DEPARTAMENTO DE CIENCIAS DE LA COMPUTACIÓN
UNIVERSIDAD DE CHILE

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Fair Exchange of Short Signatures without Trusted Third Party

Philippe Camacho
University of Chile

Digital Goods Economy

iTunes

Enforcing Secure Transactions through a Trusted Third Party (TTP)

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Problems with TTP

Anonymous Claims To Have Hacked 28,000 PayPal Passwords For Guy Fawkes Day

The Huffington Post | By Cavan Sieczkowski

Posted: 11/05/2012 11:15 am EST Updated: 11/05/2012 1:01 pm EST

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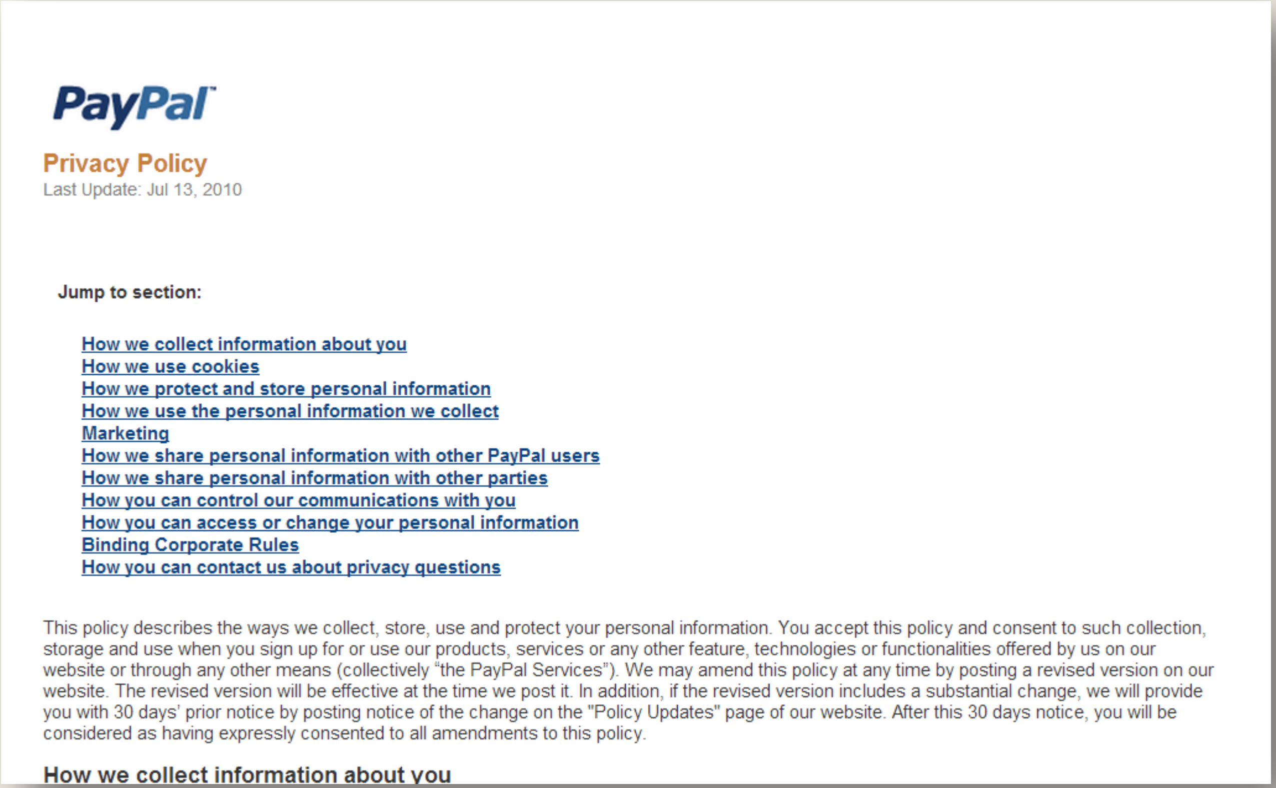
Problems with TTP



PayPal

Privacy Policy

Last Update: Jul 13, 2010

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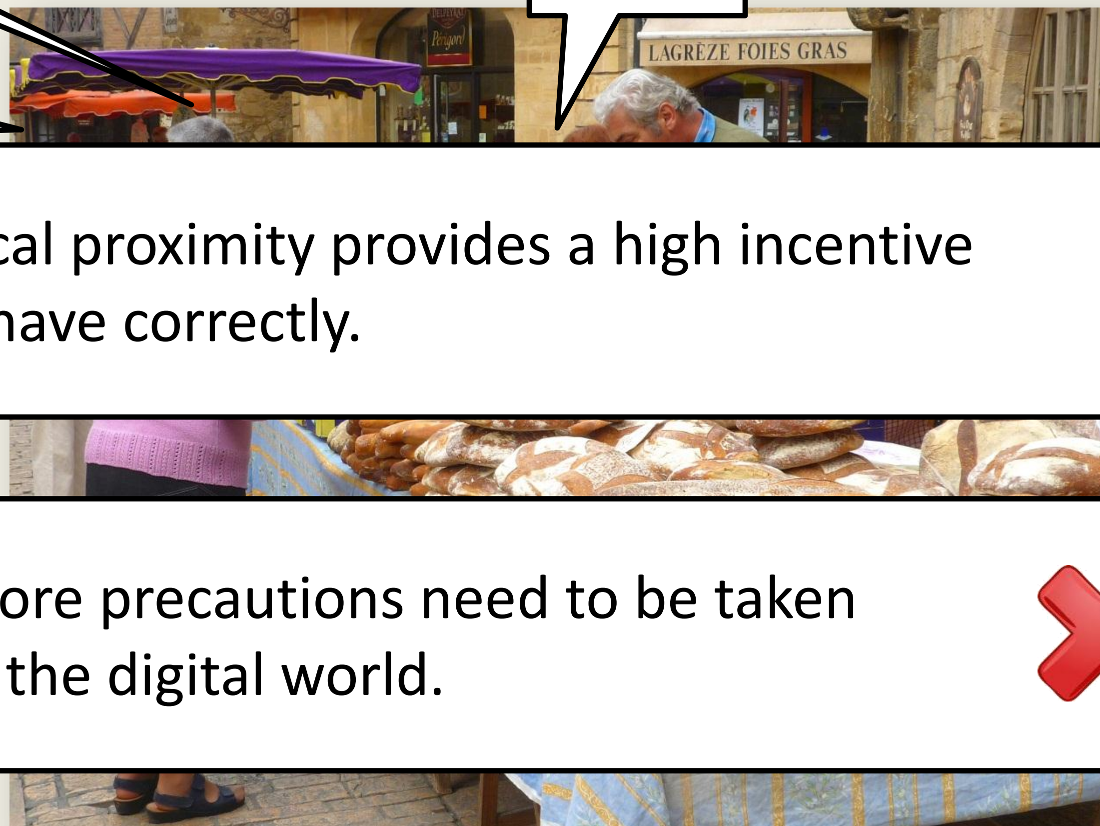
How we collect information about you

Fair Exchange in the Physical World is “easy”



Physical proximity provides a high incentive to behave correctly.

More precautions need to be taken in the digital world.

Modeling Transactions with Digital Signatures

The problem: Who starts first?
Impossibility Result **[Cleve86]**

Software License

Digital Check

Buyer

Seller

Gradual Release of a Secret

Bob’s signature

Alice’s signature

1

Allows to circumvent Cleve’s impossibility result (relaxed security definition).

0

How do I know that the bit I received is not garbage?

