Cryptanalytic Attacks on MIFARE Classic Protocol

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Outline

1. MIFARE Classic Smart Card
2. Description of MIFARE Classic Protocol
3. Attack Scenarios
4. Objectives
5. Attacks on Genuine Session or Reader
6. Attacks on Genuine Tag – General Concepts
7. Attacks on Genuine Tag – Comparison
8. Generic Differential Attack on Genuine Tag
9. Multiple Sector Attack
10. Conclusions

1. MIFARE Classic Smart Card (1/2)

- **MIFARE Classic smart card is claimed to be the most widely used contactless smart card in the world, especially for access control to buildings and public transport.** It is said to cover more than 70% of market share for access control worldwide.
- It is used in RFID (Radio Frequency IDentification) and NFC (Near Field Communication) systems.
- It implements a proprietary symmetric-key mutual authentication protocol based on a proprietary stream cipher known as CRYPTO1 (1994).
- CRYPTO1 is used for the authentication protocol and also for encrypting messages on RFID/NFC channel between card and reader (without data integrity).

1. MIFARE Classic Smart Card (2/2)

- CRYPTO1 and the protocol are reverse engineered in [Nohl et al. ‘08], [Garcia et al. ‘08] and analyzed in [Nohl et al. ‘08], [de Koning Gans et al. ‘08], [Garcia et al. ‘08], [Garcia et al. ‘09], [Courtois ’09].
- CRYPTO1 uses a preshared 48-bit secret key. On a standard CPU, brute-force attack on 48-bit key can take several years, but less than an hour on FPGA board (e.g., COPACOBANA [Kumar et al. ‘06]).
- **Proposed cryptanalytic attacks in various scenarios demonstrate that MIFARE Classic smart card does not offer 48-bit security level.**
- Dedicated reader infrastructure is not easy to change.

2. Description of MIFARE Classic Protocol (1/6)

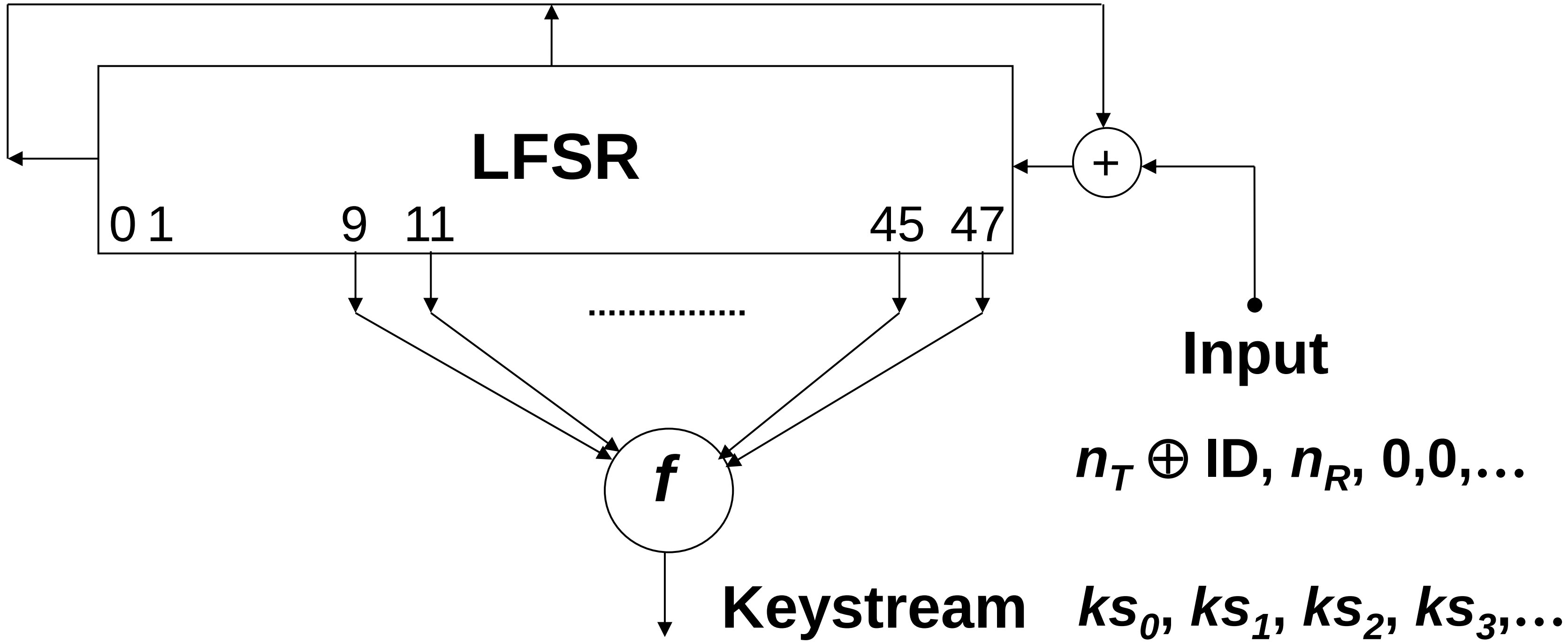
- Memory of MIFARE Classic card/tag is divided into sectors containing16-byte blocks; the last block of each sector contains 48-bit secret key (in fact, two of them); the first block of the first sector contains read-only data including 32-bit tag identifier ID and manufacturer data.
- To write or read data from a given memory block, tag and reader have to be authenticated to each other by a **challenge-response symmetric-key authentication protocol** using the key of the corresponding sector and 32-bit challenge tag and reader nonces n_T and n_R, which are generated by respective pseudorandom number generators.

