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A ROBUST AND PLAINTEXT-AWARE VARIANT OF SIGNED ELGAMAL ENCRYPTION

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ELGAMAL ENCRYPTION & BASIC CONCEPTS

CDH / DDH

- Computational Diffie-Hellman Assumption :

G generator of finite cyclic group $\mathbb{G}$. For all efficient algorithms $\mathcal{A}$:

$$\Pr\left(\mathcal{A}(G^a, G^b) = G^{ab}\right) \text{ is at most negligible.}$$

- Decisional Diffie-Hellman Assumption :

G generator of finite cyclic group $\mathbb{G}$. For all efficient algorithms $\mathcal{A}$:

$$|\Pr\left(\mathcal{A}(G^a, G^b, C) = 1\right) - \Pr\left(\mathcal{A}(G^a, G^b, G^{ab}) = 1\right)|$$

is at most negligible.

ELGAMAL PUBLIC KEY ENCRYPTION SCHEME [ElGamal84]

- G is the generator of some finite cyclic group $\mathbb{G}$ of prime order p.

$\underline{\mathtt{Encryption}}$

$$\left[\ \mathtt{pk} : X = G^x\ \right]$$

$$\begin{cases} r \leftarrow_\$ \mathbb{Z}_p^* \\ R \leftarrow G^r \\ Y \leftarrow M X^r \end{cases}$$

$$\boxed{\mathtt{Output} : \psi = (Y, R)}$$

$\underline{\mathtt{Decryption}}$

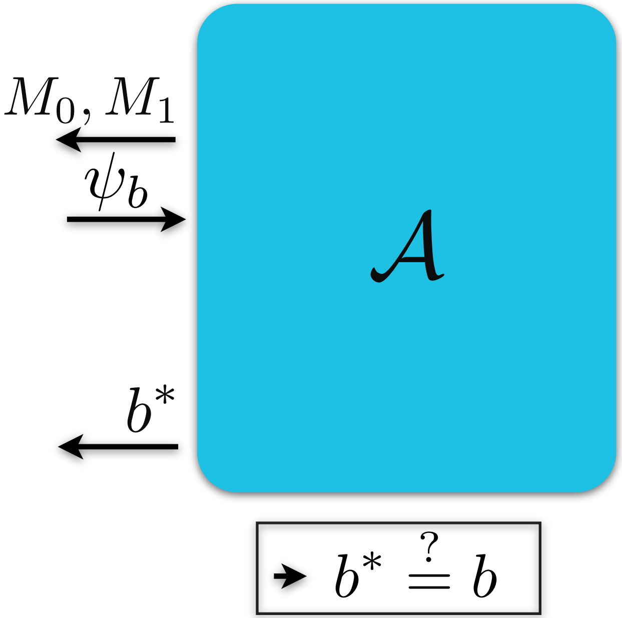
$$\left[\ \mathtt{pk} : X = G^x,\ \ \mathtt{sk} : x\ \right]$$

$$\begin{cases} R' \leftarrow R^x \end{cases}$$

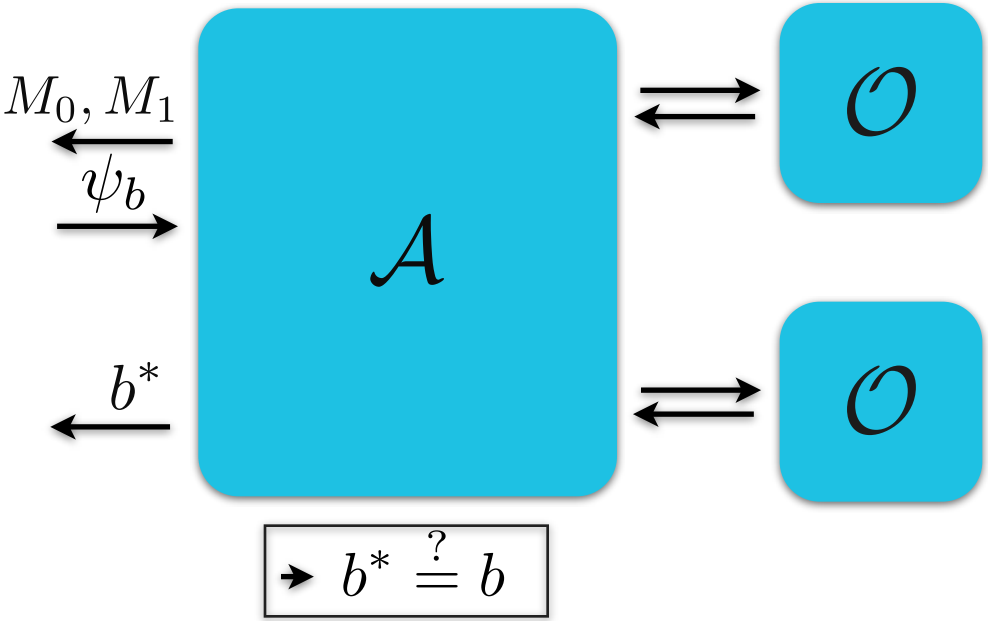
$$\boxed{\mathtt{Output} : M = Y/R'}$$

IND-CPA / IND-CCA2

- $\underline{\texttt{IND} - \texttt{CPA Game}}$



- $\underline{\texttt{IND} - \texttt{CCA2 Game}}$



IND-CCA2 security: **strongest** security notion.

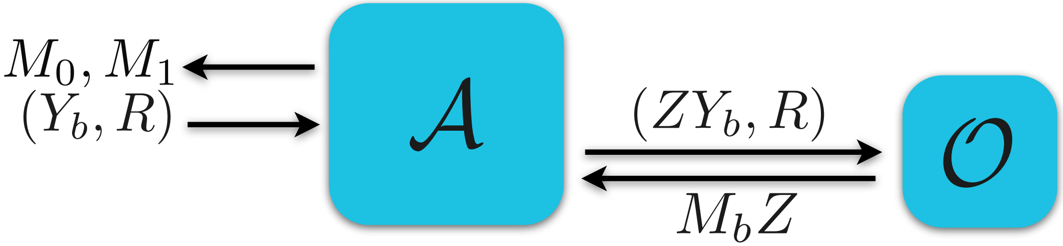
SECURITY OF ELGAMAL

[TsiounisYung98]

ElGamal is **IND-CPA** under **DDH**.

ElGamal is ***not* IND-CCA2.**

- **It is malleable**: it's easy to transform a ciphertext into another one that decrypts to a related plaintext.
- An **IND-CCA2 attacker** can ask for a decryption to win the game.



HOW TO TWEAK ELGAMAL ENCRYPTION TO REACH IND-CCA2 SECURITY?

TWO OPTIONS

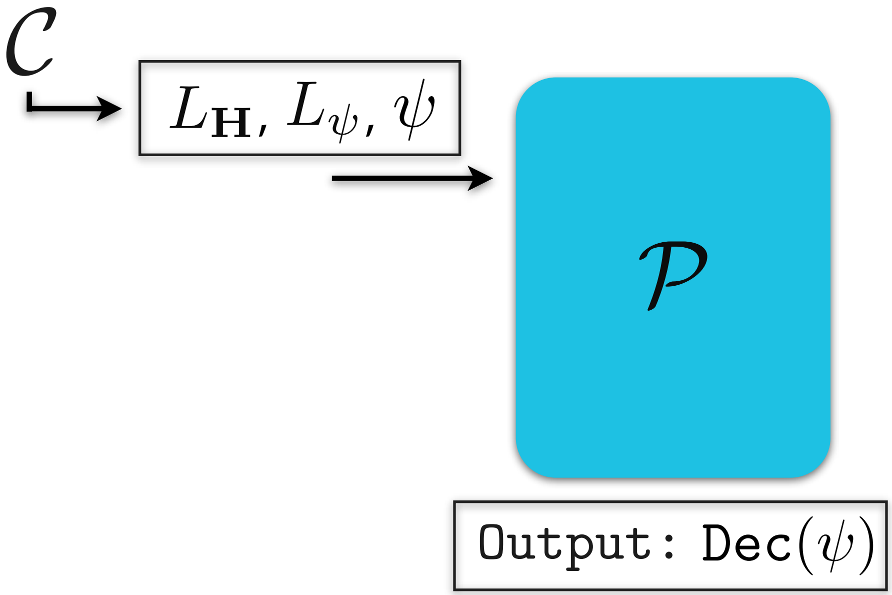
- 1st option: Include hashing [AbdallaBellareRogaway01]
  - *Example*: DHIES - R is **hashed** through some random oracle to create a **symmetric key** used to encrypt the message. $k = H(R, R')$.
  - The resulting scheme is no longer malleable.
  - It requires a **symmetric cipher**.