Our Construction

- Fair Exchange of Digital Signatures
- Boneh-Boyen [BB04] Short Signatures
- No TTP
- Practical

Contributions

- Formal definition of *Partial Fairness*

- Efficiency

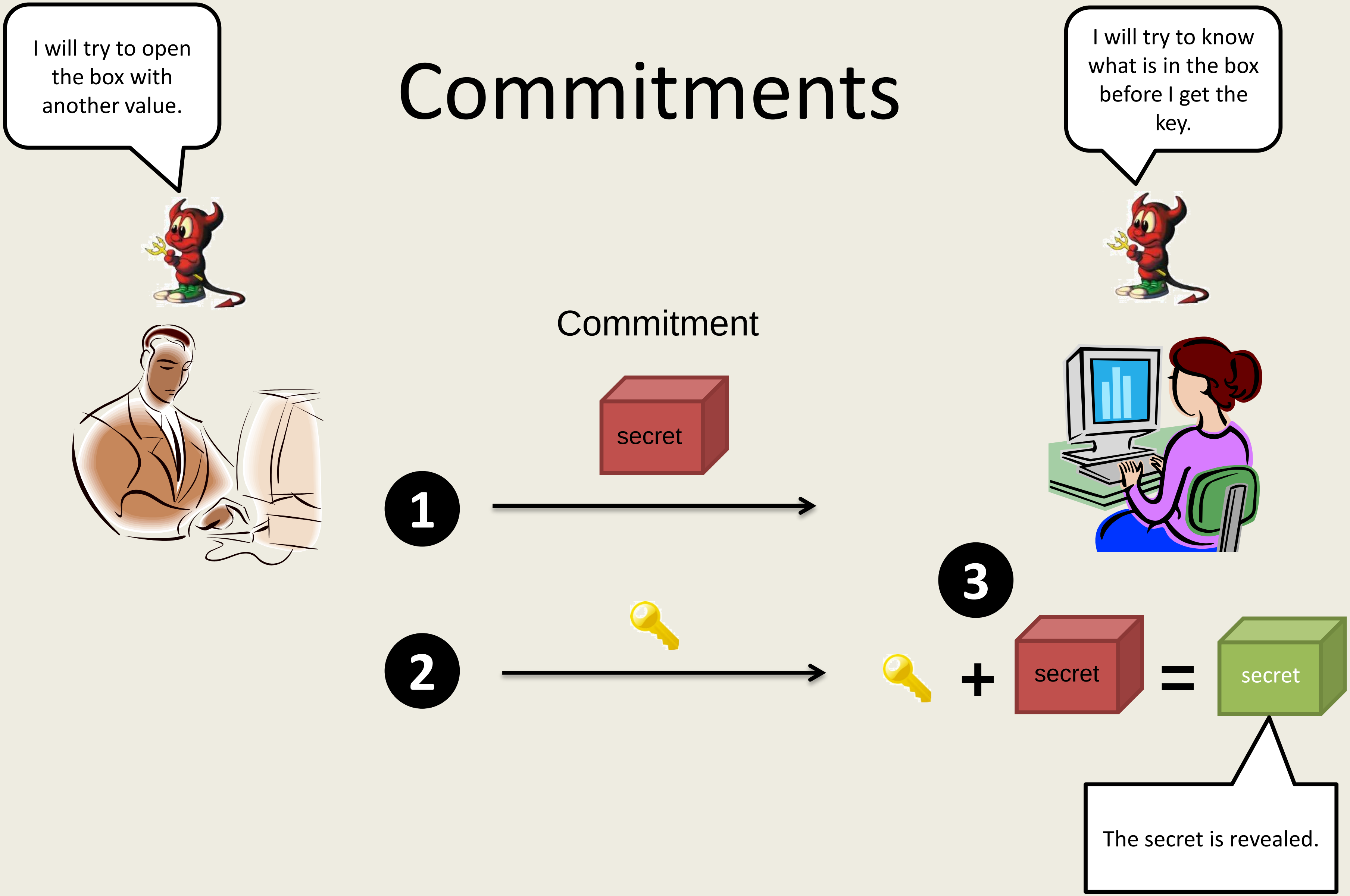
	κ: Security Parameter	$\boldsymbol{\kappa = 160}$
# Rounds	$\kappa + 1$	161
Communication	$16\kappa^2 + 12\kappa$ bits	$\approx$ 52 kB
# Crypto operations per participant	$\approx 30\kappa$	≈ 4800

- First protocol for Boneh-Boyen signatures

Contributions

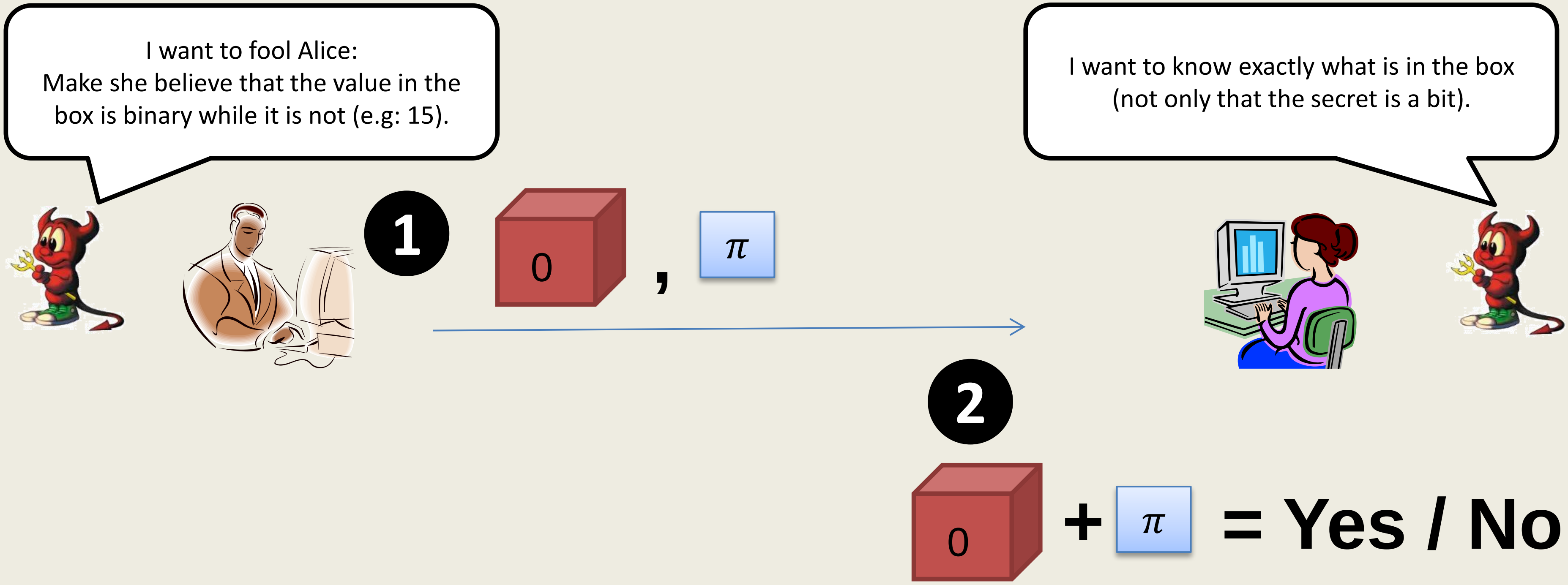
- NIZK argument to prove that a commitment encodes a **bit vector**.
- NIZK argument to prove a commitment to a **bit vector** is the **binary expansion of the discrete logarithm** θ of $D = g^{\theta}$.

Commitments



Non-Interactive
Zero-Knowledge Proofs

Prove something about the secret in the box
without opening the box.



