The *k*-BDH Assumption Family: Bilinear Cryptography from Progressively Weaker Assumptions

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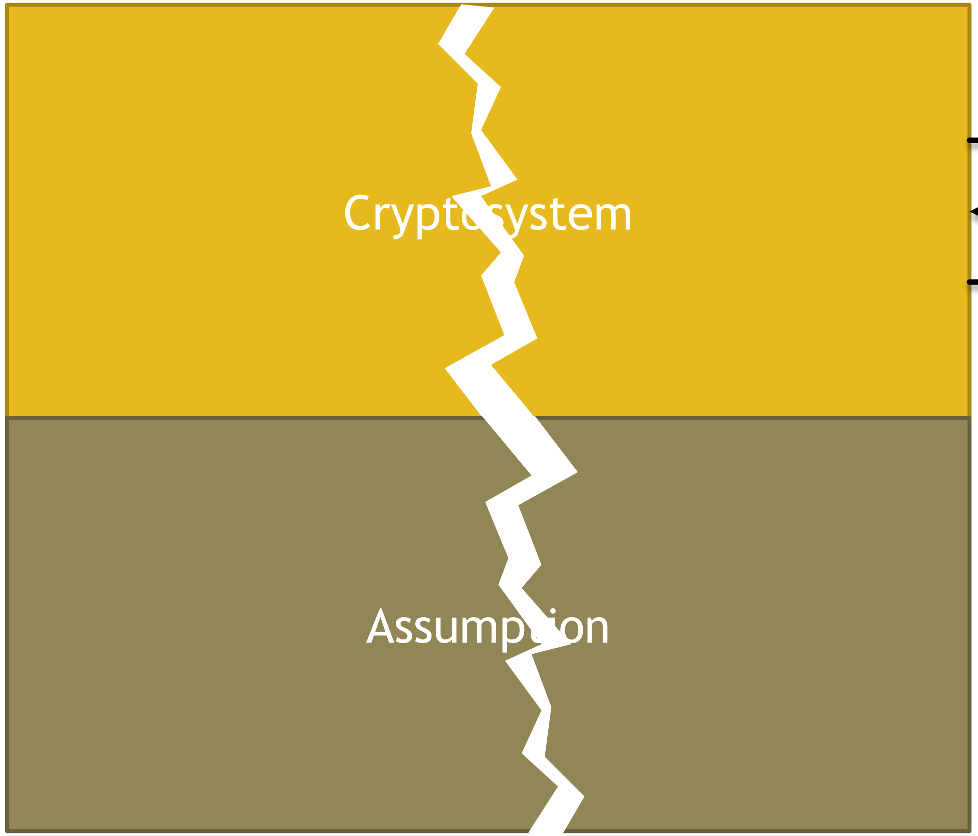
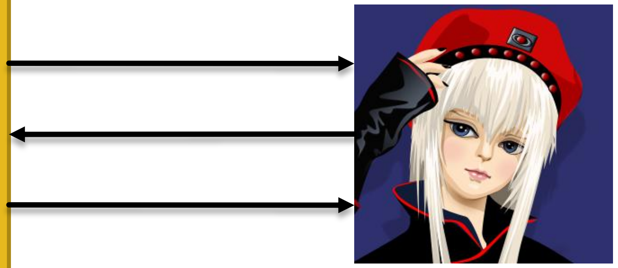
Provable Security

How to show your cryptosystem is secure:

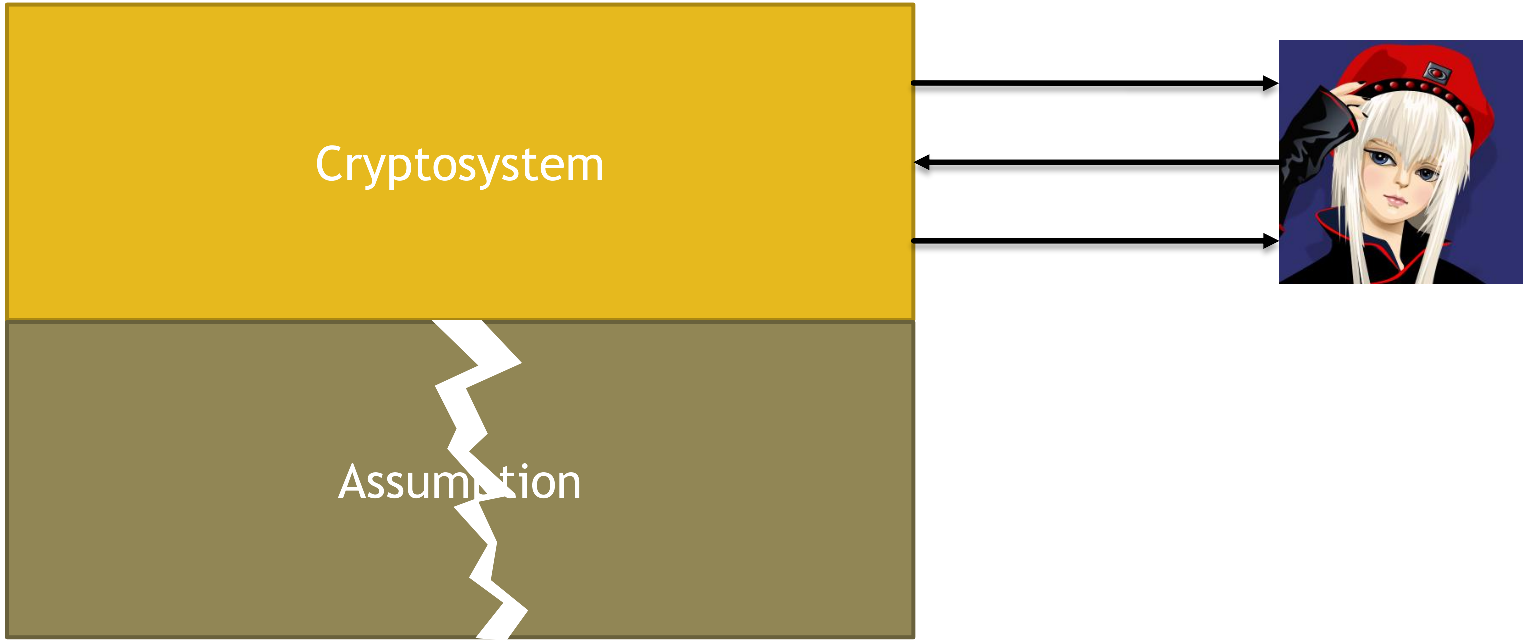




If **Assumption Holds** then **Cryptosystem Secure**

Prove contrapositive

What if the Assumption is False?

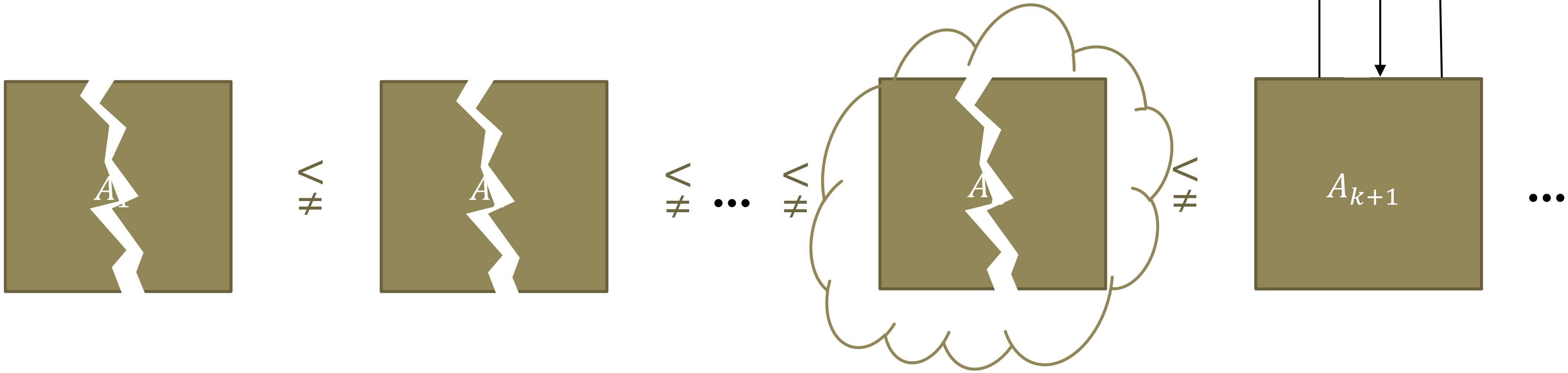


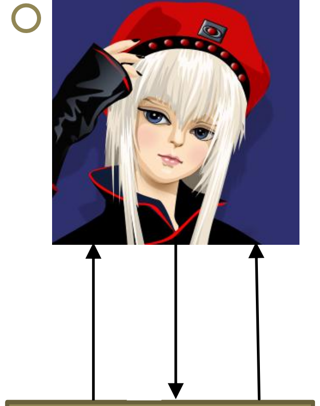
- Cannot reason about security
- Adversary can use the attack on assumption to break cryptosystem

How to Pick a Good Assumption?

- Increase the size of parameters
  - e.g., RSA assumes factoring a large number into 2 primes is hard
  - Factor: 77, 3869, 702619, …
- Use a family of assumptions
  - As you increase a parameter *k* you become more confident in the security of the assumption
  - Example: *k*-Linear [HK07, Sha07]

Family of Strictly Weaker Assumptions





- An assumption A_{k+1} is **weaker** than assumption A_k, if
  - If A_k holds then so does A_{k+1} (Breaking A_{k+1} also breaks A_k)
- The assumption, A_{k+1}, is **strictly weaker** than assumption, A_k, if
  - A_{k+1} is weaker than A_k
  - And an oracle for A_k does not help break A_{k+1}

DDH Assumption

- Given $< g, g^a, g^b, T >$
  - For some $g, g^a, g^b, T \in G$
- Does $T \stackrel{?}{=} g^{ab}$

> It is hard to compute a discrete log in a finite cyclic group G

- No polynomial time algorithm can achieve non-negligible advantage deciding

Bilinear Maps

- $\boldsymbol{e}$: $G \times G \rightarrow G_T$
  - Bilinear: $\boldsymbol{e}(g^a, g^b) = \boldsymbol{e}(g,g)^{ab}$ for all $a, b \in Z_p$
  - Non-Degenerate: If g generates G, then $\boldsymbol{e}(g,g) \neq 1$
  - Computable: $\boldsymbol{e}$ is efficiently computable on all input
- DDH does not hold for groups in which bilinear maps can be computed
  - $< g, g^a, g^b, T \stackrel{?}{=} g^{ab} >$
  - $\boldsymbol{e}(g^a, g^b) \stackrel{?}{=} \boldsymbol{e}(g, T)$

DBDH Assumption

- How can we use DDH in bilinear groups?
- Given $< g, g^a, g^b, g^c, T >$
  - For some $g, g^a, g^b, g^c \in G$ and $T \in G_T$
- Does $T \stackrel{?}{=} \boldsymbol{e}(g, g)^{abc}$

- Hard to compute discrete log: in G and G_T
- Bilinear maps have 2 inputs
- Can't undo a bilinear map

- No polynomial time algorithm can achieve non-negligible advantage deciding

Decision Linear (DLIN) Assumption

- How can we use DDH in settings where bilinear maps exist?
- Given $< g, g^{s_1}, g^{s_2}, g^{s_1 r_1}, g^{s_2 r_2}, T >$
  - $g, g^{s_1}, g^{s_2}, g^{s_1 r_1}, g^{s_2 r_2}, T \in G$
- Does $T \stackrel{?}{=} \boxed{g^{r_1}}\boxed{{}^{+r_2}}$

This is like 2 DDH problems:
$g, g^{s_1}, \boxed{g^{r_1}}, g^{s_1 r_1}$

- It is hard to compute discrete logs
- Bilinear maps only pair 2 elements (not 3: $g^{s_1}, g^{s_2}, g^{r_1+r_2}$)

- No polynomial time algorithm can achieve non-negligible advantage deciding, even in generic bilinear groups [BBS04]
- Only a decisional problem – computationally same as DDH

k-Linear Family of Assumptions

This is like k DDH problems

- k-Linear generalizes the Linear Assumption
  - 1-Linear is DDH
  - 2-Linear is Linear Assumption
- For $k \geq 1$ Given $< g, g^{s_1}, \ldots, g^{s_k}, g^{s_1 r_1}, \ldots, g^{s_k r_k}, T >$
  - $g, g^{s_1}, \ldots, g^{s_k}, g^{s_1 r_1}, \ldots, g^{s_k r_k}, T \in G$
- Does $T \stackrel{?}{=} g^{r_1 + \cdots + r_k}$
- No polynomial time algorithm can achieve non-negligible advantage deciding
- Only a decisional problem – computationally same as DDH

How are DLIN and DBDH Related?

- If DLIN holds, then so does DBDH

$$< g, g^{s_1}, g^{s_2}, g^{s_1 r_1}, g^{s_2 r_2}, T \stackrel{?}{=} g^{r_1+r_2} >$$

DLIN Instance

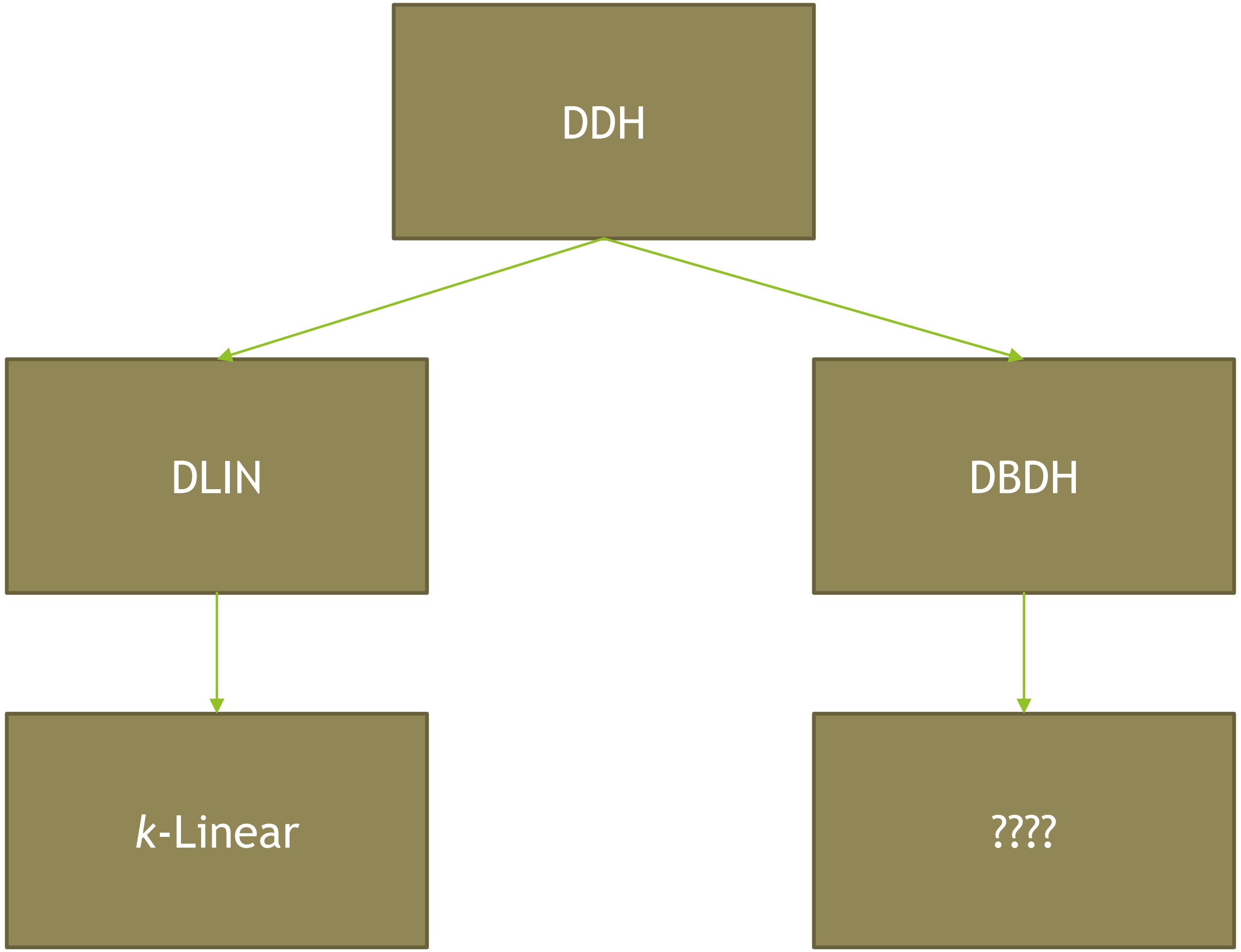
$$< g, g^{s_1}, g^{s_2}, T \stackrel{?}{=} g^{r_1+r_2}, \boldsymbol{e}(g^{s_1 r_1}, g^{s_2}) \cdot \boldsymbol{e}(g^{s_2 r_2}, g^{s_1}) > =$$

$$< g, g^{s_1}, g^{s_2}, T \stackrel{?}{=} g^{r_1+r_2}, \boldsymbol{e}(g, g)^{(s_1 s_2)(r_1+r_2)} >$$

DBDH Decider

[BW06]

Can extend DBDH to a Family of Assumptions?