- 2nd option: Add a non interactive proof of knowledge.
  - *Example*: Schnorr Signed ElGamal (SS-EG) - Add a Schnorr signature as a PoK of r. [Jakobsson98] [TsiounisYung98]

PLAINTEXT-AWARENESS

[BellareRogaway94]
[BelDesaiPointchRog98]

- ROM-PA for a public-key encryption scheme:



$\mathcal{C}$
$L_{\mathbf{H}}, L_{\psi}, \psi$
$\mathcal{P}$
$\texttt{Output: Dec}(\psi)$

$\mathcal{C}$: efficient ciphertext creator.
$\mathcal{P}$: efficient plaintext extractor.

PLAINTEXT-AWARENESS & IND-CCA2 SECURITY

[BelDesaiPointchRog98]

- Intuitively

The **only way** to produce a **valid ciphertext** is to apply the **encryption algorithm** to a public key and a message, rendering a **decryption oracle** available **to an IND-CCA2 adversary useless**.

- PA & IND-CCA2?

$$\begin{gathered}\text{IND-CPA} + \text{PA} \\ \Downarrow \\ \text{IND-CCA2}\end{gathered}$$

LIMITATIONS OF SS-EG

- So...
  - **Adding a Schnorr signature** to ElGamal encryption **would render** the scheme **IND-CCA2**.
- However...
  - ***No proof*** that SS-EG reaches **IND-CCA2** security in the ROM;
  - SS-EG is ***not* plaintext-aware**. [SeurinT13]

SCHNORR SIGNED ELGAMAL ENCRYPTION

Encryption

$[\ \mathtt{pk} : X = G^x\ ]$

$$\begin{cases} r, a \leftarrow_\$ \mathbb{Z}_p^* \\ R \leftarrow G^r,\ A \leftarrow G^a \\ Y \leftarrow M X^r \\ c \leftarrow \mathbf{H}(Y, R, A) \\ s = a + cr \end{cases}$$

$\boxed{\mathtt{Output} : \psi = (Y, R, s, c)}$

Decryption

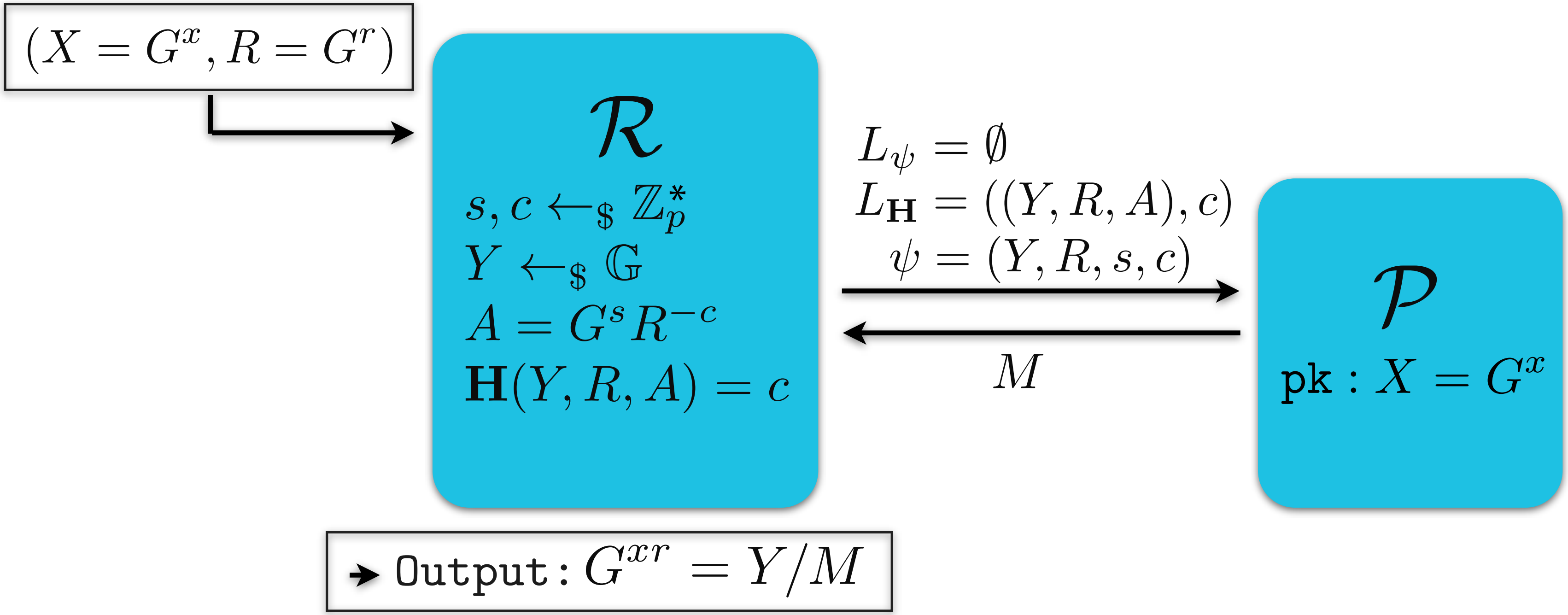
$[\ \mathtt{pk} : X = G^x \quad \mathtt{sk} : x\ ]$

$$\begin{cases} R' \leftarrow R^x \\ A \leftarrow G^s R^{-c} \\ c \overset{?}{=} \mathbf{H}(Y, R, A) \end{cases}$$

$\boxed{\mathtt{Output} : M = Y/R'}$

SS-EG IS NOT PLAINTEXT AWARE [SeurinT13]

- Sketch of the proof:
  - **Assume it is ROM-PA** secure and build a **reduction that solves the CDH problem**.



$(X = G^x, R = G^r)$

$\mathcal{R}$

$s, c \leftarrow_\$ \mathbb{Z}_p^*$

$Y \leftarrow_\$ \mathbb{G}$

$A = G^s R^{-c}$

$\mathbf{H}(Y, R, A) = c$

$L_\psi = \emptyset$

$L_{\mathbf{H}} = ((Y, R, A), c)$

$\psi = (Y, R, s, c)$

M

$\mathcal{P}$

$\mathrm{pk} : X = G^x$

$\texttt{Output} : G^{xr} = Y/M$

CPS-EG: A NEW VARIANT OF SIGNED ELGAMAL ENCRYPTION

Chaum Pedersen Signed ElGamal

[SeurinT13]

Add a CP signature as a PoK of a DL equality : $\log_G(R)=\log_X(R')$.

- CPS-EG - Definition

Encryption

$[\ \texttt{pk} : X = G^x\ ]$

$$\begin{cases} r, a \leftarrow_{\$} \mathbb{Z}_p^* \\ R \leftarrow G^r, \ A \leftarrow G^a, \ A' \leftarrow X^a \\ Y \leftarrow M X^r \\ c \leftarrow \mathbf{H}(Y, R, R' = X^r, A, A') \\ s = a + cr \end{cases}$$

$\boxed{\texttt{Output} : \psi = (Y, R, A, s)}$

Decryption

$[\ \texttt{pk} : X = G^x \quad \texttt{sk} : x\ ]$

$$\begin{cases} R' \leftarrow R^x, \ A' \leftarrow A^x \\ c \leftarrow \mathbf{H}(Y, R, R', A, A') \\ G^s \overset{?}{=} A R^c \\ X^s \overset{?}{=} A' R'^c \end{cases}$$