Abstract Protocol

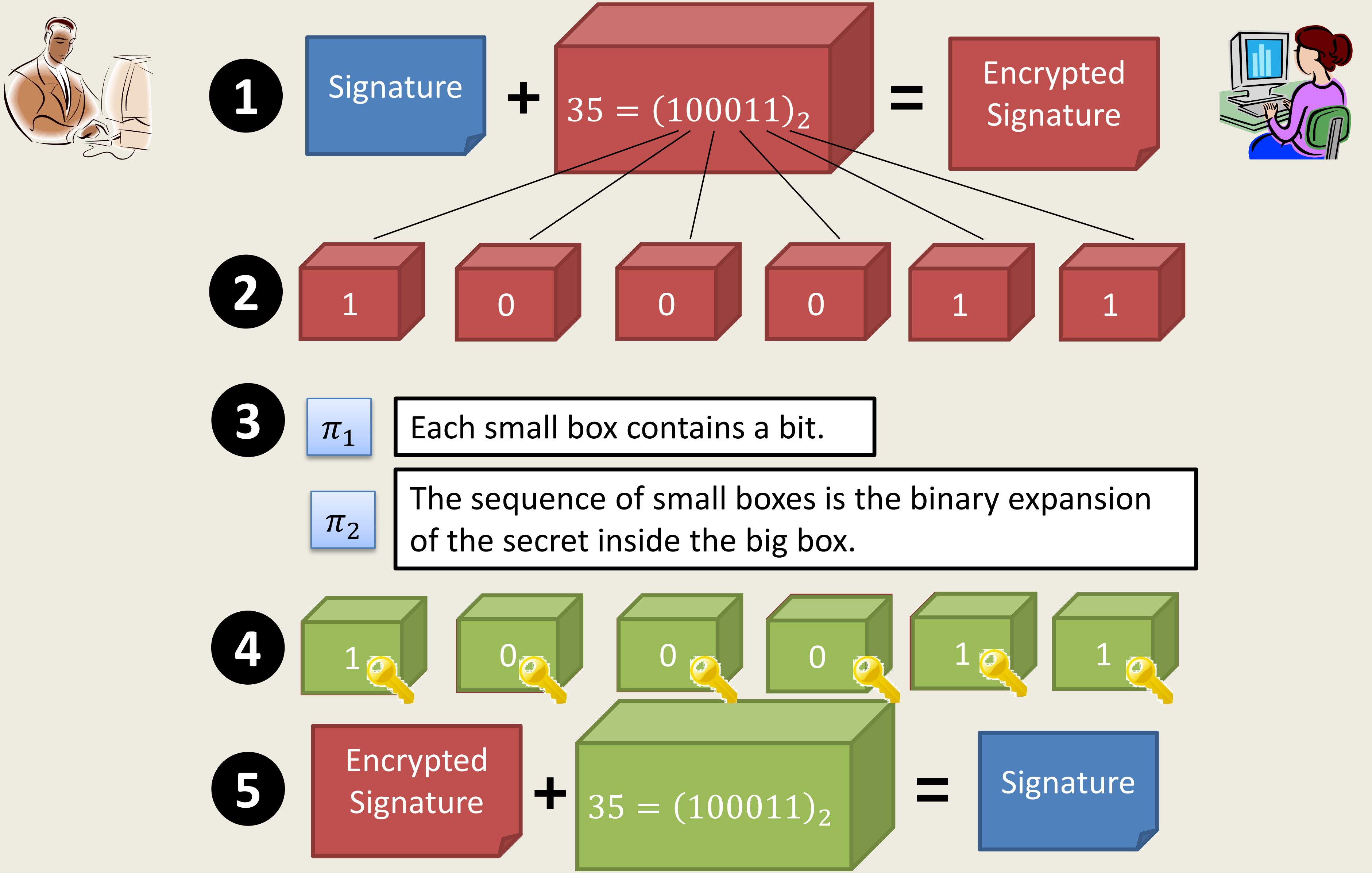
Setup

Step	$\mathcal{P}_A(\mathsf{CRS}, m_A, m_B)$		$\mathcal{P}_B(\mathsf{CRS}, m_A, m_B)$
KeyGen			
1	$(sk_A, pk_A) \leftarrow \mathsf{FEKeyGen}(1^\kappa)$		
2	pk_A	$\longrightarrow$	
3			$(sk_B, pk_B) \leftarrow \mathsf{FEKeyGen}(1^\kappa)$
4		$\longleftarrow$	pk_B
Encrypt Signature			
5	$(\theta_A, \vec{r}_A, \gamma_A) \leftarrow \mathsf{EncSigGen}(\mathsf{CRS}, sk_A, m_A)$		
6	γ_A	$\longrightarrow$	
7			$(\theta_B, \vec{r}_B, \gamma_B) \leftarrow \mathsf{EncSigGen}(\mathsf{CRS}, sk_B, m_B)$
8		$\longleftarrow$	γ_B
Verify Encrypted Signature			
10	$v \leftarrow \mathsf{EncSigCheck}(\mathsf{CRS}, pk_B, m_B, \gamma_B)$		
11	**if** $v = 0$ **then ABORT**		
12			$v \leftarrow \mathsf{EncSigCheck}(\mathsf{CRS}, pk_A, m_A, \gamma_A)$
13			**if** $v = 0$ **then ABORT**
Release Bits			
for $i = 1$ **to** κ:			
14	$\mathsf{open}_{A,i} \leftarrow \mathsf{KeyBitProofGen}(\mathsf{CRS}, \vec{r}_A, \theta_A, i)$		
15	$\mathsf{open}_{A,i}$	$\longrightarrow$	
16			$\mathsf{open}_{B,i} \leftarrow \mathsf{KeyBitProofGen}(\mathsf{CRS}, \vec{r}_B, \theta_B, i)$
17		$\longleftarrow$	$\mathsf{open}_{B,i}$
19	$v_i \leftarrow \mathsf{KeyBitCheck}(\mathsf{CRS}, \mathsf{open}_{B,i}, i)$		
20	**if** $v_i = 0$ **then ABORT**		
21			$v_i \leftarrow \mathsf{KeyBitCheck}(\mathsf{CRS}, \mathsf{open}_{A,i}, i)$
22			**if** $v_i = 0$ **then ABORT**
end for			
Recover Signature			
23	$\sigma_{m_B} \leftarrow \mathsf{EncSigDecrypt}(\gamma_B, \theta_B)$		
24			$\sigma_{m_A} \leftarrow \mathsf{EncSigDecrypt}(\gamma_A, \theta_A)$

Partial Fairness

$$\frac{\Pr[\,\mathsf{SVf}(pk_B, m_B, \sigma_A) = \mathsf{valid}\,]}{\Pr[\,\mathsf{SVf}(pk_A, m_A, \sigma_B) = \mathsf{valid}\,]} \leq Q(\kappa)$$

Protocol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Signature
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0
1
1
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Signature

Bilinear maps

- $(p, e, G, G_T, g) \leftarrow BMGen(1^k)$

- $|G| = |G_T| = p$
- $e: G \times G \rightarrow G_T$
- $e(g^a, g^b) = e(g, g)^{ab}$
- $e(g, g)$ generates G_T

Assumptions

- Given $(g, g^s, g^{s^2}, g^{s^3}, \cdots, g^{s^q})$ it's hard to compute
  - $g^{\frac{1}{s}}$ (q- Diffie-Hellman Inversion)
  - $e(g,g,)^{\frac{1}{s}}$ (q-Bilinear Diffie-Hellman Inversion)
  - $(c, g^{\frac{1}{s+c}})$ (q-Strong Diffie-Hellman)
  - $g^{s^{q+i}}$ for $1 \leq i \leq q$ ($q + i$ Diffie-Hellman Exponent)

