On the Practical Security of a Leakage Resilient Masking Scheme

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Joint work with E. Prouff and M. Rivain

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CryptoExperts

CT-RSA 2014 – Feb. 26

Side Channel Analysis

- Side Channel Attacks (SCA) appear 15 years ago
  - 1996 : Timing Attacks
  - 1998 : Power Analysis
  - 2000 : Electromagnetic Analysis
- Numerous attacks
  - 1998 : (single-bit) DPA — KocherJaffeJune 1999
  - 1999 : (multi-bit) DPA — Messerges 1999
  - 2000 : Higher-order SCA — Messerges 2000
  - 2002 : Template SCA — ChariRaoRohatgi 2002
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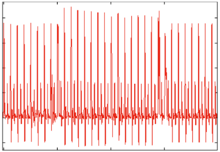

DES (ASIC), EM Radiations
http://www.dpacontest.org

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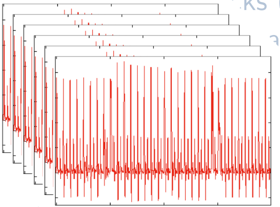

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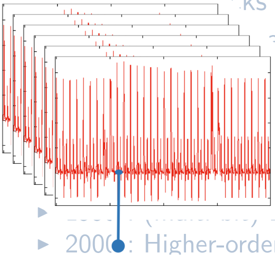
- 

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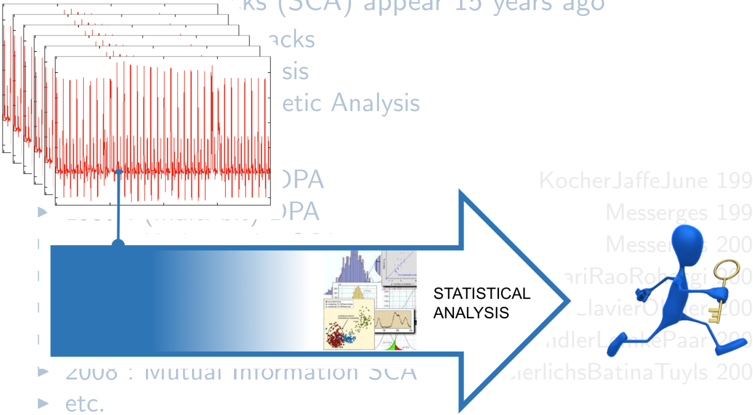

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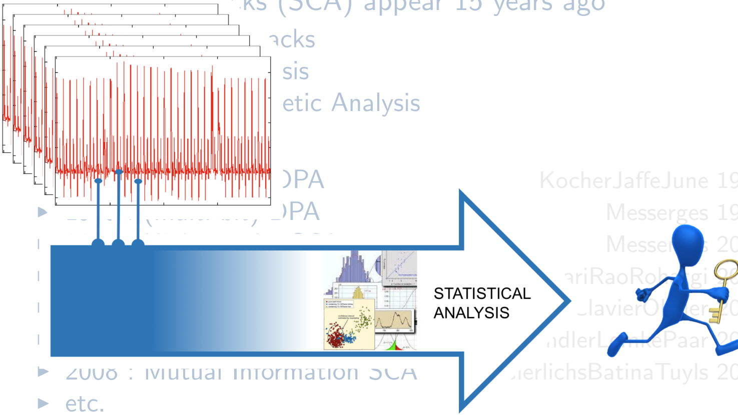
*d*th-order Side Channel Analysis



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Masking/Sharing Coutermeasures

Idea : consists in securing the implementation using secret sharing techniques.