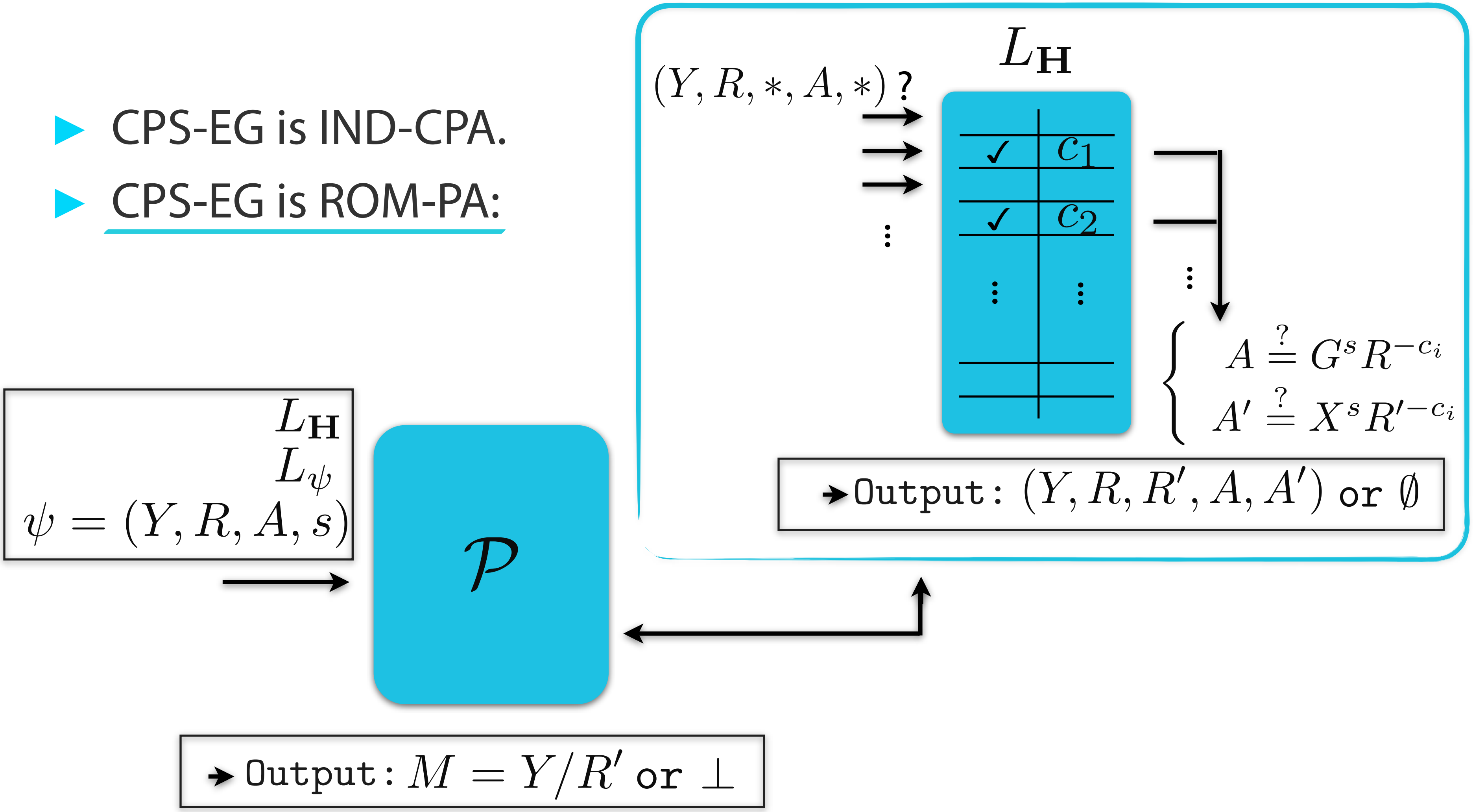
$\boxed{\texttt{Output} : M = Y/R'}$

COMPARISON - EFFICIENCY

Scheme	pub., sec. key size	exponen./ encryption	exponen./ decryption	ciphertext size
ElGamal	G, p	2	1	2G
SS-EG	G, p	3	2	2G+2p
TDH-1 [ShoupGen02]	G, p	3 (+ 2 online)	3	3G+2p
TDH-2	G, 2p	5	3	3G+2p
CPS-EG	G, p	4	4	3G+p

CPS-EG REACHES IND-CCA2 SECURITY [SeurinT13]

- CPS-EG is IND-CPA.
- CPS-EG is ROM-PA:



$L_{\mathbf{H}}$

$(Y, R, *, A, *)$?

✓	c_1
✓	c_2
$\vdots$	$\vdots$

$$\begin{cases} A \overset{?}{=} G^s R^{-c_i} \\ A' \overset{?}{=} X^s R'^{-c_i} \end{cases}$$

→ Output: (Y, R, R', A, A') or $\emptyset$

$L_{\mathbf{H}}$

L_ψ

$\psi = (Y, R, A, s)$

$\mathcal{P}$

→ Output: $M = Y/R'$ or $\perp$

CAUTION

- **Caution** when relying on Chaum & Pedersen's signature:
    - If one uses the **pair (s,c)** in the ciphertext **instead of (A,s)** the scheme does ***not* remain IND-CPA**. [Poettering]
        - $c = \mathbf{H}(Y, R, R', A, A'),\ \ R' = Y/M_b.$

          Part of the ciphertext

          M_0, M_1 known to the attacker.
        - The attacker **tries both values** for R'.
        - A' is deduced from R', s, c and X: $A' = X^s R'^{-c}$.
        - It simply needs to **test the two hash queries** corresponding to the two possible values for R'.
    - This defect is avoided when using the pair (A,s). [SeurinT13-Full]

CPS-EG IS ANONYMOUS & STRONGLY ROBUST

- Anonymity [BellareBoldyrevaDesaiPointcheval01]
  - A **ciphertext** does **not reveal** the **public key** under which it was created.

CPS-EG is ANON-CCA2 under the DDH.

- Strong Robustness [AbdallaBellareNeven10]
  - Hard to create a **ciphertext** that decrypts to a valid plaintext under **2 different secret keys**.

CPS-EG is SROB-CCA when **H** is a collision-resistant hash function.

COMPARISON - SECURITY

Scheme	IND-CCA2 under	ANON+SROB
ElGamal	(CPA under DDH)	✗
SS-EG	~ GGM	✗
TDH-1	DDH	✗
TDH-2	DDH	✗
CPS-EG	DDH	✓

TO SUM UP

- New variant of signed ElGamal encryption : CPS-EG
  - IND-CCA2-secure (plaintext-aware) under DDH assumption;
  - Reaches CCA2-anonymity and strong robustness.
- There exists a hybrid version of CPS-EG.
- Full (and *revised*) version of this paper:
  - Eprint #2012/649. [SeurinT13-Full]

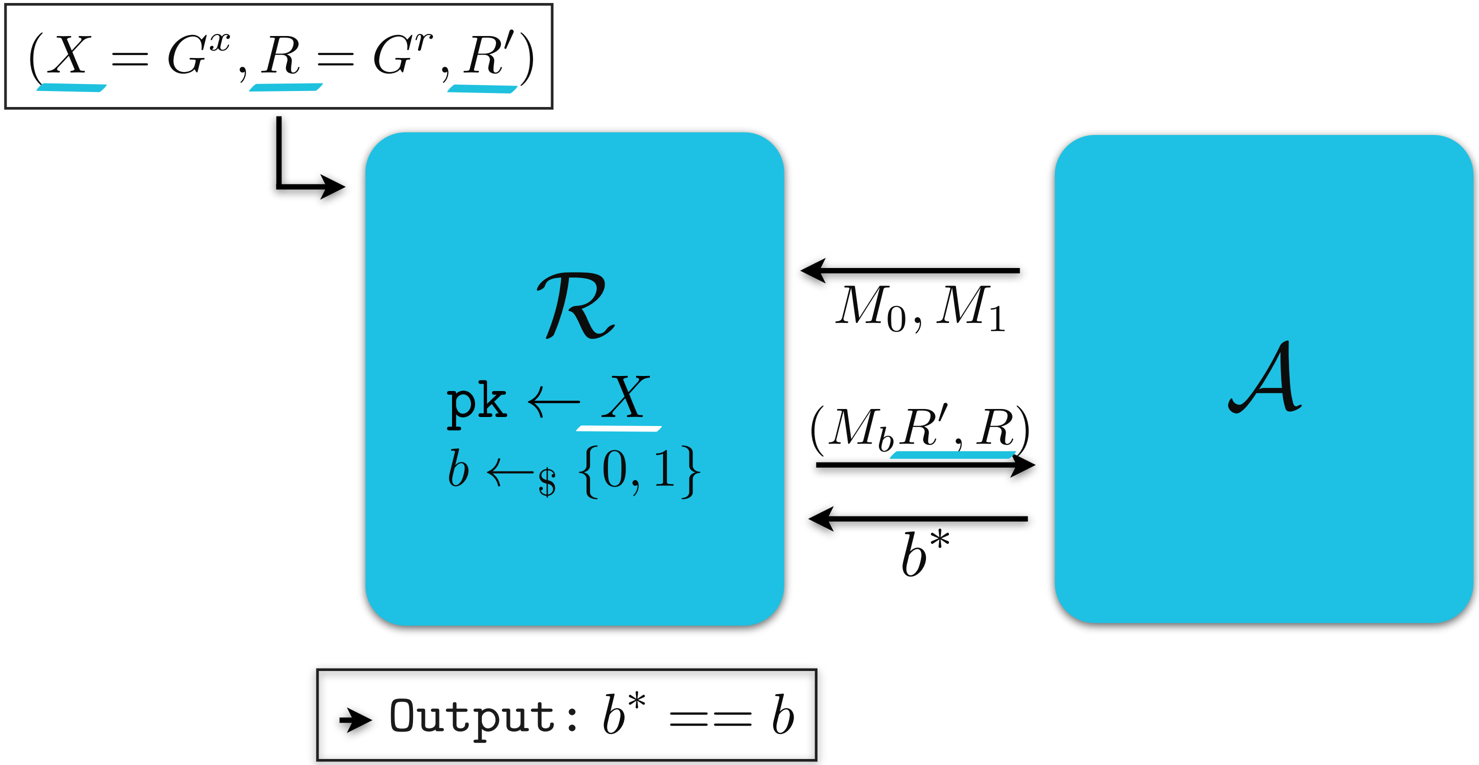
THANKS

ADDITIONAL SLIDES

ELGAMAL

ELGAMAL: IND-CPA UNDER DDH

- Sketch of the proof: [TsiounisYung98]
  - **Reduction** solving the **DDH** prob. from an **IND-CPA attacker**.