Assumptions

- **Proposition:** $q - BDHI \Rightarrow\ q + i\ - DHE$

- Our protocol is secure under
  - $q - SDH$
  - $q - BDHI$

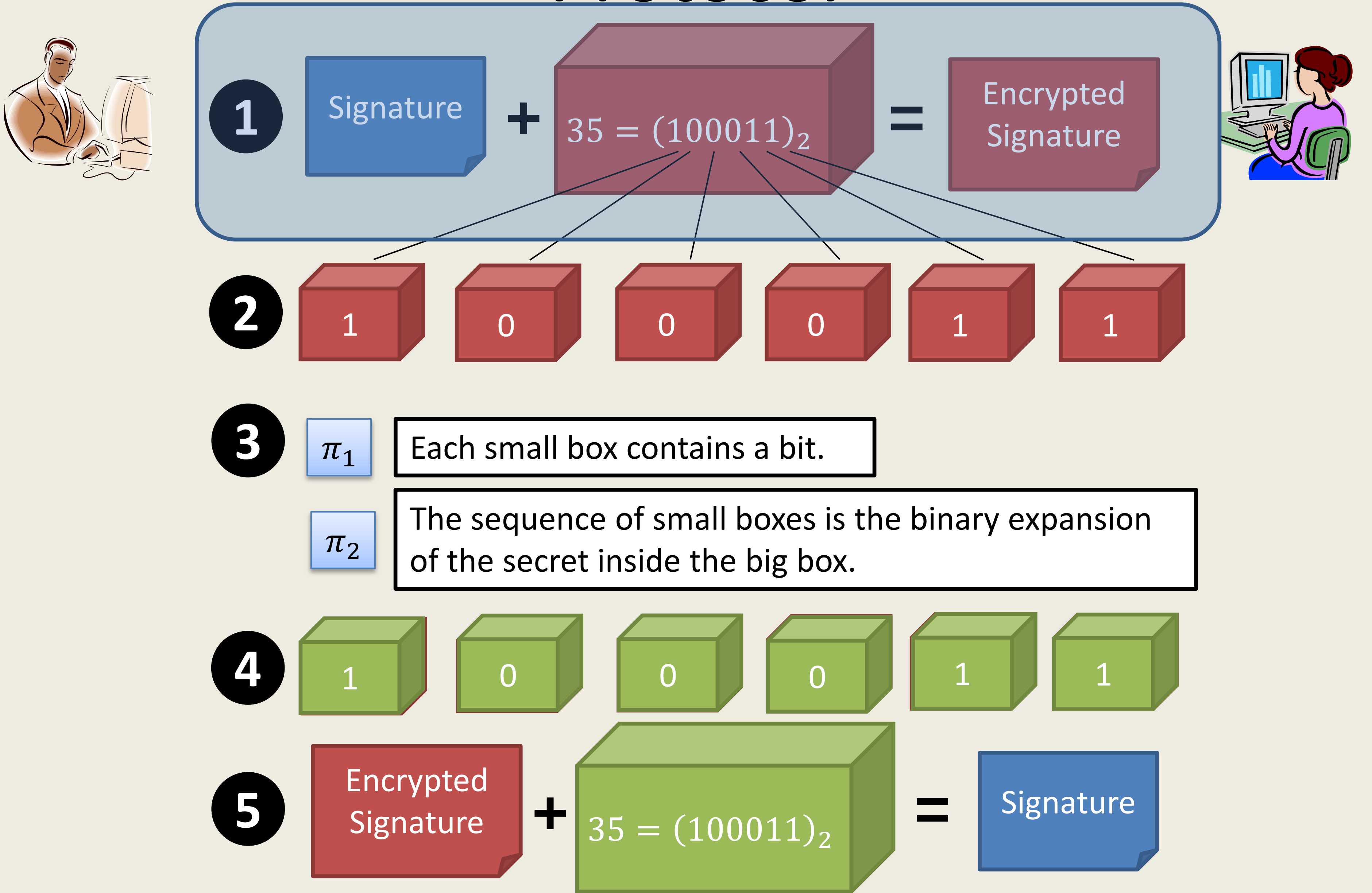
Short Signatures w/o Random Oracle [BB04]

- $\boldsymbol{KeyGen(1^k)}$
  1. $x, y \in Z_p$
  2. $u = g^x, v = g^y$
  3. $pk = (u, v), sk = (x, y)$
  4. return (sk, pk)

- $\boldsymbol{SSign(sk, m)}$
  1. $r \in Z_p$
  2. return $\sigma = (g^{\frac{1}{x+m+yr}}, r) = (\sigma_r, r)$

- $\boldsymbol{SVf(pk, m, \sigma)}$
  1. Check that $\boldsymbol{e(\sigma_r, ug^m v^r)} = e(g^{\frac{1}{x+m+yr}}, g^{x+m+yr}) = \boldsymbol{e(g, g)}$

Protocol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Signature

The Encrypted Signature

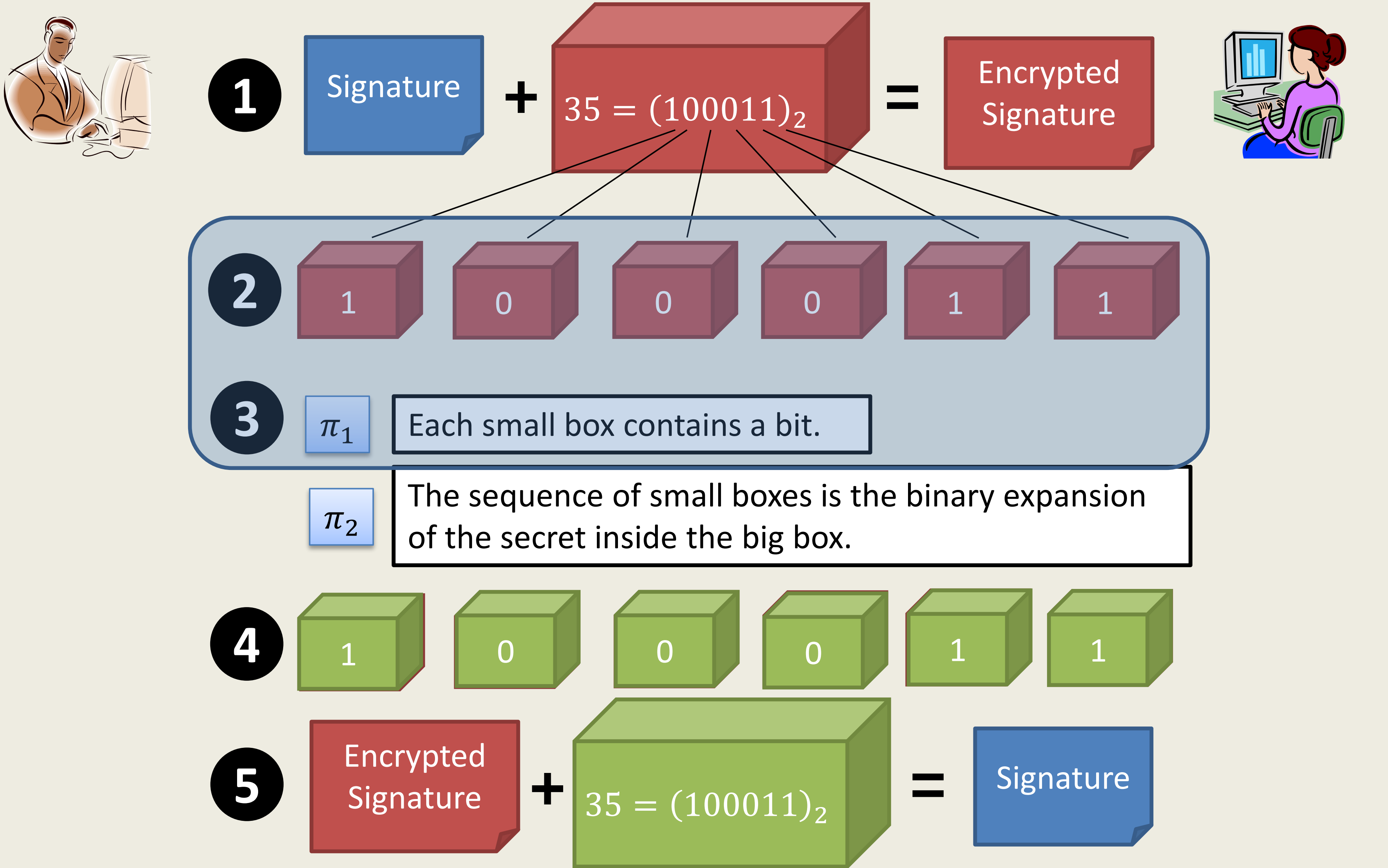
- Computing
  - $\theta \leftarrow Z_p$
  - $D = g^{\theta}$
  - $\boldsymbol{\sigma} = (\boldsymbol{g}^{\frac{\theta}{x+m+yr}}, \boldsymbol{r})$

