proof (rule MGFn-free-wt [rule-format],elim exE,rule MGFn-NormalI)
  fix T L accC
  assume ⦇prg=G, cls=accC, lcl= L⦈⊢⟨Init C⟩ₛ::T
  hence is-cls: is-class G C
    by cases simp
  show G,A⊢{Normal ((λY' s' s. s' = s ∧ normal s) ∧. G⊢init≤n)}
            .Init C.
            {λY s' s. G⊢s −⟨Init C⟩ₛ≻→ (Y, s')}
       (is G,A⊢{Normal ?P} .Init C. {?R})
  proof (rule ax-cases [where ?C=initd C])
    show G,A⊢{Normal ?P  ∧. initd C} .Init C. {?R}
      by (rule ax-derivs.Done [THEN conseq1]) (fastforce intro: init-done)
  next
    have G,A⊢{Normal (?P  ∧. Not ∘ initd C)} .Init C. {?R}
    proof (cases n)
      case 0
      with is-cls
      show ?thesis
        by − (rule ax-impossible [THEN conseq1],fastforce dest: nyinitcls-emptyD)
    next
      case (Suc m)
      with mgf-hyp have mgf-hyp': ⋀ t. G,A⊢{=:m} t≻ {G→}
        by simp
      from is-cls obtain c where c: the (class G C) = c
        by auto
      let ?Q= (λY s' (x,s) .
          G⊢ (x,init-class-obj G C s)
             − (if C=Object then Skip else Init (super (the (class G C))))→ s'
          ∧ x=None ∧ ¬inited C (globs s)) ∧. G⊢init≤m
      from c
      show ?thesis
      proof (rule ax-derivs.Init [where ?Q=?Q])
        let ?P' = Normal ((λY s' s. s' = supd (init-class-obj G C) s
                           ∧ normal s ∧ ¬ initd C s) ∧. G⊢init≤m)
        show G,A⊢{Normal (?P ∧. Not ∘ initd C ;. supd (init-class-obj G C))}
                  .(if C = Object then Skip else Init (super c)).
                  {?Q}
        proof (rule conseq1 [where ?P'=?P'])
          show G,A⊢{?P'} .(if C = Object then Skip else Init (super c)). {?Q}
          proof (cases C=Object)
            case True
            have G,A⊢{?P'} .Skip. {?Q}
              by (rule ax-derivs.Skip [THEN conseq1])
                 (auto simp add: True intro: eval.Skip)
            with True show ?thesis
              by simp
          next
            case False
            from mgf-hyp'
            have G,A⊢{?P'} .Init (super c). {?Q}
              by (rule MGFnD' [THEN conseq12]) (fastforce simp add: False c)
            with False show ?thesis
              by simp
          qed
        next
          from Suc is-cls
          show Normal (?P ∧. Not ∘ initd C ;. supd (init-class-obj G C))
                 ⇒ ?P'
            by (fastforce elim: nyinitcls-le-SucD)
```

```
        qed
      next
        from mgf-hyp′
        show ∀ l. G,A⊢{?Q ∧. (λs. l = locals (snd s)) ;. set-lvars Map.empty}
                    .init c.
                    {set-lvars l .; ?R}
          apply (rule MGFnD′ [THEN conseq12, THEN allI])
          apply (clarsimp simp add: split-paired-all)
          apply (rule eval.Init [OF c])
          apply (insert c)
          apply auto
          done
      qed
    qed
    thus G,A⊢{Normal ?P  ∧. Not ∘ initd C} .Init C. {?R}
      by clarsimp
  qed
qed
lemmas MGFn-InitD = MGFn-Init [THEN MGFnD, THEN ax-NormalD]


lemma MGFn-Call:
  assumes mgf-methds:
          ∀ C sig. G,(A::state triple set)⊢{=:n} ⟨(Methd C sig)⟩e≻ {G→}
  and mgf-e: G,A⊢{=:n} ⟨e⟩e≻ {G→}
  and mgf-ps: G,A⊢{=:n} ⟨ps⟩l≻ {G→}
  and wf: wf-prog G
  shows G,A⊢{=:n} ⟨{accC,statT,mode}e·mn({pTs′}ps)⟩e≻ {G→}
proof (rule MGFn-free-wt-da-NormalConformI [rule-format],clarsimp)
  note inj-term-simps [simp]
  fix T L accC′ E
  assume wt: (|prg=G,cls=accC′,lcl = L|)⊢⟨({accC,statT,mode}e·mn( {pTs′}ps))⟩e::T
  then obtain pTs statDeclT statM where
               wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−RefT statT and
            wt-args: (|prg=G,cls=accC,lcl=L|)⊢ps::≐pTs and
              statM: max-spec G accC statT (|name=mn,parTs=pTs|)
                     = {((statDeclT,statM),pTs′)} and
               mode: mode = invmode statM e and
                  T: T =Inl (resTy statM) and
        eq-accC-accC′: accC=accC′
        by cases fastforce+
  let ?Q=(λY s1 (x,s) . x = None ∧
             (∃ a. G⊢Norm s −e−≻a→ s1 ∧
                  (normal s1 ⟶ G, store s1⊢a::≼RefT statT)
                  ∧ Y = In1 a) ∧
             (∃ P. normal s1
                 ⟶ (|prg=G,cls=accC′,lcl=L|)⊢dom (locals (store s1))»⟨ps⟩l»P))
         ∧. G⊢init≤n ∧. (λ s. s::≼(G, L))::state assn
  let ?R=λa. ((λY (x2,s2) (x,s) . x = None ∧
               (∃ s1 pvs. G⊢Norm s −e−≻a→ s1 ∧
                        (normal s1 ⟶ G, store s1⊢a::≼RefT statT)∧
                        Y = ⌊pvs⌋l ∧ G⊢s1 −ps≐≻pvs→ (x2,s2)))
            ∧. G⊢init≤n ∧. (λ s. s::≼(G, L)))::state assn

  show G,A⊢{Normal ((λY′ s′ s. s′ = s ∧ abrupt s = None) ∧. G⊢init≤n ∧.
                  (λs. s::≼(G, L)) ∧.
                  (λs. (|prg=G, cls=accC′,lcl=L|) ⊢ dom (locals (store s))
                       » ⟨{accC,statT,mode}e·mn( {pTs′}ps)⟩e» E))}
          {accC,statT,mode}e·mn( {pTs′}ps)−≻
```

```
        {λY s′ s. ∃ v. Y = ⌊v⌋e ∧
                        G⊢s −{accC,statT,mode}e·mn( {pTs′}ps)−≻v→ s′}
   (is G,A⊢{Normal ?P} {accC,statT,mode}e·mn( {pTs′}ps)−≻ {?S})
 proof (rule ax-derivs.Call [where ?Q=?Q and ?R=?R])
   from mgf-e
   show G,A⊢{Normal ?P} e−≻ {?Q}
   proof (rule MGFnD′ [THEN conseq12],clarsimp)
     fix s0 s1 a
     assume conf-s0: Norm s0::≼(G, L)
     assume da: (|prg=G,cls=accC′,lcl=L|)⊢
                  dom (locals s0) »⟨{accC,statT,mode}e·mn( {pTs′}ps)⟩e» E
     assume eval-e: G⊢Norm s0 −e−≻a→ s1
     show (abrupt s1 = None ⟶ G,store s1⊢a::≼RefT statT) ∧
           (abrupt s1 = None ⟶
             (∃ P. (|prg=G,cls=accC′,lcl=L|)⊢ dom (locals (store s1)) »⟨ps⟩l» P))
           ∧ s1::≼(G, L)
     proof −
       from da obtain C where
         da-e: (|prg=G,cls=accC,lcl=L|)⊢
                    dom (locals (store ((Norm s0)::state)))»⟨e⟩e» C and
         da-ps: (|prg=G,cls=accC,lcl=L|)⊢ nrm C »⟨ps⟩l» E
         by cases (simp add: eq-accC-accC′)
       from eval-e conf-s0 wt-e da-e wf
       obtain (abrupt s1 = None ⟶ G,store s1⊢a::≼RefT statT)
         and s1::≼(G, L)
         by (rule eval-type-soundE) simp
       moreover
       {
         assume normal-s1: normal s1
         have ∃ P. (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨ps⟩l» P
         proof −
           from eval-e wt-e da-e wf normal-s1
           have nrm C ⊆ dom (locals (store s1))
             by (cases rule: da-good-approxE′) iprover
           with da-ps show ?thesis
             by (rule da-weakenE) iprover
         qed
       }
       ultimately show ?thesis
         using eq-accC-accC′ by simp
     qed
   qed
 next
   show ∀ a. G,A⊢{?Q←In1 a} ps≐≻ {?R a} (is ∀ a. ?PS a)
   proof
     fix a
     show ?PS a
     proof (rule MGFnD′ [OF mgf-ps, THEN conseq12],
              clarsimp simp add: eq-accC-accC′ [symmetric])
       fix s0 s1 s2 vs
       assume conf-s1: s1::≼(G, L)
       assume eval-e: G⊢Norm s0 −e−≻a→ s1
       assume conf-a: abrupt s1 = None ⟶ G,store s1⊢a::≼RefT statT
       assume eval-ps: G⊢s1 −ps≐≻vs→ s2
       assume da-ps: abrupt s1 = None ⟶
                        (∃ P. (|prg=G,cls=accC,lcl=L|)⊢
                               dom (locals (store s1)) »⟨ps⟩l» P)
       show (∃ s1. G⊢Norm s0 −e−≻a→ s1 ∧
                (abrupt s1 = None ⟶ G,store s1⊢a::≼RefT statT) ∧
```