$(\underline{X} = G^x, \underline{R} = G^r, \underline{R'})$

$\mathcal{R}$

$\mathrm{pk} \leftarrow \underline{X}$

$b \leftarrow_{\$} \{0,1\}$

M_0, M_1

$(M_b \underline{R'}, R)$

b^*

$\mathcal{A}$

$\texttt{Output:}\ b^* == b$

DHIES

1ST OPTION: HASHED VARIANT

- Example: DHIES [AbdallaBellareRogaway01]

Encryption

$$\left[\ \mathtt{pk} : X = G^x\ \right]$$

$$\begin{cases} r \leftarrow_\$ \mathbb{Z}_p \\ R \leftarrow G^r,\ R' = X^r \\ k = H(R, R') \end{cases}$$

$$\boxed{\mathtt{Output} : \psi = (R, c = \mathrm{E}_k(M))}$$

Decryption

$$\left[\ \mathtt{pk} : X = G^x \quad \mathtt{sk} : x\ \right]$$

$$\begin{cases} R' \leftarrow R^x \\ k = H(R, R') \end{cases}$$

$$\boxed{\mathtt{Output} : \ M = \mathrm{D}_k(c)}$$

- The resulting scheme is no longer malleable.
- It requires a **symmetric cipher**.

SECURITY OF DHIES

- Strong Diffie-Hellman Assumption

It is hard to compute DH(Y,X), even when additional access to a decision oracle is given, which on input (Z,T) returns 1 if DH(X,Z)=T and 0 otherwise.

LEMMA

CPS-EG - USEFUL LEMMA

Fix $X = G^x$, $x \in \mathbb{F}_p$. Let $R = G^r, R', A = G^a, A' \in \mathbb{G}$ be four group elements such that $r, a \neq 0 \mod p$, and $R' \neq R^x$ or $A' \neq A^x$.
Then there is at most one integer $c \in \mathbb{Z}_p$ such that there exists $s \in \mathbb{Z}_p$ satisfying both $G^s = AR^c$ and $X^s = A'R'^c$.

NO PROOF OF IND-CCA2 SECURITY FOR SS-EG

SCHNORR SIGNATURES & FORKING LEMMA

- Schnorr signatures are not *online-extractable*
  - Suppose **there exists a forger** and build a **reduction** that **solves the DL problem**.
  - The reduction **has to rewind the forger** to obtain two signatures of the same message under the same public key.

$$R = G^s Y^{-c} = G^{s'} Y^{-c'}$$
$$\mathrm{dlog}(Y) = \frac{s - s'}{c - c'}$$

NO PROOF THAT SS-EG IS IND-CCA2

- IND-CCA2 security & SS-EG
  - Schnorr signatures are **not online-extractable**.
  - Setting: We consider a(ny) reduction that would use an IND-CCA2 adversary.
  - The **reduction has to answer** hash queries and decryption queries.
  - **Possible** for the adversary to construct a **list of specific ciphertexts and hash queries**, where one ciphert./h. query is obtained **from the previous ones**.
  - If the attacker then sends its list of queries to the reduction, in the exact **reverse order**, then the reduction has to **rewind** the forger **exponentially**.

ROBUSTNESS AND ANONYMITY

WEAK ROBUSTNESS AND STRONG ROBUSTNESS

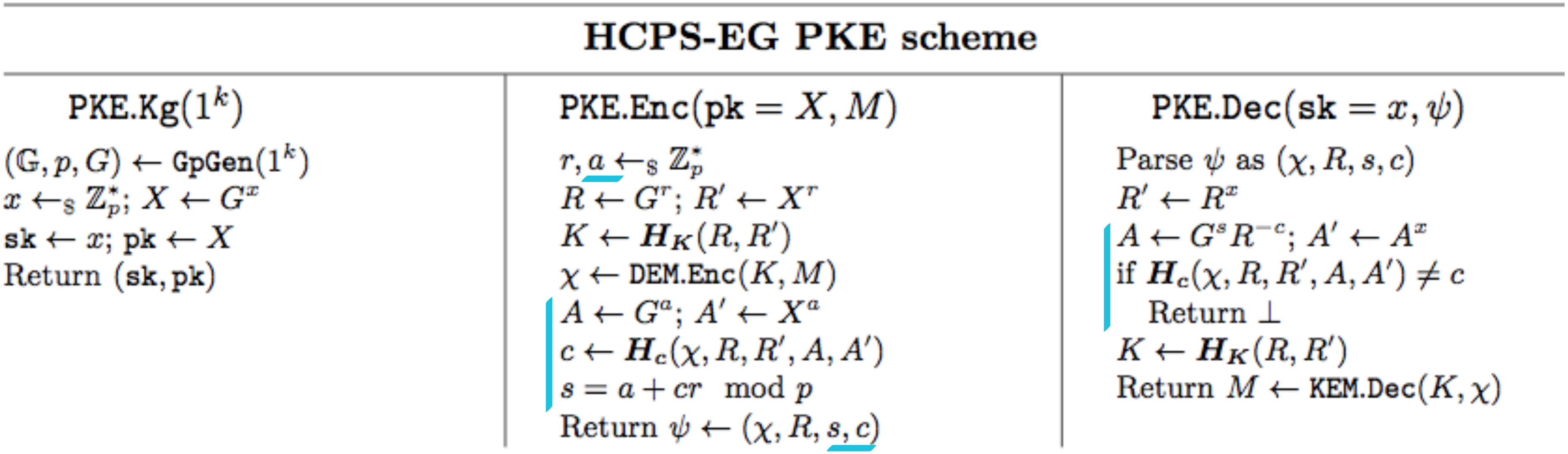
- Strong and Weak Robustness [AbdallaBellareNeven10]
  - *Definition* [Weak robustness]: Hard to create a **ciphertext** that decrypts under **2 different secret keys**.
  - *Definition* [Strong Robustness]: Hard to produce a **plaintext** that, **once encrypted** under some public key, decrypts to a **valid plaintext under another secret key**.
- Robustness can always be achieved
  - How? Append public key of the receiver to the ciphertext.

ROBUSTNESS COMBINED WITH ANONYMITY

- Robustness:
  - *Definition* [Robustness]: Hard to create a **ciphertext** that decrypts under **2 different secret keys**.
- Anonymity:
  - *Definition* [Anonymity]: A **ciphertext** does **not reveal** the **public key** under which it was created.
- Anonymity + Robustness:
  - Help make encryption **more resistant to misuse**.
  - *Ex*. **Anonymously** send a ciphertext to a **particular target of a larger group**. **Who**'s the target? Solution: **Robustness**.
  - **First solution** for robustness **not ok**.

HYBRID VARIANT HCPS-EG

HYBRID VERSION



HCPS-EG PKE scheme

```
$\mathtt{PKE.Kg}(1^k)$
$(\mathbb{G}, p, G) \leftarrow \mathtt{GpGen}(1^k)$
$x \leftarrow_\$ \mathbb{Z}_p^*;\ X \leftarrow G^x$
$\mathbf{sk} \leftarrow x;\ \mathbf{pk} \leftarrow X$
Return $(\mathbf{sk}, \mathbf{pk})$
```

```
$\mathtt{PKE.Enc}(\mathbf{pk} = X, M)$
$r, \underline{a} \leftarrow_\$ \mathbb{Z}_p^*$
$R \leftarrow G^r;\ R' \leftarrow X^r$
$K \leftarrow \boldsymbol{H_K}(R, R')$
$\chi \leftarrow \mathtt{DEM.Enc}(K, M)$
$A \leftarrow G^a;\ A' \leftarrow X^a$
$c \leftarrow \boldsymbol{H_c}(\chi, R, R', A, A')$
$s = a + cr \mod p$
Return $\psi \leftarrow (\chi, R, \underline{s, c})$
```

```
$\mathtt{PKE.Dec}(\mathbf{sk} = x, \psi)$
Parse $\psi$ as $(\chi, R, s, c)$
$R' \leftarrow R^x$
$A \leftarrow G^s R^{-c};\ A' \leftarrow A^x$
if $\boldsymbol{H_c}(\chi, R, R', A, A') \neq c$
    Return $\perp$
$K \leftarrow \boldsymbol{H_K}(R, R')$
Return $M \leftarrow \mathtt{KEM.Dec}(K, \chi)$
```

PROPERTIES OF HCPS-EG

- IND-CCA2 secure in the ROM under CDH.
  - Using the weakest form of security for the data encapsulation mechanism (DEM): ciphertext indistinguishability under one-time attacks (IND-OT).
  - *Example*: With AES in counter mode.
- Anonymous against CCA2-attacks under CDH.
- Strongly robust.