Secret key / "blinding" factor

Boneh-Boyen signature "blinded" by θ

- Checking
  - Given (D, σ, pk, m) parse σ and pk as
    - $\sigma = (\sigma_\theta, r)$
    - $pk = (g, u = g^x, v = g^y)$
  - $\boldsymbol{e}(\boldsymbol{\sigma_\theta}, \boldsymbol{u}\boldsymbol{g}^{\boldsymbol{m}}\boldsymbol{v}^{\boldsymbol{r}}) = e(g^{\frac{\theta}{\cancel{x+m+yr}}}, g^{\cancel{x+m+yr}}) = \boldsymbol{e}(\boldsymbol{D}, \boldsymbol{g})$

Protocol



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NIZK argument 1

- $CRS = (g, g^{s}, g^{s^2}, g^{s^3}, \cdots, g^{s^q}) = (g_0, g_1, g_2, g_3, \dots, g_q)$
- **Statement**

  Let $C = (C_1, C_2, \dots, C_q)$

  The prover knows $(r_i, b_i) \in (Z_p \times \{0,1\})$ such that $\boldsymbol{C_i = g^{r_i} g_i^{b_i}}$
- **Argument**
  - $A_i = g_{q-i}^{r_i} g_q^{b_i}$
  - B_i such that $e(A_i, C_i g_i^{-1}) = e(B_i, g)$
  - Return (A_i, B_i) for each $i \in [1..q]$
- **Verification**
  - $e(A_i, g) = e(C_i, g_{q-i})$
  - $e(A_i, C_i g_i^{-1}) = e(B_i, g)$

Shift C_i by $q - i$ positions to the right.

Force the product $\boldsymbol{b_i(b_i - 1)}$ to be computed in the exponent.

NIZK argument 1

- **Theorem:**

  The argument is perfectly complete, computationally sound under the $q + i$ - DHE assumption and perfectly zero-knowledge.

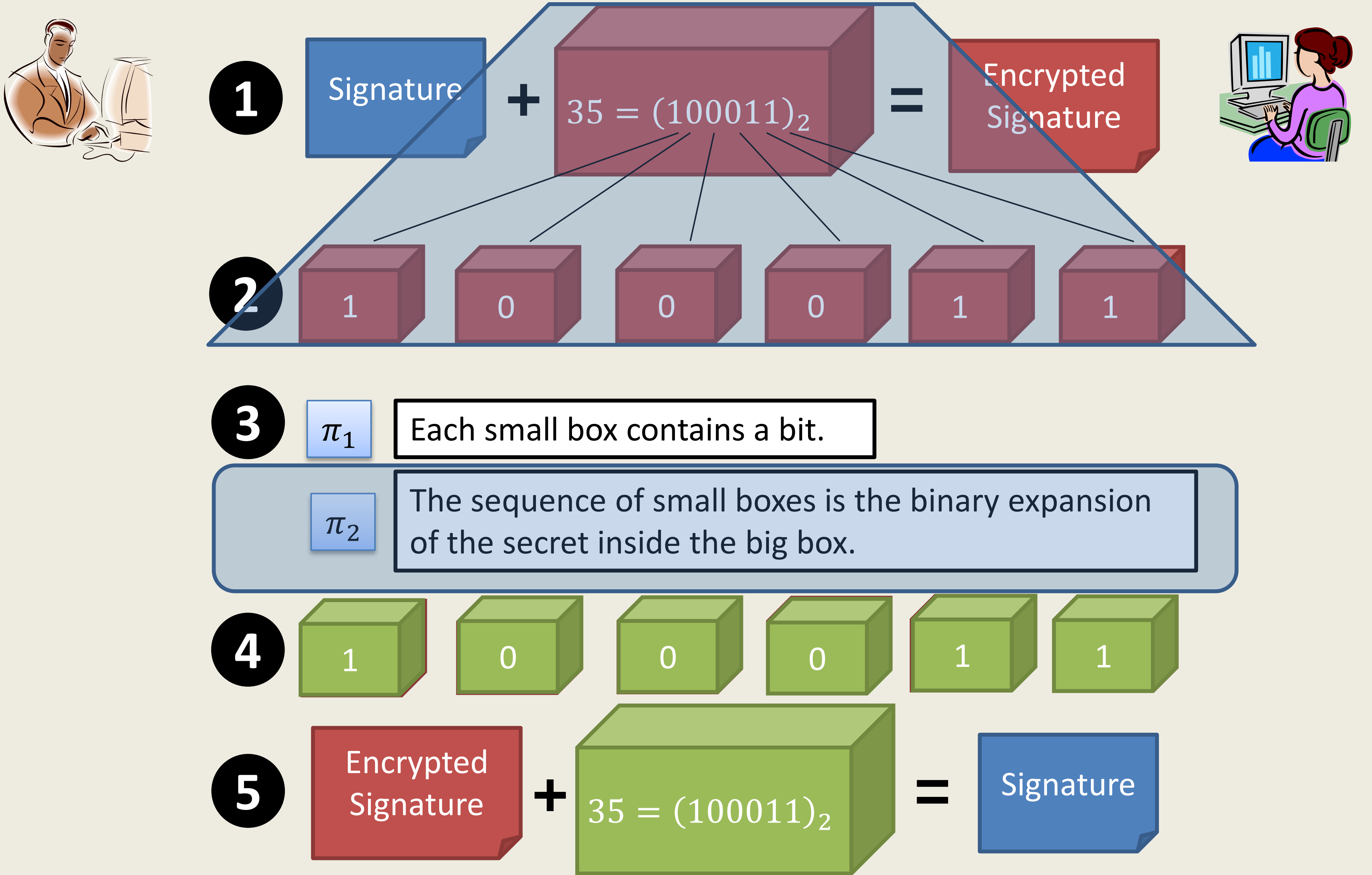
Proof (sketch).

$$e(A_i, C_i g_i^{-1}) = e(g_{q-i}^{r_i} g_q^{b_i}, g^{r_i} g_i^{b_i - 1})$$

$$= e\left(\underbrace{g_{q-i}^{r_i^2} g_q^{r_i(2b_i-1)}}_{B_i} \boldsymbol{g_{q+i}^{b_i(b_i-1)}}, g\right) = e(B_i, g)$$

If $b_i \notin \{0,1\}$, the adversary breaks the $q + i -$ DHE assumption.

Protocol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Signature

NIZK argument 2

- $CRS = \left(g, g^{s}, g^{s^2}, g^{s^3}, \cdots, g^{s^q}\right) = \left(g_0, g_1, g_2, g_3, \ldots, g_q\right)$

- We set $q = \kappa$ (security parameter)

- **Statement**
  - The prover knows $(r_i, b_i) \in \ (Z_p \times \ \{0,1\})$ and θ such that $C_i = g^{r_i} g_i^{b_i}$, $D = g^{\theta}$ and

$$\theta = \sum_{i=1}^{\kappa} b_i 2^{i-1}$$

NIZK argument 2

- **Verification:** Input (C

  - Parse $\pi = (r', U, V)$

  - Check that $e(\frac{\prod_{i=1}^{k} C_i}{g^{r'}}, g) = e(U, g_1)$

  - Check that $e(\frac{U}{D}, g) = e(V, g_1 g^{-2})$

$$\prod_{i=1}^{k} C_i = \prod_{i=1}^{k} g^{r_i} g_i^{b_i} \Leftrightarrow [r', b_1, b_2, \ldots, b_\kappa]$$

$$U = \left(\prod_{i=1}^{k} g_i^{b_i}\right)^{1/s} = \prod_{i=1}^{k} g_{i-1}^{b_i} \Leftrightarrow [b_1, b_2, \ldots, b_\kappa]$$

$$r' = \sum_i r_i$$

θ

$U \Leftrightarrow P(s)$ (i.e. $U = g^{P(s)}$)

