Inpreima2

FL

Tuesday 5 February 2012

Theorem: The preimage of the intersection of a non-empty set $A$ is the intersection of the preimage of the elements of $A$.

Remark: I write $F(x)$ when $x$ is an element of the domain of $F$ and $F\langle x\rangle$ when $x$ is a part of the domain of $F$.

---

Assume: 1. $A \neq \varnothing$

2. $F$ is a function

Prove: $F^{-1}\langle\bigcap A\rangle = \bigcap_{x\in A} F^{-1}\langle x\rangle$

Let: y be a set

$\langle 1\rangle 1$. Suffices: $y \in F^{-1}\langle\bigcap A\rangle$ iff $y \in \bigcap_{x\in A} F^{-1}\langle x\rangle$

By definition of equality (see eqriv).

$\langle 1\rangle 2$. $y \in \bigcap_{x\in A} F^{-1}\langle x\rangle$ iff $\forall x \in A\ y \in F^{-1}\langle x\rangle$

By definition of an intersection (eliin).

$\langle 1\rangle 3$. $y \in F^{-1}\langle\bigcap A\rangle$ iff $\forall x \in A\ y \in F^{-1}\langle x\rangle$

$\langle 2\rangle 1$. If $y \in F^{-1}\langle\bigcap A\rangle$ then $\forall x \in A\ y \in F^{-1}\langle x\rangle$.

$\langle 3\rangle 1$. Assume: $y \in F^{-1}\langle\bigcap A\rangle$

Prove: $\forall x \in A\ y \in F^{-1}\langle x\rangle$

$\langle 4\rangle 1$. $F(y) \in \bigcap A$

By fvimacnv and assumption $\langle 3\rangle 1$.

$\langle 4\rangle 2$. $\forall x \in A\ F(y) \in x$ by $\langle 4\rangle 1$ and definition of an intersection (elint).

$\langle 4 \rangle 3$. Q.E.D.

By fvimacnv and $\langle 4 \rangle 2$.

$\langle 3 \rangle 2$. Q.E.D.

By deduction theorem.

$\langle 2 \rangle 2$. If $\forall x \in A \; y \in F^{-1}\langle x \rangle$ then $y \in F^{-1}\langle \bigcap A \rangle$.

$\langle 3 \rangle 1$. Assume: $\forall x \in A \; y \in F^{-1}\langle x \rangle$

Prove: $y \in F^{-1}\langle \bigcap A \rangle$

$\langle 4 \rangle 1$. $\forall x \in A \; F(y) \in x$ (by assumption $\langle 3 \rangle 1$ and fvimacnvi.)

$\langle 4 \rangle 2$. $F(y) \in \bigcap A$ (by $\langle 4 \rangle 1$ and definition of an intersection (elint2).)

$\langle 4 \rangle 3$. Q.E.D.

By $\langle 4 \rangle 2$ and fvimacnv.

$\langle 3 \rangle 2$. Q.E.D.

By deduction theorem.

$\langle 2 \rangle 3$. Q.E.D.

By $\langle 2 \rangle 1$ and $\langle 2 \rangle 2$.

$\langle 1 \rangle 4$. Q.E.D.

By $\langle 1 \rangle 2$ and $\langle 1 \rangle 3$.

---

Theorem fvimacnvi: If $F$ is a function and $A \in F^{-1}\langle B \rangle$ then $F(A) \in B$.

Theorem fvimacnv: If $F$ is a function and $A$ an element of its domain then $A \in F^{-1}\langle B \rangle$ iff $F(A) \in B$.

Theorem elint2: If $A$ is a set, and $B$ a class, $A \in \bigcap B$ iff $\forall x \in B\ A \in x$.

---

Simple propositional calculus tautologies may be used without being explicitely mentionned. The ``deduction'' theorem may be used without being explicitely mentionned.

The keyword ``suffices'' changes the goal. The goal is always mention at the upper level. When ``suffices'' is used this goal changes and the proof behind the ``suffices'' step ensures the new goal and the former one are equivalent. The other steps at the same level behind the ``suffices'' are used to prove the new goal. The ``Q.E.D'' at the end of the level refers to the new goal.

The system used is in the Gentzen style. Let's recall set.mm is in the Hilbert style.

---

Modified on 3 June 2013 and again 13 July 2016.