```
          G⊢s1 −ps≐≻vs→ s2) ∧
        s2::⪯(G, L)
    proof (cases normal s1)
      case True
      with da-ps obtain P where
       (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨ps⟩l» P
        by auto
      from eval-ps conf-s1 wt-args this wf
      have s2::⪯(G, L)
        by (rule eval-type-soundE)
      with eval-e conf-a eval-ps
      show ?thesis
        by auto
    next
      case False
      with eval-ps have s2=s1 by auto
      with eval-e conf-a eval-ps conf-s1
      show ?thesis
        by auto
    qed
  qed
qed
next
  show ∀ a vs invC declC l.
    G,A⊢{?R a←⌊vs⌋l ∧.
           (λs. declC =
                invocation-declclass G mode (store s) a statT
                    (|name=mn, parTs=pTs'|) ∧
                invC = invocation-class mode (store s) a statT ∧
                l = locals (store s)) ;.
           init-lvars G declC (|name=mn, parTs=pTs'|) mode a vs ∧.
           (λs. normal s ⟶ G⊢mode→invC⪯statT)}
        Methd declC (|name=mn,parTs=pTs'|)−≻
        {set-lvars l .; ?S}
    (is ∀ a vs invC declC l. ?METHD a vs invC declC l)
  proof (intro allI)
    fix a vs invC declC l
    from mgf-methds [rule-format]
    show ?METHD a vs invC declC l
    proof (rule MGFnD' [THEN conseq12],clarsimp)
      fix s4 s2 s1::state
      fix s0 v
      let ?D= invocation-declclass G mode (store s2) a statT
                (|name=mn,parTs=pTs'|)
      let ?s3= init-lvars G ?D (|name=mn, parTs=pTs'|) mode a vs s2
      assume inv-prop: abrupt ?s3=None
          ⟶ G⊢mode→invocation-class mode (store s2) a statT⪯statT
      assume conf-s2: s2::⪯(G, L)
      assume conf-a: abrupt s1 = None ⟶ G,store s1⊢a::⪯RefT statT
      assume eval-e: G⊢Norm s0 −e−≻a→ s1
      assume eval-ps: G⊢s1 −ps≐≻vs→ s2
      assume eval-mthd: G⊢?s3 −Methd ?D (|name=mn,parTs=pTs'|)−≻v→ s4
      show G⊢Norm s0 −{accC,statT,mode}e·mn( {pTs'}ps)−≻v
                   → (set-lvars (locals (store s2))) s4
      proof −
        obtain D where D: D=?D by simp
        obtain s3 where s3: s3=?s3 by simp
        obtain s3' where
          s3': s3' = check-method-access G accC statT mode
```

```
                        (|name=mn,parTs=pTs'|) a s3
        by simp
      have eq-s3'-s3: s3'=s3
      proof -
        from inv-prop s3 mode
        have normal s3 ⟹
          G⊢invmode statM e→invocation-class mode (store s2) a statT≼statT
          by auto
        with eval-ps wt-e statM conf-s2 conf-a [rule-format]
        have check-method-access G accC statT (invmode statM e)
                (|name=mn,parTs=pTs'|) a s3 = s3
          by (rule error-free-call-access) (auto simp add: s3 mode wf)
        thus ?thesis
          by (simp add: s3' mode)
      qed
      with eval-mthd D s3
      have G⊢s3' −Methd D (|name=mn,parTs=pTs'|)−≻v→ s4
        by simp
      with eval-e eval-ps D - s3'
      show ?thesis
        by (rule eval-Call) (auto simp add: s3 mode D)
    qed
   qed
  qed
 qed
qed
```

```
lemma eval-expression-no-jump':
  assumes eval: G⊢s0 −e−≻v→ s1
  and   no-jmp: abrupt s0 ≠ Some (Jump j)
  and      wt: (|prg=G, cls=C,lcl=L|)⊢e::−T
  and      wf: wf-prog G
shows abrupt s1 ≠ Some (Jump j)
using eval no-jmp wt wf
by − (rule eval-expression-no-jump
        [where ?Env=(|prg=G, cls=C,lcl=L|),simplified],auto)
```

To derive the most general formula for the loop statement, we need to come up with a proper loop invariant, which intuitively states that we are currently inside the evaluation of the loop. To define such an invariant, we unroll the loop in iterated evaluations of the expression and evaluations of the loop body.

```
definition
  unroll :: prog ⇒ label ⇒ expr ⇒ stmt ⇒ (state × state) set where
  unroll G l e c = {(s,t). ∃ v s1 s2.
                        G⊢s −e−≻v→ s1 ∧ the-Bool v ∧ normal s1 ∧
                        G⊢s1 −c→ s2 ∧ t=(abupd (absorb (Cont l)) s2)}
```

```
lemma unroll-while:
  assumes unroll: (s, t) ∈ (unroll G l e c)*
  and     eval-e: G⊢t −e−≻v→ s'
  and     normal-termination: normal s' ⟶ ¬ the-Bool v
  and     wt: (|prg=G,cls=C,lcl=L|)⊢e::−T
  and     wf: wf-prog G
  shows G⊢s −l· While(e) c→ s'
using unroll
```

```
proof (induct rule: converse-rtrancl-induct)
  show G⊢t −l• While(e) c→ s′
  proof (cases normal t)
    case False
    with eval-e have s′=t by auto
    with False show ?thesis by auto
  next
    case True
    note normal-t = this
    show ?thesis
    proof (cases normal s′)
      case True
      with normal-t eval-e normal-termination
      show ?thesis
        by (auto intro: eval.Loop)
    next
      case False
      note abrupt-s′ = this
      from eval-e - wt wf
      have no-cont: abrupt s′ ≠ Some (Jump (Cont l))
        by (rule eval-expression-no-jump′) (insert normal-t,simp)
      have
        if the-Bool v
             then (G⊢s′ −c→ s′ ∧
                    G⊢(abupd (absorb (Cont l)) s′) −l• While(e) c→ s′)
             else s′ = s′
      proof (cases the-Bool v)
        case False thus ?thesis by simp
      next
        case True
        with abrupt-s′ have G⊢s′ −c→ s′ by auto
        moreover from abrupt-s′ no-cont
        have no-absorb: (abupd (absorb (Cont l)) s′)=s′
          by (cases s′) (simp add: absorb-def split: if-split)
        moreover
        from no-absorb abrupt-s′
        have G⊢(abupd (absorb (Cont l)) s′) −l• While(e) c→ s′
          by auto
        ultimately show ?thesis
          using True by simp
      qed
      with eval-e
      show ?thesis
        using normal-t by (auto intro: eval.Loop)
    qed
  qed
next
  fix s s3
  assume unroll: (s,s3) ∈ unroll G l e c
  assume while: G⊢s3 −l• While(e) c→ s′
  show G⊢s −l• While(e) c→ s′
  proof −
    from unroll obtain v s1 s2 where
      normal-s1: normal s1 and
      eval-e: G⊢s −e−≻v→ s1 and
      continue: the-Bool v and
      eval-c: G⊢s1 −c→ s2 and
      s3: s3=(abupd (absorb (Cont l)) s2)
      by (unfold unroll-def) fast
```

from *eval-e normal-s1* **have**
normal s
by (*rule eval-no-abrupt-lemma* [*rule-format*])
with *while eval-e continue eval-c s3* **show** *?thesis*
by (*auto intro*!: *eval.Loop*)
qed
qed

lemma *MGFn-Loop*:
assumes *mfg-e*: G,(A::*state triple set*)$\vdash\{=:n\}\ \langle e\rangle_e\succ\ \{G\rightarrow\}$
and *mfg-c*: $G,A\vdash\{=:n\}\ \langle c\rangle_s\succ\ \{G\rightarrow\}$
and *wf*: *wf-prog G*
shows $G,A\vdash\{=:n\}\ \langle l\cdot\ \mathit{While}(e)\ c\rangle_s\succ\ \{G\rightarrow\}$
proof (*rule MGFn-free-wt* [*rule-format*],*elim exE*)
fix *T L C*
assume *wt*: $(\!|\mathit{prg} = G,\ \mathit{cls} = C,\ \mathit{lcl} = L|\!)\vdash\langle l\cdot\ \mathit{While}(e)\ c\rangle_s::T$
then obtain *eT* **where**
wt-e: $(\!|\mathit{prg} = G,\ \mathit{cls} = C,\ \mathit{lcl} = L|\!)\vdash e::-eT$
by *cases simp*
show *?thesis*
proof (*rule MGFn-NormalI*)
show $G,A\vdash\{\mathit{Normal}\ ((\lambda Y'\ s'\ s.\ s' = s \land \mathit{normal}\ s) \land.\ G\vdash \mathit{init}\le n)\}$
$.l\cdot\ \mathit{While}(e)\ c.$
$\{\lambda Y\ s'\ s.\ G\vdash s\ -\mathit{In1r}\ (l\cdot\ \mathit{While}(e)\ c)\succ\rightarrow\ (Y,\ s')\}$
proof (*rule conseq12*
[**where** $?P'$=$(\lambda\ Y\ s'\ s.\ (s,s') \in (\mathit{unroll}\ G\ l\ e\ c)^*\ ) \land.\ G\vdash \mathit{init}\le n$
and $?Q'$=$((\lambda\ Y\ s'\ s.\ (\exists\ t\ b.\ (s,t) \in (\mathit{unroll}\ G\ l\ e\ c)^* \land$
$Y=\lfloor b\rfloor_e \land G\vdash t\ -e-\succ b\rightarrow\ s'))$
$\land.\ G\vdash \mathit{init}\le n)\leftarrow=\mathit{False}\!\downarrow=\Diamond$])
show $G,A\vdash\{(\lambda Y\ s'\ s.\ (s,\ s') \in (\mathit{unroll}\ G\ l\ e\ c)^*) \land.\ G\vdash \mathit{init}\le n\}$
$.l\cdot\ \mathit{While}(e)\ c.$
$\{((\lambda Y\ s'\ s.\ (\exists\, t\ b.\ (s,\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^* \land$
$Y = \mathit{In1}\ b \land G\vdash t\ -e-\succ b\rightarrow\ s'))$
$\land.\ G\vdash \mathit{init}\le n)\leftarrow=\mathit{False}\!\downarrow=\Diamond\}$
proof (*rule ax-derivs.Loop*)
from *mfg-e*
show $G,A\vdash\{(\lambda Y\ s'\ s.\ (s,\ s') \in (\mathit{unroll}\ G\ l\ e\ c)^*) \land.\ G\vdash \mathit{init}\le n\}$
$e-\succ$
$\{(\lambda Y\ s'\ s.\ (\exists\, t\ b.\ (s,\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^* \land$
$Y = \mathit{In1}\ b \land G\vdash t\ -e-\succ b\rightarrow\ s'))$
$\land.\ G\vdash \mathit{init}\le n\}$
proof (*rule MGFnD'* [*THEN conseq12*],*clarsimp*)
fix *s Z s' v*
assume $(Z,\ s) \in (\mathit{unroll}\ G\ l\ e\ c)^*$
moreover
assume $G\vdash s\ -e-\succ v\rightarrow\ s'$
ultimately
show $\exists\, t.\ (Z,\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^* \land G\vdash t\ -e-\succ v\rightarrow\ s'$
by *blast*
qed
next
from *mfg-c*
show $G,A\vdash\{\mathit{Normal}\ (((\lambda Y\ s'\ s.\ \exists\, t\ b.\ (s,\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^* \land$
$Y = \lfloor b\rfloor_e \land G\vdash t\ -e-\succ b\rightarrow\ s')$
$\land.\ G\vdash \mathit{init}\le n)\leftarrow=\mathit{True})\}$
$.c.$
$\{\mathit{abupd}\ (\mathit{absorb}\ (\mathit{Cont}\ l))\ .;$
$((\lambda Y\ s'\ s.\ (s,\ s') \in (\mathit{unroll}\ G\ l\ e\ c)^*) \land.\ G\vdash \mathit{init}\le n)\}$