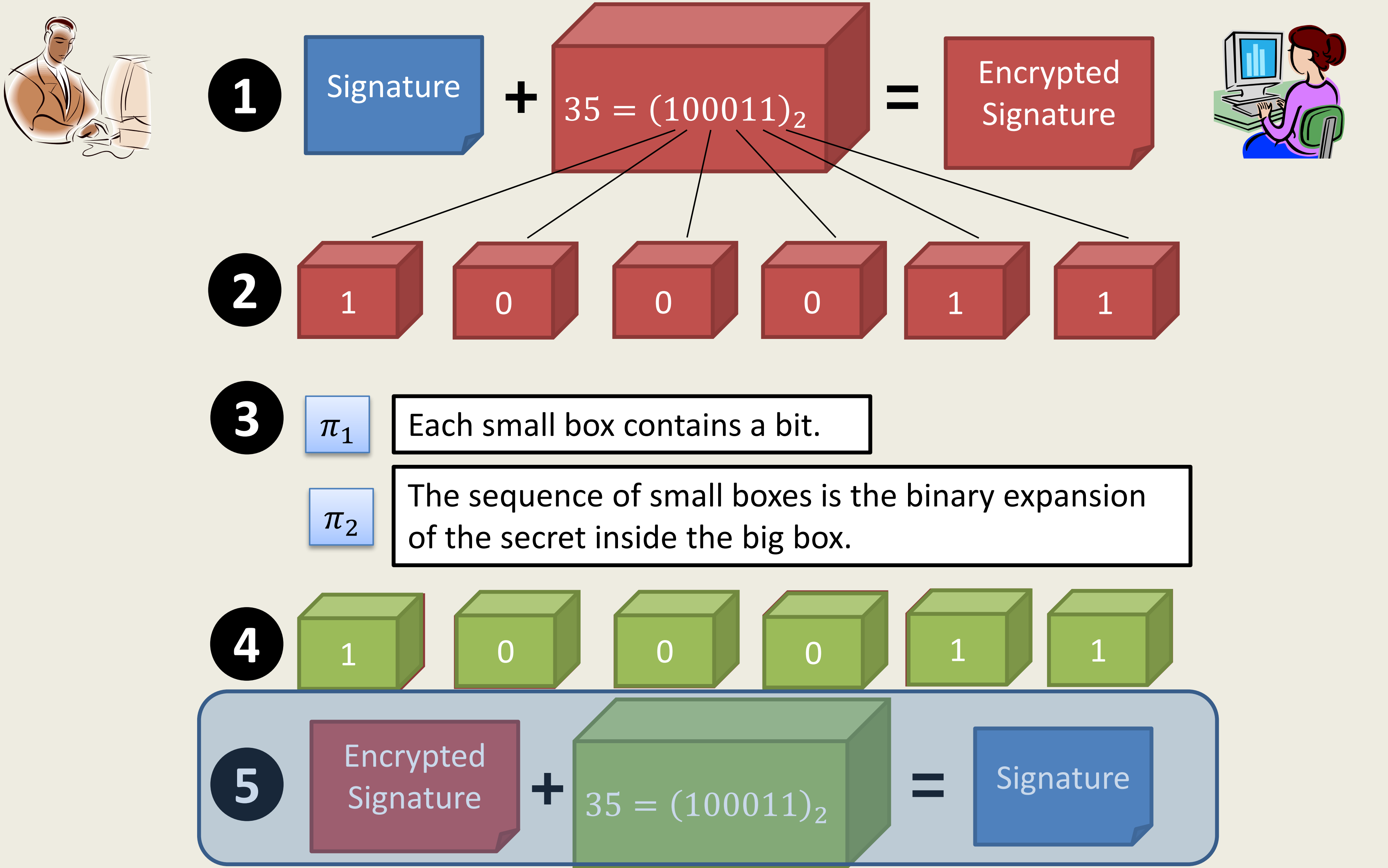
$V \Leftrightarrow W(s)$ s.t. $P(s) - P(2) = W(s)(s-2)$

NIZK argument 2

- **Theorem:**

  The argument is perfectly complete, computationally sound under the $q-SDH$ assumption and perfectly zero-knowledge.

Protocol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Signature

Recovering the Signature

- All the bits b_i are revealed
- Compute $\theta = \sum_{i=1}^{\kappa} b_i \, 2^{i-1}$
- We have $\sigma = \left(g^{\frac{\cancel{\theta}}{x+m+yr}}, r\right) = (\sigma_\theta, r)$
- Compute $\boldsymbol{\sigma} = (\sigma_\theta{}^{\mathbf{1/\theta}}, \boldsymbol{r})$

Proofs of Knowledge

- Discrete logarithm θ of
  - $D = g^{\theta}$
- r_i, b_i such that
  - $C_i = g^{r_i} g_i^{b_i}$

Needed in order to simulate the adversary despite it aborts early.

Simultaneous Hardness of Bits for Discrete Logarithm

Holds in the generic group model **[Schnorr98]**

An adversary cannot distinguish between a **random sequence** of $\boldsymbol{\kappa - l}$ bits and the **first** $\boldsymbol{\kappa - l}$ bits **of** $\boldsymbol{\theta}$ given $\boldsymbol{g^{\theta}}$.

$$Adv^{SHDL}(\mathcal{A}, \kappa) = \left|\Pr\left[\begin{array}{c}\theta \stackrel{R}{\leftarrow} \mathbb{Z}_p : \\ 1 \leftarrow \mathcal{A}(g^{\theta}, \theta[1..\kappa - l])\end{array}\right] - \Pr\left[\begin{array}{c}\theta, \alpha \stackrel{R}{\leftarrow} \mathbb{Z}_p : \\ 1 \leftarrow \mathcal{A}(g^{\theta}, \alpha[1..\kappa - l])\end{array}\right]\right|$$

$l = \omega(\log \kappa)$

Conclusion

- Fair exchange protocol for short signatures [BB04] without TTP
- Practical
- Two new NIZK arguments

Thank you!

Partial Fairness

Only contract signing

- A randomized protocol for signing contracts [EGL85]
- Gradual release of a secret [BCDB87]
- Practically and Provably secure release of a secret and exchange of signatures [Damgard95]
- Resource Fairness and Composability of Cryptographic protocols [GMPY06]

RSA, Rabin, ElGamal signatures

"Time-line" assumptions, Generic construction

- **Theorem:**

  The protocol is partially fair under the $\kappa - SDH$ and the $\kappa - BDHI$ assumption.

Proof (Sketch)

- Type I
  - Does not forge values but aborts «early»
  - => He has to break the signature scheme

  - **Careful:**
    What happens if A detects he is simulated?
    - The simulator will try to break the SHDL assumption
    - If few bits remain, it does not win, everything is OK!

Proof (Sketch)

- Type II
  - Forge values
  - The simulator can extract all values computed by adversary and break the soundness of the NIZK arguments or binding property of commitment scheme.

Fully Secure Attribute-Based Systems with Short Ciphertexts/Signatures and Threshold Access Structures

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Dengguo Feng[1] San Ling[2] Huaxiong Wang[2]

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[2]Nanyang Technological University, Singapore

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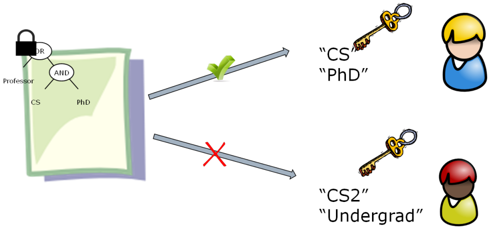
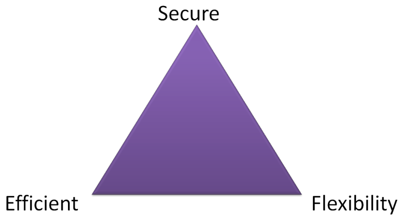
Attribute-based systems [SW05,GPSW06,MPR11]

- Policies and credentials are labeled with attributes
- Highly expressive, fine grained access policy
- Non-interactive role based access control



Performance tradeoff

- Efficiency: communication, computation costs
- Security: adaptive vs selective, CPA vs CCA
- Flexibility: expressiveness



Current status

- Most existing ABE and ABS schemes have linear-size ciphertexts and signatures.
- Some recent proposals focused on reducing the overhead, but achieved better efficiency at the expense of weaker security.
- None work achieve both adaptive security and constant-size ciphertexts and signatures for a relatively expressive access policy.

