# An Efficient Protocol for Oblivious DFA Evaluation and Applications 

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## Deterministic Finite Automaton

- Other names
- Finite State Machines (FSM)
- Finite State Automaton (FSA)
- A simple model of computation
- Digital logic
- Computer programs
- Pattern matching


The machine accepts a string if the process ends in a double circle

## Formal DFA Representation

- $M=\left(Q, \Sigma, \delta, q_{0}, F\right)$
- $Q$ is the set of states (finite)
- $\Sigma$ is the alphabet (finite)
- $\delta: Q \times \Sigma \rightarrow Q$ is the transition function
- $q_{0} \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states


## Oblivious DFA Evaluation

$$
X=\begin{array}{l|l|l|l|l}
a & b & e & a & h
\end{array}
$$



$\qquad$
$\qquad$

$M(x)$

IDS application

## Oblivious DFA Evaluation



$M(x)$

## Pattern matching

## Security Requirement

- Secure two-party computation
- Hide the DFA
- Hide the input string
- Only reveal the output
- Malicious input holder
- Guarantee Idea/real world simulation
- Malicious DFA holder
- Guarantee input privacy


## General-Purpose Solutions

- General two-party computation
- Garbled circuit approach
- Drawbacks
- Circuits get big quickly
- Circuit creation is sometimes cumbersome
- Not suitable for the client/server model


## Special-Purpose Solutions

- A number of constructions
- For oblivious DFA evaluation
- Or oblivious branching program
- Drawbacks
- DFA holder's public-key ops is large
- Proportional to DFA size


## A Yao-like Approach to ODFA

- DFA holder
- DFA $\rightarrow$ DFA matrix
- DFA matrix $\rightarrow$ Garbled DFA matrix
- Permute and encrypt the matrix
- Oblivious transfer
- Receive garbled inputs
- Input holder
- Evaluate/ungarble a single transit path


## (1) DFA $\rightarrow$ DFA Matrix



## DFA Matrix

- DFA matrix size
$-\mathrm{nx}|\mathrm{Q}|$ matrix
- Each cell holds 2 index
$-2 n|Q| \log |Q|$ bits to represent
- DFA evaluation
- Traverse a single transit path on the matrix


## Problems

- We should hide the state number
- We should make sure he has come from the correct last state
- We have to make sure he is not able to decrypt more than one cell in each row


## (2) DFA Matrix $\rightarrow$ Permuted DFA Matrix

$$
n=1: \operatorname{Per}[1]=\{3,2,4,5,1\}
$$

| $Q=3$ | $Q=2$ | $\ldots$ | $Q=1$ |
| :---: | :---: | :---: | :---: |
| $\delta\left(s_{3}, 0\right), \delta\left(s_{3}, 1\right)$ | $\delta\left(s_{2}, 0\right), \delta\left(s_{2}, 1\right)$ | $\ldots$ | $\delta\left(s_{1}, 0\right), \delta\left(s_{1}, 1\right)$ |

## (1) Permute

 each row$$
n=0: \operatorname{Per}[0]=\{2,1,5,4,3\}
$$

| $Q=2$ | $Q=1$ | ... | $Q=3$ |
| :---: | :---: | :---: | :---: |
| $\delta\left(s_{2}, 0\right), \delta\left(s_{2}, 1\right)$ | $\delta\left(s_{1}, 0\right), \delta\left(s_{1}, 1\right)$ | ... | $\delta\left(s_{3}, 0\right), \delta\left(s_{3}, 1\right)$ |
|  | $(2,3)$ |  |  |
| the | $(2,1)$ |  |  |

## Problems

- We should hide the state number
- We should make sure he has come from the correct last state
- We have to make sure he is not able to decrypt more than one cell in each row


## (3) Encrypt with PAD Matrix

State q'

| O 0 | State q' |  | 1 |
| :---: | :---: | :---: | :---: |
| Next PAD 0 | Next 0 | Next PAD 1 | Next 1 |
| XOR with Expand(Next PAD 0) |  |  |  |

Problem: He is able to decrypt both PADS and

Nexts!