```
      proof (rule MGFnD' [THEN conseq12],clarsimp)
        fix Z s' s v t
        assume unroll: (Z, t) ∈ (unroll G l e c)*
        assume eval-e: G⊢t −e−≻v→ Norm s
        assume true: the-Bool v
        assume eval-c: G⊢Norm s −c→ s'
        show (Z, abupd (absorb (Cont l)) s') ∈ (unroll G l e c)*
        proof −
          note unroll
          also
          from eval-e true eval-c
          have (t,abupd (absorb (Cont l)) s') ∈ unroll G l e c
            by (unfold unroll-def) force
          ultimately show ?thesis ..
        qed
      qed
    qed
  next
    show
     ∀ Y s Z.
      (Normal ((λY' s' s. s' = s ∧ normal s) ∧. G⊢init≤n)) Y s Z
      ⟶ (∀ Y' s'.
             (∀ Y Z'.
               ((λY s' s. (s, s') ∈ (unroll G l e c)*) ∧. G⊢init≤n) Y s Z'
               ⟶ (((λY s' s. ∃ t b. (s,t) ∈ (unroll G l e c)*
                                      ∧ Y=⌊b⌋e ∧ G⊢t −e−≻b→ s')
                   ∧. G⊢init≤n)←=False↓=♢) Y' s' Z')
             ⟶ G⊢Z −⟨l· While(e) c⟩s≻→ (Y', s'))
    proof (clarsimp)
      fix Y' s' s
      assume asm:
        ∀ Z'. (Z', Norm s) ∈ (unroll G l e c)*
               ⟶ card (nyinitcls G s') ≤ n ∧
                   (∃ v. (∃ t. (Z', t) ∈ (unroll G l e c)* ∧ G⊢t −e−≻v→ s') ∧
                   (fst s' = None ⟶ ¬ the-Bool v)) ∧ Y' = ♢
      show Y' = ♢ ∧ G⊢Norm s −l· While(e) c→ s'
      proof −
        from asm obtain v t where
          — Z' gets instantiated with Norm s
          unroll: (Norm s, t) ∈ (unroll G l e c)* and
          eval-e: G⊢t −e−≻v→ s' and
          normal-termination: normal s' ⟶ ¬ the-Bool v and
           Y': Y' = ♢
          by auto
        from unroll eval-e normal-termination wt-e wf
        have G⊢Norm s −l· While(e) c→ s'
          by (rule unroll-while)
        with Y'
        show ?thesis
          by simp
      qed
    qed
  qed
 qed
qed
```

lemma *MGFn-FVar*:
  fixes *A* :: *state triple set*

```
 assumes mgf-init: G,A⊢{=:n} ⟨Init statDeclC⟩s≻ {G→}
  and    mgf-e: G,A⊢{=:n} ⟨e⟩e≻ {G→}
  and    wf: wf-prog G
  shows G,A⊢{=:n} ⟨{accC,statDeclC,stat}e..fn⟩v≻ {G→}
proof (rule MGFn-free-wt-da-NormalConformI [rule-format],clarsimp)
  note inj-term-simps [simp]
  fix T L accC' V
  assume wt: ⦇prg = G, cls = accC', lcl = L⦈⊢⟨{accC,statDeclC,stat}e..fn⟩v::T
  then obtain statC f where
    wt-e: ⦇prg=G, cls=accC', lcl=L⦈⊢e::−Class statC and
    accfield: accfield G accC' statC fn = Some (statDeclC,f ) and
    eq-accC: accC=accC' and
    stat: stat=is-static  f
    by (cases) (auto simp add: member-is-static-simp)
  let ?Q=(λY s1 (x,s) . x = None ∧
               (G⊢Norm s −Init statDeclC→ s1) ∧
               (∃ E. ⦇prg=G,cls=accC',lcl=L⦈⊢dom (locals (store s1)) »⟨e⟩e» E))
               ∧. G⊢init≤n ∧. (λ s. s::≼(G, L))
  show G,A⊢{Normal
            ((λY' s' s. s' = s ∧ abrupt s = None) ∧. G⊢init≤n ∧.
             (λs. s::≼(G, L)) ∧.
             (λs. ⦇prg=G,cls=accC',lcl=L⦈
                ⊢ dom (locals (store s)) » ⟨{accC,statDeclC,stat}e..fn⟩v» V))
            } {accC,statDeclC,stat}e..fn=≻
            {λY s' s. ∃ vf. Y = ⌊vf⌋v ∧
                        G⊢s −{accC,statDeclC,stat}e..fn=≻vf→ s'}
    (is G,A⊢{Normal ?P} {accC,statDeclC,stat}e..fn=≻ {?R})
  proof (rule ax-derivs.FVar [where ?Q=?Q ])
    from mgf-init
    show G,A⊢{Normal ?P} .Init statDeclC. {?Q}
    proof (rule MGFnD' [THEN conseq12],clarsimp)
      fix s s'
      assume conf-s: Norm s::≼(G, L)
      assume da: ⦇prg=G,cls=accC',lcl=L⦈
                 ⊢ dom (locals s) »⟨{accC,statDeclC,stat}e..fn⟩v» V
      assume eval-init: G⊢Norm s −Init statDeclC→ s'
      show (∃ E. ⦇prg=G, cls=accC', lcl=L⦈⊢ dom (locals (store s')) »⟨e⟩e» E) ∧
            s'::≼(G, L)
      proof −
        from da
        obtain E where
          ⦇prg=G, cls=accC', lcl=L⦈⊢ dom (locals s) »⟨e⟩e» E
          by cases simp
        moreover
        from eval-init
        have dom (locals s) ⊆ dom (locals (store s'))
          by (rule dom-locals-eval-mono [elim-format]) simp
        ultimately obtain E' where
          ⦇prg=G, cls=accC', lcl=L⦈⊢ dom (locals (store s')) »⟨e⟩e» E'
          by (rule da-weakenE)
        moreover
        have s'::≼(G, L)
        proof −
          have wt-init: ⦇prg=G, cls=accC, lcl=L⦈⊢(Init statDeclC)::√
          proof −
            from wf wt-e
            have iscls-statC: is-class G statC
              by (auto dest: ty-expr-is-type type-is-class)
            with wf accfield
```