2. Description of MIFARE Classic Protocol (2/6)

- Tag nonce, n_T, is generated by a 16-bit linear feedback shift register (LFSR), which starts from the same state after powering up the tag and has period 618 ms
    - At most 16 bits of entropy
    - True randomness – variable generation time of tag nonce.
- Reader nonce n_R is generated by a pseudorandom number generator, which starts from the same state after reader restart and generates a nonce upon invocation by authentication protocol
    - True randomness – variable number of invocations after restart.
- **Fresh nonces protect against replay attacks**.

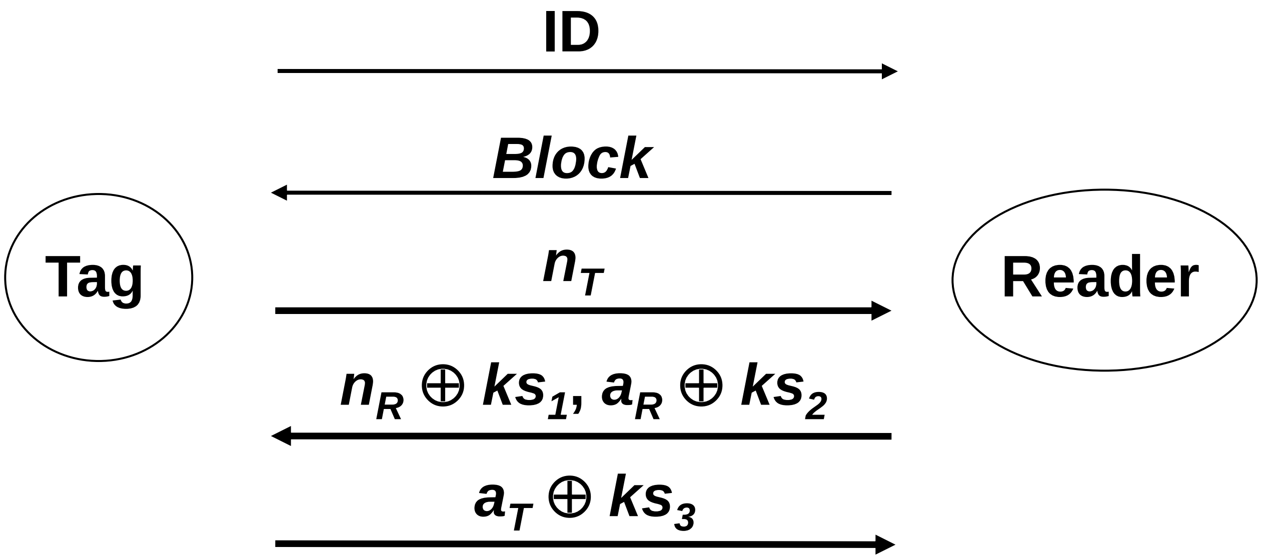
2. Description of MIFARE Classic Protocol (3/6)

- CRYPTO1 – nonlinear filter generator → 48-bit LFSR, 20-bit nonlinear filter function *f*
  - Initialized by secret key and then refreshed by nonces n_T and n_R.



2. Description of MIFARE Classic Protocol (5/6)

- Three-pass authentication protocol for one sector



Tag

Reader

ID

Block

n_T

$n_R \oplus ks_1, a_R \oplus ks_2$

$a_T \oplus ks_3$

$a_R = \mathrm{suc}^2(n_T)$

$a_T = \mathrm{suc}^3(n_T)$

- For multiple sectors, for each new sector, ***Block*** is encrypted by using previous sector key and n_T is sent encrypted with new key as $n_T \oplus ks_0$.

2. Description of MIFARE Classic Protocol (4/6)

- **suc** function maps 32 bits into next 32 bits of 16-bit LFSR.
- First $n_T \oplus$ ID and then n_R are bitwise XORed into LFSR
    - ks_0 is bitwise generated while XORing $n_T \oplus$ ID
    - ks_1 is bitwise generated while XORing n_R
    - ks_2, ks_3, and subsequent keystream are generated by clocking LFSR autonomously.
- Tag and reader generate the same keystream, clocking LFSR forwards, with a difference that reader uses n_R directly, whereas tag first decrypts encrypted n_R bit-by-bit, as the keystream bit encrypting a bit of n_R depends only on previous bits of n_R.
- For multiple sectors, n_T is treated analogously.

2. Description of MIFARE Classic Protocol (6/6)

- **Error detection in MIFARE Classic communication protocol**
  - Every plaintext byte is followed by one parity bit computed as the binary complement of the XOR of all 8 bits **(ISO/IEC 14443-A)**
  - Each parity bit is encrypted with the same keystream bit used to encrypt the next bit of plaintext.
- Due to linear coding preceding the encryption, ciphertext reveals linear relations among keystream bits (1 relation over 8+1 keystream bits per 8+1 ciphertext bits).
- Due to reused keystream bits, ciphertext reveals linear relations among plaintext bits (1 relation over 8+1 plaintext bits per 1+1 ciphertext bits).