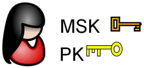
The motive of this work: full security and constant-size overhead

Offer solutions that achieve both full security and constant-size ABE ciphertexts or ABS signatures:

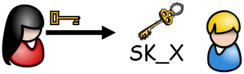
- Give formal definitions and security models for predicate encryption (PE) and predicate signatures (PS).
- Propose a generic construction of attribute-based systems supporting threshold access policies from inner-product systems.
- The resulting attribute-based constructions preserve the properties from underlying inner-product schemes.
- Present concrete constructions of fully secure ABE/ABS with constant-size ciphertexts/signatures from the IPE/IPS schemes tailored to our needs.

Background: predicate encryption (PE)

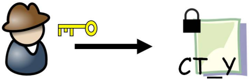
- Setup$(1^\kappa) \rightarrow (PP, Msk)$



- KeyGen$(PP, Msk, X) \rightarrow sk_X$



- Enc$(PP, Y, Msg) \rightarrow CT_Y$



- Dec$(PP, sk_X, CT) \rightarrow Msg'$

$$Dec(PP, sk_X, Enc(PP, Y, Msg)) = Msg \quad \Longleftrightarrow \quad R(X, Y) = 1$$

Security: ciphertext indistinguishability

Experiment $Exp^{ind}_{\mathcal{PE}}(\kappa)$:

$$\boxed{Y \longleftarrow \mathcal{A}}$$

$$b \xleftarrow{R} \{0,1\}$$

$$\mathsf{PP}, \mathsf{MSK} \xleftarrow{R} \mathbf{Setup}$$

$$(Msg_0, Msg_1, Y) \xleftarrow{R} \mathcal{A}^{\mathbf{KeyGen}(\cdot)}(\mathsf{PP})$$

$$\mathsf{CT} \xleftarrow{R} \mathbf{Enc}(\mathsf{PP}, Y, Msg_b)$$

$$b' \longleftarrow \mathcal{A}^{\mathbf{KeyGen}(\cdot)}(\mathsf{PP}, \mathsf{CT})$$

If $b = b'$ and $R(X, Y) \neq 1$ return 1 else return 0

Variants of PE

There exist many public key primitives that can be viewed as special cases of PE:

- ABE: ciphertext-policy (CP) & key-policy (KP)

$$X :\longrightarrow S \subseteq \{att_1, \ldots, att_n\}, \quad Y :\longrightarrow \phi,\ \phi \textit{ is an access structure}$$

$$R(X,Y) = \begin{cases} 1 & \textit{if} \quad S \in \phi \\ 0 & \textit{if} \quad S \notin \phi \end{cases}$$

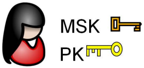
- Inner-product encryption (IPE):

$$X :\longrightarrow \vec{v} \in \mathbb{Z}_p^n, \quad Y :\longrightarrow \vec{x} \in \mathbb{Z}_p^n$$

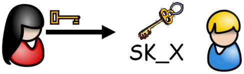
$$R(X,Y) = \begin{cases} 1 & \textit{if} \quad \langle \vec{v}, \vec{x} \rangle = 0 \\ 0 & \textit{if} \quad \langle \vec{v}, \vec{x} \rangle \neq 0 \end{cases}$$

Predicate signature (PS)

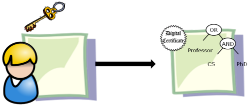
- Setup$(1^\kappa) \to (PP, Msk)$



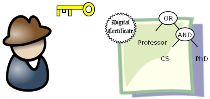
- KeyGen$(PP, Msk, X) \to sk_X$



- Sign$(PP, Y, sk_X, Msg) \to \sigma$



- Verify$(PP, \sigma, Y) \to \{0, 1\}$



$$Verify(PP, Sign(PP, KeyGen(PP, Msk, X), Msg), Y) = 1 \iff R(X, Y) = 1$$

Security: unforgeability

Experiment $Exp^{unf}_{\mathcal{PS}}(\kappa)$:

$Y \longleftarrow \mathcal{A}$

$\mathsf{PP}, \mathsf{MSK} \xleftarrow{R} \textbf{Setup}$

$(Msg, Y, \sigma) \xleftarrow{R} \mathcal{A}^{\textbf{KeyGen}(\cdot),\textbf{Sign}(\cdot)}(\mathsf{PP})$

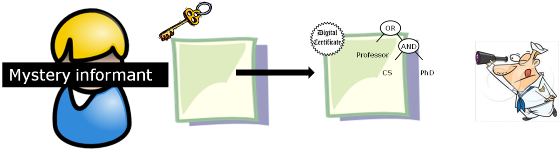
If $\textbf{Verify}(PP, \sigma, Y) = 1, R(X, Y) \neq 1$

and (Msg, Y) has not been made as

signature queries return 1 else return 0

Security: perfect privacy

A predicate signature ensures the verifier only knows that the signer's role can satisfy the specified signing policy.



For any Msg, X_1, X_2 and Y such that $R(X_1, Y) = R(X_2, Y) = 1$, we have

$$\textbf{Sign}(\mathsf{PP}, \textbf{KeyGen}(\mathsf{PP}, \mathsf{MSK}, X_1), Y, Msg) \equiv \textbf{Sign}(\mathsf{PP}, \textbf{KeyGen}(\mathsf{PP}, \mathsf{MSK}, X_2), Y, Msg)$$

Variants of PS

There exist many signature primitives that can be viewed as special cases of PS:

- ABS:

$$X :\longrightarrow S \subseteq \{att_1, \ldots, att_n\}, \quad Y :\longrightarrow \phi,\ \phi \textit{ is an access structure}$$

$$R(X,Y) = \begin{cases} 1 & \textit{if} \quad S \in \phi \\ 0 & \textit{if} \quad S \notin \phi \end{cases}$$

- Inner-product signature (IPS):

$$X :\longrightarrow \vec{v} \in \mathbb{Z}_p^n, \quad Y :\longrightarrow \vec{x} \in \mathbb{Z}_p^n$$

$$R(X,Y) = \begin{cases} 1 & \textit{if} \quad \langle \vec{v}, \vec{x} \rangle = 0 \\ 0 & \textit{if} \quad \langle \vec{v}, \vec{x} \rangle \neq 0 \end{cases}$$

Intuitions of generic constructions: exact threshold policy [KSW08]

Express an attribute subset S as a vector $\vec{x}_S$:

$$\vec{x}_S := (\overbrace{b_1}^{att_1}, \ldots, \overbrace{b_i}^{att_i}, \ldots), \quad for \quad i = 1, 2, \ldots \quad b_i = \begin{cases} 1 & if \quad att_i \in S \\ 0 & if \quad att_i \notin S \end{cases}$$

If S_1 and S_2 have t attributes overlap, we have

$$\langle \vec{x}_{S_1}, \vec{x}_{S_2} \rangle = t$$

Exact threshold policy from inner-product policy

- **Setup**(κ, U): $\mathsf{IPE.Setup}(\kappa, n+1) \rightarrow (\mathsf{PP}, \mathsf{MSK})$;
- **Enc**$(\mathsf{PP}, \Gamma := (\Omega, t), Msg)$: $\mathsf{IPE.Enc}(\mathsf{PP}, (t, \vec{x}_\Omega), M) \rightarrow \mathsf{CT}_\Gamma$;
- **KeyGen**$(\mathsf{PP}, \mathsf{MSK}, S)$: $\mathsf{IPE.KeyGen}(\mathsf{PP}, \mathsf{MSK}, (-1, \vec{x}_S)) \rightarrow \mathsf{SK}_S$;
- **Dec**$(\mathsf{PP}, \mathsf{CT}_\Gamma, \mathsf{SK}_S)$: $\mathsf{IPE.Dec}(\mathsf{PP}, \mathsf{CT}_\Gamma, \mathsf{SK}_S) \rightarrow Msg$.

Correctness. $\langle(-1, \vec{x}_S), (t, \vec{x}_\Omega)\rangle = 0$ if $|\Omega \cap S| = t$.