- Why?
  - For $k > 2$ unclear how *k*-Linear and DBDH are related
  - *k*-Linear only operates in the source group
  - Example: Boneh-Boyen IBE – the message is hidden in target group

This is should be like *k* DBDH problems

Failed Attempt

- Given $g, g^a, g^b, g^{s_1}, \ldots, g^{s_k}, g^{s_1 r_1}, \ldots, g^{s_k r_k} \epsilon$ and $T \epsilon G_T$
- Does $T \stackrel{?}{=} \prod_i \boldsymbol{e}(g, g^{s_i})^{abr_i} = \prod_i \boldsymbol{e}(g, g\ )^{abs_i r_i} = \boldsymbol{e}(g, g)^{ab(s_1 r_1 + \cdots + s_k r_k)}$
- Embeds k DBDH instances: $(g, g^a, g^b, g^{s_i r_i}, \boldsymbol{e}(g, g)^{ab(s_i r_i)})$
- … But is equivalent to DBDH:
  $(g, g^a, g^b, \prod_i g^{s_i r_i} = g^{(s_1 r_1 + \cdots + s_k r_k)}, T \stackrel{?}{=} \prod_i \boldsymbol{e}(g, g\ )^{abs_i r_i})$

k-BDH Assumption

- Given $g, g^a, g^b, g^{s_1}, \dots, g^{s_k}, g^{s_1 r_1}, \dots, g^{s_k r_k} \epsilon\ G$ and $T \epsilon\ G_T$
- Does $T \stackrel{?}{=} \prod_i \boldsymbol{e}(g,g)^{abr_i} = \prod_i \boldsymbol{e}(g^{s_i}, g^{s_i})^{(a/s_i)(b/s_i)r_i} = \boldsymbol{e}(g,g)^{ab(r_1+\cdots+r_k)}$
- Embeds k DBDH instances: $(g^{s_i}, g^a, g^b, g^{s_i r_i}, \boldsymbol{e}(g^{s_i}, g^{s_i})^{(a/s_i)(b/s_i)r_i})$
- … And is a family of strictly weaker assumptions!

A Family of Weaker Assumptions

- If the *k*-BDH assumption holds, so does the *(k+1)*-BDH assumption

$$g, g^x, g^y, v_1, \dots, v_k, v_1^{r_1}, \dots, v_k^{r_k}, T \stackrel{?}{=} \prod_{1\le i\le k} \boldsymbol{e}(g,g)^{xyr_i}$$

k-BDH Instance

$$v_{k+1}, r_{k+1}$$

Random Values

$$< g, g^x, g^y, v_1, \dots, v_{k+1}, v_1^{r_1}, \dots, v_{k+1}^{r_{k+1}},$$

$$T \cdot \boldsymbol{e}(g^x, g^y)^{r_{k+1}} \stackrel{?}{=} \prod_{1\le i\le k+1} \boldsymbol{e}(g,g)^{xyr_i} >$$

(k+1)-BDH Decider

Evidence of a Family of Strictly Weaker Assumptions

- An oracle for k-BDH does not help in deciding a (k+1)-BDH instance
- Similar to the separation proof of k-Linear [Sha07]
- Generic Group Model [BS84,Nechaev94,Shoup97]
  - Interact with adversary using an idealized version of groups
  - Bound the probability of finding an inconsistency if actual groups were used

- Oracle to k-BDH is implemented as a modified k-multilinear map
  - maps k elements in G and one element G_T to an element group G_M

Application: IBE

- Fits in the Boneh-Boyen Framework
- Base switching techniques needed

- Setup:
  - Public parameters: $g, u = g^x, v_1 = g^{s_1}, \ldots, v_k = g^{s_k}, v_1^{\hat{r}_1}, \ldots, v_k^{\hat{r}_k}, w_1, \ldots, w_k$
  - Master key: $s_1, \ldots, s_k, \hat{r}_1, \ldots, \hat{r}_k, x$
- KeyGen(ID):
  - Select random $n_1, \ldots, n_k \in Z_p^*$
  - For each $1 \le i \le k$ output $\left(K_{A,i}, K_{B,i}\right) = (g^{x\hat{r}_i}\left(w_i u^{\mathrm{ID}}\right)^{n_i}, v_i^{n_i})$
- Encrypt (m, ID):
  - Select random $y_1, \ldots, y_k \in Z_p^*$
  - Output $C_0 = m \prod_{1 \le i \le k} \boldsymbol{e}\left(g^x, v_i{}^{\hat{r}_i}\right)^{y_i}$
  - For each $1 \le i \le k$ output $\left(C_{A,i}, C_{B,i}\right) = (v_i^{y_i}, \left(w_i u^{\mathrm{ID}}\right)^{y_i})$
- Decrypt(c):
  - $\frac{C_0 \cdot \prod_{1 \le i \le k} \boldsymbol{e}(K_{B,i}, C_{B,i})}{\prod_{1 \le i \le k} \boldsymbol{e}(K_{A,i}, C_{A,i})} = \frac{m \prod_{1 \le i \le k} \boldsymbol{e}\left(g^x, v_i{}^{\hat{r}_i}\right)^{y_i} \cdot \prod_{1 \le i \le k} \boldsymbol{e}(v_i^{n_i}, \left(w_i u^{\mathrm{ID}}\right)^{y_i})}{\prod_{1 \le i \le k} \boldsymbol{e}(g^{x\hat{r}_i}\left(w_i u^{\mathrm{ID}}\right)^{n_i}, v_i^{y_i})} = m$

Conclusions

- Goal: Introduction the *k*-BDH Family of Assumptions
  - Relationship to standard assumptions (DDH, *k*-Linear, DBDH)
- It is a family of strictly weaker assumptions
- Usable: We construct an IBE in the Boneh-Boyen Framework

- Future Work
  - IBE construction grows with *k* (public parameters, keys, encryption)
  - Different applications
- http://eprint.iacr.org/2012/687

Security in knowledge

Efficient Delegation of Key Generation and Revocation Functionalities in Identity-Based Encryption

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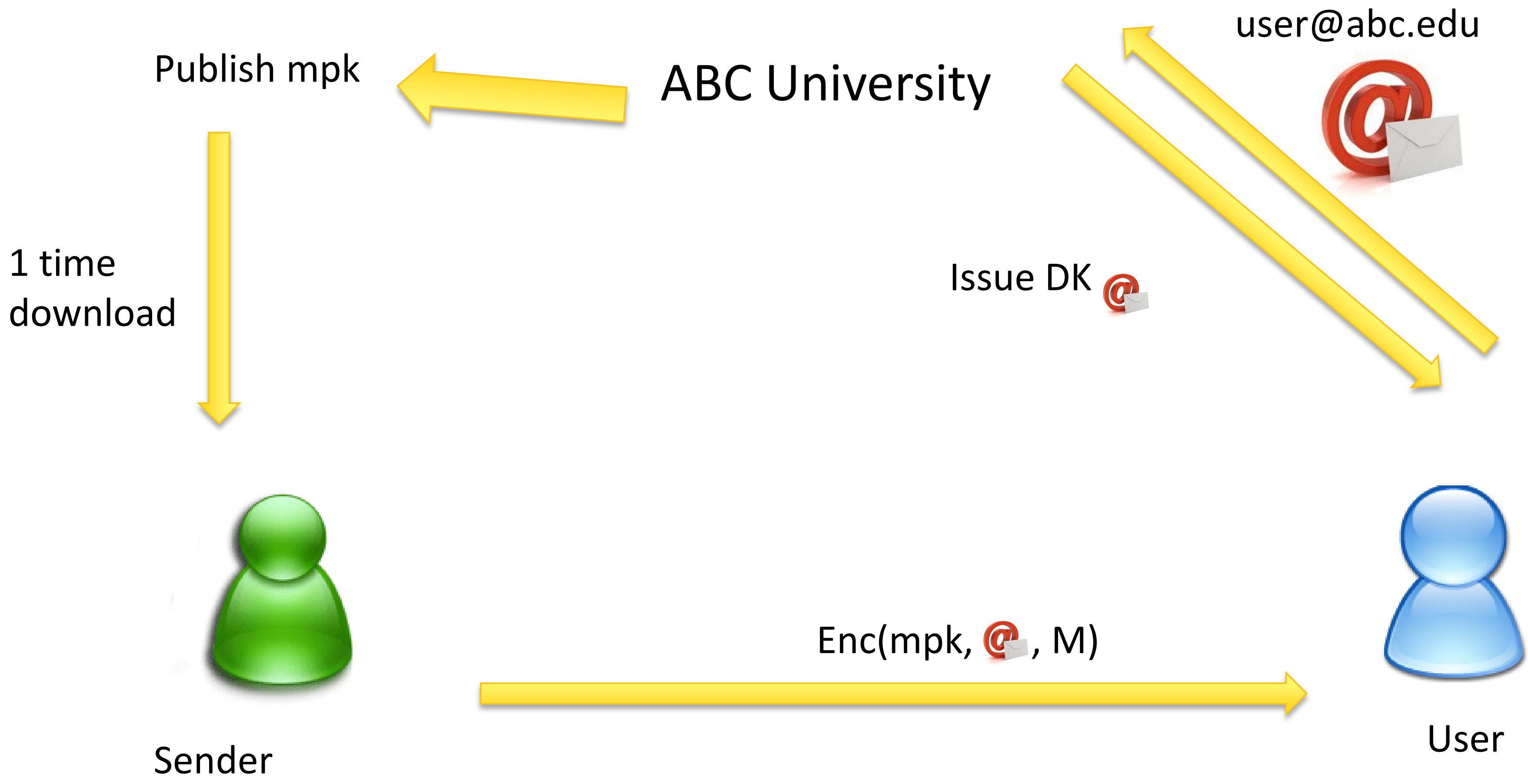
RSACONFERENCE2013