3. Attack Scenarios

- Passive, genuine session scenario
  - Attacker intercepts genuine session and aims at decrypting some recorded traces (possibly also reconstructing the key)
  - Impractical, due to very short distances needed.
- Active, genuine reader scenario
  - Attacker uses fake/emulated tag to initiate fake authentication sessions with genuine reader, in order to reconstruct the key
  - Impractical, due to very short distances needed.
- Active, genuine tag scenario, also called tag-only scenario
  - Attacker uses fake/emulated reader to initiate fake authentication sessions with genuine tag (**queries**), in order to reconstruct the key
  - **Easy to implement, provided that the on-line stage is short**.

4. Objectives

- Present a critical comprehensive survey of currently known attacks on MIFARE Classic, in various attack scenarios, and put them into the right perspective in light of the prior art in cryptanalysis.
- Propose improvements of known attacks, if possible.
- **In particular, in tag-only scenario, significantly reduce required on-line time, while keeping off-line time practical**
  - Few seconds or less is very practical, a few tens of seconds is much less practical, a few minutes is impractical, ten or more minutes is very impractical.
- Discuss design drawbacks and design principles.

5. Attacks on Genuine Session or Reader (1/2)

- Objective of two attacks [Garcia et al. ‘08] is to reconstruct the 48-bit key from known keystream
  - 64 bits of ks_2 and ks_3 from genuine session, passive scenario
  - 32 bits of ks_2 and, possibly, additional 32 bits of ks_3 from genuine reader (encrypting known command, such as ‘halt’).
- Internal state, e.g., 48-bit LFSR state S_{64} from which the first bit of ks_2 is generated is first recovered and, then, the key is reconstructed by clocking LFSR backwards (LFSR rollback)
  - Possible even if encrypted $\{n_R\}=n_R \oplus ks_1$ is known instead of n_R
  - Not only for f that does not depend on the left-most LFSR state bit (by branching [Golić et al. ‘00]).

5. Attacks on Genuine Session or Reader (2/2)

- Time-memory-data tradeoff (TMDT) attack in genuine reader scenario requires multiple fake authentication sessions (e.g., 4096 ≈ 10 min) in on-line stage ($T \cdot M = 2^{48}$)
  - Adaptation – success probability equal to 1 instead of being high
  - On-line time can be reduced while keeping off-line time and memory, at the expense of increasing pre-computation time.
- Inversion attack in genuine session or reader scenario requires one/two genuine or fake authentication sessions in on-line stage, for 64/32 known keystream bits
  - Adaptation of generic inversion attack and decimation technique from [Golić '96], [Golić et al. '00], to deal with short keystream
  - Bad tap positions (attack takes $2^{17}/2^{20}$ steps instead of 2^{47} steps).

6. Attacks on Genuine Tag – General Concepts (1/2)

- Attacker can obtain known keystream only because of **a peculiar property of the authentication protocol**
  - In the second pass, fake reader sends fake 64-bit ciphertext, i.e., encrypted values $\{n_R\}$ and $\{a_R\}$, and 8 encrypted parity bits p, $\{p\}$, of n_R and a_R, without knowing the key
  - Authentication by tag will fail, but, if all 8 decrypted bits in p are correct, tag sends back 4-bit ciphertext of a fixed 4-bit error message, encrypted with first 4 bits of ks_3, which reveals 4 keystream bits **(such a query is called successful)**
  - In addition, correct parity bits reveal 8 independent linear relations among keystream bits, i.e., 8 keystream parity bits
  - **Altogether, each sussessful query reveals 12 bits of information about the key**.

6. Attacks on Genuine Tag – General Concepts (2/2)

- Queries can use random or fixed tag nonces. According to [Garcia et al. ‘09], attacker can perform about
  - 1500 queries/sec with a random tag nonce, q_r
  - 30 queries/sec with a fixed tag nonce, q_f, by reset query technique (switch-off the field, power-up passive tag, and start a query at a fixed time after reset; at most ten attempts to get the same n_T).
- **Query strategies in on-line stage:**
  - For random n_T, if $\{n_R\}$, $\{a_R\}$, $\{p\}$ are randomly chosen, then, on average, 256 queries q_r are required to get one successful query, since p is then random (here $\{a_R\}=a_R \oplus ks_2$)
  - For fixed n_T, if $\{n_R\}$, $\{a_R\}$ are fixed and $\{p\}$ is varied, then, on average, 128 queries q_f are required to get one successful query, since p is then fixed.

7. Attacks on Genuine Tag – Comparison (1/2)

- Three attacks proposed in [Garcia et al. ‘09] are here denoted as Att. 1, 2, and 3
  - Att. 1 has impractical off-line time (brute force)
  - Att. 2 has impractical on-line time (15 min)
  - Att. 3 has impractical setup time (brute force).
- The best previously known attack is differential attack proposed in [Courtois ‘09] here denoted as Att. 4
  - Att. 4 makes better use of differential properties of *f* than Att. 2, thus significantly reducing on-line time
  - **In this work, Att. 4 is corrected&optimized into Att. 4***.
- New differential attack, Att. 5, significantly reduces on-line time of Att. 4/4*, while keeping off-line time practical.