## Problems

- We should hide the state number
- We should make sure he has come from the correct last state
- We have to make sure he is not able to decrypt more than one cell in each row


## (4) Encrypt with Keys



## (4) Encrypt with Keys



## Protocol



## Complexity

- Public-Key ops
- O(n) for both parties
- Can be reduced to $k$ using OT extension
- DFA holder's symmetric-key ops
- n|Q| PRG evaluations
- Input holder's symmetric-key ops
- n PRG evaluations
- Communication
- $2 n|Q|(\log |Q|+k)$ bits
- Round complexity
- 1 round


## Secure Pattern Matching

$$
T=a b d r \sec a \operatorname{ct}
$$

$$
p=a b e a
$$



- Does a pattern p exist in text T
- Locations of occurrences of $p$ in $T$
- Number of occurrences of $p$ in $T$


## A Different Presentation of Protocol

- Pointed out by reviewers
- Can be viewed as a generalization of Yao's Garbled Circuit Protocol
- Each gate takes non-boolean inputs and returns non-boolean outputs



## Comparison

|  | Round <br> Complexity | client Computations |  | server Computations |  | Communication |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Asymmetric | Symmetric | Asymmetric | Symmetric | Complexity |  |
| Troncoso [21] | $O(n)$ | $O(n\|Q\|)$ | None | $O(n\|Q\|)$ | $O(n\|Q\|)$ | $O(n\|Q\| k)$ |
| Frikken [3] | 2 | $O(n+\|Q\|)$ | $O(n\|Q\|)$ | $O(n+\|Q\|)$ | $O(n\|Q\|)$ | $O(n\|Q\| k)$ |
| Gennaro [4] | $\min (O(\|Q\|), O(n))$ | $O(n\|Q\|)$ | None | $O(n\|Q\|)$ | None | $O(n\|Q\| k)$ |
| Yao's protocol $[22]$ | 1 | $O(n)$ | $O(n\|Q\| \log \|Q\|)$ | $O(n)$ | $O(n\|Q\| \log \|Q\|)$ | $O(n\|Q\| k)$ |
| Ishai [12] | 1 | $O(n)$ | None | $O(n\|Q\|)$ | None | $O\left(k n^{2}\right)$ |
| Protocol 1(PRG) | 1 | $O(n)$ | $O(n)$ | $O(n)$ | $O(n\|Q\|)$ | $O(n\|Q\| k)$ |
| Protocol 1 <br> $($ PRG+Extended OT) | 1.5 | $O(k)$ | $O(n)$ | $O(k)$ | $O(n\|Q\|)$ | $O(n\|Q\| k)$ |

## Implementation

- Complete C++ implementation
- Experiments on Intel Core i7, 4GB RAM



## Future Work

- IDS DFAs are not too dense
- Can we do better?
- We will do many DFA evaluations
- Better batch evaluations?
- Better communication particularly
- (reusing part of the DFA matrix?)

Secure Multi-Party Computation of Boolean Circuits with Applications to Privacy in On-Line Marketplaces

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Joint work with
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Jonathan Katz (Uning-Wook Hwang, Tal Malkin, Dan Rubenstein (Columbia University) Session ID: CRYP-403
Session Classification: Advanced
RSNCONFERENCE2012

## Motivation: Online Marketplace



Find best match

with minimal price

## Online Marketplace

- Participants
- Providers: have resources and associated metric
- Customer: has preference
- Desired result
- For each resource r, compute its score according to the input of the customer and the providers
- Output the resource with best score


## Example: P2P Content Distribution

- Resources = Peers
- Providers: ISPs
- Know bandwidth info of peers
- Customer
- Knows which peer has the desired file
- Wants to find a suitable peer with highest bandwidth
- Result
- Score for a peer: if the peer has the file, output its bandwidth; otherwise, output 0
- Output: the peer with the highest score


## Other Examples

- Cloud computing
- Find the best-quality cloud service within price limit
- Find the cheapest cloud service of desired quality
- Mobile social network
- Find the closet user within enough matching interests
- Find the user with most matching interests within a certain distance.