```
          have iscls-statDeclC: is-class G statDeclC
            by (auto dest!: accfield-fields dest: fields-declC)
          thus ?thesis by simp
        qed
        obtain I where
          da-init: (|prg=G,cls=accC,lcl=L|)
             ⊢ dom (locals (store ((Norm s)::state))) »⟨Init statDeclC⟩s» I
          by (auto intro: da-Init [simplified] assigned.select-convs)
        from eval-init conf-s wt-init da-init  wf
        show ?thesis
          by (rule eval-type-soundE)
      qed
      ultimately show ?thesis by iprover
    qed
  qed
next
  from mgf-e
  show G,A⊢{?Q} e−≻ {λVal:a:. fvar statDeclC stat fn a ..; ?R}
  proof (rule MGFnD′ [THEN conseq12],clarsimp)
    fix s0 s1 s2 E a
    let ?fvar = fvar statDeclC stat fn a s2
    assume eval-init: G⊢Norm s0 −Init statDeclC→ s1
    assume eval-e: G⊢s1 −e−≻a→ s2
    assume conf-s1: s1::≼(G, L)
    assume da-e: (|prg=G,cls=accC′,lcl=L|)⊢ dom (locals (store s1)) »⟨e⟩e» E
    show G⊢Norm s0 −{accC,statDeclC,stat}e..fn=≻fst ?fvar→ snd ?fvar
    proof −
      obtain v s2′ where
        v: v=fst ?fvar and s2′: s2′=snd ?fvar
        by simp
      obtain s3 where
        s3: s3= check-field-access G accC′ statDeclC fn stat a s2′
        by simp
      have eq-s3-s2′: s3=s2′
      proof −
        from eval-e conf-s1 wt-e da-e wf obtain
          conf-s2: s2::≼(G, L)  and
          conf-a: normal s2 ⟹ G,store s2⊢a::≼Class statC
          by (rule eval-type-soundE) simp
        from accfield wt-e eval-init eval-e conf-s2 conf-a - wf
        show ?thesis
          by (rule  error-free-field-access
                   [where ?v=v and ?s2′=s2′,elim-format])
            (simp add: s3 v s2′ stat)+
      qed
      from eval-init eval-e
      show ?thesis
        apply (rule eval.FVar [where ?s2′=s2′])
        apply  (simp add: s2′)
        apply  (simp add: s3 [symmetric]   eq-s3-s2′ eq-accC s2′ [symmetric])
        done
    qed
  qed
qed
qed
```

lemma *MGFn-Fin*:

```
  assumes wf: wf-prog G
  and     mgf-c1: G,A⊢{=:n} ⟨c1⟩ₛ≻ {G→}
  and     mgf-c2: G,A⊢{=:n} ⟨c2⟩ₛ≻ {G→}
  shows G,(A::state triple set)⊢{=:n} ⟨c1 Finally c2⟩ₛ≻ {G→}
proof (rule MGFn-free-wt-da-NormalConformI [rule-format],clarsimp)
  fix T L accC C
  assume wt: ⦇prg=G,cls=accC,lcl=L⦈⊢In1r (c1 Finally c2)::T
  then obtain
    wt-c1: ⦇prg=G,cls=accC,lcl=L⦈⊢c1::√ and
    wt-c2: ⦇prg=G,cls=accC,lcl=L⦈⊢c2::√
    by cases simp
  let ?Q = (λY' s' s. normal s ∧ G⊢s −c1→ s' ∧
              (∃ C1. ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s)) »⟨c1⟩ₛ» C1)
              ∧ s::≼(G, L))
            ∧. G⊢init≤n
  show G,A⊢{Normal
             ((λY' s' s. s' = s ∧ abrupt s = None) ∧. G⊢init≤n ∧.
             (λs. s::≼(G, L)) ∧.
             (λs. ⦇prg=G,cls=accC,lcl =L⦈
                    ⊢dom (locals (store s)) »⟨c1 Finally c2⟩ₛ» C))}
            .c1 Finally c2.
            {λY s' s. Y = ♢ ∧ G⊢s −c1 Finally c2→ s'}
    (is G,A⊢{Normal ?P} .c1 Finally c2. {?R})
  proof (rule ax-derivs.Fin [where ?Q=?Q])
    from mgf-c1
    show G,A⊢{Normal ?P} .c1. {?Q}
    proof (rule MGFnD' [THEN conseq12],clarsimp)
      fix s0
      assume ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals s0) »⟨c1 Finally c2⟩ₛ» C
      thus ∃ C1. ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals s0) »⟨c1⟩ₛ» C1
        by cases (auto simp add: inj-term-simps)
    qed
  next
    from mgf-c2
    show ∀ abr. G,A⊢{?Q ∧. (λs. abr = abrupt s) ;. abupd (λabr. None)} .c2.
          {abupd (abrupt-if (abr ≠ None) abr) .; ?R}
    proof (rule MGFnD' [THEN conseq12, THEN allI],clarsimp)
      fix s0 s1 s2 C1
      assume da-c1:⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals s0) »⟨c1⟩ₛ» C1
      assume conf-s0: Norm s0::≼(G, L)
      assume eval-c1: G⊢Norm s0 −c1→ s1
      assume eval-c2: G⊢abupd (λabr. None) s1 −c2→ s2
      show G⊢Norm s0 −c1 Finally c2
              → abupd (abrupt-if (∃ y. abrupt s1 = Some y) (abrupt s1)) s2
      proof −
        obtain abr1 str1 where s1: s1=(abr1,str1)
          by (cases s1)
        with eval-c1 eval-c2 obtain
          eval-c1': G⊢Norm s0 −c1→ (abr1,str1) and
          eval-c2': G⊢Norm str1 −c2→ s2
          by simp
        obtain s3 where
          s3: s3 = (if ∃ err. abr1 = Some (Error err)
                      then (abr1, str1)
                      else abupd (abrupt-if (abr1 ≠ None) abr1) s2)
          by simp
        from eval-c1' conf-s0 wt-c1 - wf
        have error-free (abr1,str1)
          by (rule eval-type-soundE) (insert da-c1,auto)
```

```
        with s3 have eq-s3: s3=abupd (abrupt-if (abr1 ≠ None) abr1) s2
          by (simp add: error-free-def)
        from eval-c1' eval-c2' s3
        show ?thesis
          by (rule eval.Fin [elim-format]) (simp add: s1 eq-s3)
      qed
    qed
  qed
qed


lemma Body-no-break:
 assumes eval-init: G⊢Norm s0 −Init D→ s1
   and      eval-c: G⊢s1 −c→ s2
   and       jmpOk: jumpNestingOkS {Ret} c
   and        wt-c: (|prg=G, cls=C, lcl=L|)⊢c::√
   and        clsD: class G D=Some d
   and          wf: wf-prog G
  shows ∀ l. abrupt s2 ≠ Some (Jump (Break l)) ∧
             abrupt s2 ≠ Some (Jump (Cont l))
proof
  fix l show abrupt s2 ≠ Some (Jump (Break l)) ∧
             abrupt s2 ≠ Some (Jump (Cont l))
  proof −
    fix accC from clsD have wt-init: (|prg=G, cls=accC, lcl=L|)⊢(Init D)::√
      by auto
    from eval-init wf
    have s1-no-jmp: ⋀ j. abrupt s1 ≠ Some (Jump j)
      by − (rule eval-statement-no-jump [OF - - - wt-init],auto)
    from eval-c - wt-c wf
    show ?thesis
      apply (rule jumpNestingOk-eval [THEN conjE, elim-format])
      using jmpOk s1-no-jmp
      apply auto
      done
  qed
qed


lemma MGFn-Body:
  assumes wf: wf-prog G
  and      mgf-init: G,A⊢{=:n} ⟨Init D⟩s≻ {G→}
  and      mgf-c: G,A⊢{=:n} ⟨c⟩s≻ {G→}
  shows  G,(A::state triple set)⊢{=:n} ⟨Body D c⟩e≻ {G→}
proof (rule MGFn-free-wt-da-NormalConformI [rule-format],clarsimp)
  fix T L accC E
  assume wt: (|prg=G, cls=accC,lcl=L|)⊢⟨Body D c⟩e::T
  let ?Q=(λY' s' s. normal s ∧ G⊢s −Init D→ s' ∧ jumpNestingOkS {Ret} c)
         ∧. G⊢init≤n
  show G,A⊢{Normal
             ((λY' s' s. s' = s ∧ fst s = None) ∧. G⊢init≤n ∧.
              (λs. s::≼(G, L)) ∧.
              (λs. (|prg=G,cls=accC,lcl=L|)
                     ⊢ dom (locals (store s)) »⟨Body D c⟩e» E))}
            Body D c−≻
            {λY s' s. ∃ v. Y = In1 v ∧ G⊢s −Body D c−≻v→ s'}
    (is G,A⊢{Normal ?P} Body D c−≻ {?R})
  proof (rule ax-derivs.Body [where ?Q=?Q])
    from mgf-init
```

```
    show G,A⊢{Normal ?P} .Init D. {?Q}
    proof (rule MGFnD' [THEN conseq12],clarsimp)
      fix s0
      assume da: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals s0) »⟨Body D c⟩e» E
      thus jumpNestingOkS {Ret} c
        by cases simp
    qed
  next
    from mgf-c
    show G,A⊢{?Q}.c.{λs.. abupd (absorb Ret) .; ?R←⌊the (locals s Result)⌋e}
    proof (rule MGFnD' [THEN conseq12],clarsimp)
      fix s0 s1 s2
      assume eval-init: G⊢Norm s0 −Init D→ s1
      assume eval-c: G⊢s1 −c→ s2
      assume nestingOk: jumpNestingOkS {Ret} c
      show G⊢Norm s0 −Body D c−≻the (locals (store s2) Result)
              → abupd (absorb Ret) s2
      proof −
        from wt obtain d where
          d: class G D=Some d and
          wt-c: (|prg = G, cls = accC, lcl = L|)⊢c::√
          by cases auto
        obtain s3 where
          s3: s3= (if ∃ l. fst s2 = Some (Jump (Break l)) ∨
                          fst s2 = Some (Jump (Cont l))
                    then abupd (λx. Some (Error CrossMethodJump)) s2
                    else s2)
          by simp
        from eval-init eval-c nestingOk wt-c d wf
        have eq-s3-s2: s3=s2
          by (rule Body-no-break [elim-format]) (simp add: s3)
        from eval-init eval-c s3
        show ?thesis
          by (rule eval.Body [elim-format]) (simp add: eq-s3-s2)
      qed
    qed
  qed
qed


lemma MGFn-lemma:
  assumes mgf-methds:
          ⋀ n. ∀ C sig. G,(A::state triple set)⊢{=:n} ⟨Methd C sig⟩e≻ {G→}
  and wf: wf-prog G
  shows ⋀ t. G,A⊢{=:n} t≻ {G→}
proof (induct rule: full-nat-induct)
  fix n t
  assume hyp: ∀ m. Suc m ≤ n ⟶ (∀ t. G,A⊢{=:m} t≻ {G→})
  show G,A⊢{=:n} t≻ {G→}
  proof −
  {
    fix v e c es
    have G,A⊢{=:n} ⟨v⟩v≻ {G→} and
      G,A⊢{=:n} ⟨e⟩e≻ {G→} and
      G,A⊢{=:n} ⟨c⟩s≻ {G→} and
      G,A⊢{=:n} ⟨es⟩l≻ {G→}
    proof (induct rule: compat-var.induct compat-expr.induct compat-stmt.induct compat-expr-list.induct)
      case (LVar v)
      show G,A⊢{=:n} ⟨LVar v⟩v≻ {G→}
```