7. Attacks on Genuine Tag – Comparison (2/2)

Attacks	**Att. 1**	**Att. 2**	**Att. 3**	**Att. 4***	**New Att. 5**
Setup time	0	0	**2^{48}**	0	0
Setup memory	0	0	$48\cdot 2^{36}$ bit 384 GByte	0	0
On-line time	$1280q_r$ 1 sec	**$28500q_f$** **15 min**	$4230q_r+128q_f$ 7 sec	**$3(256q_r+112q_f)$** **11.7 sec**	**$2(256q_r+48q_f)$** **3.5 sec**
Off-line time	**$5\cdot 2^{48}$** **3 year**	$2^{32.8}$ 10 min	2^{24} TLU 20 sec	$2^{16}+2^{26}f_{ev}$ ≈0	$2^{32}+2^{25}f_{ev}$ 5 min
Off-line memory	≈0	≈0	≈0	168 Byte	42 KByte
Success Prob	100%	100%	100%	99.4%	99.1%

8. Generic Differential Attack on Genuine Tag (1/5)

- **Objective:**
  - From a number of successful queries for different (n_T, {n_R}), recover a set of candidate LFSR states at a given time, for some (n_T, {n_R})
  - By LFSR rollback, then recover a set of candidate keys and, finally, get the unique key possibly using additional successful queries
  - **Problem: the values of n_R are unknown to the attacker**.
- **First on-line phase:** Get one successful query (random n_T).
- **Second on-line phase:** Modify last m bits of {n_R} and then fix n_T, {n_R}, {a_R} and vary last 5 bits of {p} until a successful query occurs; on average, 16 such queries with a fixed n_T, for each of 2^m-1 modifications of {n_R} are needed.
- On-line stage thus yields 2^m successful queries.

8. Generic Differential Attack on Genuine Tag (2/5)

- Since $n_R = \{n_R\} \oplus ks_1$, where the last m bits of ks_1 depend on the last m bits of $\{n_R\}$, each m-bit change δ_m of $\{n_R\}$ can result in the same change of n_R with some probability depending on the filter function f.
- This is the probability that the last m keystream bits of ks_1, used for encrypting the last m bits of $\{n_R\}$, are independent of the last m bits of $\{n_R\}$
  - The first of these m bits is already independent, as it depends on previous bits of $\{n_R\}$ only
  - The remaining m-1 bits are output bits of (m-1)-bit augmented filter function of all LFSR state bits as input bits; these m-1 output bits are independent of the last m-1 input bits with probability π_{m-1}

8. Generic Differential Attack on Genuine Tag (3/5)

  - It follows that $\pi_0=1$, $\pi_1=29/32\approx0.906$, $\pi_2=(29/32)^2\approx0.821$
  - Worst design case: $\pi_{m-1}=1$ would hold if *f* were independent of the last *m* LFSR state bits
  - Best design case: $\pi_{m-1}=0$ would hold if *f* were linear in the last LFSR state bit (a sufficient condition from [Golić '96]).
- Since LFSR sequence depends linearly on n_R, for each value of δ_m, subsequent LFSR sequence can be expressed as bitwise XOR of unknown LFSR sequence corresponding to initial value of $\{n_R\}$ and a known binary sequence determined by known value of δ_m **(this holds with probability π_{m-1})**.
- 4 keystream bits from each successful query are used.

8. Generic Differential Attack on Genuine Tag (4/5)

- Attacker thus obtains $4\cdot 2^m$ equations of the form $z_i=f(S_i\oplus\Delta_i)$, for 2^m values of δ_m and $96\leq i\leq 99$, where S_i is unknown LFSR state for $\delta_m=0$ at time i, and z_i and Δ_i depend only on δ_m (to be solved in off-line stage).
- Nonlinear system of equations can be solved for S_{99} by an adapted variant of the well-known resynchronization attack [Daemen et al. '94] (essentially, exhaustive search) and using the fact that
  - f depends only on 20 bits of LFSR state and after 2 steps, f depends on the same 19 bits and 1 new bit (bad tap positions)
  - Accordingly, z_{96} and z_{98} depend on 21 LFSR bits of S_{99} (as well as z_{97} and z_{99} depend on other 21 LFSR bits of S_{99}).

8. Generic Differential Attack on Genuine Tag (5/5)

- Att. 4* is corrected&optimized Att. 4 from [Courtois '09], which is a differential attack for m=3
  - Queries with random/fixed tag nonces are better handled
  - For one run, success probability $\pi_2 \approx 0.821$ (instead of $\approx$0.75) and average on-line time $\approx$3.9 sec (instead of $\approx$8.52 sec for first run and $\approx$5.33 sec for other runs), and off-line time $\approx$0
  - For 3 independent runs of Att. 4*, success probability $\approx$0.994, average on-line time $\approx$11.7 sec, and average off-line time $\approx$0.
- **Att. 5 is a new differential attack, for *m*=2**
  - For one run, success probability $\pi_1 \approx 0.906$, average on-line time $\approx$1.77 sec, and off-line time $\approx$5 min
  - For 2 independent runs of Att. 5, success probability $\approx$0.991, average on-line time $\approx$3.5 sec, and average off-line time $\approx$5 min.

9. Multiple Sector Attack

- **Assumption:** Key for the first sector has been already recovered in any described attack scenario.
- Main feature is that n_T is sent encrypted by tag
  - This should prevent known keystream attacks, unless
  - **Entropy of n_T is small!** (Repeated keystream bits, timing.)
- By effectively guessing n_T, attacker gets 96 keystream bits in genuine session scenario [Garcia et al. ‘08] and 32 keystream bits in tag-only scenario [Garcia et al. ‘09] and then applies adapted inversion attack.
- **Consequently, tag-only attack scenario is much easier for other sectors than for the first sector**.