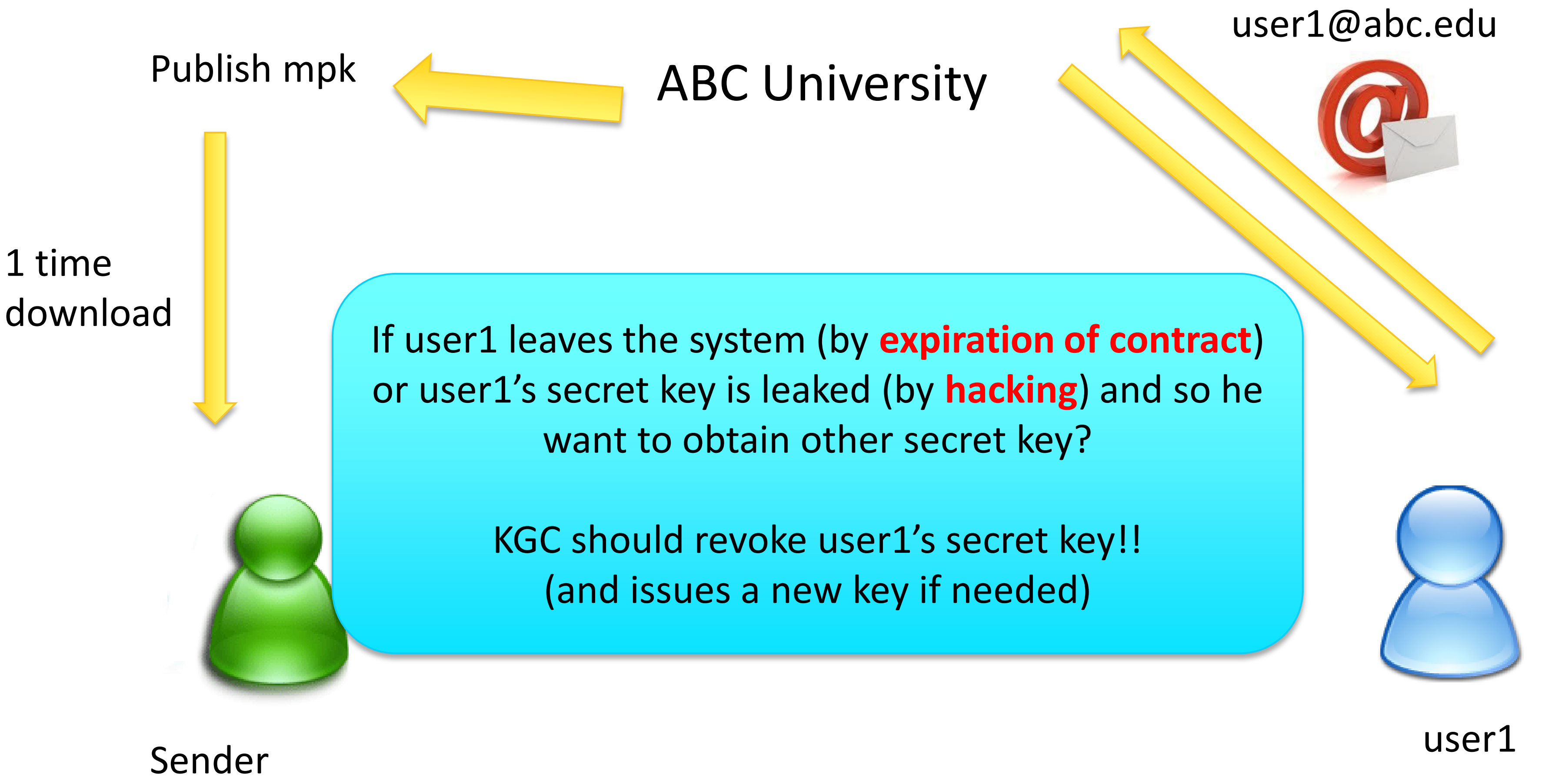
Session ID: CRYP-F41

Session Classification: Intermediate

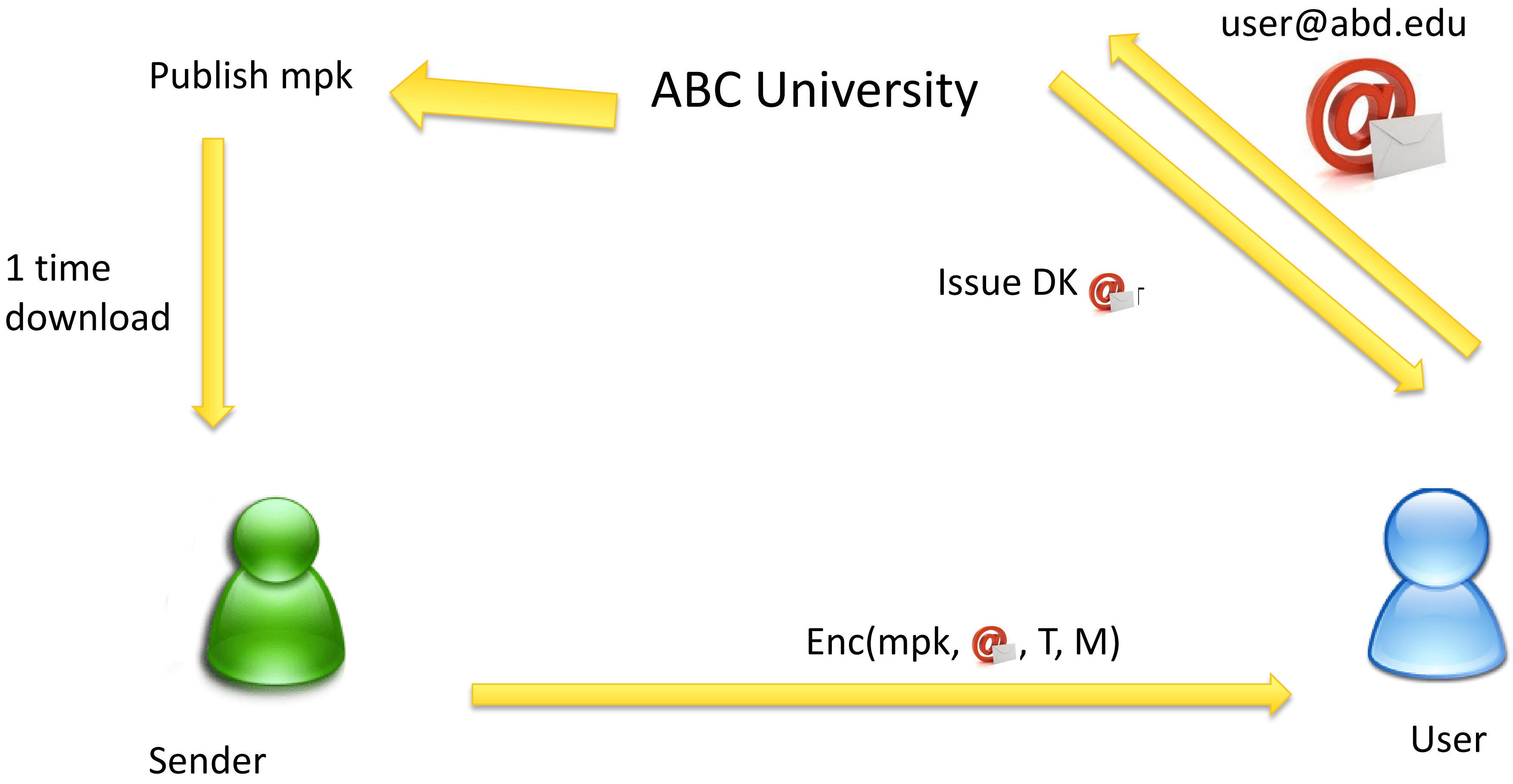
Identity Based Encryption



Revocation Functionality in Identity-Based Encryption



Trivial Approach for Revocation Functionality in IBE



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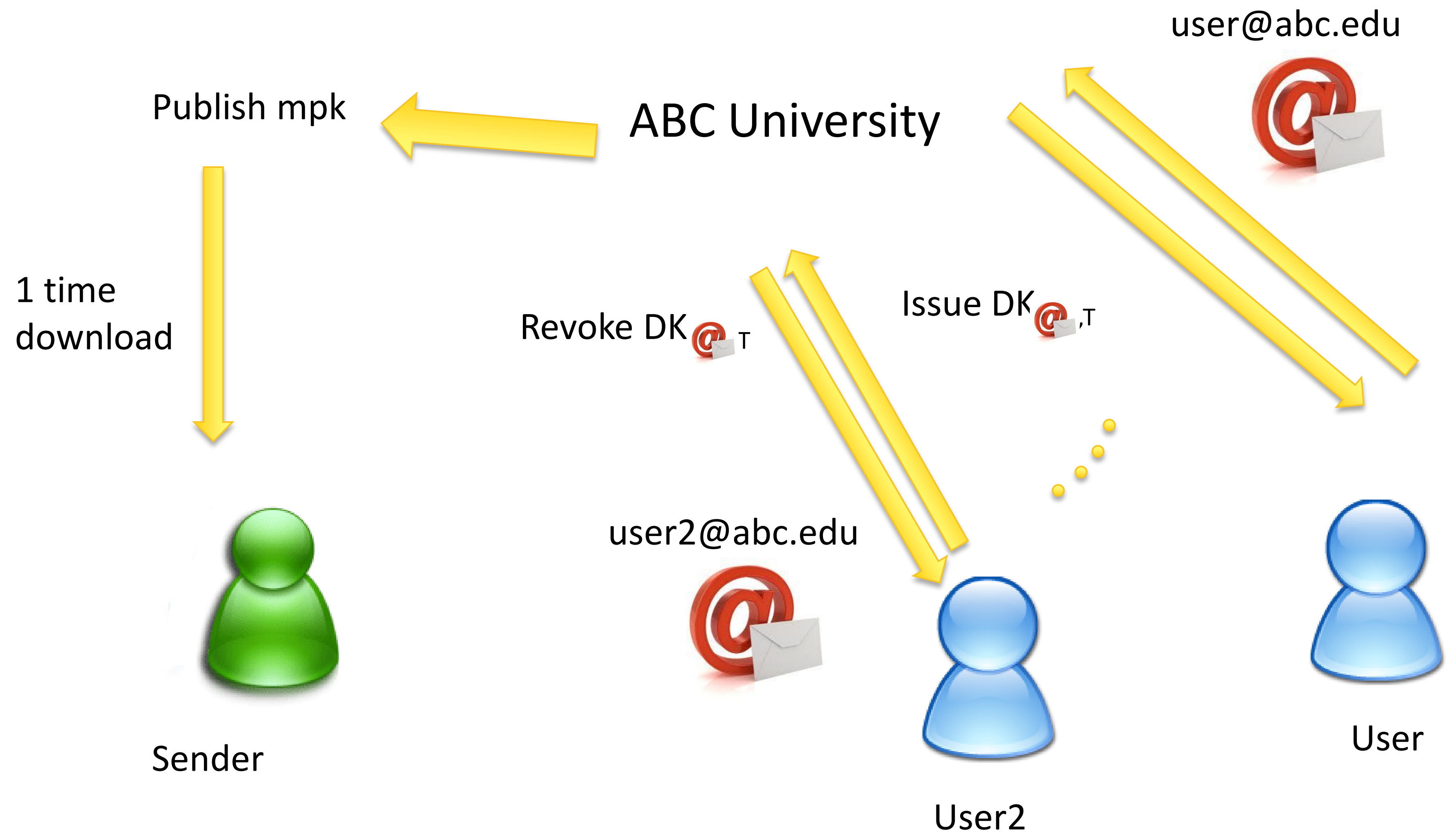

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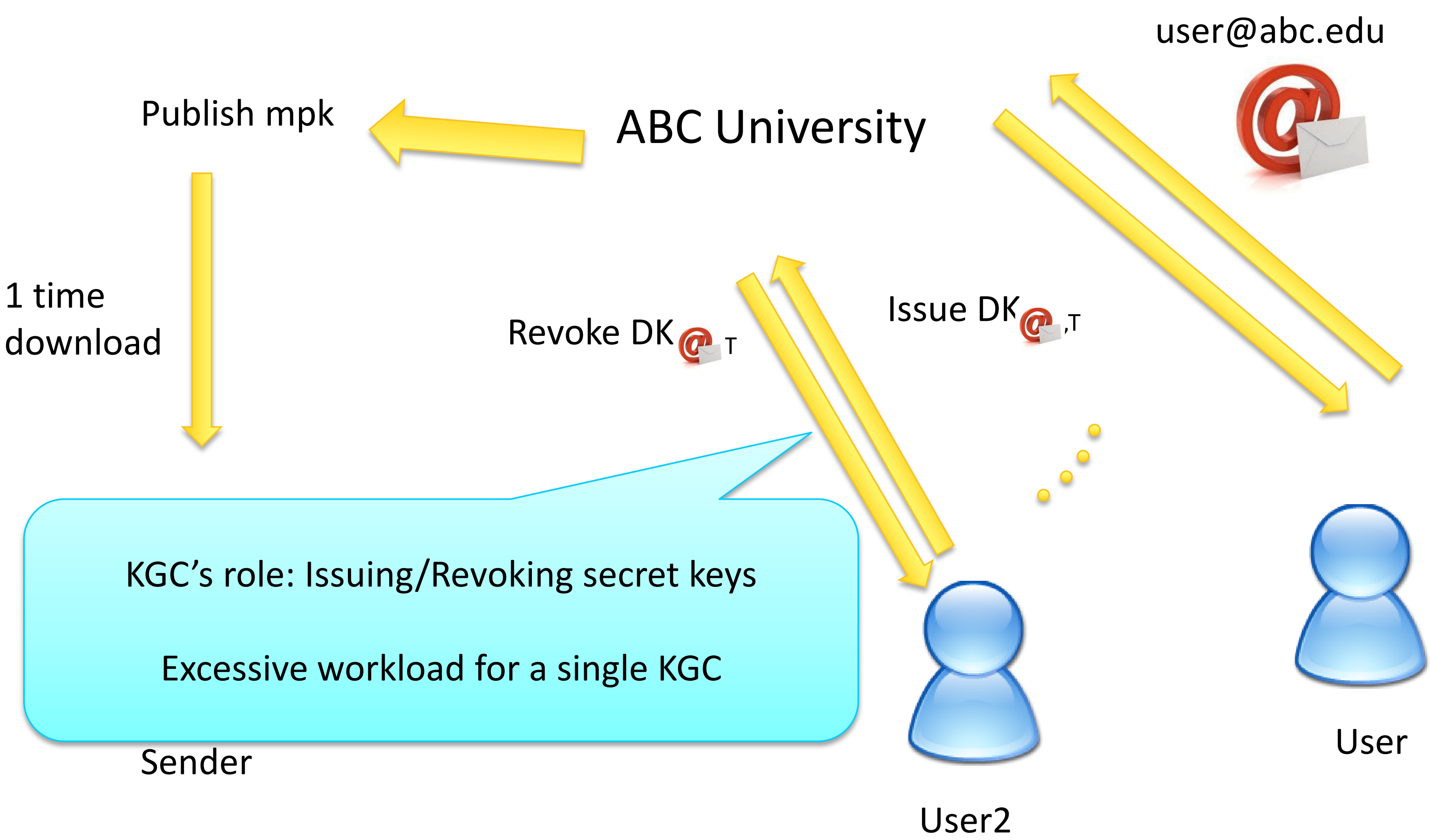
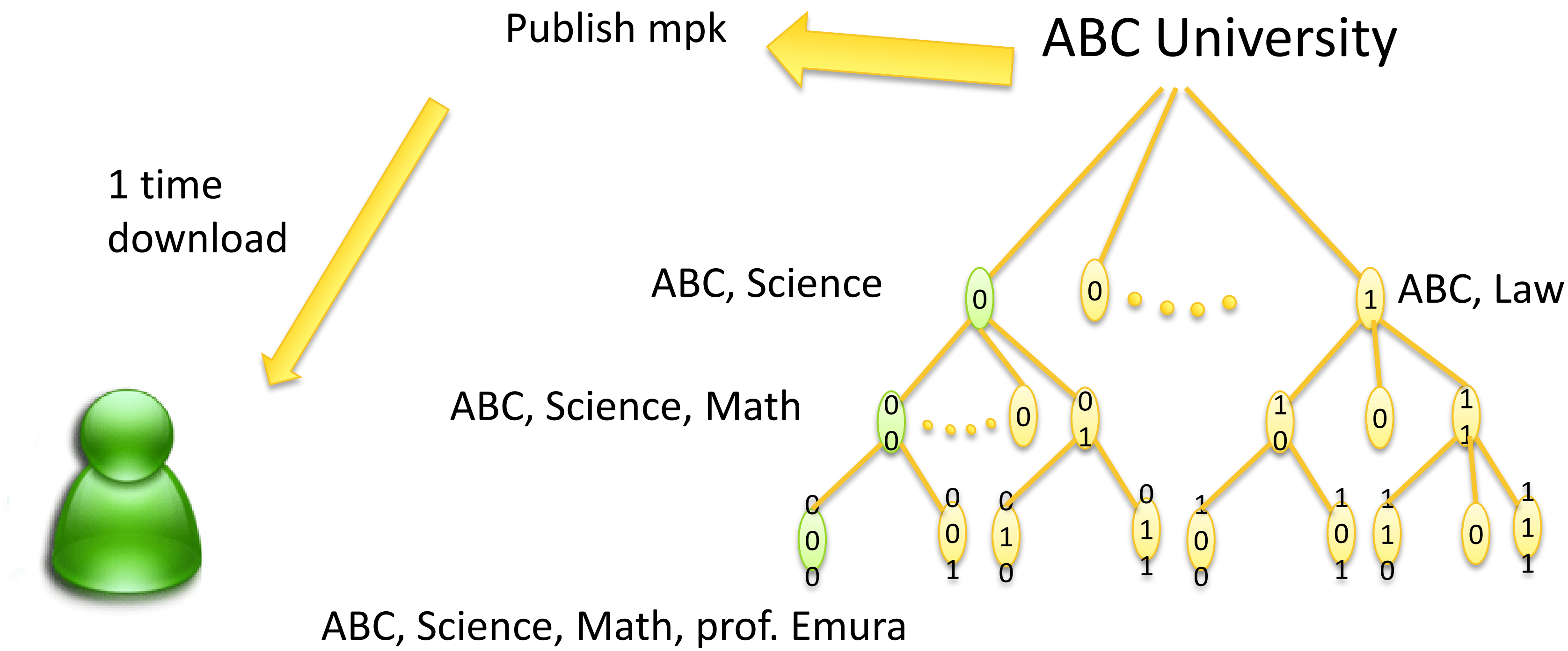


Our Goal:
Delegation of KGC's Roles (Key Generation & Revocation)



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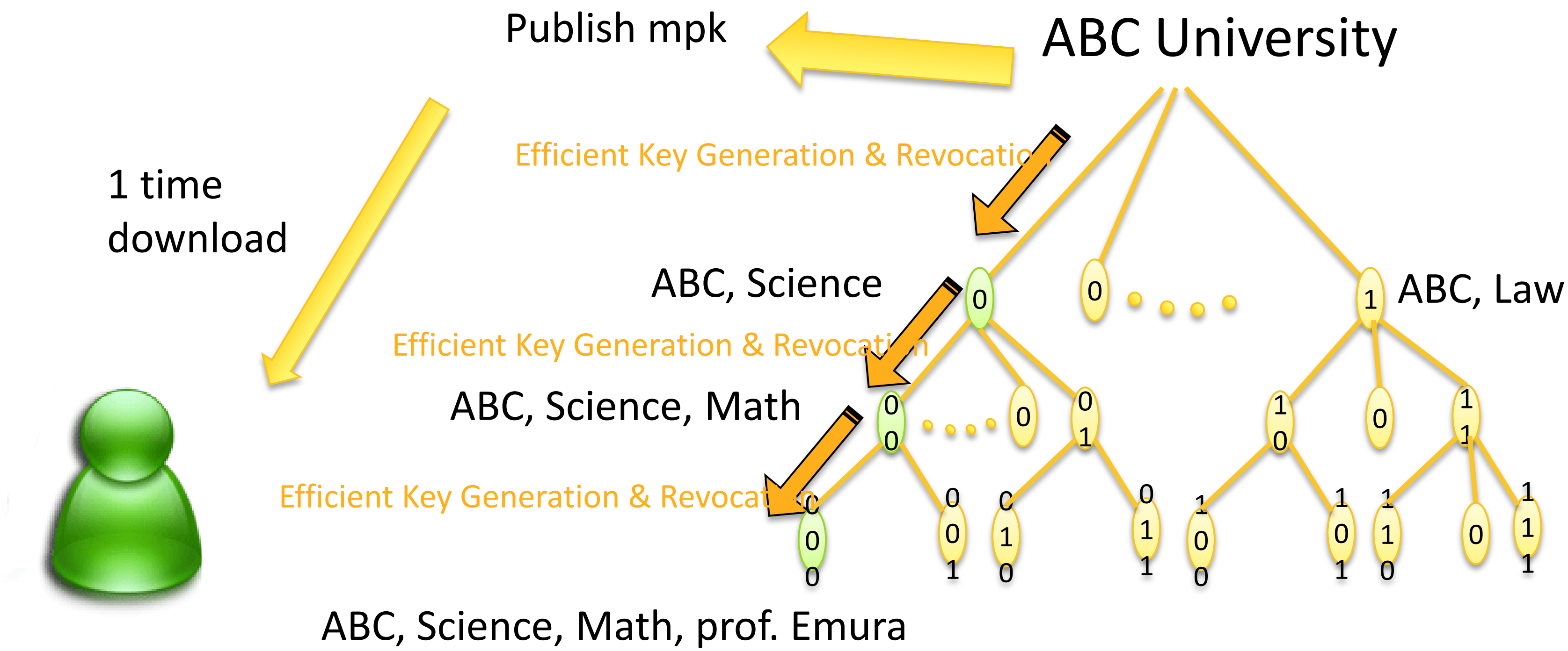
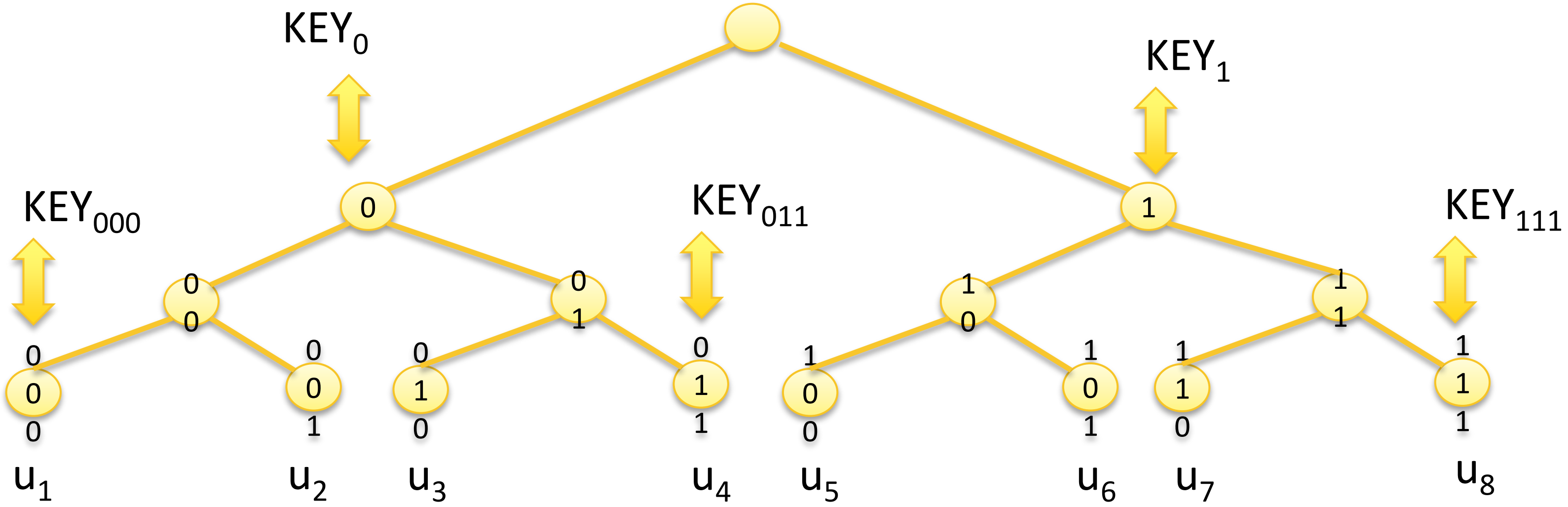