```
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.LVar [THEN conseq1])
      apply (clarsimp)
      apply (rule eval.LVar)
      done
  next
    case (FVar accC statDeclC stat e fn)
    from MGFn-Init [OF hyp] and ‹G,A⊢{=:n} ⟨e⟩ₑ≻ {G→}› and wf
    show ?case
      by (rule MGFn-FVar)
  next
    case (AVar e1 e2)
    note mgf-e1 = ‹G,A⊢{=:n} ⟨e1⟩ₑ≻ {G→}›
    note mgf-e2 = ‹G,A⊢{=:n} ⟨e2⟩ₑ≻ {G→}›
    show G,A⊢{=:n} ⟨e1.[e2]⟩ᵥ≻ {G→}
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.AVar)
      apply  (rule MGFnD [OF mgf-e1, THEN ax-NormalD])
      apply (rule allI)
      apply (rule MGFnD' [OF mgf-e2, THEN conseq12])
      apply (fastforce intro: eval.AVar)
      done
  next
    case (InsInitV c v)
    show ?case
      by (rule MGFn-NormalI) (rule ax-derivs.InsInitV)
  next
    case (NewC C)
    show ?case
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.NewC)
      apply (rule MGFn-InitD [OF hyp, THEN conseq2])
      apply (fastforce intro: eval.NewC)
      done
  next
    case (NewA T e)
    thus ?case
      apply −
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.NewA
               [where ?Q = (λY' s' s. normal s ∧ G⊢s −In1r (init-comp-ty T)
                            ≻→ (Y',s')) ∧. G⊢init≤n])
      apply  (simp add: init-comp-ty-def split: if-split)
      apply  (rule conjI, clarsimp)
      apply   (rule MGFn-InitD [OF hyp, THEN conseq2])
      apply   (clarsimp intro: eval.Init)
      apply  clarsimp
      apply  (rule ax-derivs.Skip [THEN conseq1])
      apply  (clarsimp intro: eval.Skip)
      apply (erule MGFnD' [THEN conseq12])
      apply (fastforce intro: eval.NewA)
      done
  next
    case (Cast C e)
    thus ?case
      apply −
      apply (rule MGFn-NormalI)
      apply (erule MGFnD'[THEN conseq12,THEN ax-derivs.Cast])
      apply (fastforce intro: eval.Cast)
```

```
    done
next
  case (Inst e C)
  thus ?case
    apply -
    apply (rule MGFn-NormalI)
    apply (erule MGFnD'[THEN conseq12,THEN ax-derivs.Inst])
    apply (fastforce intro: eval.Inst)
    done
next
  case (Lit v)
  show ?case
    apply -
    apply (rule MGFn-NormalI)
    apply (rule ax-derivs.Lit [THEN conseq1])
    apply (fastforce intro: eval.Lit)
    done
next
  case (UnOp unop e)
  thus ?case
    apply -
    apply (rule MGFn-NormalI)
    apply (rule ax-derivs.UnOp)
    apply (erule MGFnD' [THEN conseq12])
    apply (fastforce intro: eval.UnOp)
    done
next
  case (BinOp binop e1 e2)
  thus ?case
    apply -
    apply (rule MGFn-NormalI)
    apply (rule ax-derivs.BinOp)
    apply  (erule MGFnD [THEN ax-NormalD])
    apply (rule allI)
    apply (case-tac need-second-arg binop v1)
    apply  simp
    apply  (erule MGFnD' [THEN conseq12])
    apply  (fastforce intro: eval.BinOp)
    apply simp
    apply (rule ax-Normal-cases)
    apply  (rule ax-derivs.Skip [THEN conseq1])
    apply  clarsimp
    apply  (rule eval-BinOp-arg2-indepI)
    apply   simp
    apply  simp
    apply (rule ax-derivs.Abrupt [THEN conseq1], clarsimp simp add: Let-def)
    apply (fastforce intro: eval.BinOp)
    done
next
  case Super
  show ?case
    apply -
    apply (rule MGFn-NormalI)
    apply (rule ax-derivs.Super [THEN conseq1])
    apply (fastforce intro: eval.Super)
    done
next
  case (Acc v)
  thus ?case
```