10. Conclusions (1/3)

- **MIFARE Classic protocol does not offer 48-bit security level due to:**
  - Repeatable and predictable tag nonces
  - Really bad tap positions for filter function and bad choice of filter function in CRYPTO1
  - Generation&encryption of parity bits for error detection
  - Peculiar feature that tag can sent an encrypted response if authentication of reader fails.
- The most effective known attack is adapted inversion attack in genuine session or genuine reader scenarios
  - Impractical, due to very short distances to genuine static reader required.

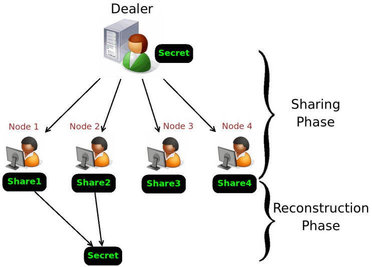
10. Conclusions (2/3)

- The most practical attack scenario is genuine tag scenario, and the new attack, requiring only about 3 sec of on-line time, is the most effective currently known attack in this scenario
  - It significantly improves on the most effective previously known attack, requiring about 10 sec of on-line time.
- **Practical countermeasures**
  - Use electromagnetic-shield covers for tags on smart cards
  - User notification or activation on demand of on-going NFC communications, for tags on mobile devices.
- **Impractical countermeasure**: Replace MIFARE Classic cards/tags and dedicated readers (costly).

10. Conclusions (3/3)

- **Light-weight cryptography**
  - Ease of implementation
  - Reduced, but guaranteed security level
  - Designs secure against known attacks
  - Reduced security margins with respect to known attacks
  - **Difficult to satisfy in practice.**
- **Obscurity in cryptography**
  - Unknown algorithms make cryptanalysis significantly more difficult, if not impossible
  - Designs should be secure against known attacks even if algorithms are compromised (e.g., reverse engineered)
  - **Typically not satisfied in practice.**

Asynchronous Computational VSS with Reduced Communication Complexity



Cryptographer's Track, RSA Conference 2013

Michael Backes
Saarland University & MPI-SWS
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Amit Datta
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Saarland University
Germany

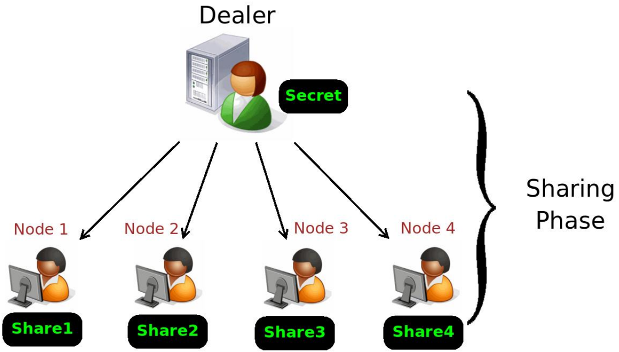
Outline

- Background
  - Secret Sharing
  - Asynchronous Verifiable Secret Sharing (AVSS)
- State-of-the-art for AVSS and Shortcomings
- Our Protocols
  - eAVSS: efficient AVSS
  - eAVSS-SC: efficient AVSS with Strong Commitment