Exact threshold to threshold: IPE to tKP-ABE

Introduce multiple IPE secret keys to achieve flexibility:

$$\mathsf{tKP.KeyGen}(\mathsf{PP}, \Gamma := (\Omega, t), \mathsf{MSK}):$$

$$\mathsf{IPE.KeyGen}(\mathsf{PP}, (t, \vec{x}_\Omega), \mathsf{MSK}) \to \mathsf{IPE.SK}_1$$

$$\mathsf{IPE.KeyGen}(\mathsf{PP}, (t+1, \vec{x}_\Omega), \mathsf{MSK}) \to \mathsf{IPE.SK}_2$$

$$\mathsf{IPE.KeyGen}(\mathsf{PP}, (t+2, \vec{x}_\Omega), \mathsf{MSK}) \to \mathsf{IPE.SK}_3$$

$$\vdots$$

$$\mathsf{KP.SK}_{(\Omega,t)} := \{\mathsf{IPE.SK}_j\}_{1 \le j \le m-t+1}$$

$$\mathsf{tKP.Enc}(\mathsf{PP}, S, Msg):$$

$$\mathsf{IPE.Enc}(\mathsf{PP}, (-1, \vec{x}_S), Msg) \to \mathsf{CT}$$

Exact threshold to threshold: IPE to tCP-ABE

$\text{tCP.KeyGen}(\text{PP}, S, \text{MSK}):$

$\text{IPE.KeyGen}(\text{PP}, (1, \vec{x}_S, 0), \text{MSK}) \rightarrow \text{IPE.SK}_1$

$\text{IPE.KeyGen}(\text{PP}, (1, \vec{x}_S, -1), \text{MSK}) \rightarrow \text{IPE.SK}_2$

$\text{IPE.KeyGen}(\text{PP}, (1, \vec{x}_S, -2), \text{MSK}) \rightarrow \text{IPE.SK}_3$

$\vdots$

$\text{CP.SK}_S := \{\text{IPE.SK}_i\}_{1 \leq i \leq |S|-1}$

$\text{tCP.Enc}(\text{PP}, \Gamma := (\Omega, t), Msg):$

$\text{IPE.Enc}(\text{PP}, (-t, \vec{x}_\Omega, 1), Msg) \rightarrow \text{CT}$

Exact threshold to threshold: IPS to tABS

$$\mathsf{tABS.KeyGen}(\mathsf{PP}, S, \mathsf{MSK}):$$

$$\mathsf{IPS.KeyGen}(\mathsf{PP}, (1, \vec{x}_S, 0), \mathsf{MSK}) \to \mathsf{IPS.SK}_1$$

$$\mathsf{IPS.KeyGen}(\mathsf{PP}, (1, \vec{x}_S, -1), \mathsf{MSK}) \to \mathsf{IPS.SK}_2$$

$$\mathsf{IPS.KeyGen}(\mathsf{PP}, (1, \vec{x}_S, -2), \mathsf{MSK}) \to \mathsf{IPS.SK}_3$$

$$\vdots$$

$$\mathsf{ABS.SK}_S := \{\mathsf{IPS.SK}_i\}_{1 \le i \le |S|-1}$$

$$\mathsf{tABS.Sign}(\mathsf{PP}, \mathsf{ABS.SK}_S, \Gamma := (\Omega, t), Msg):$$

$$\mathsf{IPS.Sign}(\mathsf{PP}, \mathsf{IPS.SK}_{k-t+1}, (-t, \vec{x}_\Omega, 1), Msg) \to \sigma$$

where $\mathsf{IPS.SK}_{k-t+1} \leftarrow \mathsf{IPS.KeyGen}(\mathsf{PP}, (-t, \vec{x}_S, t-k), \mathsf{MSK})$

$k := |S \cap \Omega| \ge t$

Concrete constructions of tABE and tABS

Basing the transformation from inner-product systems to attribute-based systems supporting threshold access structures:

- Properties-preserving:
  - full security/selective security
  - constant-size ciphertext/signature
  - perfect privacy
- Building blocks of IPE/IPS schemes tailored to our needs:
  - IPE: [AL10], but too complicated.
  - IPS: non-existent.

The properties of underlying IPE & IPS

scheme	group order	based on	size of CT or signature
[AL10]	prime	none	constant
Our IPE	composite	[AL10]	constant
Our IPS1	composite	our IPE	constant
Our IPS2	prime	our IPE & DPVS	constant

Our IPE: fully secure IPE with constant-size ciphertexts in composite order group

- $\mathsf{IPE.Setup}(\lambda, n) \to (\mathsf{PP}, \mathsf{MSK})$
  $\mathsf{PP} := \left(\mathcal{I} := (N = p_1 p_2 p_3, G, G_T, e), g, \vec{h} := (h_0, \ldots, h_n), e(g,g)^\alpha\right)$
  $\mathsf{MSK} := (\alpha, \boxed{X_3})$.
- $\mathsf{IPE.KeyGen}(\mathsf{PP}, \mathsf{MSK}, \vec{v}) \to \mathsf{IPE.SK}_{\vec{v}} := (K_0, K_1, \ldots, K_n)$

$$K_0 := g^r \cdot \boxed{R_0}, \quad K_1 := g^\alpha h_0^r \cdot \boxed{R_1}, \quad \left\{ K_i := \left(h_1^{-\frac{v_i}{v_1}} h_i \right)^r \cdot \boxed{R_i} \right\}_{i=2,\ldots,n}.$$

- $\mathsf{IPE.Enc}(\mathsf{PP}, \vec{x}, Msg) \to \mathsf{CT} := (C, C_0, C_1)$

$$C := Msg \cdot e(g,g)^{\alpha s}, \quad C_0 := g^s, \quad C_1 := \left(h_0 \prod_{j=1}^{n} h_j^{x_j} \right)^s.$$

- $\mathsf{IPE.Dec}(\mathsf{PP}, \vec{x}, \mathsf{IPE.SK}_{\vec{v}}, \mathsf{CT})$: The algorithm computes

$$Msg' = C \cdot \frac{e(C_1, K_0)}{e(C_0, K_1 \prod_{j=2}^{n} K_j^{x_j})}$$

The security of our IPE & IPS

- Dual system proof [Wat09] is applied to obtain full security.
- Some composite order complexity assumptions are introduced.
- Our IPS scheme is prefectly private because the distribution of the signature is the same.

Comparisons

	scheme	security	size of SK	size of CT or Sig	expressiveness	Pai
CP-ABE	[EM+09]	selective	$\mathcal{O}(n)$	$\mathcal{O}(1)$	(n,n)-threshold	2
	[CZF11]	selective	$\mathcal{O}(n)$	$\mathcal{O}(1)$	and-gate	2
	[HLR10]	selective	$\mathcal{O}(n)$	$\mathcal{O}(1)$	threshold	3
	[GZC11]	selective	$\mathcal{O}(n)^2$	$\mathcal{O}(1)$	threshold	3
	[OT10]	full	$\mathcal{O}(n)$	$\mathcal{O}(n)$	general	$\mathcal{O}(n)$
	Our CP-ABE	full	$\mathcal{O}(n)^2$	$\mathcal{O}(1)$	threshold	2
KP-ABE	[ABP11]	selective	$\mathcal{O}(n)^2$	$\mathcal{O}(1)$	general	3
	[OT10]	full	$\mathcal{O}(n)$	$\mathcal{O}(n)$	general	$\mathcal{O}(n)$
	Our KP-ABE	full	$\mathcal{O}(n)^2$	$\mathcal{O}(1)$	threshold	2
ABS	[HLLR12a]	selective	$\mathcal{O}(n)$	$\mathcal{O}(1)$	threshold	12
	[HLLR12b]	selective	$\mathcal{O}(n)^2$	$\mathcal{O}(1)$	threshold	3
	[OT11]	full	$\mathcal{O}(n)$	$\mathcal{O}(n)$	general	$\mathcal{O}(n)$
	Our ABS	full	$\mathcal{O}(n)^2$	$\mathcal{O}(1)$	threshold	3

Conclusion

- We define the syntax and security notions of PE/PS.
- We bridge a connection between inner-product systems and attribute-based systems.
- Our tABE/tABS schemes achieve both full security and short ciphertexts/signatures.