```
      apply -
      apply (rule MGFn-NormalI)
      apply (erule MGFnD'[THEN conseq12,THEN ax-derivs.Acc])
      apply (fastforce intro: eval.Acc simp add: split-paired-all)
      done
  next
    case (Ass v e)
    thus G,A⊢{=:n} ⟨v:=e⟩ₑ≻ {G→}
      apply -
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.Ass)
      apply  (erule MGFnD [THEN ax-NormalD])
      apply (rule allI)
      apply (erule MGFnD'[THEN conseq12])
      apply (fastforce intro: eval.Ass simp add: split-paired-all)
      done
  next
    case (Cond e1 e2 e3)
    thus G,A⊢{=:n} ⟨e1 ? e2 : e3⟩ₑ≻ {G→}
      apply -
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.Cond)
      apply  (erule MGFnD [THEN ax-NormalD])
      apply (rule allI)
      apply (rule ax-Normal-cases)
      prefer 2
      apply  (rule ax-derivs.Abrupt [THEN conseq1],clarsimp simp add: Let-def)
      apply  (fastforce intro: eval.Cond)
      apply (case-tac b)
      apply  simp
      apply  (erule MGFnD'[THEN conseq12])
      apply  (fastforce intro: eval.Cond)
      apply simp
      apply (erule MGFnD'[THEN conseq12])
      apply (fastforce intro: eval.Cond)
      done
  next
    case (Call accC statT mode e mn pTs' ps)
    note mgf-e = ‹G,A⊢{=:n} ⟨e⟩ₑ≻ {G→}›
    note mgf-ps = ‹G,A⊢{=:n} ⟨ps⟩ₗ≻ {G→}›
    from mgf-methds mgf-e mgf-ps wf
    show G,A⊢{=:n} ⟨{accC,statT,mode}e·mn({pTs'}ps)⟩ₑ≻ {G→}
      by (rule MGFn-Call)
  next
    case (Methd D mn)
    from mgf-methds
    show G,A⊢{=:n} ⟨Methd D mn⟩ₑ≻ {G→}
      by simp
  next
    case (Body D c)
    note mgf-c = ‹G,A⊢{=:n} ⟨c⟩ₛ≻ {G→}›
    from wf MGFn-Init [OF hyp] mgf-c
    show G,A⊢{=:n} ⟨Body D c⟩ₑ≻ {G→}
      by (rule MGFn-Body)
  next
    case (InsInitE c e)
    show ?case
      by (rule MGFn-NormalI) (rule ax-derivs.InsInitE)
  next
```

case (*Callee l e*)
show *?case*
by (*rule MGFn-NormalI*) (*rule ax-derivs.Callee*)
next
case *Skip*
show *?case*
apply −
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Skip* [*THEN conseq1*])
apply (*fastforce intro*: *eval.Skip*)
done
next
case (*Expr e*)
thus *?case*
apply −
apply (*rule MGFn-NormalI*)
apply (*erule MGFnD′*[*THEN conseq12*,*THEN ax-derivs.Expr*])
apply (*fastforce intro*: *eval.Expr*)
done
next
case (*Lab l c*)
thus $G,A\vdash\{=:n\}\ \langle l\cdot\ c\rangle_s\succ\ \{G\rightarrow\}$
apply −
apply (*rule MGFn-NormalI*)
apply (*erule MGFnD′* [*THEN conseq12*, *THEN ax-derivs.Lab*])
apply (*fastforce intro*: *eval.Lab*)
done
next
case (*Comp c1 c2*)
thus $G,A\vdash\{=:n\}\ \langle c1;;\ c2\rangle_s\succ\ \{G\rightarrow\}$
apply −
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Comp*)
apply (*erule MGFnD* [*THEN ax-NormalD*])
apply (*erule MGFnD′* [*THEN conseq12*])
apply (*fastforce intro*: *eval.Comp*)
done
next
case (*If′ e c1 c2*)
thus $G,A\vdash\{=:n\}\ \langle If(e)\ c1\ Else\ c2\rangle_s\succ\ \{G\rightarrow\}$
apply −
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.If*)
apply (*erule MGFnD* [*THEN ax-NormalD*])
apply (*rule allI*)
apply (*rule ax-Normal-cases*)
prefer *2*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*],*clarsimp simp add*: *Let-def*)
apply (*fastforce intro*: *eval.If*)
apply (*case-tac b*)
apply *simp*
apply (*erule MGFnD′* [*THEN conseq12*])
apply (*fastforce intro*: *eval.If*)
apply *simp*
apply (*erule MGFnD′* [*THEN conseq12*])
apply (*fastforce intro*: *eval.If*)
done
next
case (*Loop l e c*)

note *mgf-e* = $\langle G,A\vdash\{=:n\}\ \langle e\rangle_e\succ\ \{G\rightarrow\}\rangle$
note *mgf-c* = $\langle G,A\vdash\{=:n\}\ \langle c\rangle_s\succ\ \{G\rightarrow\}\rangle$
from *mgf-e mgf-c wf*
show $G,A\vdash\{=:n\}\ \langle l\bullet\ While(e)\ c\rangle_s\succ\ \{G\rightarrow\}$
by (*rule MGFn-Loop*)
next
case (*Jmp j*)
thus *?case*
apply −
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Jmp* [*THEN conseq1*])
apply (*auto intro*: *eval.Jmp*)
done
next
case (*Throw e*)
thus *?case*
apply −
apply (*rule MGFn-NormalI*)
apply (*erule MGFnD′* [*THEN conseq12*, *THEN ax-derivs.Throw*])
apply (*fastforce intro*: *eval.Throw*)
done
next
case (*TryC c1 C vn c2*)
thus $G,A\vdash\{=:n\}\ \langle Try\ c1\ Catch(C\ vn)\ c2\rangle_s\succ\ \{G\rightarrow\}$
apply −
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Try* [**where**
$?Q = \ (\lambda Y'\ s'\ s.\ normal\ s \wedge (\exists s''.\ G\vdash s\ -\langle c1\rangle_s\succ\rightarrow\ (Y',s'') \wedge$
$G\vdash s''\ -sxalloc\rightarrow\ s')) \wedge .\ G\vdash init\le n])$
apply (*erule MGFnD* [*THEN ax-NormalD*, *THEN conseq2*])
apply (*fastforce elim*: *sxalloc-gext* [*THEN card-nyinitcls-gext*])
apply (*erule MGFnD′*[*THEN conseq12*])
apply (*fastforce intro*: *eval.Try*)
apply (*fastforce intro*: *eval.Try*)
done
next
case (*Fin c1 c2*)
note *mgf-c1* = $\langle G,A\vdash\{=:n\}\ \langle c1\rangle_s\succ\ \{G\rightarrow\}\rangle$
note *mgf-c2* = $\langle G,A\vdash\{=:n\}\ \langle c2\rangle_s\succ\ \{G\rightarrow\}\rangle$
from *wf mgf-c1 mgf-c2*
show $G,A\vdash\{=:n\}\ \langle c1\ Finally\ c2\rangle_s\succ\ \{G\rightarrow\}$
by (*rule MGFn-Fin*)
next
case (*FinA abr c*)
show *?case*
by (*rule MGFn-NormalI*) (*rule ax-derivs.FinA*)
next
case (*Init C*)
from *hyp*
show $G,A\vdash\{=:n\}\ \langle Init\ C\rangle_s\succ\ \{G\rightarrow\}$
by (*rule MGFn-Init*)
next
case *Nil-expr*
show $G,A\vdash\{=:n\}\ \langle[]\rangle_l\succ\ \{G\rightarrow\}$
apply −
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Nil* [*THEN conseq1*])
apply (*fastforce intro*: *eval.Nil*)
done

next
  case (*Cons-expr e es*)
  thus $G,A \vdash \{=:n\}\ \langle e\#\ es \rangle_l \succ\ \{G \rightarrow\}$
    apply –
    apply (*rule MGFn-NormalI*)
    apply (*rule ax-derivs.Cons*)
    apply (*erule MGFnD* [*THEN ax-NormalD*])
    apply (*rule allI*)
    apply (*erule MGFnD'*[*THEN conseq12*])
    apply (*fastforce intro*: *eval.Cons*)
    done
  qed
}
thus *?thesis*
  by (*cases rule*: *term-cases*) *auto*
qed
qed

lemma *MGF-asm*:
$[\![\forall\ C\ sig.\ is\text{-}methd\ G\ C\ sig \longrightarrow G,A \vdash \{\doteq\}\ In1l\ (Methd\ C\ sig) \succ\ \{G \rightarrow\};\ wf\text{-}prog\ G]\!]$
$\Longrightarrow G,(A::state\ triple\ set) \vdash \{\doteq\}\ t \succ\ \{G \rightarrow\}$
apply (*simp* (*no-asm-use*) *add*: *MGF-MGFn-iff*)
apply (*rule allI*)
apply (*rule MGFn-lemma*)
apply (*intro strip*)
apply (*rule MGFn-free-wt*)
apply (*force dest*: *wt-Methd-is-methd*)
apply *assumption*
done

nested version

lemma *nesting-lemma'* [*rule-format* (*no-asm*)]:
assumes *ax-derivs-asm*: $\bigwedge A\ ts.\ ts \subseteq A \Longrightarrow P\ A\ ts$
and *MGF-nested-Methd*: $\bigwedge A\ pn.\ \forall\ b \in bdy\ pn.\ P\ (insert\ (mgf\text{-}call\ pn)\ A)\ \{mgf\ b\}$