Outline

- Previous Approaches
  - Revocable Symmetric Key Encryption: Broadcast Encryption
  - Revocable Identity-Based Encryption
- Trivial Approach – Exponentially large secret key
- Our Approach – Asymmetric trade
- Further Study

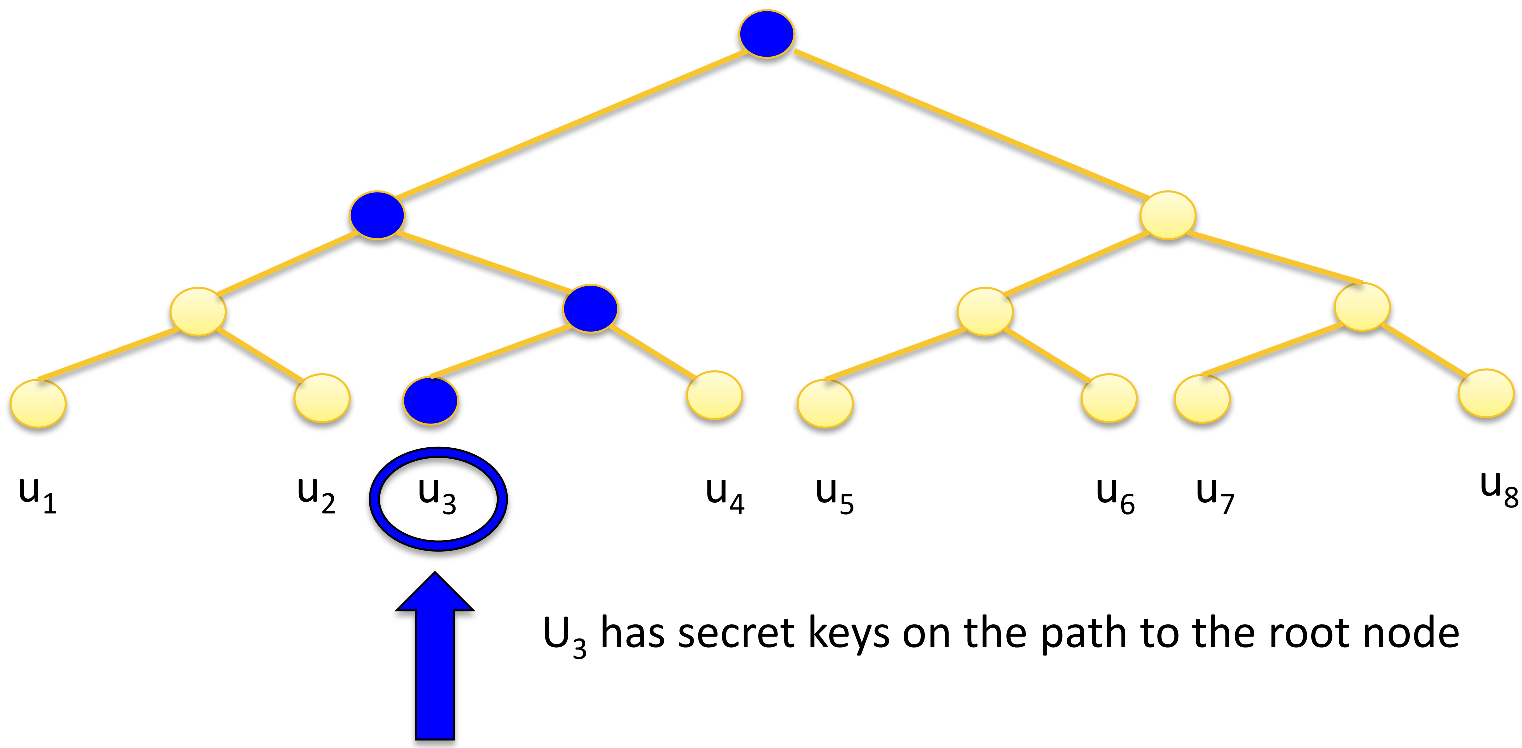
Broadcast Encryption (BE) Technique

Each node has the corresponding key.



We consider a binary tree kept by KGC

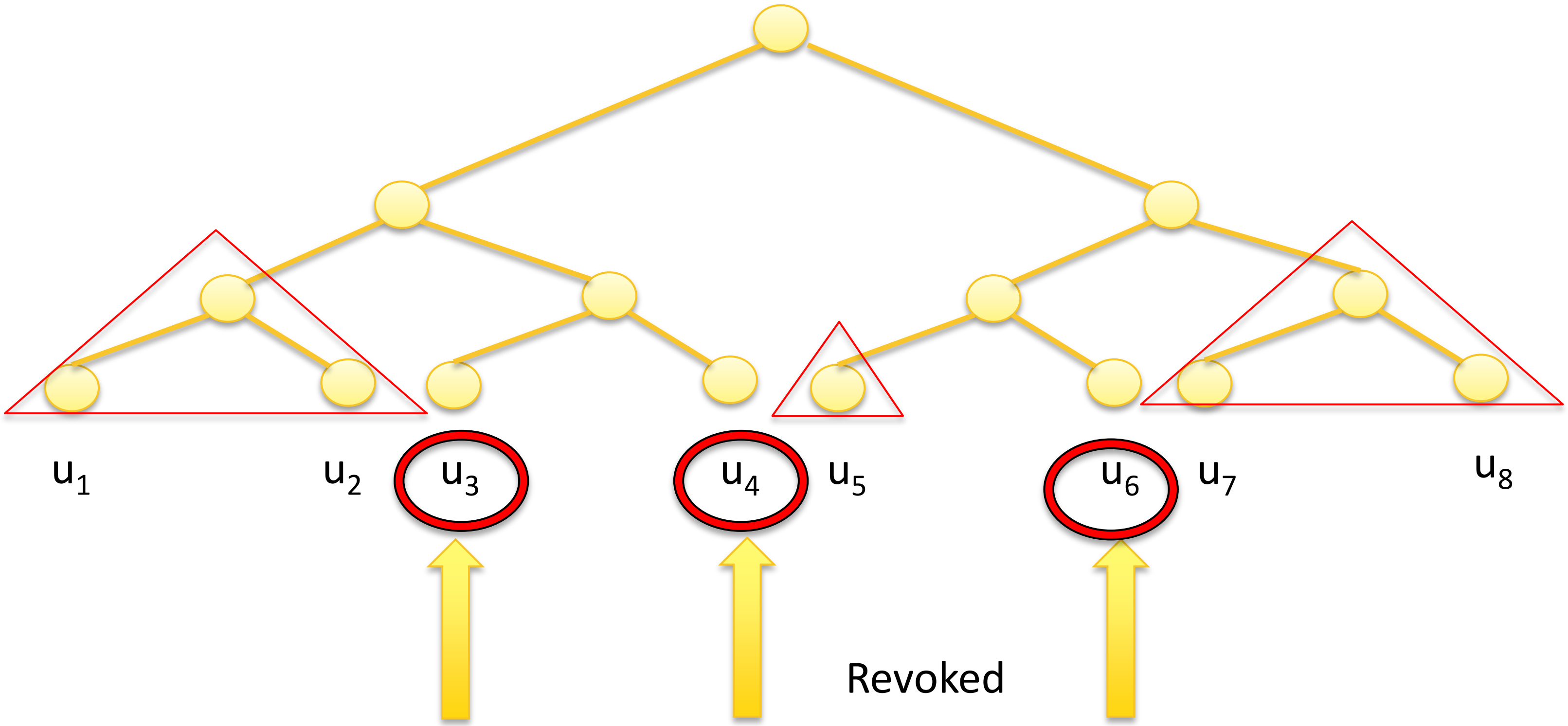
Broadcast Encryption (BE) Technique



U_3 has secret keys on the path to the root node

Broadcast Encryption (BE) Technique

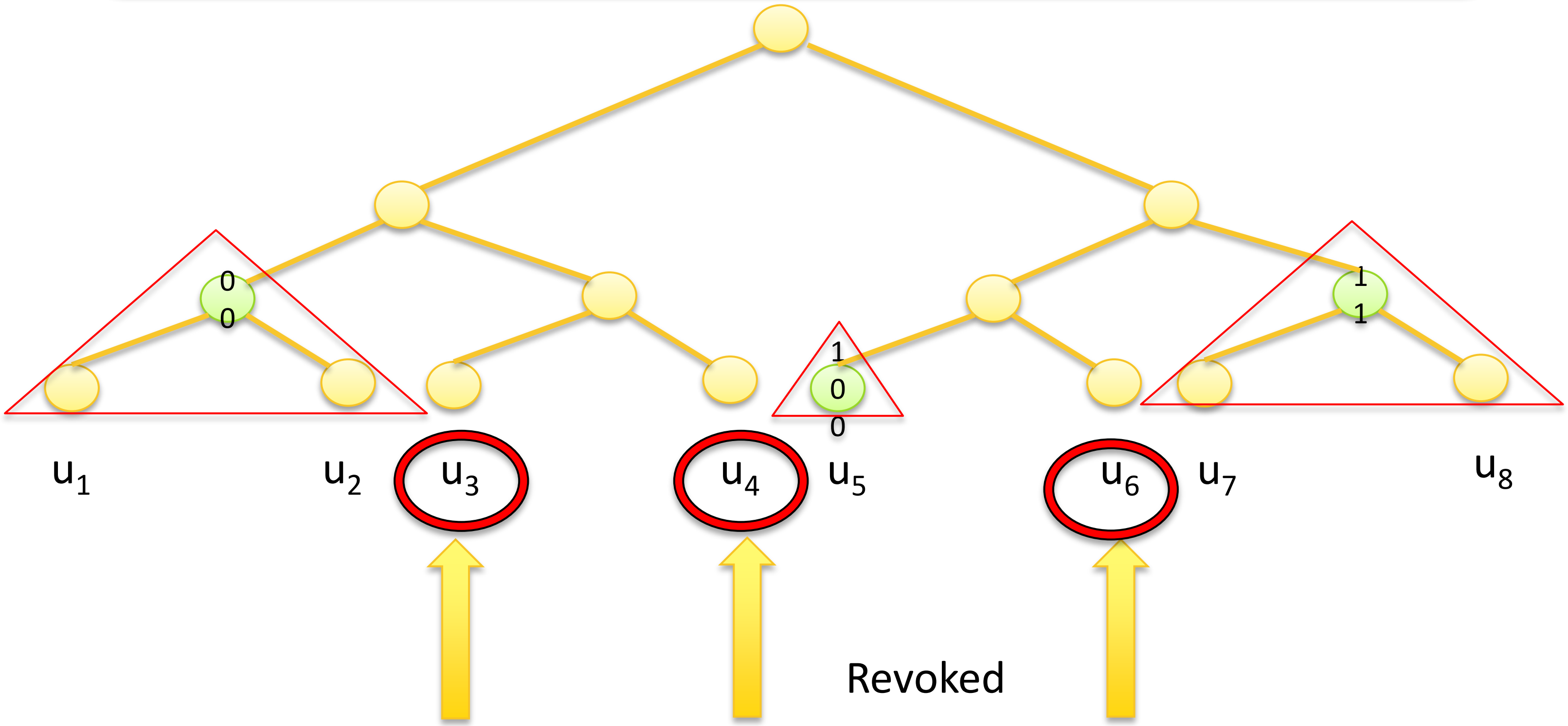
If u_3, u_4, and u_6 are revoked, first compute triangles containing only non-revoked users.



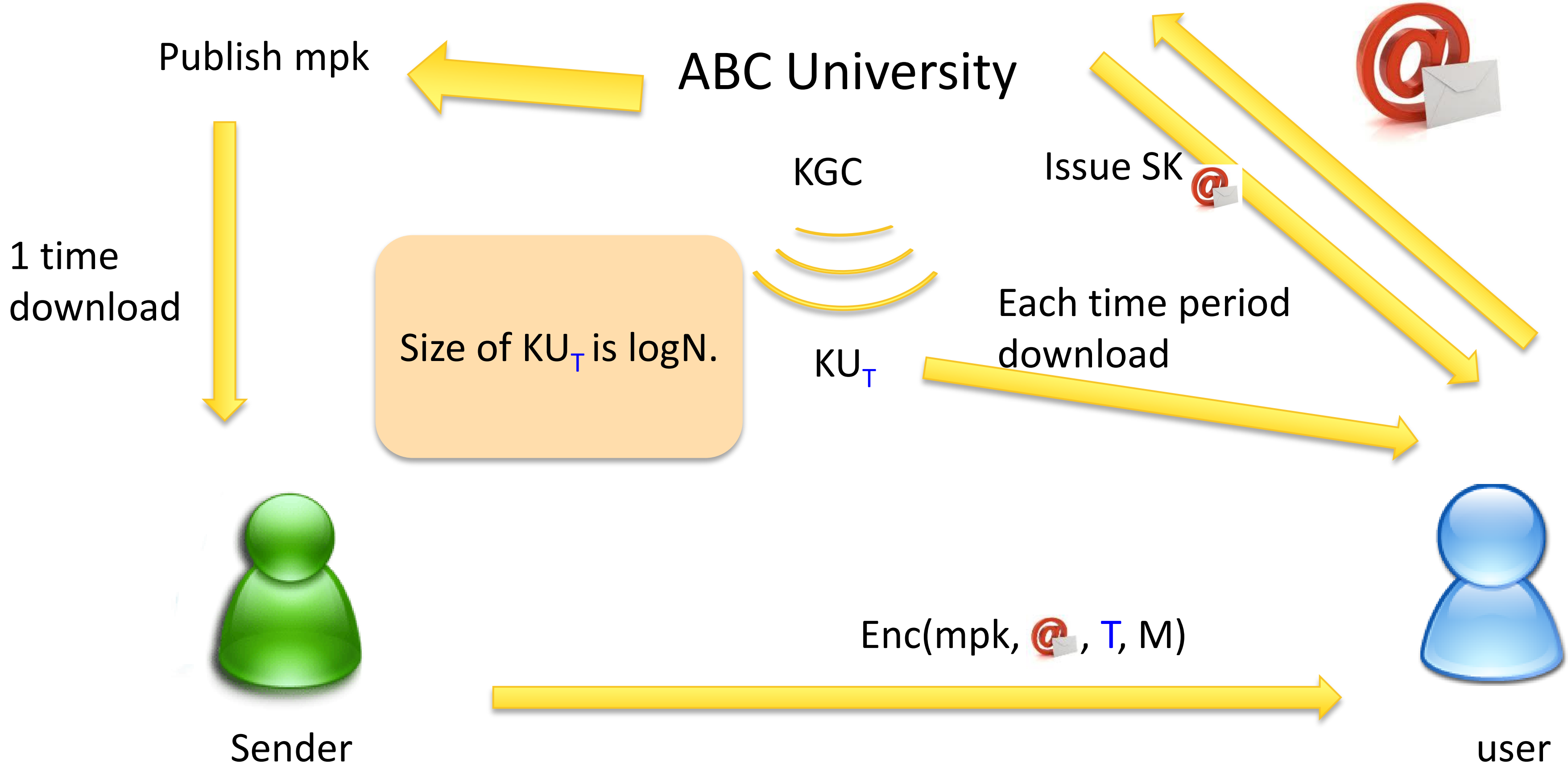
Broadcast Encryption (BE) Technique

Encrypt a session key using KEY_{00}, KEY_{101}, KEY_{11}.
Then, only non-revoked user can recover the session key.