Efficient Public Key Cryptosystem Resilient to Key Leakage Chosen Ciphertext Attacks

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[2]Jinan University(cryptjweng@gmail.com)
[3]Fudan University(ylzhao@fudan.edu.cn)

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Introduction and Motivation

Introduction to leakage-resilient PKE

- IND-CCA2 security assumes that the secret key sk is totally randomly distributed, i.e., $Entropy(sk) = |sk|$.

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- Side-channel attack, i.e., "memory attacks", may leak information about the secret key sk, i.e., $Entropy(sk) \leq |sk|$.
- Leakage-resilient public key encryption (PKE) schemes are designed to resist key leakage.
- We will consider IND-CCA2 security in **bounded key-leakage model**, where the total amount of leaked information about the key is bounded by some parameter λ (bits).

Related Works

- In 2005, Akavia et.al.[AGV09] showed that the lattice-based PKE scheme proposed by Regev [R05] is resilient to any leakage of $L/\text{polylog}(L)$ bits, with L the bit length of the secret key.

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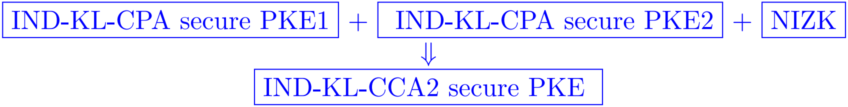

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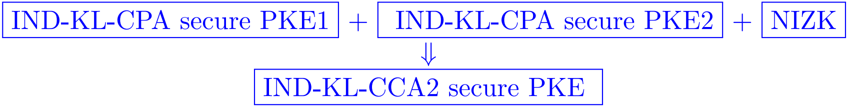

IND-KL-CPA secure PKE1 + IND-KL-CPA secure PKE2 + NIZK

⇓

IND-KL-CCA2 secure PKE

The key leakage λ is also up to $L(1 - o(1))$.

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IND-KL-CPA secure PKE + strong f-*true-simulation extractable* NIZK

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$$\Downarrow$$

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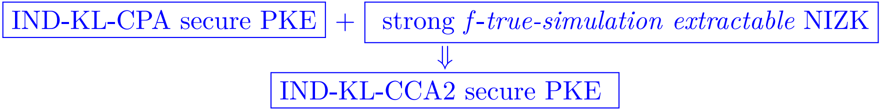
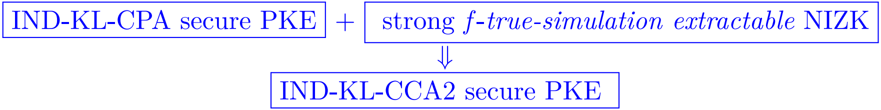


IND-KL-CPA secure PKE + strong f-*true-simulation extractable* NIZK

$\Downarrow$

IND-KL-CCA2 secure PKE

- In 2010, Dodis et.al. [DHLW10] exploited more efficient ways to achieve IND-KL-CCA2 security.
- They proposed a new concept of *true-simulation extractable* NIZK arguments, and gave a construction of IND-KL-CCA2-secure PKE from an IND-KL-CPA secure one together with a strong f-*true-simulation extractable* NIZK argument. The key leakage λ is also up to $L(1 - o(1))$.



IND-KL-CPA secure PKE + strong f-*true-simulation extractable* NIZK

$\Downarrow$

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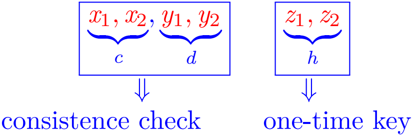

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Encryption	Decryption
$u_1 = g_1^r, u_2 = g_2^r, r \in \mathbb{Z}_q^*, s \in \{0,1\}^t$;	$\alpha = T(u_1, u_2, e, s)$;
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Output (u_1, u_2, s, e, v).	Output M.

Figure: The KL-CS-PKE Scheme by Naor-Segev.

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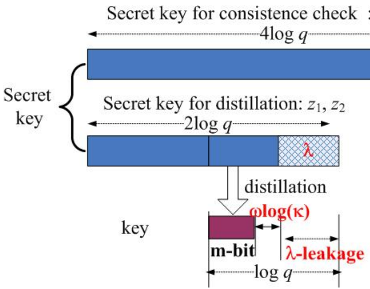
- It uses extractors to deal with key leakage.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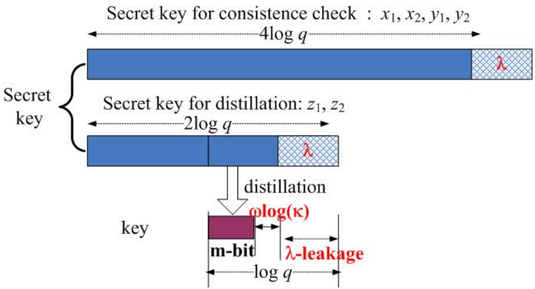
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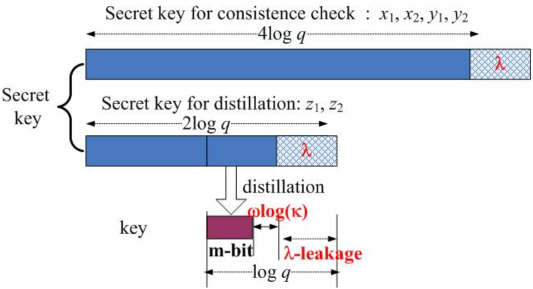
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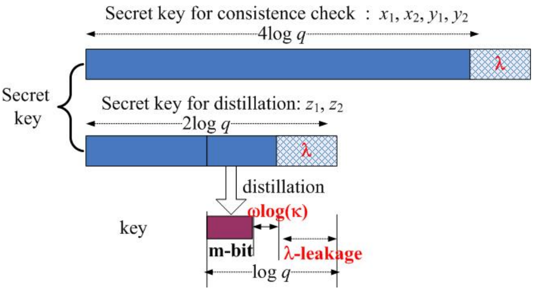
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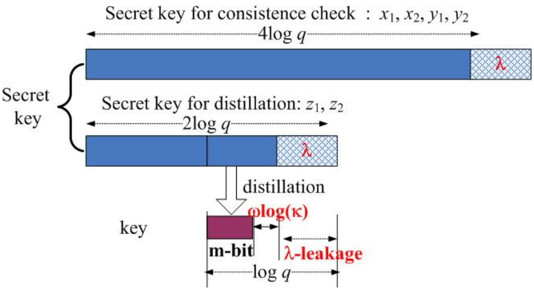
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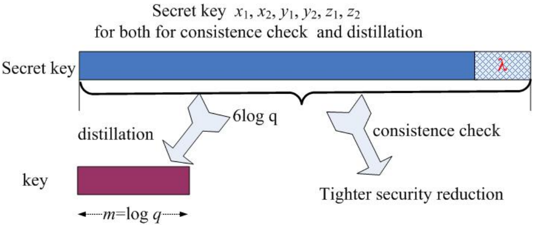
Naor and Segev noted this and called for further refinement.

Our contributions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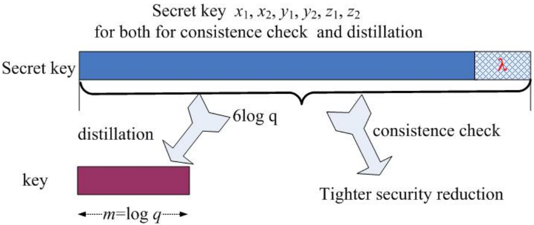

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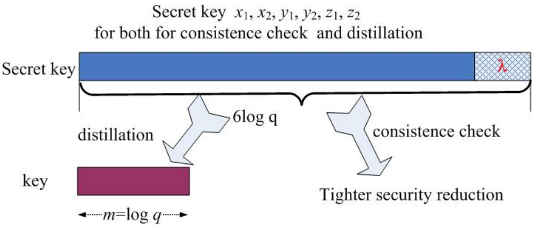

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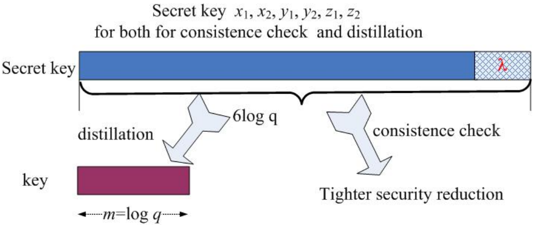

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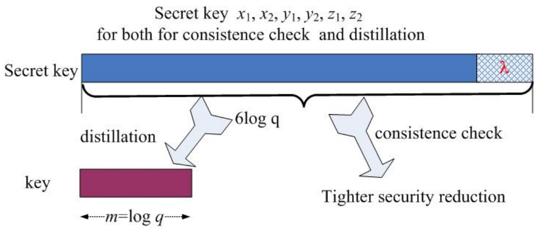
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    - The security reduction is tighter than that of KL-CS-PKE [NS09].