$\Longrightarrow P\ A\ \{mgf\text{-}call\ pn\}$
and *MGF-asm*: $\bigwedge A\ t.\ \forall\ pn \in U.\ P\ A\ \{mgf\text{-}call\ pn\} \Longrightarrow P\ A\ \{mgf\ t\}$
and *finU*: *finite U*
and *uA*: *uA* = *mgf-call‘U*
shows $\forall\ A.\ A \subseteq uA \longrightarrow n \leq card\ uA \longrightarrow card\ A = card\ uA - n$
$\longrightarrow (\forall\ t.\ P\ A\ \{mgf\ t\})$
using *finU uA*
apply –
apply (*induct-tac n*)
apply (*tactic ALLGOALS* (*clarsimp-tac* ***context***))
apply (*tactic ‹dresolve-tac* ***context*** [*Thm.permute-prems 0 1* @{*thm card-seteq*}] *1›*)
apply *simp*
apply (*erule finite-imageI*)
apply (*simp add*: *MGF-asm ax-derivs-asm*)
apply (*rule MGF-asm*)
apply (*rule ballI*)
apply (*case-tac mgf-call pn* ∈ *A*)
apply (*fast intro*: *ax-derivs-asm*)
apply (*rule MGF-nested-Methd*)
apply (*rule ballI*)
apply (*drule spec*, *erule impE*, *erule-tac* [*2*] *impE*, *erule-tac* [*3*] *spec*)
apply *hypsubst-thin*
apply *fast*

apply (*drule finite-subset*)
apply (*erule finite-imageI*)
apply *auto*
done

lemma *nesting-lemma* [*rule-format* (*no-asm*)]:
assumes *ax-derivs-asm*: $\bigwedge A\ ts.\ ts \subseteq A \Longrightarrow P\ A\ ts$
and *MGF-nested-Methd*: $\bigwedge A\ pn.\ \forall b \in bdy\ pn.\ P\ (insert\ (mgf\ (f\ pn))\ A)\ \{mgf\ b\}$
$\Longrightarrow P\ A\ \{mgf\ (f\ pn)\}$
and *MGF-asm*: $\bigwedge A\ t.\ \forall pn \in U.\ P\ A\ \{mgf\ (f\ pn)\} \Longrightarrow P\ A\ \{mgf\ t\}$
and *finU*: *finite U*
shows $P\ \{\}\ \{mgf\ t\}$
using *ax-derivs-asm MGF-nested-Methd MGF-asm finU*
by (*rule nesting-lemma'*) (*auto intro*!: *le-refl*)

lemma *MGF-nested-Methd*: $[\![$
$G,insert\ (\{Normal \doteq\}\ \langle Methd\ \ C\ sig\rangle_e \succ\{G\rightarrow\})\ A$
$\vdash\{Normal \doteq\}\ \langle body\ G\ C\ sig\rangle_e \succ\{G\rightarrow\}$
$]\!] \Longrightarrow\ G,A\vdash\{Normal \doteq\}\ \ \langle Methd\ \ C\ sig\rangle_e \succ\{G\rightarrow\}$
apply (*unfold MGF-def*)
apply (*rule ax-MethdN*)
apply (*erule conseq2*)
apply *clarsimp*
apply (*erule MethdI*)
done

lemma *MGF-deriv*: $wf\text{-}prog\ G \Longrightarrow G,(\{\}::state\ triple\ set)\vdash\{\doteq\}\ t\succ\ \{G\rightarrow\}$
apply (*rule MGFNormalI*)
apply (*rule-tac* $mgf = \lambda t.\ \{Normal \doteq\}\ t\succ\ \{G\rightarrow\}$ **and**
$bdy = \lambda\ (C,sig)\ .\{\langle body\ G\ C\ sig\rangle_e\ \}$ **and**
$f = \lambda\ (C,sig)\ .\ \langle Methd\ C\ sig\rangle_e$ **in** *nesting-lemma*)
apply (*erule ax-derivs.asm*)
apply (*clarsimp simp add*: *split-tupled-all*)
apply (*erule MGF-nested-Methd*)
apply (*erule-tac* [*2*] *finite-is-methd* [*OF wf-ws-prog*])
apply (*rule MGF-asm* [*THEN MGFNormalD*])
apply (*auto intro*: *MGFNormalI*)
done

simultaneous version

lemma *MGF-simult-Methd-lemma*: *finite ms* $\Longrightarrow$
$G,A \cup (\lambda(C,sig).\ \{Normal \doteq\}\ \langle Methd\ \ C\ sig\rangle_e\succ\ \{G\rightarrow\})\ `\ ms$
$|\vdash(\lambda(C,sig).\ \{Normal \doteq\}\ \langle body\ G\ C\ sig\rangle_e\succ\ \{G\rightarrow\})\ `\ ms \Longrightarrow$
$G,A|\vdash(\lambda(C,sig).\ \{Normal \doteq\}\ \langle Methd\ \ C\ sig\rangle_e\succ\ \{G\rightarrow\})\ `\ ms$
apply (*unfold MGF-def*)
apply (*rule ax-derivs.Methd* [*unfolded mtriples-def*])
apply (*erule ax-finite-pointwise*)
prefer *2*
apply (*rule ax-derivs.asm*)
apply *fast*
apply *clarsimp*
apply (*rule conseq2*)
apply (*erule* (*1*) *ax-methods-spec*)

apply *clarsimp*
apply (*erule eval-Methd*)
done

lemma *MGF-simult-Methd*: *wf-prog* $G \Longrightarrow$
G,({}::*state triple set*)|⊢(λ(C,*sig*). {*Normal* $\doteq$} ⟨*Methd C sig*⟩$_e$≻ {G→})
' *Collect* (*case-prod* (*is-methd G*))
apply (*frule finite-is-methd* [*OF wf-ws-prog*])
apply (*rule MGF-simult-Methd-lemma*)
apply *assumption*
apply (*erule ax-finite-pointwise*)
prefer *2*
apply (*rule ax-derivs.asm*)
apply *blast*
apply *clarsimp*
apply (*rule MGF-asm* [*THEN MGFNormalD*])
apply (*auto intro*: *MGFNormalI*)
done

corollaries

lemma *eval-to-evaln*: ⟦G⊢s −t≻→ (Y', s');*type-ok G t s*; *wf-prog G*⟧
$\Longrightarrow$ $\exists$ n. G⊢s −t≻−n→ (Y', s')
apply (*cases normal s*)
apply (*force simp add*: *type-ok-def intro*: *eval-evaln*)
apply (*force intro*: *evaln.Abrupt*)
done

lemma *MGF-complete*:
assumes *valid*: G,{}|=$\{P\}$ t≻ $\{Q\}$
and *mgf*: G,({}::*state triple set*)⊢{$\doteq$} t≻ {G→}
and *wf*: *wf-prog G*
shows G,({}::*state triple set*)⊢{P::*state assn*} t≻ $\{Q\}$
proof (*rule ax-no-hazard*)
from *mgf*
have G,({}::*state triple set*)⊢{$\doteq$} t≻ {λY s' s. G⊢s −t≻→ (Y, s')}
by (*unfold MGF-def*)
thus G,({}::*state triple set*)⊢{P ∧. *type-ok G t*} t≻ $\{Q\}$
proof (*rule conseq12*,*clarsimp*)
fix Y s Z Y' s'
assume *P*: P Y s Z
assume *type-ok*: *type-ok G t s*
assume *eval-t*: G⊢s −t≻→ (Y', s')
show Q Y' s' Z
proof −
from *eval-t type-ok wf*
obtain n **where** *evaln*: G⊢s −t≻−n→ (Y', s')
by (*rule eval-to-evaln* [*elim-format*]) *iprover*
from *valid* **have**
valid-expanded:
$\forall$ n Y s Z. P Y s Z ⟶ *type-ok G t s*
⟶ ($\forall$ Y' s'. G⊢s −t≻−n→ (Y', s') ⟶ Q Y' s' Z)
by (*simp add*: *ax-valids-def triple-valid-def*)
from *P type-ok evaln*
show Q Y' s' Z
by (*rule valid-expanded* [*rule-format*])
qed

qed
qed

theorem *ax-complete*:
assumes *wf*: *wf-prog G*
and *valid*: G,{}$\models$\{P::*state assn*\} $t\succ$ \{Q\}
shows G,({}::*state triple set*)$\vdash$\{P\} $t\succ$ \{Q\}
proof −
from *wf* **have** G,({}::*state triple set*)$\vdash$\{$\doteq$\} $t\succ$ \{$G\rightarrow$\}
by (*rule MGF-deriv*)
from *valid this wf*
show *?thesis*
by (*rule MGF-complete*)
qed

end

Chapter 25

AxExample

1 Example of a proof based on the Bali axiomatic semantics

theory *AxExample*
imports *AxSem Example*
begin

definition
  arr-inv :: *st* ⇒ *bool* **where**
 arr-inv = (λ*s*. ∃ *obj a T el*. *globs s* (*Stat Base*) = *Some obj* ∧
      values obj (*Inl* (*arr*, *Base*)) = *Some* (*Addr a*) ∧
      heap s a = *Some* (⦇*tag*=*Arr T 2*,*values*=*el*⦈))

lemma *arr-inv-new-obj*:
⋀*a*. ⟦*arr-inv s*; *new-Addr* (*heap s*)=*Some a*⟧ ⟹ *arr-inv* (*gupd*(*Inl a*↦*x*) *s*)
apply (*unfold arr-inv-def*)
apply (*force dest*!: *new-AddrD2*)
done

lemma *arr-inv-set-locals* [*simp*]: *arr-inv* (*set-locals l s*) = *arr-inv s*
apply (*unfold arr-inv-def*)
apply (*simp* (*no-asm*))
done

lemma *arr-inv-gupd-Stat* [*simp*]:
  Base ≠ *C* ⟹ *arr-inv* (*gupd*(*Stat C*↦*obj*) *s*) = *arr-inv s*
apply (*unfold arr-inv-def*)
apply (*simp* (*no-asm-simp*))
done

lemma *ax-inv-lupd* [*simp*]: *arr-inv* (*lupd*(*x*↦*y*) *s*) = *arr-inv s*
apply (*unfold arr-inv-def*)
apply (*simp* (*no-asm*))
done

declare *if-split-asm* [*split del*]
declare *lvar-def* [*simp*]

ML ‹
fun inst1-tac ctxt s t xs st =