Secret Sharing

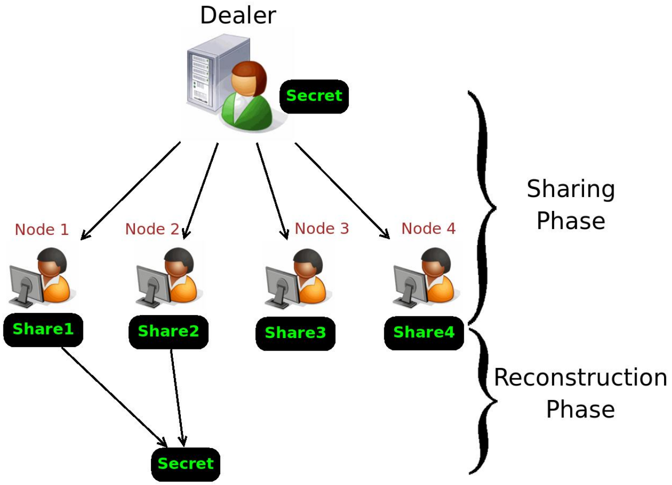
Secret Sharing

Sharing Phase



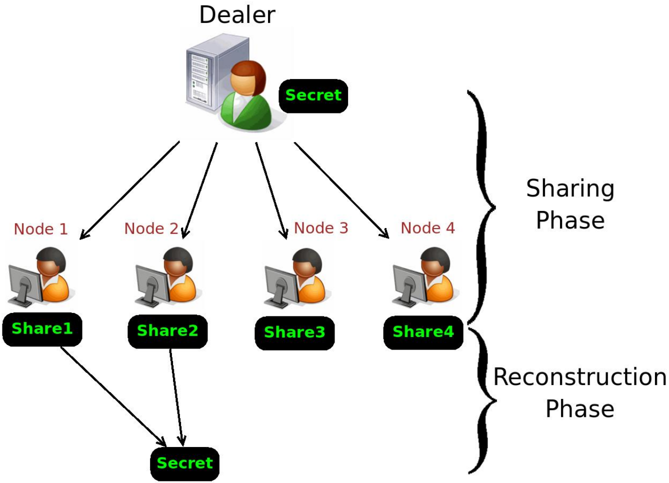
Secret Sharing

Reconstruction Phase



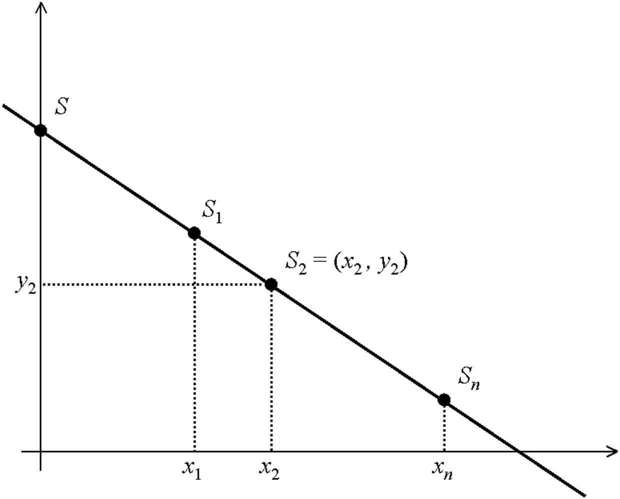
Secret Sharing

$(4, 1)$-Secret Sharing



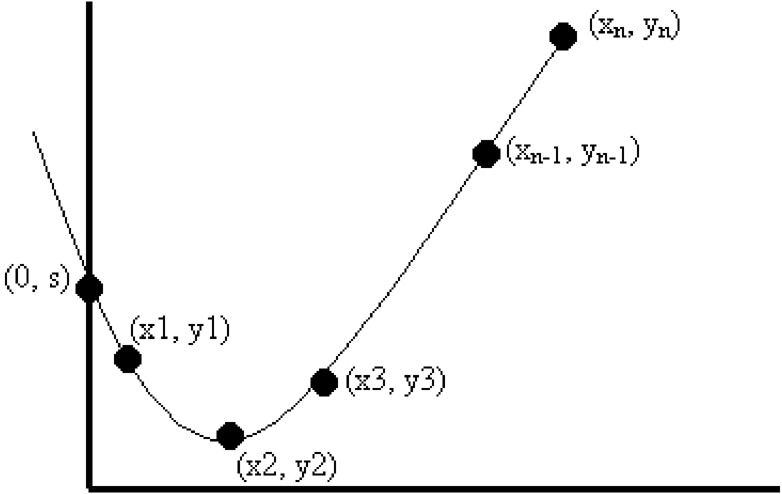
Polynomial-Based Shamir Secret Sharing

$(n, 1)$-Secret Sharing for secret S



Polynomial-Based Shamir Secret Sharing

Not an $(n, 1)$-Secret Sharing for secret S



Verifiable Secret Sharing—VSS

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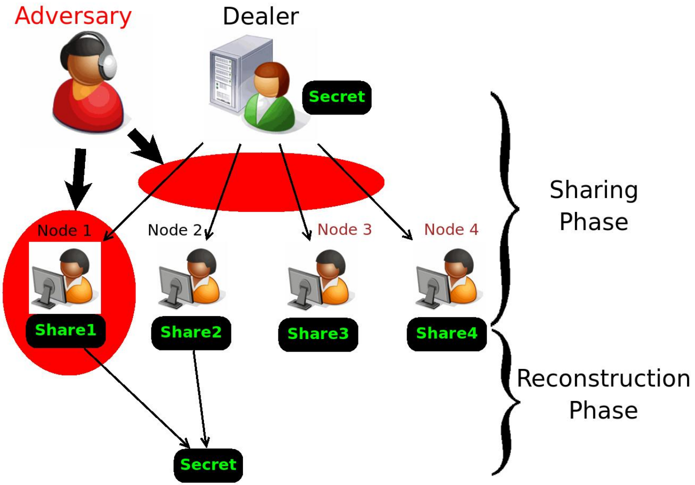
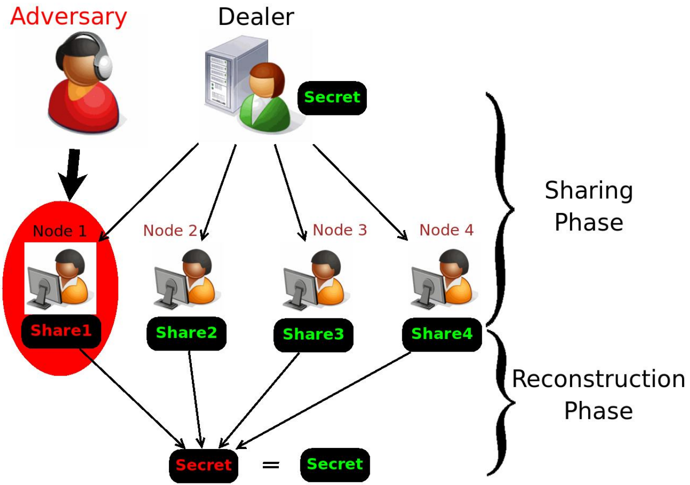
Privacy Property