Preliminaries

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$$H_{k_1,k_2,\cdots,k_l}(g_0, g_1, \cdots, g_l) = g_0 \cdot g_1^{k_1} \cdot \ldots \cdot g_l^{k_l} (= g^{x_0 + k_1 x_1 + \cdots + k_l x_l}),$$

with $g_i = g^{x_i}$ for $i = 0, 1, \cdots, l$.

Leftover Hash Lemma[Dodis08]

Lemma

Assume $\{H_k : \mathcal{X} \to \mathcal{Y}\}_{k \in \mathcal{K}}$ *is a family of universal hash functions. For any random variables* $X \in \mathcal{X}$, $Z \in \mathcal{Z}$, *and* $K \leftarrow \mathcal{K}$,

$$SD((H_k(X), K), (U_{\mathcal{Y}}, K)) \leq \frac{1}{2}\sqrt{P_c(X)|\mathcal{Y}|} \leq \frac{1}{2}\sqrt{2^{-\boldsymbol{H}_\infty(X)}|\mathcal{Y}|},$$

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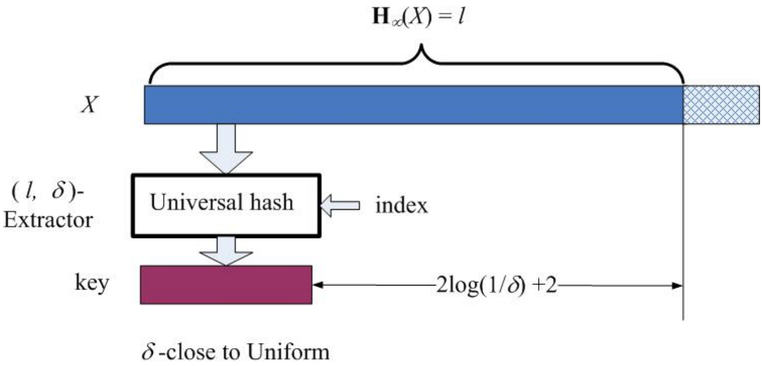
$$SD((H_k(X), K, Z), (U_{\mathcal{Y}}, K, Z)) \leq \frac{1}{2}\sqrt{2^{-\tilde{\boldsymbol{H}}_\infty(X|Z)}|\mathcal{Y}|},$$

where $U_{\mathcal{Y}}$ *denotes a uniform distribution over* $\mathcal{Y}$.

Extractors with Universal Hashing [Dodis08]



IND-KL-CCA2 Security

KL-CCA2 Game: *Adversary* $\mathcal{A}$ v.s. its *environment*

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$$\mathsf{Adv}_{\mathcal{A}} = |\Pr[\sigma = \hat{\sigma}] - 1/2|.$$

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$$\mathsf{PK} = (\mathbb{G}, q, g, \mathsf{T}, c, d, h), \mathsf{SK} = (x_1, x_2, y_1, y_2, z_1, z_2).$$

Overview of KL-CS-PKE by Naor and Segev [CRYPTO09]

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Encryption	Decryption
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$e = M \oplus \mathsf{Ext}(h^r, s)$;	If $v \neq u_1^{x_1 + y_1\alpha} u_2^{x_2 + y_2\alpha}$, output $\perp$;
$\alpha = T(u_1, u_2, e, s)$; $\quad v = c^r d^{r\alpha}$;	otherwise $M = e \oplus \mathsf{Ext}(u_1^{z_1} u_2^{z_2}, s)$;
Output (u_1, u_2, s, e, v).	Output M.

Figure: The KL-CS-PKE Scheme by Naor-Segev.

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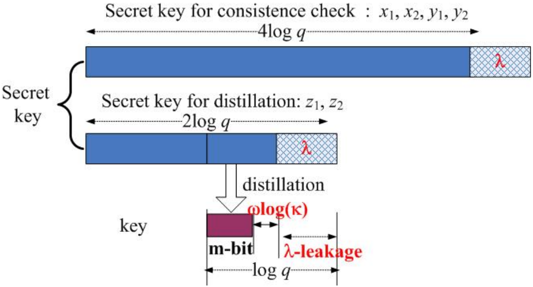

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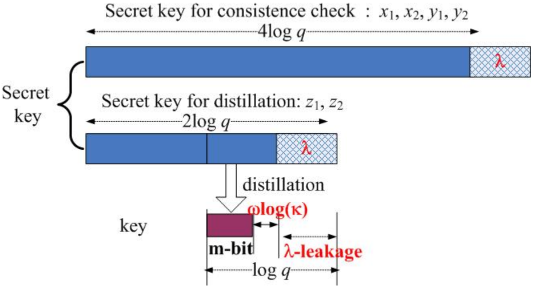
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λ
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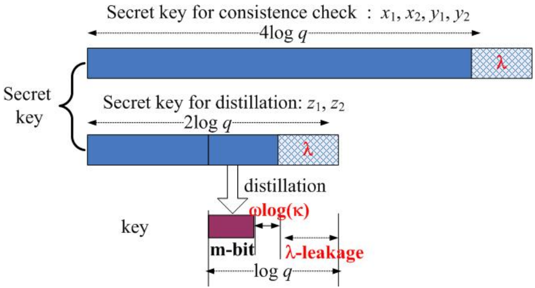
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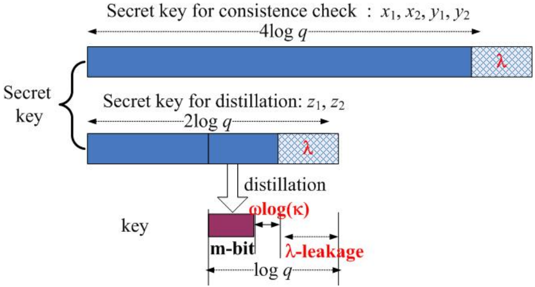
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- $m \leq \log_2 q - \lambda - \omega(\log \kappa)$.

Our Proposal

New Idea and Key Observation

- The new idea is that all the three parts of secret key, namely (x_1, x_2), (y_1, y_2) and (z_1, z_2), are involved both in the ciphertext consistence check and the random distillation to mask plaintexts.

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    - On the other hand, we use a special universal hash function (i.e., $H_s(a, b) = a \cdot b^s$, for $a, b \in \mathbb{G}$ and $s \in \mathbb{Z}_q^*$) as an extractor, where $a = (cd)^r$ and $b = h^r$ for $r \in \mathbb{Z}_q^*$ with our proposal, which allows plaintext space to be $\mathbb{G}$, and makes the security proof neat and tighter.

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- The actual design of our proposal was also carefully guided by the underlying analysis, particularly for ensuring non-zero matrix determinant.

Overview of Our Proposal

Encryption	Decryption
$u_1 = g_1^r, u_2 = g_2^r, \quad r, s \in \mathbb{Z}_q^*;$	$\alpha = T(u_1, u_2, e, s);$
$\boxed{e = M \cdot (cd)^r \cdot h^{rs};}$	If $\boxed{v \neq u_1^{x_1+y_1\alpha+z_1} u_2^{x_2+y_2\alpha+z_2},}$ outpu
$\alpha = T(u_1, u_2, e, s);$	otherwise
$\boxed{v = (c \cdot h)^r \cdot d^{r\alpha};}$	$\boxed{M = e \cdot u_1^{-(x_1+y_1+z_1 s)} u_2^{-(x_2+y_2+z_2}}$
Output (u_1, u_2, s, e, v)	Output $M.$

Figure: Our proposal

Proof Outline

Main Theorem

Theorem

The above scheme is $(\log q, \lambda, \epsilon)$*-IND-KL-CCA2 secure public key encryption scheme. Here* q *is the prime order of the group* $\mathbb{G}$ *that* PKE *is based on,* $\lambda \leq \log q - \omega(\log \kappa)$ *(more precisely,* $\lambda \leq \log q - 2\log\frac{1}{\delta} + 2$ *where* $\delta = \frac{2^{\lambda/2-1}}{\sqrt{q}}$*)) and*

$$\epsilon \leq \mathbf{Adv}^{DDH}(\kappa) + \mathbf{Adv}^{TCR}(\kappa) + \frac{2^{\lambda} Q(\kappa)}{q - Q(\kappa)} + \frac{2^{\lambda/2-1}}{\sqrt{q}},$$

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- Plaintext length $m = \log q$;
- Leakage parameter $\lambda = \log q - o(1)$.