## Privacy in Online Marketplaces

- Privacy
- The providers and the customer should learn nothing about anyone's inputs (beyond the output)
- Semi-honest security: corrupted parties follow the protocol honestly, but try to infer secret information from the protocol transcript.
- Protocol?
- One could attempt to construct a specific protocol...
- How well would a generic secure multi-party computation (MPC) protocol work?


## Secure Multi-Party Computation



## Generic Solutions?

- "Generic" = a protocol for any function, specified as a boolean/arithmetic circuit
- Good news: generic solutions exist
- Bad news: relatively inefficient (?)


## However, in the Past Few Years

- Growing interest in research community
- Optimizing efficiency of protocols
- Increased capability of modern computers

Several generic solutions have been implemented

## Two Types of Generic MPC Solutions

- Boolean circuits
- A circuit with boolean gates e.g., XOR and AND.
- Input of each party: represented as bits
- Arithmetic circuits
- A circuit with addition/multiplication gates in some field, e.g., GF(p) or GF(2n)
- Input of each party: an element in the given field


## Boolean or Arithmetic?

| Function | Boolean (Bit) | Arithmetic (Field) |
| :--- | :--- | :--- |
| Statistics <br> (e.g., average) | Large circuit | Small circuit |
| Comparison <br> (e.g., less than) | Small circuit | Large circuit |

## Boolean circuits better suited for addressing the private marketplaces problem

## Previous Work on MPC Solutions

| Circuit | Language | Circuit <br> Scalability | \# Corrupted <br> parties |  |
| :---: | :---: | :---: | :---: | :---: |
| FairplayMP <br> [BNP08] | Boolean | Java | No <br> (~4000 gates) | $<n / 2$ |
| VIFF <br> [DGKN09] | Arithmetic | Python | Yes | $<n / 2$ |
| SEPIA <br> [BSMD10] | Arithmetic | Java | Yes | $<n / 2$ |
| Ours | Boolean | C++ | Yes | $<n$ |

Not satisfactory for our purpose

## Our Contributions

- We provide the first scalable implementation of multi-party computation for boolean circuits, with optimal resilience
- We apply our implementation to the problem of online marketplaces
- Performance better than what is obtained using previous solutions (VIFF, SEPIA)
- Another indication that generic secure MPC can be useful in solving practical problems



## Overview of the Protocol

- We implement the [GMW87] protocol
- The function is given as a boolean circuit
- With XOR and AND gates
- Evaluate the circuit in a gate-by-gate manner
- Invariant: the actual value of each wire is secretshared.


## Overview of the GMW Protocol

$$
x>y=x(1 \oplus y)=x \oplus x y
$$



Evaluate the circuit gate-by-gate .
The value of each wire is secret-shared.

## 1-out-of-4 Oblivious Transfer (OT)


input $\left(x_{0}, x_{1}, x_{2}, x_{3}\right)$
input $\sigma$
OT
output $x_{\sigma}$

## GMW - Evaluating AND gates


$a_{1}, b_{1}$


Run oblivious transfer

$a_{2}, b_{2}$

$$
\begin{aligned}
& x_{0}=\left(a_{1}+0\right)\left(b_{1}+0\right)+r \\
& x_{1}=\left(a_{1}+0\right)\left(b_{1}+1\right)+r \\
& x_{2}=\left(a_{1}+1\right)\left(b_{1}+0\right)+r \\
& x_{3}=\left(a_{1}+1\right)\left(b_{1}+1\right)+r
\end{aligned}
$$

$$
\sigma=2 a_{2}+b_{2}
$$

$$
\underbrace{\text { OT }}_{x_{\sigma}}
$$

$$
c_{1}=r
$$

$$
c_{2}=x_{\sigma}
$$

## GMW Protocol: Multi-Party Setting

- Input wires: XOR of all shares are the actual value.
- XOR gate: same as before (i.e., $c_{i}=a_{i}+b_{i}$ )
- AND gate: use OT between all pairs of parties
- Details omitted