# of triangles ~ log N (where N is # of leaf nodes)



Revocable IBE:
Combining BE technique with IBE scheme



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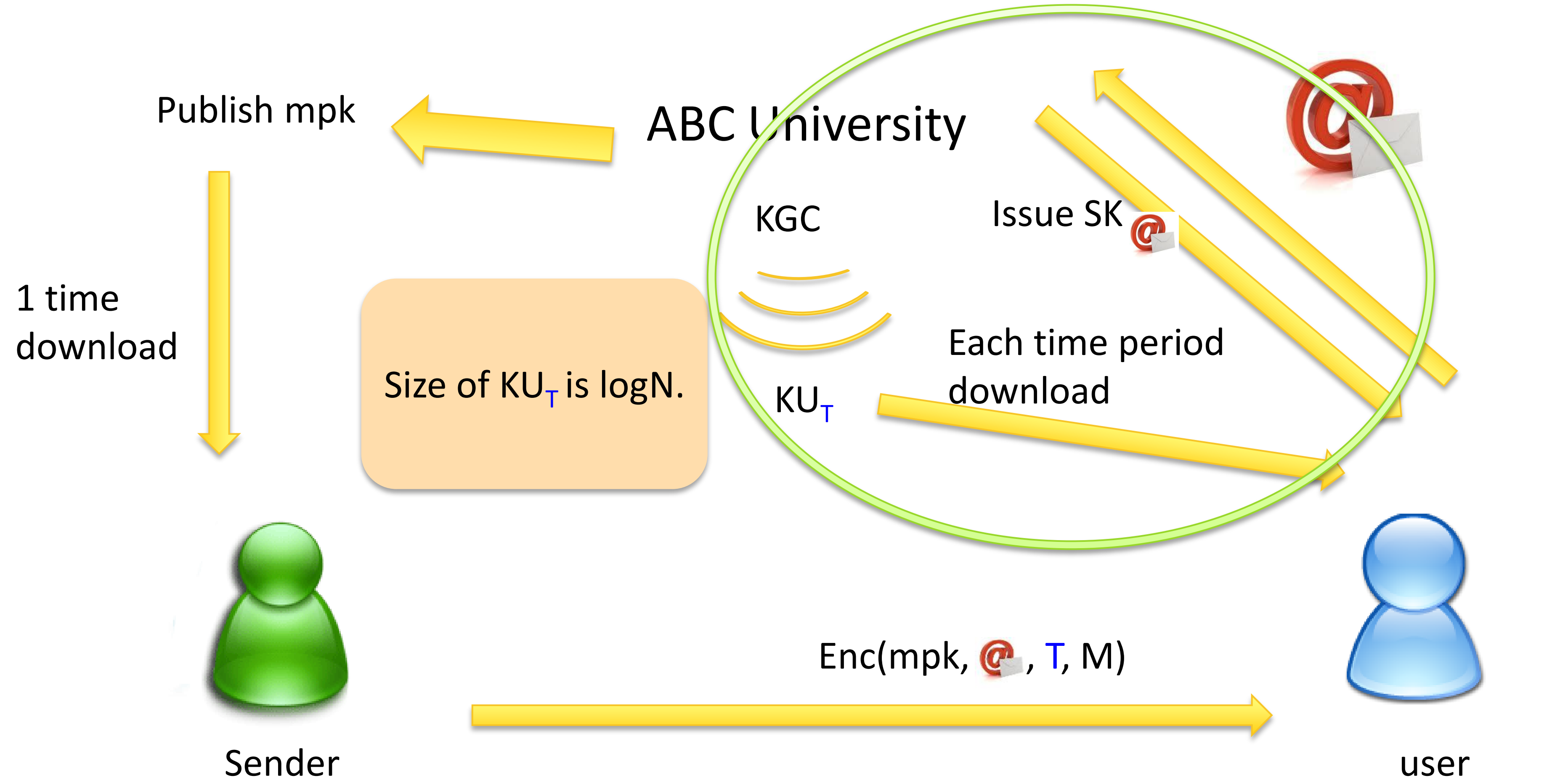
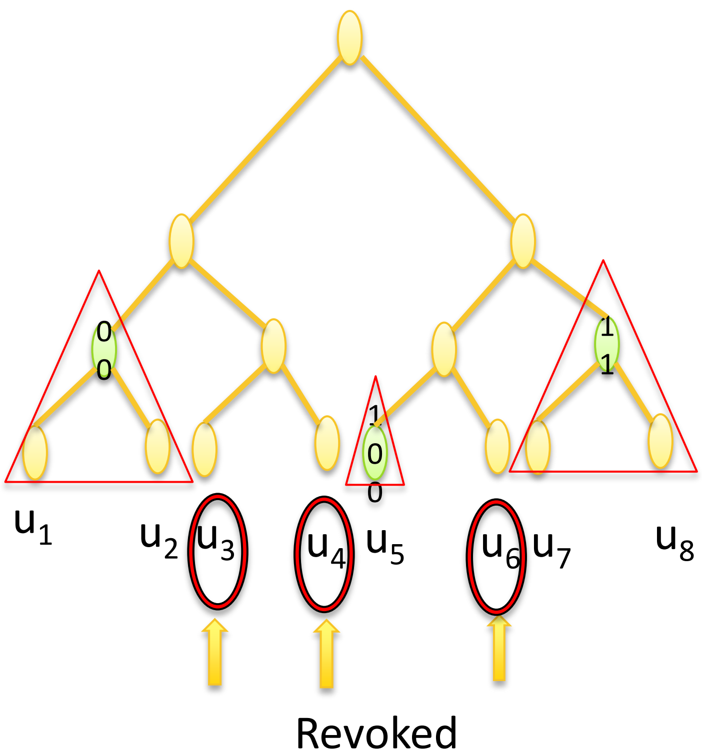
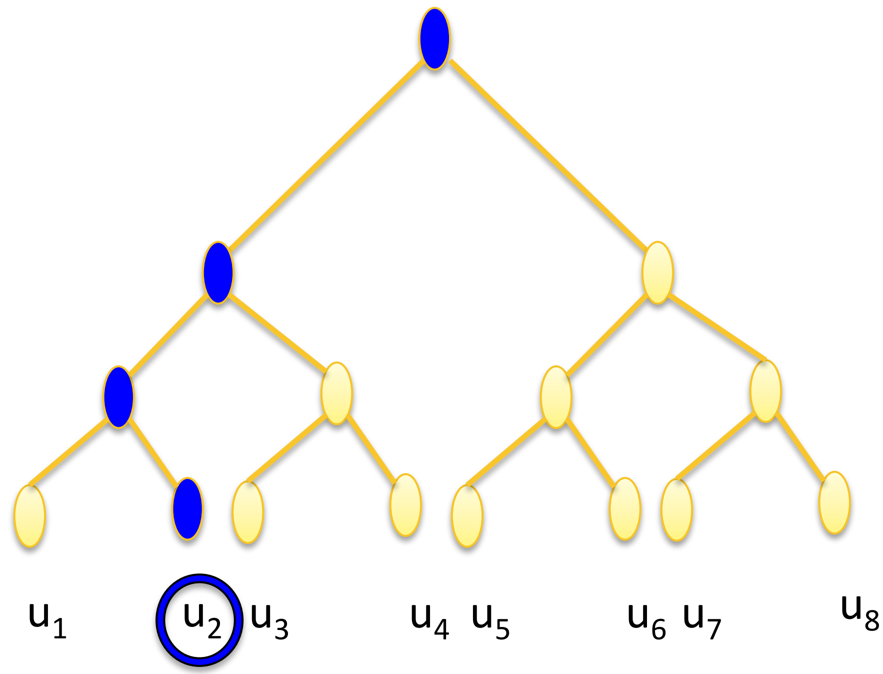


Revocable IBE:
Combining BE technique with IBE scheme

Key Revocation

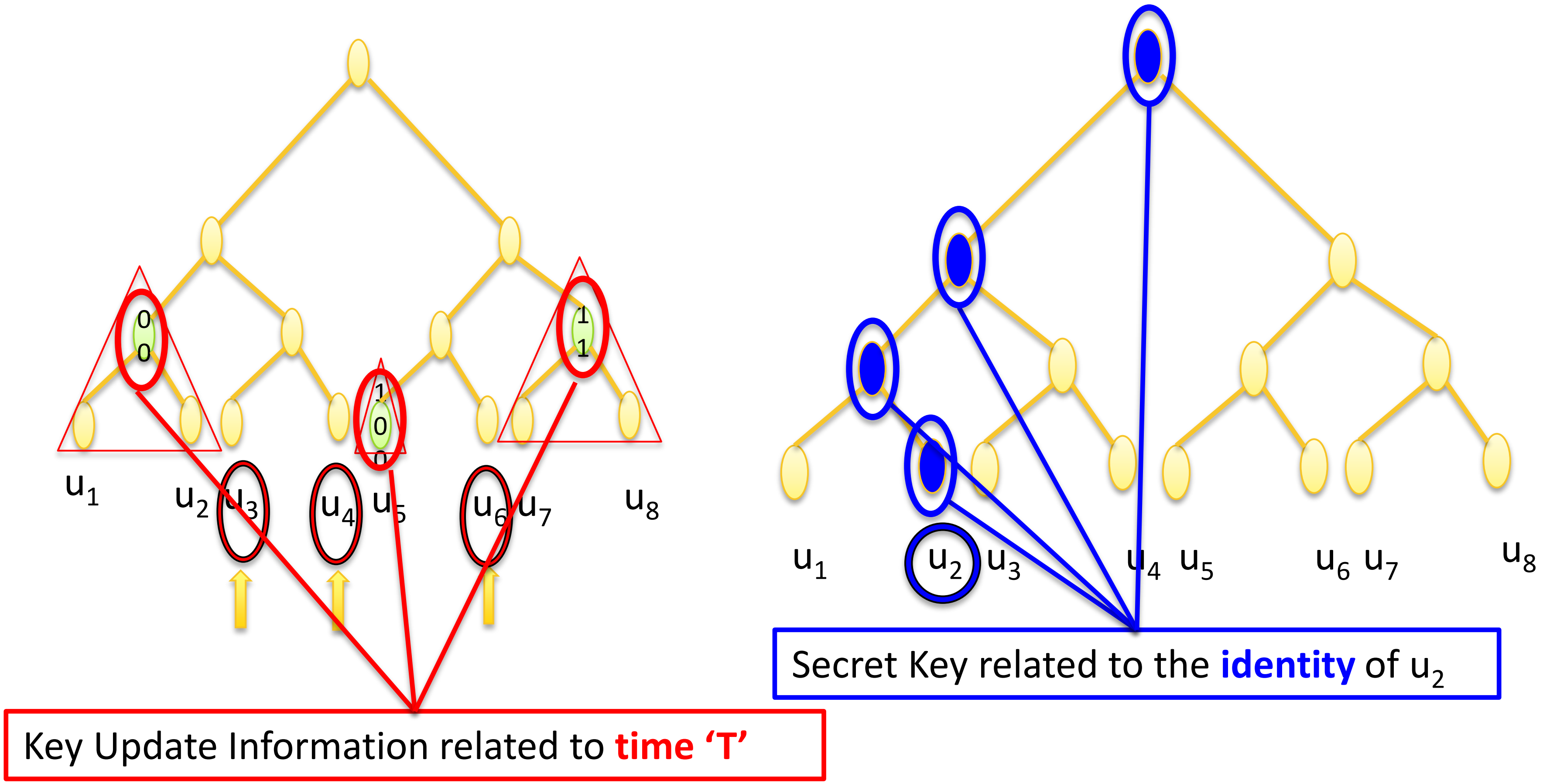


Key Generation



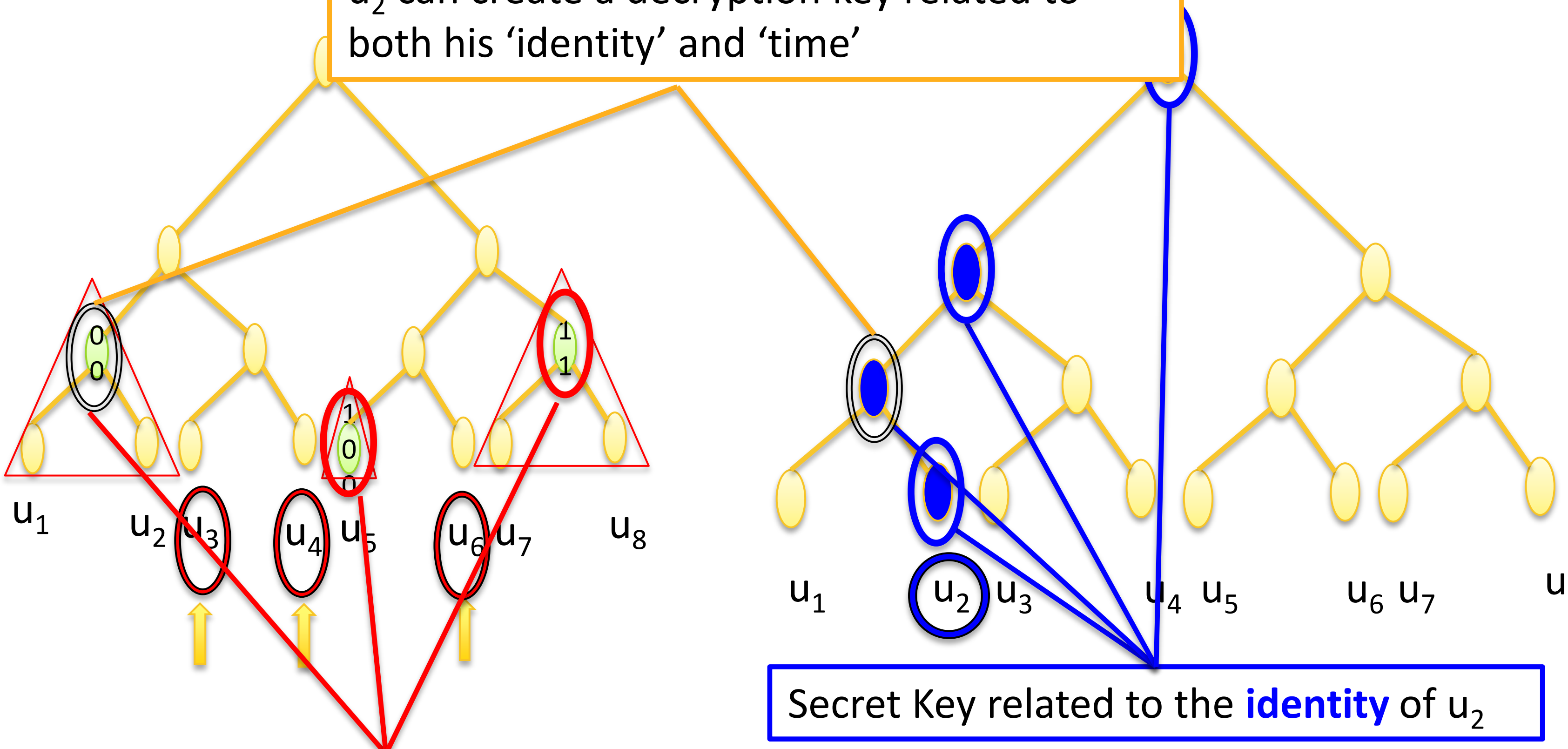
Whenever a user is registered in the system,
the user is assigned in the leaf node of binary tree.