Security Proof

We proceed with a series of games played between a simulator $\mathcal{D}$ and an adversary $\mathcal{A}$, and show that Game i and Game $i+1$ are indistinguishable except with negligible probability, $i = 0, 1, 2, 3, 4, 5$. We define S_i as the event that the adversary $\mathcal{A}$ outputs a correct guess of b.

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The change is only conceptual, and thus $\Pr[S_1] = \Pr[S_0]$.

Game 2: Same as Game 1 except for the generation of the challenge ciphertext $C^* = (u_1^*, u_2^*, e^*, s^*, v^*)$, where $u_1^* = g_1^{r_1^*}, u_2^* = g_2^{r_2^*}$, with r_1^*, r_2^* chosen uniformly at random from $\mathbb{Z}_q^*$.

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By the DDH assumption, $|\Pr[S_2] - \Pr[S_1]|$ is negligible.

Game 3: Same as Game 2 except that the simulator applies a special rejection rule. Let F denote the event that there exists a decryption query of the form $C = (u_1, u_2, e, s, v)$ such that $C \neq C^*$ but $T(u_1, u_2, e, s) = T(u_1^*, u_2^*, e^*, s^*)$, which means a hash collision occurs. The simulator $\mathcal{D}$ rejects the corresponding queried ciphertext C when F occurs.

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By the TCR property of T, $|\Pr[S_3] - \Pr[S_2]|$ is negligible.

Game 4: Same as Game 3, except that the simulator applies a special rejection rule. If $\mathcal{A}$ asks for decryption of an invalid ciphertext $C = (u_1, u_2, e, s, v)$, i.e., (g_1, g_2, u_1, u_2) is not a DDH tuple, the simulator $\mathcal{D}$ rejects with $\perp$ and the game aborts.

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Let F' be the event that $\mathcal{D}$ outputs $\perp$ for a consistent but valid ciphertext, we have:
$|\Pr[S_4| \neq F'] - \Pr[S_3| \neq F']| \leq \Pr[F']$,
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Below, we analyze the probability that the event F' occurs.

Analyzing the Event F'

Let $\beta = \log_{g_1} g_2$. Firstly, note that by submitting valid ciphertexts to the decryption oracle, the adversary only learns linear combinations of the constraints $\log_{g_1} c = x_1 + \beta x_2$, $\log_{g_1} d = y_1 + \beta y_2$ and $\log_{g_1} h = z_1 + \beta z_2$, which are already known from the public-keys. Also note that $q, g_1, g_2, T, u_1^*, u_2^*, s^*, \sigma$ all are independent of the secret key.

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$$\begin{array}{rcl}
\log_{g_1} c & = & x_1 + \beta x_2 \\
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\log_{g_1} h & = & z_1 + \beta z_2 \\
\log_{g_1} v^* & = & r_1^* x_1 + r_2^* \beta x_2 + \alpha^* r_1^* y_1 + \alpha^* r_2^* \beta y_2 + r_1^* z_1 + r_2^* \beta z_2 \\
\log_{g_1} e^*/M_b & = & r_1^* x_1 + r_2^* \beta x_2 + r_1^* y_1 + r_2^* \beta y_2 + s^* r_1^* z_1 + s^* r_2^* \beta z_2; \\
 & & \lambda\text{-bit leakage of } (x_1, x_2, y_1, y_2, z_1, z_2).
\end{array}$$

As the secret key elements $(x_1, x_2, y_1, y_2, z_1, z_2)$ are uniformly chosen from $\mathbb{Z}_q^6$, according to the average min-entropy theory, we have

$$\begin{aligned}
& \tilde{\mathrm{H}}_\infty((x_1, x_2, y_1, y_2, z_1, z_2)|c, d, h, C^*, M_b, \lambda\text{-leakage}) \\
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\end{aligned}$$

Let $C = (u_1, u_2, e, s, v)$ be the first invalid ciphertext submitted by $\mathcal{A}$. Let $r_1 = \log_{g_1} u_1$ and $r_2 = \log_{g_1} u_2$, then $r_1 \neq r_2$. We must have:

$$\begin{array}{rcl}
\log_{g_1} c & = & x_1 + \beta x_2; \\
\log_{g_1} d & = & y_1 + \beta y_2; \\
\log_{g_1} h & = & z_1 + \beta z_2; \\
\log_{g_1} v^* & = & r_1^* x_1 + r_2^* \beta x_2 + \alpha^* r_1^* y_1 + \alpha^* r_2^* \beta y_2 + r_1^* z_1 + r_2^* \beta z_2; \\
\log_{g_1} e^*/M_b & = & r_1^* x_1 + r_2^* \beta x_2 + r_1^* y_1 + r_2^* \beta y_2 + s^* r_1^* z_1 + s^* r_2^* \beta z_2; \\
\log_{g_1} v & = & r_1 x_1 + r_2 \beta x_2 + \alpha r_1 y_1 + \alpha r_2 \beta y_2 + r_1 z_1 + r_2 \beta z_2.
\end{array} \quad (3)$$

Let $\mathbf{A}=\begin{pmatrix} 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \beta \\ r_1^* & r_2^*\beta & \alpha^* r_1^* & \alpha^* r_2^*\beta & r_1^* & r_2^*\beta \\ r_1^* & r_2^*\beta & r_1^* & r_2^*\beta & s^* r_1^* & s^* r_2^*\beta \\ r_1 & r_2\beta & \alpha r_1 & \alpha r_2\beta & r_1 & r_2\beta \end{pmatrix}$. This is distilled into:

$$\begin{pmatrix} 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \beta \\ r_1^* & r_2^*\beta & \alpha^* r_1^* & \alpha^* r_2^*\beta & r_1^* & r_2^*\beta \\ r_1^* & r_2^*\beta & r_1^* & r_2^*\beta & s^* r_1^* & s^* r_2^*\beta \\ r_1 & r_2\beta & \alpha r_1 & \alpha r_2\beta & r_1 & r_2\beta \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \log_{g_1}\ c \\ \log_{g_1}\ d \\ \log_{g_1}\ h \\ \log_{g_1}\ v^* \\ \log_{g_1}\ e^*/M_b \\ \log_{g_1}\ v \end{pmatrix}. \tag{4}$$

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The determinant of matrix $\mathbf{A}$ is given by

$$det(\mathbf{A}) = \beta^3(r_1^* - r_2^*)^2(r_1 - r_2)(\alpha^* - \alpha)(s^* - 1).$$

Then $det(\mathbf{A}) \neq 0$ except with a negligible probability $1/q$. Hence Eq. (4) is an injective function.

An injective function preserves its min-entropy. Then

$$\tilde{\mathrm{H}}_\infty((x_1, x_2, y_1, y_2, z_1, z_2)|c, d, h, C^*, M_b, \lambda\text{-leakage})$$

$$= \tilde{\mathrm{H}}_\infty((c, d, h, v^*, e^*/M_b, v)|c, d, h, C^*, M_b, \lambda\text{-leakage})$$

$$= \tilde{\mathrm{H}}_\infty(v|c, d, h, C^*, M_b, \lambda\text{-leakage}) \leq \log q - \lambda.$$

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Similarly, the i-th invalid ciphertext is accepted by $\mathcal{D}$ with probability at most $2^\lambda/(q - i + 1) \leq 2^\lambda/(q - Q(\kappa))$, where $Q(\kappa)$ is the total number of decryption queries.
By the union bound, we have

$$\Pr[F'] \leq \frac{2^\lambda Q(\kappa)}{q - Q(\kappa)}$$

and

$$|\Pr[S_4] - \Pr[S_3]| \leq \Pr[F'] \leq \frac{2^\lambda Q(\kappa)}{q - Q(\kappa)}.$$

Game 5: Same as Game 4 except for the generation of the challenge ciphertext $C^* = (u_1^*, u_2^*, e^*, s^*, v^*)$. The only change is that e^* is replaced with an element $\hat{e}$ chosen uniformly at random from G.

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What left to establish in the rest is to show: $|\Pr[S_5] - \Pr[S_4]|$ is negligible.

$|\Pr[S_5] - \Pr[S_4]|$ Is Negligible

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- Next, we will show that e^*/M_b is in fact the output a $(2\log_2 q - \lambda, \delta)$ extractor with $\underbrace{(u_1^*)^{x_1+y_1}(u_2^*)^{x_2+y_2}}_{a}$ and $\underbrace{(u_1^*)^{z_1}(u_2^*)^{z_2}}_{b}$ as input.