## Implementation of the GMW Protocol

- Critically depends on efficiency of OT protocol
- Basic OT [NP01]
- Multi-threading: two-threads for each pair-wise OT
- Number-theory package: NTL http://shoup.net (modified for MT)
- OT extension [IKNP03]
- Several (e.g., 80) basic OTs with long inputs $\rightarrow$ many bit OTs
- Small overhead: four hash functions per OT
- Use SHA-1 implementation from PolarSSL
- OT preprocessing [Bea95]
- Preprocess OTs on random input
- Use them for OTs on actual input: tiny overhead (a few bits)



## Circuit: P2P Content Distribution


$\sharp$ AND gates $\approx 3 r \ell+r \log r$
( $r=\sharp$ resources, $\ell=$ bit-length of an integer)

## Experiments in LAN: P2P Content Distribution



## Running-Time Ratio: VIFF/Ours



## Our implementation is 10-30x faster

## Running-Time Ratio: SEPIA/ours



## Our implementation is $\sim 10 x$ faster

## Experiments in PlanetLab

- Similar results
- With somewhat bigger deviation
- Details are in the paper


## Summary

- Generic MPC implementation
- Boolean circuit representation, optimal corruption thereshold
- Source code:
http://www.ee.columbia.edu/~kwhwang/projects/gmw.html
- Application to privacy in online marketplaces
- Generic MPC can be practical
- Explore generic solutions before designing new protocols


Thank you

## OT Extension [IKNP03,LXX05]

- Several long string OTs $\rightarrow$ many bit OTs



Very efficient: four additional hashes per OT

## OT Preprocessing [Bea95]

- bit OTs on random input $\rightarrow$ bit OTs on actual input

input $\left(x_{0}, x_{1}, x_{2}, x_{3}\right)$
input 1

3

$$
\left(r_{3}+x_{0}, r_{2}+x_{1}, r_{1}+x_{2}, r_{0}+x_{3}\right)
$$

## Two-Party Computation? (Not in This Talk)

- Initial work
- Fairplay [MNPS04]
- Rather slow and not scalable
- Subsequent work
- Improves performance and scalability
- [LPS08,PSSW09,HEKM11, M11]
- With semi-honest security
- Corrupted parties follow the protocol honestly, but try to infer secret information from the protocol transcript.


## GMW - Evaluating AND gates

$$
\begin{aligned}
& x_{0}=\left(a_{1}+0\right)\left(b_{1}+0\right)+r \\
& x_{1}=\left(a_{1}+0\right)\left(b_{1}+1\right)+r \\
& x_{2}=\left(a_{1}+1\right)\left(b_{1}+0\right)+r \\
& x_{3}=\left(a_{1}+1\right)\left(b_{1}+1\right)+r \\
& c_{1}+c_{2}=r+x_{\sigma}=\left(a_{1}+a_{2}\right)\left(b_{1}+b_{2}\right)=a b=c
\end{aligned}
$$

$$
\text { Check } x_{\sigma}=\left(a_{1}+a_{2}\right)\left(b_{1}+b_{2}\right)+r
$$

$$
\begin{array}{lll}
a_{2}=0, b_{2}=0: & \sigma=0, & x_{0}=\left(a_{1}+a_{2}\right)\left(b_{1}+b_{2}\right)+r \\
a_{2}=0, b_{2}=1: & \sigma=1, & x_{1}=\left(a_{1}+a_{2}\right)\left(b_{1}+b_{2}\right)+r \\
a_{2}=1, b_{2}=0: & \sigma=2, & x_{2}=\left(a_{1}+a_{2}\right)\left(b_{1}+b_{2}\right)+r \\
a_{2}=1, b_{2}=1: & \sigma=3, & x_{3}=\left(a_{1}+a_{2}\right)\left(b_{1}+b_{2}\right)+r
\end{array}
$$