- First Ideas in GoubinPatarin99 and ChariJutlaRaoRohatgi99.
- Soundness based on the following remark :

ChariJutlaRaoRohatgi, CRYPTO 1999

- Bit x masked $\mapsto x_0, x_1, \ldots, x_d$
- Leakage : $L_i \sim x_i + \mathcal{N}(\mu, \sigma^2)$
- # of leakage samples to test $\left((L_i)_i | x = 0\right) = \left((L_i)_i | x = 1\right)$:

$$q \geq O(1)\sigma^d$$

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extended to *continuous leakage* by ProufRivain, EUROCRYPT 2013

DucDziembowskiFaust, to appear EUROCRYPT 2014

Probing Adversary

- Notion introduced in IshaiSahaiWagner, CRYPTO 2003
- A d^{th}-order probing adversary is allowed to observe **at most** d intermediate results during the overall algorithm processing.
  - Hardware interpretation : d is the maximum of wires observed in the circuit.
  - Software interpretation : d is the maximum of different timings during the processing.
- d^{th}-order probing adversary = d^{th}-order SCA as introduced in Messerges99.
- Countermeasures proved to be secure against a d^{th}-order probing adv. :
  - $d = 1, 2$: KocherJaffeJune99, BlömerGuajardoKrummel04, ProuffRivain07, RivainDottaxProuff08.
  - $d \geq 1$: IshaiSahaiWagner03, ProuffRoche11, GenelleProuffQuisquater11, CarletGoubinProuffQuisquaterRivain12, Coron14.

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Higher-Order Masking Schemes

Achieving security in the probing adversary model

Definition

A *dth-order masking scheme* for an encryption algorithm $c \leftarrow \mathcal{E}(m, k)$ is an algorithm

$$(c_0, c_1, \ldots, c_d) \leftarrow \mathcal{E}'\big((m_0, m_1, \ldots, m_d), (k_0, k_1, \ldots, k_d)\big)$$

- Completeness : there exists R s.t. :

$$R(c_0, \cdots, c_d) = \mathcal{E}(m, k)$$

- Security : $\forall\{iv_1, iv_2, \ldots, iv_d\} \subseteq \{\text{intermediate var. of } \mathcal{E}'\}$:

$$\Pr\big(k \mid iv_1, iv_2, \ldots, iv_d\big) = \Pr\big(k\big)$$

State Of The Art

dth-order masking schemes

- Boolean Masking $n = 2d + 1, O(d^2)$
  - [Ishai *et al.* 03] (hardware oriented)
  - $\hookrightarrow$ [Rivain-Prouff 10] [Kim *et al.* 11]
  - [Coron 14 *to appear*] (table re-computation)
- Multiplicative Masking $n = d + 1, O(d^2)$
  - [Genelle *et al.* 11]
  - (alternating Boolean and Multiplicative Masking)
- Polynomial Masking $\widetilde{O}(d^2)$
  - [Prouff-Roche 11] ($n = 2d + 1$, Glitches Resitance)
- Inner-Product Masking $O(d^2)$
  - [Balasch *et al.* 12] ($n = 2(d + 1)$, Glitches Resistance)

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Mutual Information Evaluation

Hamming Weight Model and Additive Gaussian Noise

$$\mathcal{O}(Z) = HW(Z) + \mathcal{B}$$

$\mathcal{B} \leftarrow \mathcal{N}(0, \sigma)$

In this idealized model, the success rate of an optimal multi-query (HO-)SCA targeting $(Z_0, \cdots, Z_d)$ is a monotonously increasing function of

$$\mathcal{I}(\mathcal{O}(Z_0), \cdots, \mathcal{O}(Z_d); Z)$$

[Standaert *et al.* 09]

Boolean Sharing

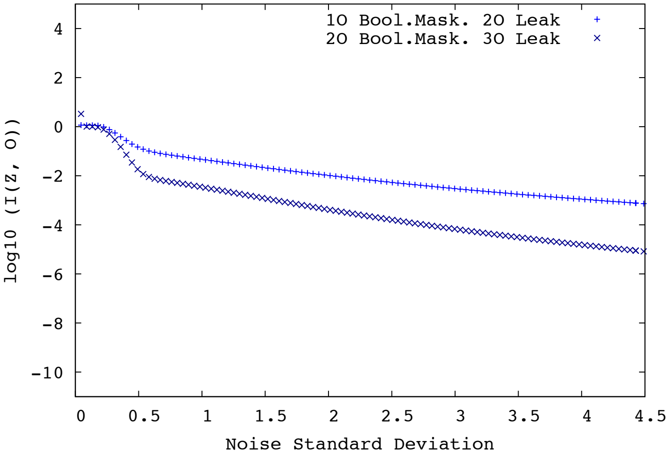
Manipulation of randomized variable

$$z \xrightarrow{\$} (z \oplus r_1 \oplus \cdots \oplus r_d, r_1, \cdots, r_d) \ ,$$

where r_i are randomly generated in $\mathrm{GF}(2^\ell)$.