Verifiable Secret Sharing—VSS

Correctness Property



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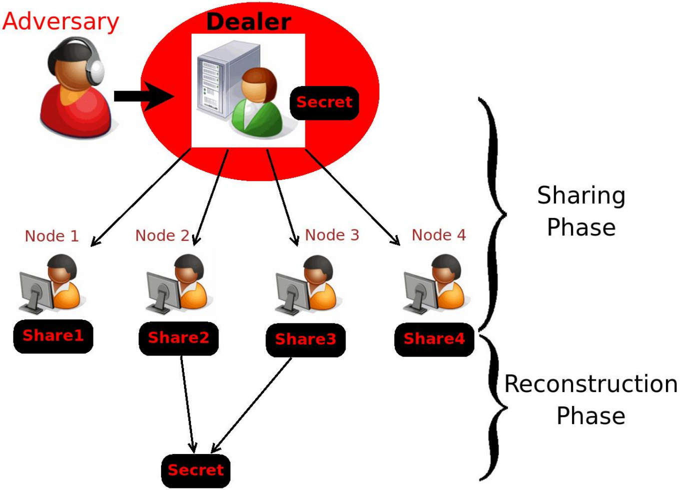
Commitment Property



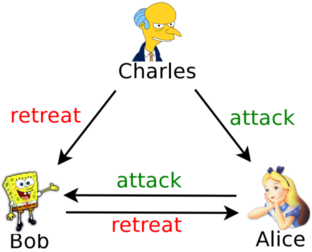
VSS in the Literature

Applications

1. Multi-party Computation
2. Threshold Cryptography

$$\lambda_i = \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

3. Byzantine Agreement



4. Coin-tossing

VSS in the Literature

Communication Assumption

Most of the VSS protocols in the literature assume that the network to be synchronous. However,

- the Internet does not always function synchronously, and
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State-of-the-art

- Asynchronous VSS (AVSS) protocols are inefficient in terms of communication complexity
- The best known AVSS protocol [Cachin et al., ACM CCS ’02] communicates $O(\kappa n^3)$ bits

Our Contributions

- Communication complexity for AVSS can be reduced using the concept of (constant-size) polynomial commitments
- We present two AVSS protocols with $O(\kappa n^2)$ communication complexity:
  - Our first protocol satisfies the standard VSS definition
    Applications:
    1. Stand-alone VSS scenarios
    2. Byzantine agreement
  - Our second protocol satisfies a stronger VSS definition
    Applications:
    1. multiparty computation (MPC)
    2. threshold cryptography

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Asynchronous Message Passing Setting

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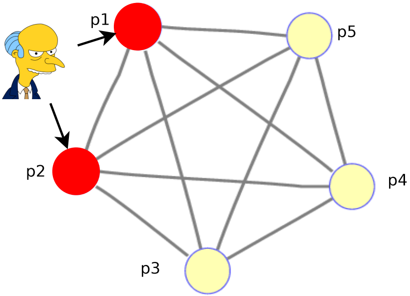


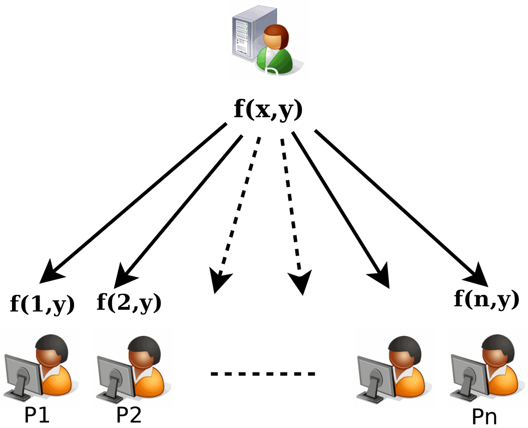
processes: $p_1, \ldots, p_n$

- pairwise connected by an asynchronous channel
    - messages can be arbitrarily delayed, or reordered
    - however, messages are eventually delivered

- at most t of n processes may exhibit faulty behavior In this setting, the optimal resiliency bound is $n \geq 3t + 1$

State-of-the-art Protocol for AVSS

Sharing Phase



State-of-the-art Protocol for AVSS

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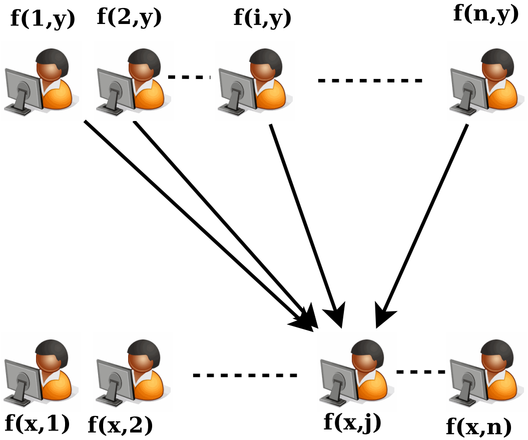

State-of-the-art Protocol for AVSS

Message complexity: $O(n^2)$

For verification:

$$\text{Commitment matrix } \mathbf{C} = \{C_{jl}\} = g^{f_{jl}}$$

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Communication complexity: $O(\kappa n^3)$,

where κ is the security parameter

Reference :

C. Cachin, K. Kursawe, A.Lysyanskaya, and R. Strobl.
Asynchronous Verifiable Secret Sharing and Proactive Cryptosystems.
In ACM CCS ’02.

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What We Want to Achieve

Message complexity: $O(n^2)$,

Communication complexity: $O(\kappa n^2)$,

where κ is the security parameter

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Solution : Constant-size Polynomial Commitments
Helps commit to a univariate polynimial by publishing just one value

Reference:
A.Kate, G. M. Zaverucha, and I. Goldberg.
Constant-Size Commitments to Polynomials and Their Applications.
In Proceedings of ASIACRYPT '10.