Revocable IBE: Combining BE technique with IBE scheme



Revocable IBE:
Combining BE technique with IBE scheme

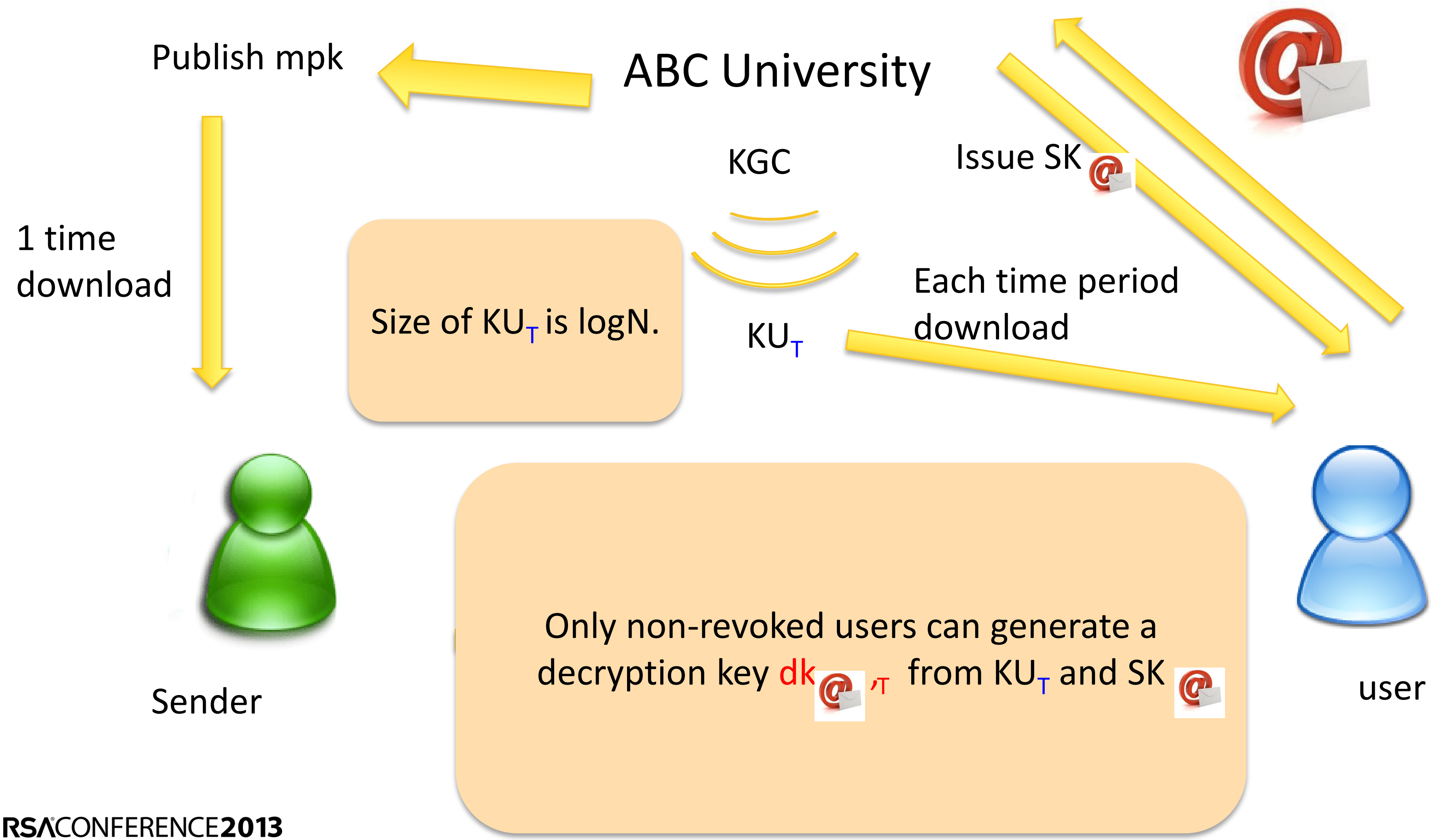
Since u_2 has both related to the same node, u_2 can create a decryption key related to both his 'identity' and 'time'



Secret Key related to the **identity** of u_2

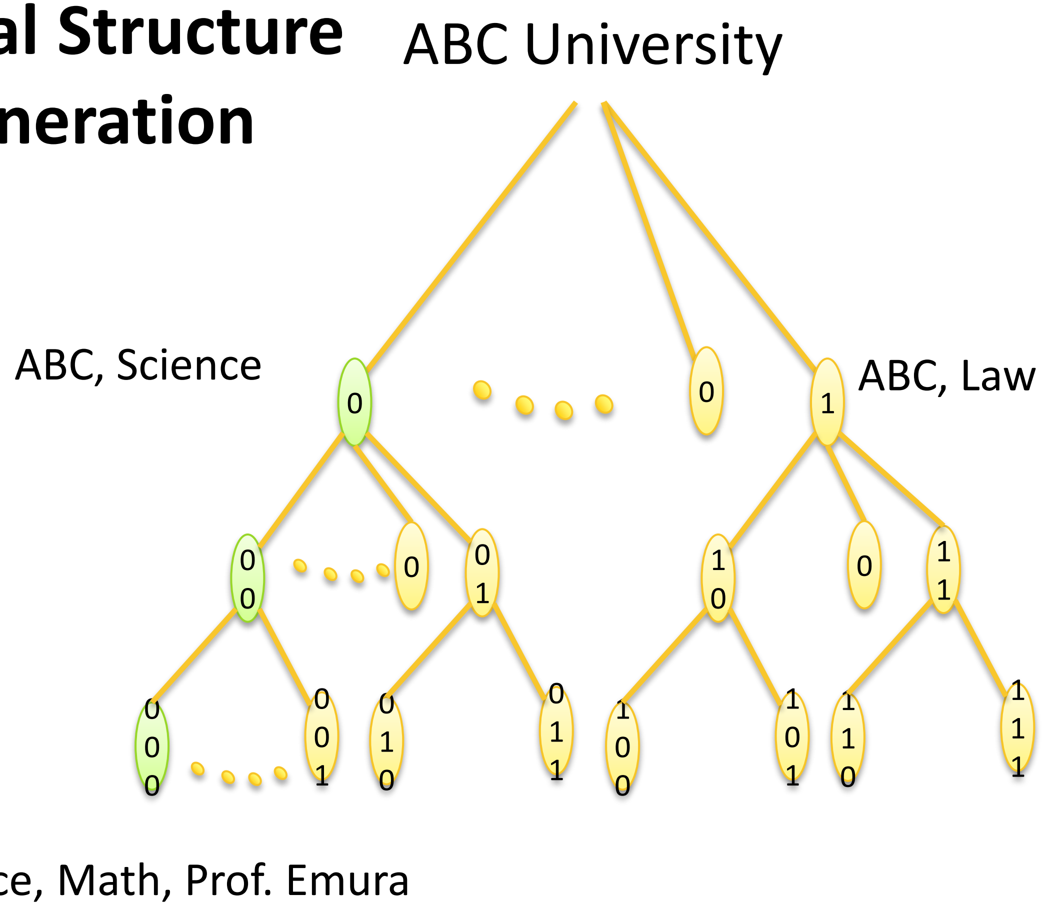
Key Update Information related to **time 'T'**

Revocable IBE: Combining BE technique with IBE scheme



Trivial Approach for Our Goal

Hierarchical Structure for Key Generation



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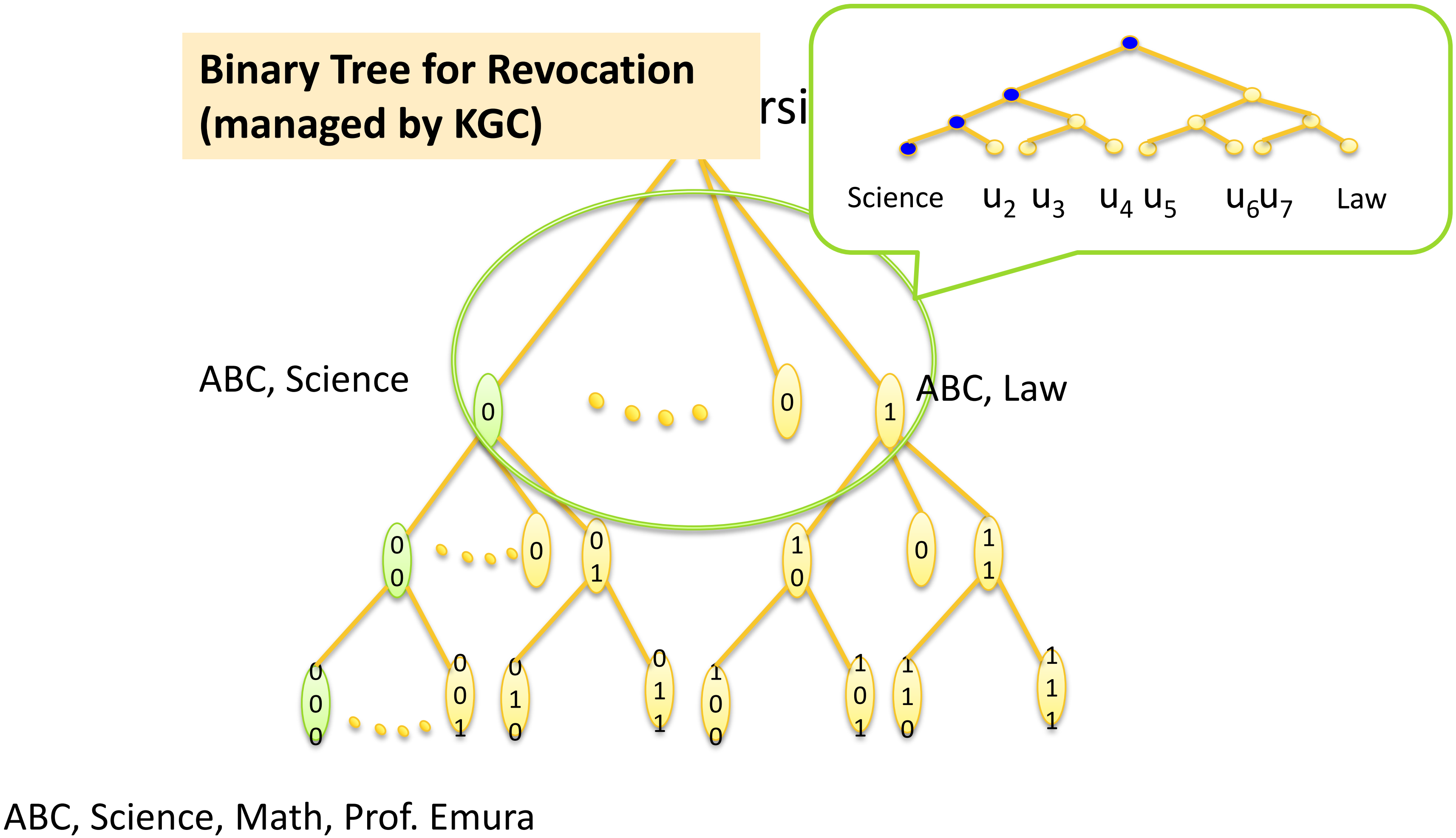

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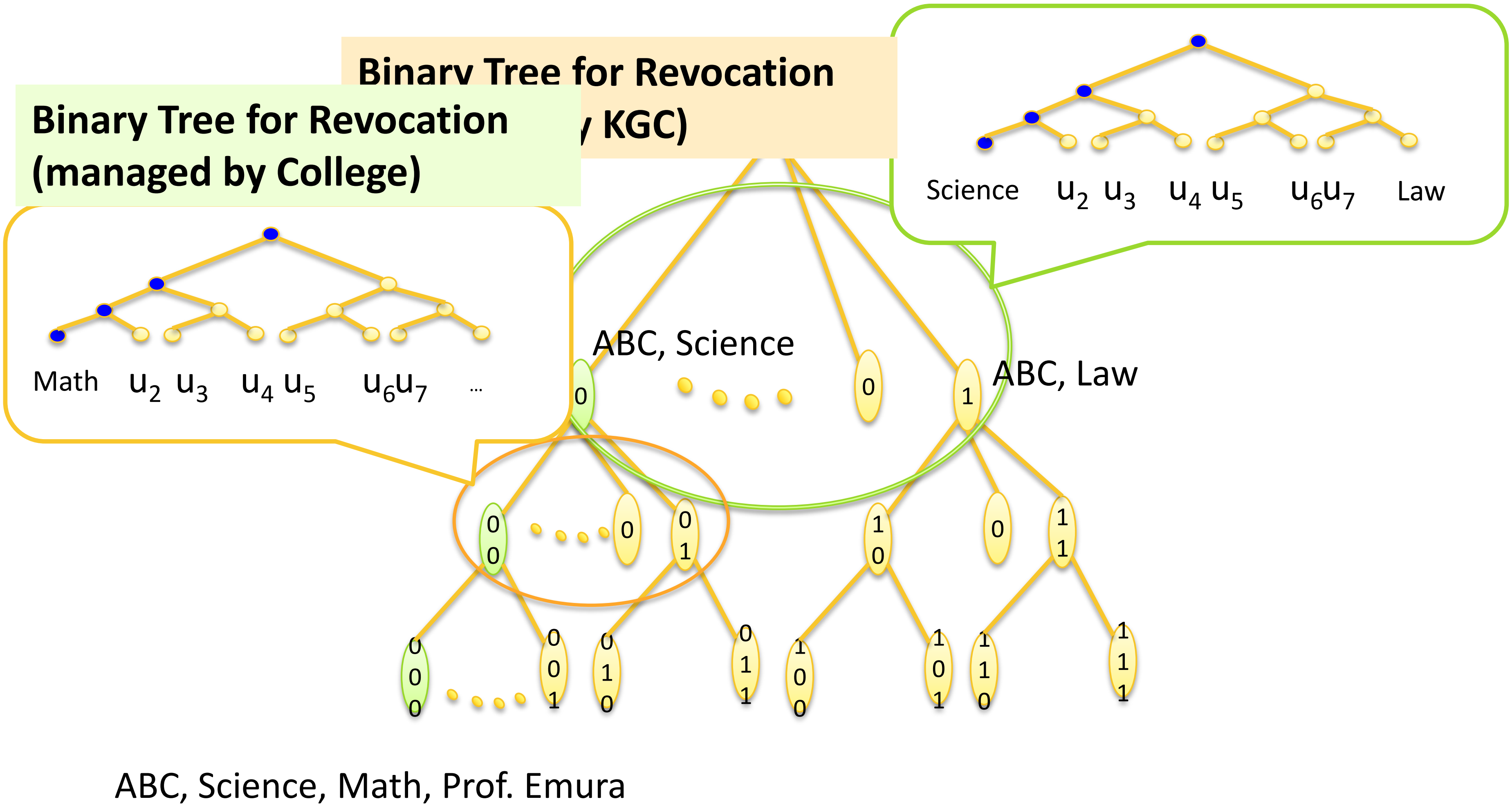

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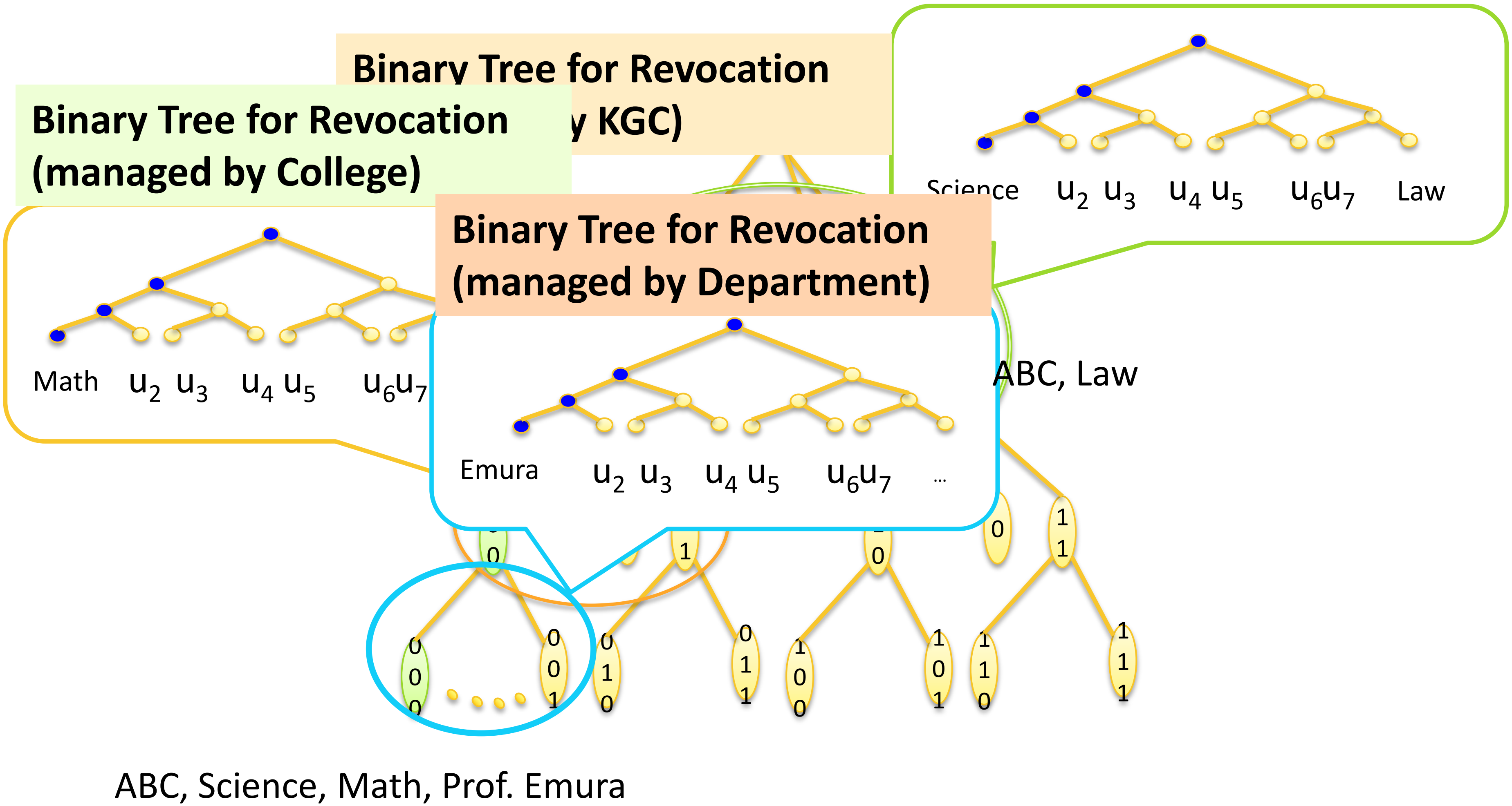