```
  (case AList.lookup (op =) (rev (Term.add-var-names (Thm.prop-of st) [])) s of
    SOME i => PRIMITIVE (Rule-Insts.read-instantiate ctxt [(((s, i), Position.none), t)] xs) st
  | NONE => Seq.empty);

fun ax-tac ctxt =
  REPEAT o resolve-tac ctxt [allI] THEN'
  resolve-tac ctxt
    @{thms ax-Skip ax-StatRef ax-MethdN ax-Alloc ax-Alloc-Arr ax-SXAlloc-Normal ax-derivs.intros(8-)};
›
```

theorem *ax-test*: *tprg*,({}::'*a triple set*)⊢
  {*Normal* (λ*Y s Z*::'*a*. *heap-free four s* ∧ ¬*initd Base s* ∧ ¬ *initd Ext s*)}
  .*test* [*Class Base*].
  {λ*Y s Z*. *abrupt s* = *Some* (*Xcpt* (*Std IndOutBound*))}
apply (*unfold test-def arr-viewed-from-def*)
apply (*tactic ax-tac* ***context*** *1*)
defer
apply (*tactic ax-tac* ***context*** *1*)
defer
apply (*tactic* ‹*inst1-tac* ***context*** *Q*
  λ*Y s Z*. *arr-inv* (*snd s*) ∧ *tprg*,*s*⊢*catch SXcpt NullPointer* []›)
prefer *2*
apply *simp*
apply (*rule-tac P'* = *Normal* (λ*Y s Z*. *arr-inv* (*snd s*)) **in** *conseq1*)
prefer *2*
apply *clarsimp*
apply (*rule-tac Q'* = (λ*Y s Z*. *Q Y s Z*)←=*False*↓=◇ **and** *Q* = *Q* **for** *Q* **in** *conseq2*)
prefer *2*
apply *simp*
apply (*tactic ax-tac* ***context*** *1*)
prefer *2*
apply (*rule ax-impossible* [*THEN conseq1*], *clarsimp*)
apply (*rule-tac P'* = *Normal P* **and** *P* = *P* **for** *P* **in** *conseq1*)
prefer *2*
apply *clarsimp*
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)
prefer *2*
apply (*rule ax-subst-Val-allI*)
apply (*tactic* ‹*inst1-tac* ***context*** *P'* λ*a*. *Normal* (*PP a*←*x*) [*PP*, *x*]›)
apply (*simp del*: *avar-def2 peek-and-def2*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)

apply (*rule-tac Q'* = *Normal* (λ*Var*:(*v*, *f*) *u ua*. *fst* (*snd* (*avar tprg* (*Intg 2*) *v u*)) = *Some* (*Xcpt* (*Std IndOutBound*))) **in** *conseq2*)
prefer *2*
apply (*clarsimp simp add*: *split-beta*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *2*)
apply (*rule ax-derivs.Done* [*THEN conseq1*])
apply (*clarsimp simp add*: *arr-inv-def inited-def in-bounds-def*)
defer
apply (*rule ax-SXAlloc-catch-SXcpt*)
apply (*rule-tac Q'* = (λ*Y* (*x*, *s*) *Z*. *x* = *Some* (*Xcpt* (*Std NullPointer*)) ∧ *arr-inv s*) ∧. *heap-free two* **in** *conseq2*)
prefer *2*
apply (*simp add*: *arr-inv-new-obj*)

apply (*tactic ax-tac* ***context*** *1*)
apply (*rule-tac C* = *Ext* **in** *ax-Call-known-DynT*)
apply (*unfold DynT-prop-def*)
apply (*simp* (*no-asm*))
apply (*intro strip*)
apply (*rule-tac P′* = *Normal P* **and** *P* = *P* **for** *P* **in** *conseq1*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*rule ax-thin* [*OF - empty-subsetI*])
apply (*simp* (*no-asm*) *add*: *body-def2*)
apply (*tactic ax-tac* ***context*** *1*)

defer
apply (*simp* (*no-asm*))
apply (*tactic ax-tac* ***context*** *1*)

apply (*rule-tac* [*2*] *ax-derivs.Abrupt*)

apply (*rule ax-derivs.Expr*)
apply (*tactic ax-tac* ***context*** *1*)
prefer *2*
apply (*rule ax-subst-Var-allI*)
apply (*tactic* ‹*inst1-tac* ***context*** *P′ λa vs l vf*. *PP a vs l vf*←*x* ∧. *p* [*PP*, *x*, *p*]›)
apply (*rule allI*)
apply (*tactic* ‹*simp-tac* (***context*** *delloop split-all-tac delsimps* [@{*thm peek-and-def2*}, @{*thm heap-def2*}, @{*thm subst-res-def2*}, @{*thm normal-def2*}]) *1*›)
apply (*rule ax-derivs.Abrupt*)
apply (*simp* (*no-asm*))
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *2*, *tactic ax-tac* ***context*** *2*, *tactic ax-tac* ***context*** *2*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic* ‹*inst1-tac* ***context*** *R λa′*. *Normal* ((*λVals*:*vs* (*x*, *s*) *Z*. *arr-inv s* ∧ *inited Ext* (*globs s*) ∧ *a′* ≠ *Null* ∧ *vs* = [*Null*]) ∧. *heap-free two*) []›)
apply *fastforce*
prefer *4*
apply (*rule ax-derivs.Done* [*THEN conseq1*],*force*)
apply (*rule ax-subst-Val-allI*)
apply (*tactic* ‹*inst1-tac* ***context*** *P′ λa*. *Normal* (*PP a*←*x*) [*PP*, *x*]›)
apply (*simp* (*no-asm*) *del*: *peek-and-def2 heap-free-def2 normal-def2 o-apply*)
apply (*tactic ax-tac* ***context*** *1*)
prefer *2*
apply (*rule ax-subst-Val-allI*)
apply (*tactic* ‹*inst1-tac* ***context*** *P′ λaa v*. *Normal* (*QQ aa v*←*y*) [*QQ*, *y*]›)
apply (*simp del*: *peek-and-def2 heap-free-def2 normal-def2*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)

apply (*simp* (*no-asm*))

apply (*rule-tac Q′* = *Normal* ((*λY* (*x*, *s*) *Z*. *arr-inv s* ∧ (∃ *a*. *the* (*locals s* (*VName e*)) = *Addr a* ∧ *obj-class* (*the* (*globs s* (*Inl a*))) = *Ext* ∧
invocation-declclass tprg IntVir s (*the* (*locals s* (*VName e*))) (*ClassT Base*)
(⦇*name* = *foo*, *parTs* = [*Class Base*]⦈ = *Ext*)) ∧. *initd Ext* ∧. *heap-free two*)
in *conseq2*)
prefer *2*
apply *clarsimp*
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)

defer
apply (*rule ax-subst-Var-allI*)
apply (*tactic* ‹*inst1-tac* ***context*** *P′ λvf. Normal* (*PP vf* ∧. *p*) [*PP*, *p*]›)
apply (*simp* (*no-asm*) *del*: *split-paired-All peek-and-def2 initd-def2 heap-free-def2 normal-def2*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)

apply (*rule-tac Q′* = *Normal* ((*λY s Z. arr-inv* (*store s*) ∧ *vf*=*lvar* (*VName e*) (*store s*)) ∧. *heap-free three* ∧. *initd Ext*) **in** *conseq2*)
prefer *2*
apply (*simp add*: *invocation-declclass-def dynmethd-def*)
apply (*unfold dynlookup-def*)
apply (*simp add*: *dynmethd-Ext-foo*)
apply (*force elim*!: *arr-inv-new-obj atleast-free-SucD atleast-free-weaken*)

apply (*rule ax-InitS*)
apply *force*
apply (*simp* (*no-asm*))
apply (*tactic* ‹*simp-tac* (***context*** *delloop split-all-tac*) *1*›)
apply (*rule ax-Init-Skip-lemma*)
apply (*tactic* ‹*simp-tac* (***context*** *delloop split-all-tac*) *1*›)
apply (*rule ax-InitS* [*THEN conseq1*])
apply *force*
apply (*simp* (*no-asm*))
apply (*unfold arr-viewed-from-def*)
apply (*rule allI*)
apply (*rule-tac P′* = *Normal P* **and** *P* = *P* **for** *P* **in** *conseq1*)
apply (*tactic* ‹*simp-tac* (***context*** *delloop split-all-tac*) *1*›)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*rule-tac* [*2*] *ax-subst-Var-allI*)
apply (*tactic* ‹*inst1-tac* ***context*** *P′ λvf l vfa. Normal* (*P vf l vfa*) [*P*]›)
apply (*tactic* ‹*simp-tac* (***context*** *delloop split-all-tac delsimps* [@{*thm split-paired-All*}, @{*thm peek-and-def2*}, @{*thm heap-free-def2*}, @{*thm initd-def2*}, @{*thm normal-def2*}, @{*thm supd-lupd*}]) *2*›)
apply (*tactic ax-tac* ***context*** *2*)
apply (*tactic ax-tac* ***context*** *3*)
apply (*tactic ax-tac* ***context*** *3*)
apply (*tactic* ‹*inst1-tac* ***context*** *P λvf l vfa. Normal* (*P vf l vfa*←◇) [*P*]›)
apply (*tactic* ‹*simp-tac* (***context*** *delloop split-all-tac*) *2*›)
apply (*tactic ax-tac* ***context*** *2*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *2*)
apply (*rule ax-derivs.Done* [*THEN conseq1*])
apply (*tactic* ‹*inst1-tac* ***context*** *Q λvf. Normal* ((*λY s Z. vf*=*lvar* (*VName e*) (*snd s*)) ∧. *heap-free four* ∧. *initd Base* ∧. *initd Ext*) []›)
apply (*clarsimp split del*: *if-split*)
apply (*frule atleast-free-weaken* [*THEN atleast-free-weaken*])
apply (*drule initedD*)
apply (*clarsimp elim*!: *atleast-free-SucD simp add*: *arr-inv-def*)
apply *force*
apply (*tactic* ‹*simp-tac* (***context*** *delloop split-all-tac*) *1*›)
apply (*rule ax-triv-Init-Object* [*THEN peek-and-forget2*, *THEN conseq1*])
apply (*rule wf-tprg*)
apply *clarsimp*
apply (*tactic* ‹*inst1-tac* ***context*** *P λvf. Normal* ((*λY s Z. vf* = *lvar* (*VName e*) (*snd s*)) ∧. *heap-free four* ∧. *initd Ext*) []›)
apply *clarsimp*
apply (*tactic* ‹*inst1-tac* ***context*** *PP λvf. Normal* ((*λY s Z. vf* = *lvar* (*VName e*) (*snd s*)) ∧. *heap-free four* ∧. *Not* ∘ *initd Base*) []›)

apply *clarsimp*

apply (*rule conseq1*)
apply (*tactic ax-tac* ***context*** *1*)
apply *clarsimp*
done

lemma *Loop-Xcpt-benchmark*:
$Q = (\lambda Y\ (x,s)\ Z.\ x \neq None \longrightarrow the\text{-}Bool\ (the\ (locals\ s\ i))) \Longrightarrow$
$G,(\{\}::'a\ triple\ set)\vdash\{Normal\ (\lambda Y\ s\ Z::'a.\ True)\}$
$.lab1\cdot\ While(Lit\ (Bool\ True))\ (If(Acc\ (LVar\ i))\ (Throw\ (Acc\ (LVar\ xcpt)))\ Else$
$(Expr\ (Ass\ (LVar\ i)\ (Acc\ (LVar\ j))))).\ \{Q\}$
apply (*rule-tac* $P' = Q$ **and** $Q' = Q\leftarrow{=}False\downarrow{=}\diamondsuit$ **in** *conseq12*)
apply *safe*
apply (*tactic ax-tac* ***context*** *1*)
apply (*rule ax-Normal-cases*)
prefer *2*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*], *clarsimp simp add*: *Let-def*)
apply (*rule conseq1*)
apply (*tactic ax-tac* ***context*** *1*)
apply *clarsimp*
prefer *2*
apply *clarsimp*
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic*
‹*inst1-tac* ***context*** P' *Normal* $(\lambda s..\ (\lambda Y\ s\ Z.\ True)\downarrow{=}Val\ (the\ (locals\ s\ i)))$ []›)
apply (*tactic ax-tac* ***context*** *1*)
apply (*rule conseq1*)
apply (*tactic ax-tac* ***context*** *1*)
apply *clarsimp*
apply (*rule allI*)
apply (*rule ax-escape*)
apply *auto*
apply (*rule conseq1*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)
apply *clarsimp*
apply (*rule-tac* $Q' = Normal\ (\lambda Y\ s\ Z.\ True)$ **in** *conseq2*)
prefer *2*
apply *clarsimp*
apply (*rule conseq1*)
apply (*tactic ax-tac* ***context*** *1*)
apply (*tactic ax-tac* ***context*** *1*)
prefer *2*
apply (*rule ax-subst-Var-allI*)
apply (*tactic* ‹*inst1-tac* ***context*** P' $\lambda b\ Y\ ba\ Z\ vf.\ \lambda Y\ (x,s)\ Z.\ x{=}None \wedge snd\ vf = snd\ (lvar\ i\ s)$ []›)
apply (*rule allI*)
apply (*rule-tac* $P' = Normal\ P$ **and** $P = P$ **for** P **in** *conseq1*)
prefer *2*
apply *clarsimp*
apply (*tactic ax-tac* ***context*** *1*)
apply (*rule conseq1*)
apply (*tactic ax-tac* ***context*** *1*)
apply *clarsimp*
apply (*tactic ax-tac* ***context*** *1*)
apply *clarsimp*

done

end