A PolyCommit scheme

Setup$(1^\kappa, t)$ generates an appropriate algebraic structure $\mathcal{G} = \langle e, \mathbb{G}, \mathbb{G}_T \rangle$ and the system parameter PK

Commit$(PK, f(x))$ outputs a commitment $\mathcal{C}$ to a polynomial $f(x)$

CreateWitness$(PK, f(x), i)$ outputs $\langle i, f(i), w_i \rangle$, where w_i is a witness for the evaluation $f(i)$ of $f(x)$

VerifyEval$(PK, \mathcal{C}, i, f(i), w_i)$ verifies that $f(i)$ is indeed the evaluation of the polynomial committed in $\mathcal{C}$

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Construction

- Single element commitments for univariate polynomials
- However, the scheme does not work for multi-variate polynomials required in AVSS schemes!!

Our eAVSS Protocol

Dealer D

- Select a polynomial $f(x)$, such that $f(0) = s$
- $\mathcal{C} = \text{Commit}(PK, f(x))$, $w_i = \text{CreateWitness}((PK, f(x), i))$
- Send ($\mathcal{C}$, w_i, $f(i)$) to every party P_i

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Party P_i

- If VerifyEval(PK, $\mathcal{C}$, i, $f(i)$, w_i) succeeds, send (`echo`, $\mathcal{C}$)
- On receiving $(n - t)$ (`echo`, $\mathcal{C}$), send (`ready`, `share`, $\mathcal{C}$)
- Otherwise:
  - (a) On receiving $(n - 2t)$ (`ready`,*, $\mathcal{C}$) signals, send (`ready`, `share`, $\mathcal{C}$) to every party P_j.
  - (b) On receiving $(n - 2t)$ (`ready`,*, $\mathcal{C}'$) signals, send (`ready`, `no-share`, $\mathcal{C}'$) to every party P_j.
- On receiving $(n - t)$ (`ready`, $\mathcal{C}$) signals, and at least $(n - 2t)$ contain `share`, terminate

Salient Points

- There are at least $n - 2t \geq 3t + 1 - 2t = t + 1$ honest parties with correct shares

- There are at most n `send`, n^2 `echo`, and n^2 `ready` messages

Properties of eAVSS

Liveness. If the dealer D is honest, then all honest parties complete sharing.

Secrecy. If D is honest, then the adversary has no information about s.

Agreement. If some honest party completes the sharing phase, then all honest parties will eventually complete the sharing phase.

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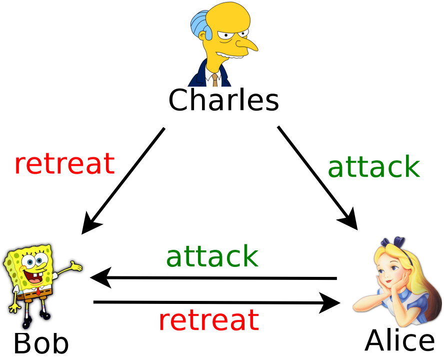
Correctness. Once all honest parties complete sharing, there exists a fixed value $z \in \mathbb{Z}_p$, such that the following holds:

(a) If an honest dealer has shared the secret s, then $s = z$.

(b) If each of the honest servers P_i reconstructs some z_i, then $z_i = z$

Suitable for

Byzantine Agreement



Charles

retreat

attack

attack

retreat

Bob

Alice

Coin-tossing

Not Suitable for

Multi-party Computation

Threshold Cryptography

$$\lambda_i = \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{x-x_j}{x_i-x_j}$$

Properties of Stronger AVSS

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Secrecy. If D is honest, then the adversary has no information about s.

Agreement. If some honest party completes the sharing phase, then all honest parties will eventually complete the sharing phase.

Strong Correctness. Once $t + 1$ honest parties complete sharing, there exists a fixed value $z \in \mathbb{Z}_p$, such that the following holds:

(a) If an honest dealer has shared the secret s, then $s = z$.

(b) If each of the honest servers P_i reconstructs some z_i, then $z_i = z$

Protocol for eAVSS-SC

- Dealer `sends` polynomials $f^0(x), f^1(x), ... f^n(x)$, with $f^k(x) = F(x, k)$, $F(x, y)$ is of degree $\leq t$. Commitments: $\mathcal{C}^0, \mathcal{C}^1, ..., \mathcal{C}^n$.

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- There will be at least $t+1$ honest parties with correct polynomials $f^k(x) = F(x,k)$, and they compute their shares $s_k = f^k(0) = F(0,k) = F(k,0) = f^0(k)$

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- These $t+1$ parties can enable any P_i to reconstruct its polynomial $f^i(x)$ by sending $f^k(i) = f^i(k)$

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- These $t+1$ parties can enable any P_i to reconstruct its polynomial $f^i(x)$ by sending $f^k(i) = f^i(k)$

Problem: Have to send a vector of commitments in the `echo` and `ready` messages.

Solution: Perform another round of PolyCommit on hash values of the commitments.

Take Away

- Constant-size polynomial commitments help to obtain an AVSS protocol with reduced communication complexity

- We have presented two efficient AVSS schemes (eAVSS and eAVSS-SC) with $O(n^2\kappa)$ communication complexity
- They reduce the communication complexity of various AVSS applications by a linear (in n) factor

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Thanks!