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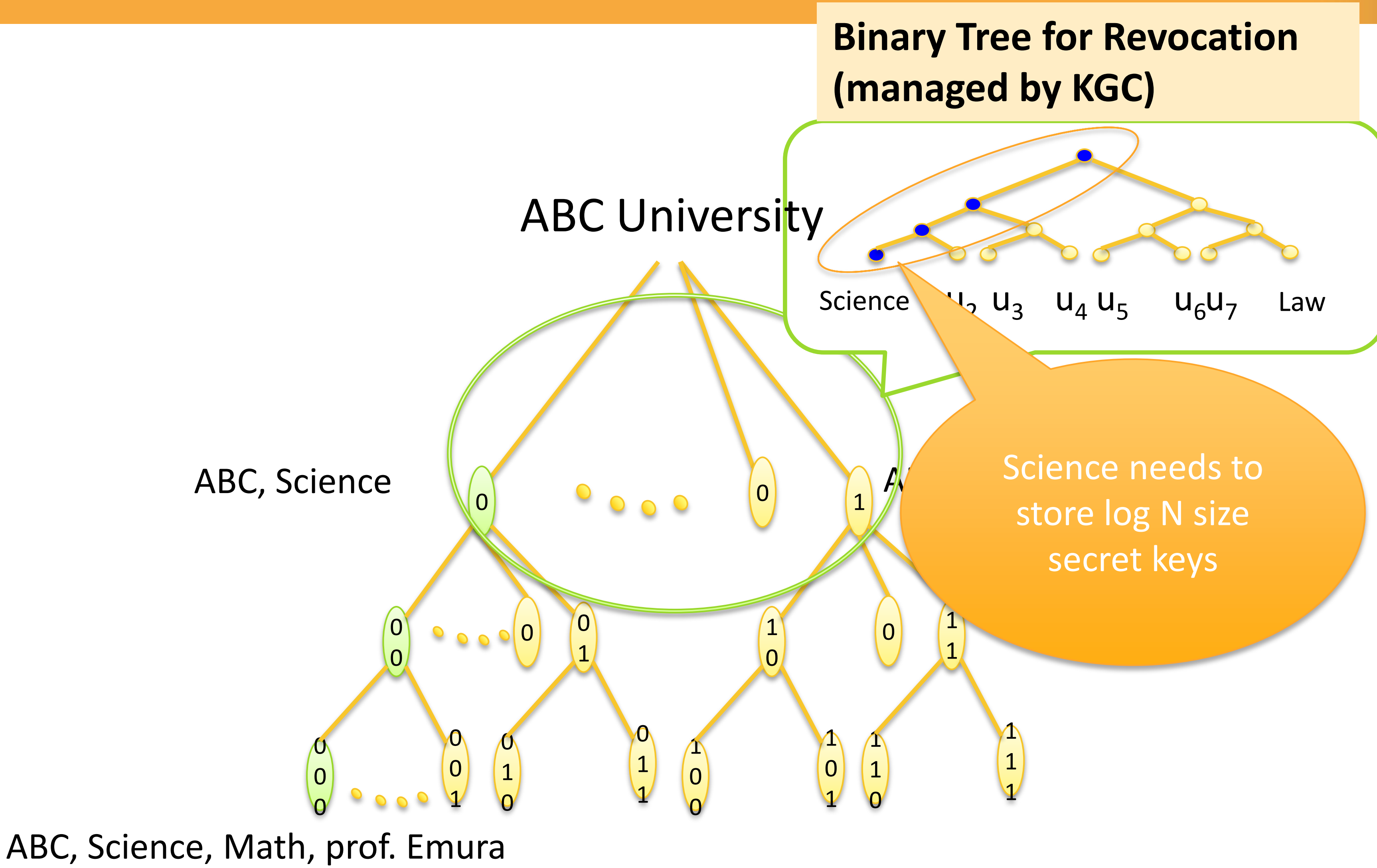
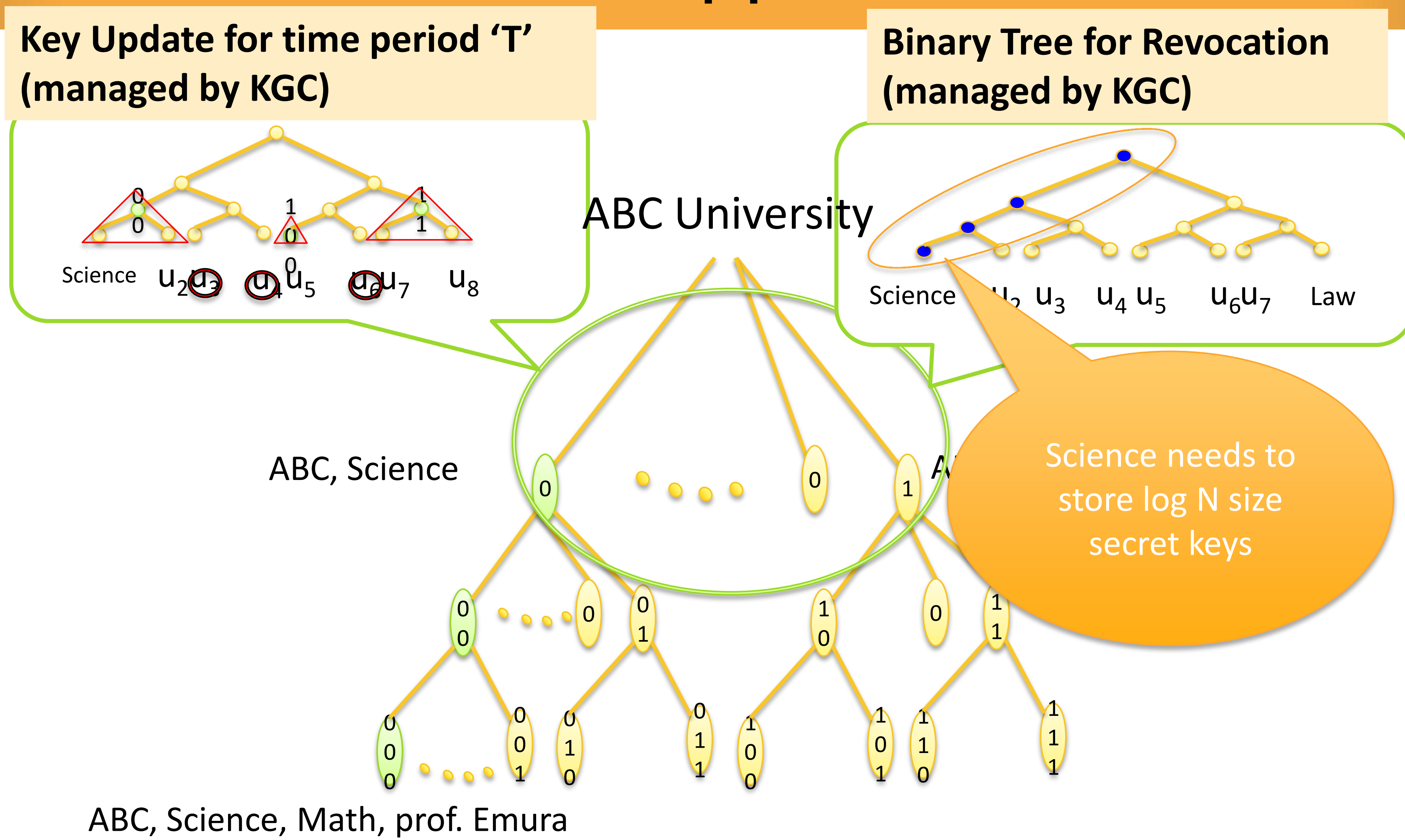


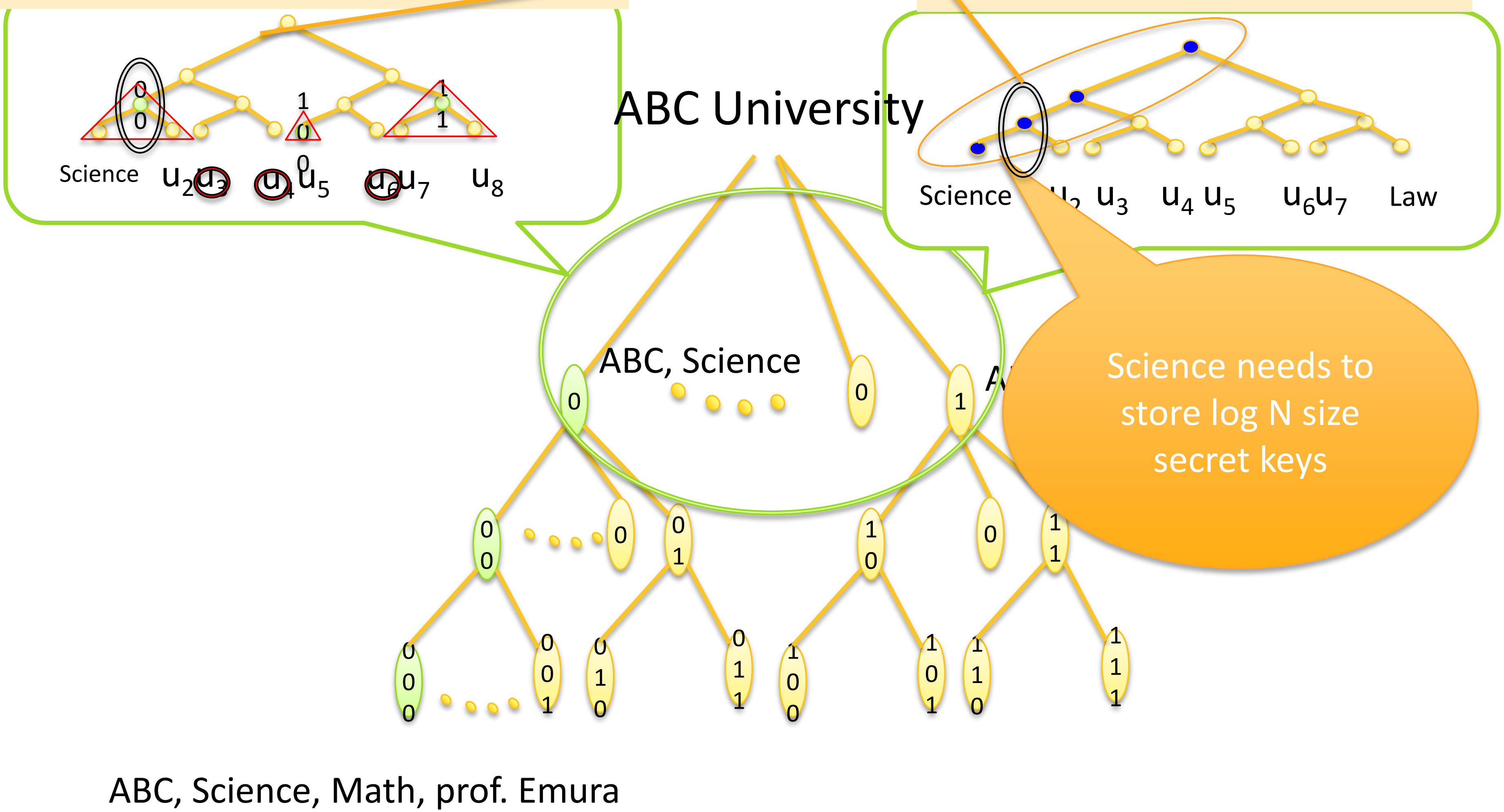
Trivial Approach



Science is not revoked on time ‘T’.
(1) It can create a decryption key relate to its identity & time ‘T’.
(2) It can generate Key Update for its children.

Key Update for tim
(managed by KGC)

(managed by KGC)



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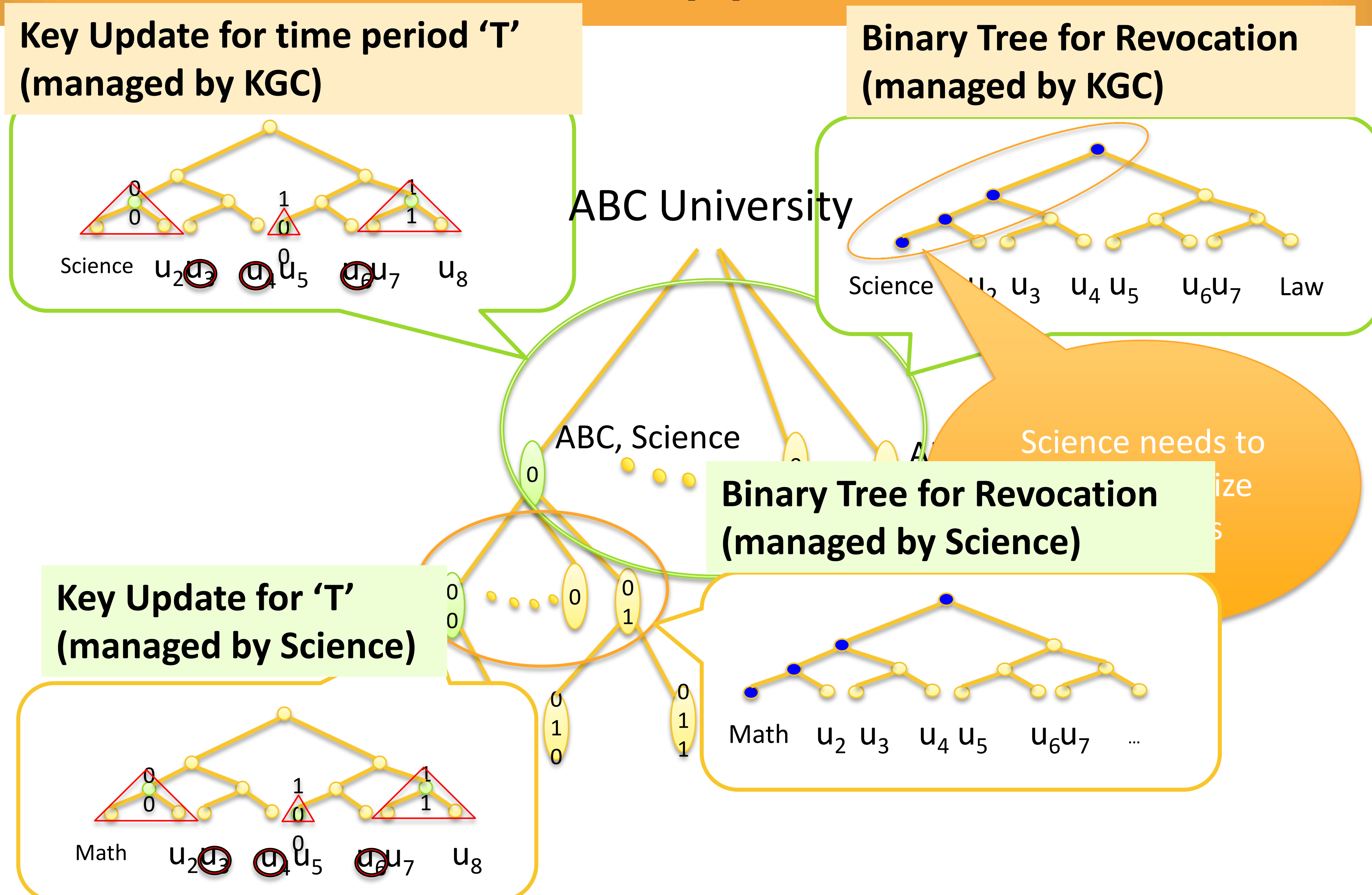