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- Given the information c, d, h, v^* and the λ-bit leakage, we determine the average min-entropy

$$\tilde{\mathrm{H}}_\infty\left(\underbrace{(u_1^*)^{x_1+y_1}(u_2^*)^{x_2+y_2}}_{a}, \underbrace{(u_1^*)^{z_1}(u_2^*)^{z_2}}_{b} \mid c, d, h, v^*, \lambda\text{-leakage}\right).$$

Let us check the following equations in $x_1, x_2, y_1, y_2, z_1, z_2$.

$$\begin{array}{rcl}
\log_{g_1} c & = & x_1 + \beta x_2; \\
\log_{g_1} d & = & y_1 + \beta y_2; \\
\log_{g_1} h & = & z_1 + \beta z_2; \\
\log_{g_1} v^* & = & r_1^* x_1 + r_2^* \beta x_2 + \alpha^* r_1^* y_1 + \alpha^* r_2^* \beta y_2 + r_1^* z_1 + \\
\log_{g_1} \left(\underbrace{(u_1^*)^{x_1+y_1} (u_2^*)^{x_2+y_2}}_{a} \right) & = & r_1^* x_1 + r_2^* \beta x_2 + r_1^* y_1 + r_2^* \beta y_2; \\
\log_{g_1} \left(\underbrace{(u_1^*)^{z_1} (u_2^*)^{z_2}}_{b} \right) & = & r_1^* z_1 + r_2^* \beta z_2.
\end{array} \tag{5}$$

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\log_{g_1} \left(\underbrace{(u_1^*)^{x_1+y_1} (u_2^*)^{x_2+y_2}}_{a} \right) &= r_1^* x_1 + r_2^* \beta x_2 + r_1^* y_1 + r_2^* \beta y_2; \\
\log_{g_1} \left(\underbrace{(u_1^*)^{z_1} (u_2^*)^{z_2}}_{b} \right) &= r_1^* z_1 + r_2^* \beta z_2.
\end{aligned} \tag{5}$$

Equivalently,

$$\underbrace{\begin{pmatrix} 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \beta \\ r_1^* & r_2^* \beta & \alpha^* r_1^* & \alpha^* r_2^* \beta & r_1^* & r_2^* \beta \\ r_1^* & r_2^* \beta & r_1^* & r_2^* \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & r_1^* & r_2^* \beta \end{pmatrix}}_{B} \cdot \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \log_{g_1} c \\ \log_{g_1} d \\ \log_{g_1} h \\ \log_{g_1} v^* \\ \log_{g_1} a \\ \log_{g_1} b \end{pmatrix}. \tag{6}$$

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$$\begin{aligned}
& \tilde{\mathrm{H}}_\infty\left(a, b \mid c, d, h, v^*, \lambda\text{-leakage}\right) && (7)\\
= \; & \tilde{\mathrm{H}}_\infty\left((u_1^*)^{x_1+y_1}(u_2^*)^{x_2+y_2}, (u_1^*)^{z_1}(u_2^*)^{z_2} | c, d, h, v^*, \lambda\text{-leakage}\right) && (8)\\
= \; & \tilde{\mathrm{H}}_\infty((x_1, x_2, y_1, y_2, z_1, z_2) | c, d, h, v^*, \lambda\text{-leakage}) && (9)\\
\geq \; & \tilde{\mathrm{H}}_\infty((x_1, x_2, y_1, y_2, z_1, z_2) | c, d, h, v^*) - \lambda \geq 2\log q - \lambda, . && (10)
\end{aligned}$$

- Applying the universal hash function $H_{s^*} : \mathbb{G} \times \mathbb{G} \to \mathbb{G}$ (i.e, $H_{s^*}(a, b) = a \cdot b^{s^*}$ where $a = (u_1^*)^{x_1+y_1}(u_2^*)^{x_2+y_2}$ and $b = (u_1^*)^{z_1}(u_2^*)^{z_2}$) as a $(2\log_2 q - \lambda, \delta)$ extractor to the two variables $(u_1^*)^{x_1+y_1}(u_2^*)^{x_2+y_2}, (u_1^*)^{z_1}(u_2^*)^{z_2}$, we have

$$\begin{aligned} e^*/M_b &= H_{s^*}\left((u_1^*)^{x_1+y_1}(u_2^*)^{x_2+y_2}, (u_1^*)^{z_1}(u_2^*)^{z_2}\right) & (11)\\ &= (u_1^*)^{x_1+y_1}(u_2^*)^{x_2+y_2}\left((u_1^*)^{z_1}(u_2^*)^{z_2}\right)^{s^*} & (12)\\ &= (u_1^*)^{x_1+y_1+z_1 s^*} \cdot (u_2^*)^{x_2+y_2+z_2 s^*}. & (13) \end{aligned}$$

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- According to the generalized leftover lemma, $SD(e^*, U) \leq \frac{1}{2}\sqrt{q \cdot \frac{2^\lambda}{q^2}} = \frac{2^{\lambda/2-1}}{\sqrt{q}}$.

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- Hence, $|\Pr[S_5] - \Pr[S_4]| \leq \delta = \frac{2^{\lambda/2-1}}{\sqrt{q}}$

Performance

Parameters of CS-PKE, KL-CS-PKE and Our Proposal

Let $\epsilon_1 = \mathbf{Adv}^{DDH}(\kappa), \epsilon_2 = \mathbf{Adv}^{\mathbf{TCR}}(\kappa)$, and $|M|$ denote the plaintext bit-length. Let λ be the amount of leakage bits, and $Q(\kappa)$ be the number of decryption queries.

Scheme	$\|M\|$	leakage	$\mathsf{Adv}^{\mathbf{IND\text{-}KL\text{-}CCA2}}_{\mathsf{PKE},\mathcal{A}}(1^\kappa)$
CS [CS03]	$\log q$	—	$\epsilon_1 + \epsilon_2 + \frac{Q(\kappa)}{q-Q(\kappa)}$
KL-CS [NS09] $(m + \lambda \leq \log q - \omega(\log \kappa))$	m	λ	$\epsilon_1 + \epsilon_2 + \frac{2^\lambda Q(\kappa)}{q-Q(\kappa)} + \frac{2^{(\lambda+m)/2-1}}{\sqrt{q}}$
Ours $(\lambda \leq \log q - \omega(\log \kappa))$	$\log q$	λ	$\epsilon_1 + \epsilon_2 + \frac{2^\lambda Q(\kappa)}{q-Q(\kappa)} + \frac{2^{\lambda/2-1}}{\sqrt{q}}$

Table: Parameters of CS-PKE, KL-CS-PKE and our proposals

Efficiency and Ciphertext Sizes of CS-PKE, KL-CS-PKE and Our Proposal

Scheme	KeyGen	Enc	Dec	Ciphertext Size
CS [CS03]	3 SE	3 Ex +1 S	2 SE	$4\mathbb{G}$
KL-CS [NS09]	3 SE	3 Ex + 1 SE+ 1 Ext	2 SE	$4\mathbb{G} + t$-bit
Ours	3 SE	2 Ex + 2 SE	2 SE	$4\mathbb{G} + \log q$-bit

Table: Efficiency and ciphertext sizes of CS-PKE, KL-CS-PKE and our proposal

Conclusion

- A response to Naor and Segev's calling for further refinement of key leakage resilient variant of Cramer-Shoup Cryptosystem in order to get rid of the dependency between plaintext length m and leakage parameter λ.

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- With some careful observations and a calculation guided design, our proposal follows a new line:
  - The whole secret key is involved in both ciphertext consistence checking and randomness distillation;
  - A special universal hashing based extractor is employed; alternatively, randomness extractor is only implicitly used with our proposal).

- Our scheme is IND-KL-CCA2 secure with a tighter reduction, $\lambda = \log q - \omega(\log \kappa)$ leakage resilient, and the plaintext space is the whole group that the scheme is based on and is independent of the leakage parameter. The performance of our proposal is comparable to the original Cramer-Shoup cryptosystem.

- Our scheme is IND-KL-CCA2 secure with a tighter reduction, $\lambda = \log q - \omega(\log \kappa)$ leakage resilient, and the plaintext space is the whole group that the scheme is based on and is independent of the leakage parameter. The performance of our proposal is comparable to the original Cramer-Shoup cryptosystem.
- To the best of our knowledge, the first leakage-resilient CS-type cryptosystem whose plaintext length is independent of the key leakage parameter, and is also the most efficient IND-CCA2 PKE scheme resilient to up to $\log q - \omega(\log \kappa)$ leakage.

Thanks