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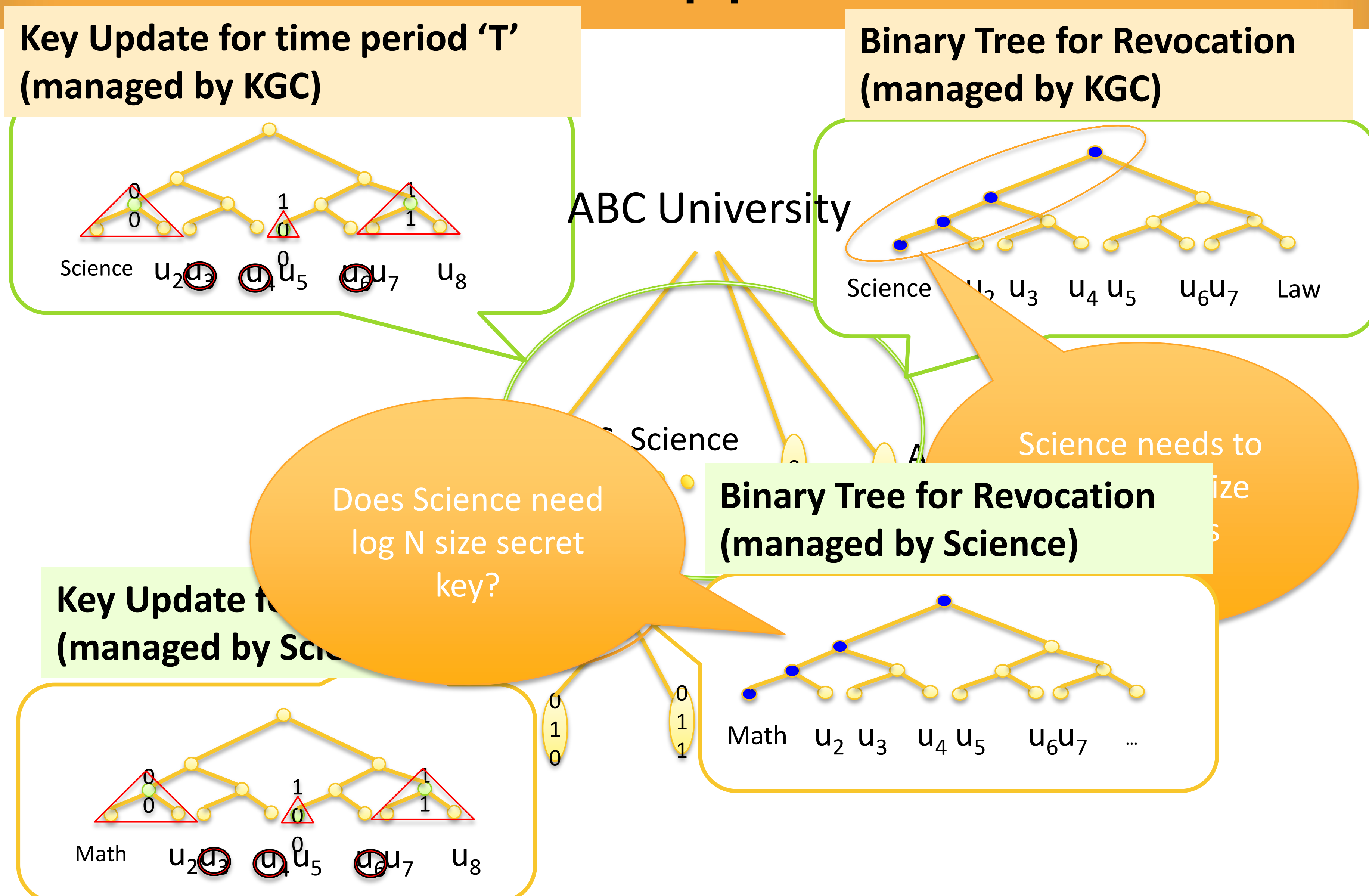
Trivial Approach



Approach

No!

The parent (Science) has log N size secret key and one subkey is used for each time period.

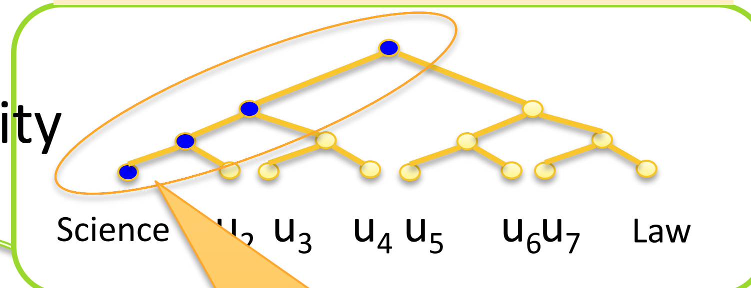
A child (Math) does not know which subkey will be used for each time period.

Therefore, children should have $(\log N)^2$ subkeys.

…

n-th level user has $(\log N)^n$ size secret keys

Binary Tree for Revocation (managed by KGC)



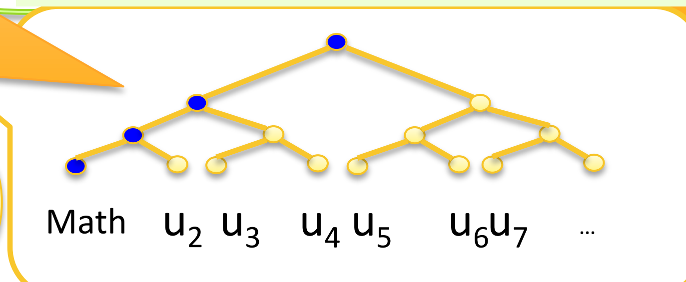
University

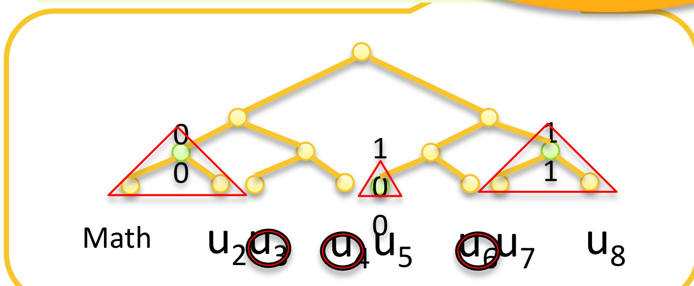
cience

Science needs to

ize

Binary Tree for Revocation (managed by Science)





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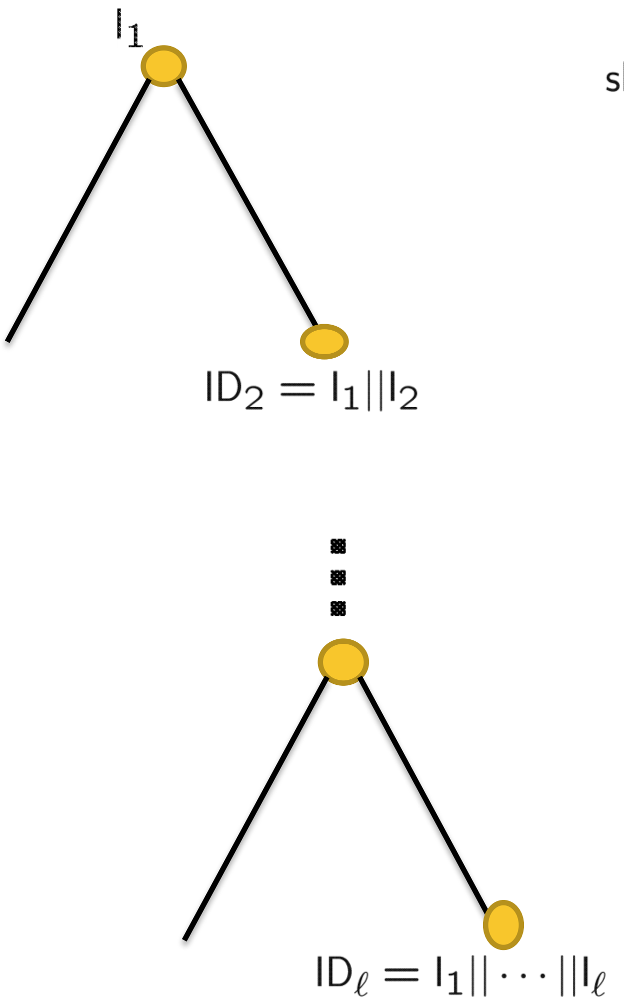
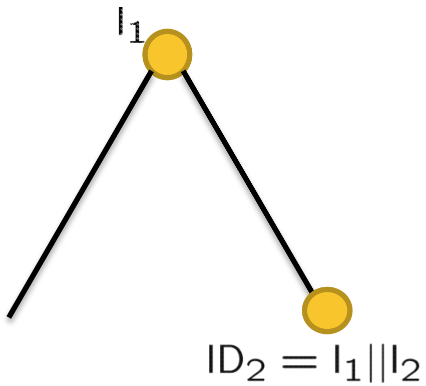


$$\mathsf{sk}_{\mathsf{ID}_{|1}} = \left\{ \begin{array}{l} \mathsf{d}_{\mathsf{I}_1}^{1} \\ \vdots \\ \mathsf{d}_{\mathsf{I}_1}^{\log N} \end{array} \right\} \log N \text{ subkeys}$$

$$\mathsf{sk}_{\mathsf{ID}_{|2}} = \left\{ \begin{array}{l} \mathsf{d}_{\mathsf{I}_1}^{1} \cdot \mathsf{d}_{\mathsf{I}_2}^{1} \\ \mathsf{d}_{\mathsf{I}_1}^{1} \cdot \mathsf{d}_{\mathsf{I}_2}^{2} \\ \vdots \\ \mathsf{d}_{\mathsf{I}_1}^{1} \cdot \mathsf{d}_{\mathsf{I}_2}^{\log N} \\ \mathsf{d}_{\mathsf{I}_1}^{2} \cdot \mathsf{d}_{\mathsf{I}_2}^{1} \\ \vdots \\ \mathsf{d}_{\mathsf{I}_1}^{\log N} \cdot \mathsf{d}_{\mathsf{I}_2}^{\log N} \end{array} \right\} (\log N)^2 \text{ subkeys}$$

$$\mathsf{sk}_{\mathsf{ID}_{|\ell}} = \left\{ \vdots \right\} \quad (\log N)^\ell \text{ subkeys}$$

Our Approach – Asymmetric Trade



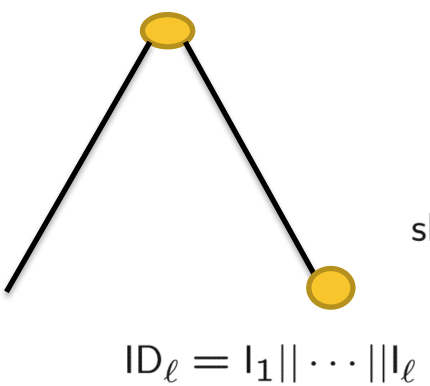
$$sk_{ID_{|1}} = \left\{ \begin{array}{c} d_{I_1}^{1} \\ \vdots \\ d_{I_1}^{\log N} \end{array} \right\}$$

$\log N$ subkeys

$$sk_{ID_{|2}} = \left[\begin{array}{c} \left\{ \begin{array}{c} d_{I_1}^{1} \\ d_{I_1}^{2} \\ \vdots \\ d_{I_1}^{\log N} \end{array} \right\} \\ \\ \left\{ \begin{array}{c} d_{I_2}^{1} \\ d_{I_2}^{2} \\ \vdots \\ d_{I_2}^{\log N} \end{array} \right\} \end{array} \right] \rightarrow \text{Product} \Rightarrow (\log N)^2 \text{ subkeys}$$

$2 \log N$ subkeys

$\vdots$



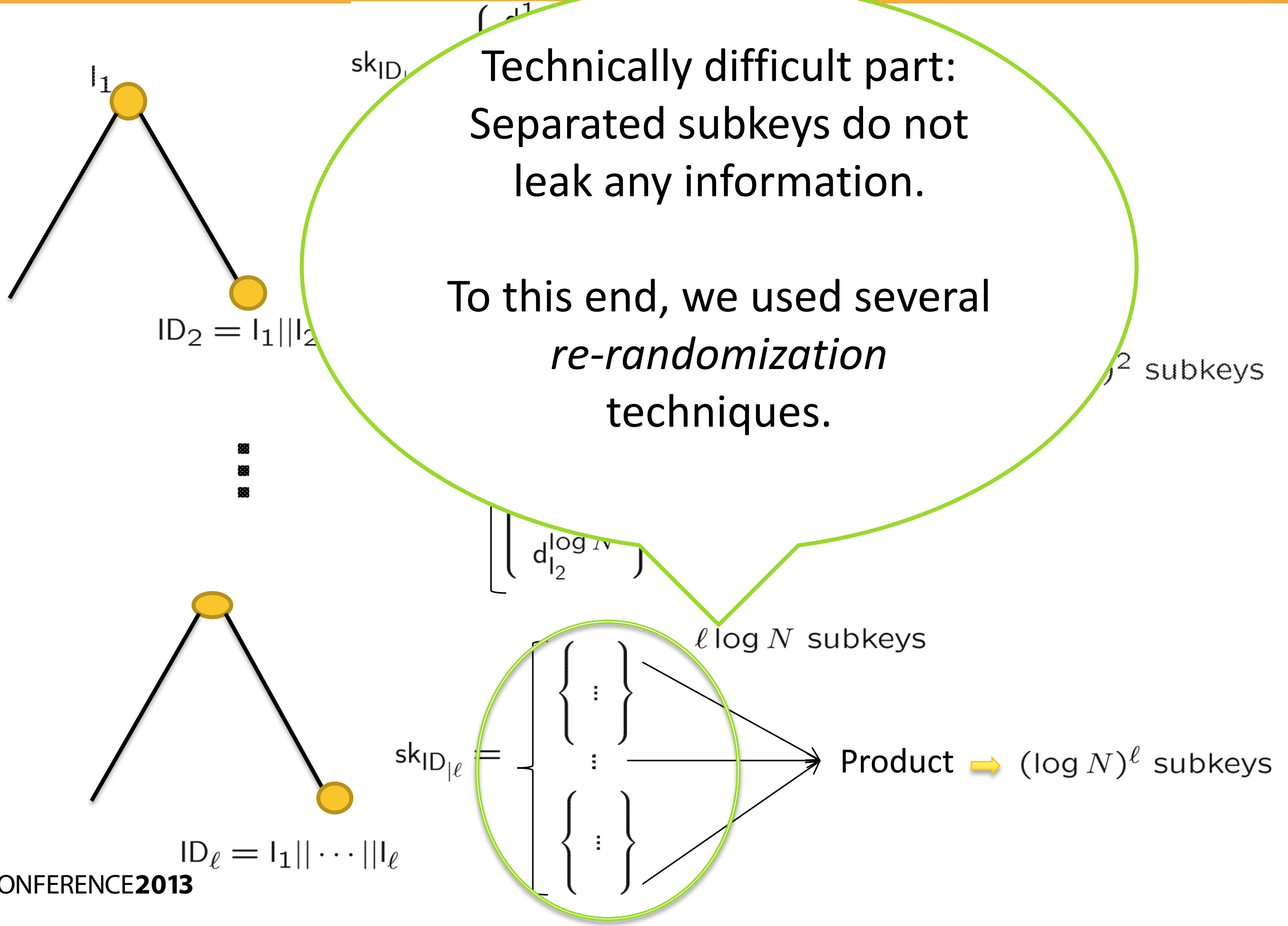
$\ell \log N$ subkeys

$$sk_{ID_{|\ell}} = \left[\begin{array}{c} \left\{ \vdots \right\} \\ \vdots \\ \left\{ \vdots \right\} \end{array} \right] \rightarrow \text{Product} \Rightarrow (\log N)^\ell \text{ subkeys}$$

Our Approach – Asymmetric Trade

Technically difficult part:
Separated subkeys do not leak any information.

To this end, we used several *re-randomization* techniques.



Our Result

- We propose the first practical RHIBE scheme
  - Our scheme is based on Boneh-Boyen HIBE scheme
  - The size of secret key is $O(l^2 \log N)$, where l is user's level.
  - We proved that the proposed scheme satisfies a weaker security notion such as *selective* security notion.

Further Study

- Fully secure RHIBE
- Different revocation method, such as *Subset Difference*
- Revocation methodology in functional encryption

- Thanks!

Security in knowledge