Information Leaked by a d^{th}-order Boolean Sharing

8-bit variables



IP-masking DziembowskiFaust, TCC 2012

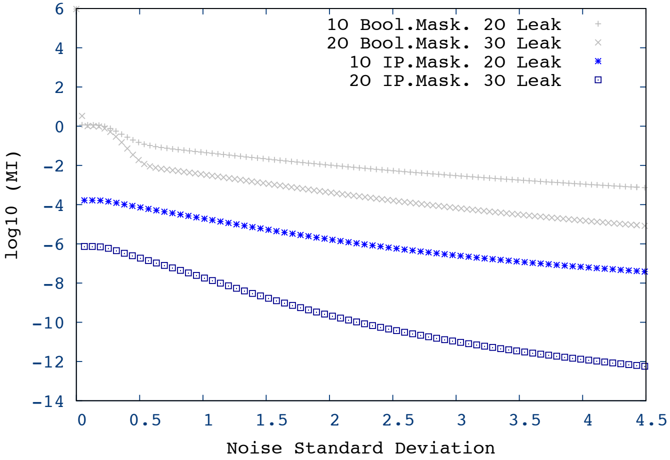
Manipulation of randomized variable

$$z \xrightarrow{\$} \left(L_1, \cdots, L_n, \frac{z \oplus \sum_{i=2}^{n} L_i R_i}{L_1}, R_2, \cdots, R_n\right)$$

where L_i are randomly generated in $\mathrm{GF}(2^\ell)^\star$
and R_i are randomly generated in $\mathrm{GF}(2^\ell)$.

Information Leaked by a d^{th}-order IP sharing

8-bit variables



IP-masking Scheme BalaschFaustGierlichsVerbauwhede, ASIACRYPT 2012

Practical Leakage Resilient Masking Scheme

- $2n$ shares for $(n-1)$ probing security
- (HO-)Glitches Attack resistant masking scheme
- Weak information leakage assuming standard Leakage Functions *e.g. HW*
- Complexity $O(n^2)$
- Proofs in the continuous bounded-range leakage model
  - $\mathcal{O}() : \{0,1\}^\ell \mapsto \{0,1\}^\lambda$ $\lambda << \ell$
  - no limit in the number of observations

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- Weak information leakage assuming standard Leakage Functions *e.g. HW*
- Complexity $O(n^2)$
- Proofs in the continuous bounded-range leakage model only if $n \geq 130$
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Inner-Product Sharing Scheme

$$z \xrightarrow{\$} (L_1, \cdots, L_n, \frac{z \oplus \sum_{i=2}^{n} L_i R_i}{L_1}, R_2, \cdots, R_n) = (\mathbf{L}_z, \mathbf{R}_z)$$

R_i in $\mathrm{GF}(2^\ell)$, L_i in $\mathrm{GF}(2^\ell)^\star$.

IP-Masking Scheme

$$\text{inputs} : \{(\mathbf{L}_A, \mathbf{R}_A), (\mathbf{L}_B, \mathbf{R}_B)\}$$

- $\texttt{RefreshMasks}(A)$: $O(n)$
- $A + B$: $O(n)$
- $xA + y$: $O(n)$
- $A \times B$: $O(n^2)$

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Algorithm RefreshMasks

$\langle \mathbf{L}, \mathbf{R} \rangle$ denotes the scalar product.

Input : the $(2n, d)$-sharing $(\mathbf{L}, \mathbf{R})$ of Z.
Output: the $(2n, d)$-sharing $(\mathbf{L}^\star, \mathbf{R}^\star)$ such that $\langle \mathbf{L}^\star, \mathbf{R}^\star \rangle = \langle \mathbf{L}, \mathbf{R} \rangle$.

```
/* Refresh Masks                                                    */
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$\mathbf{L}^\star \leftarrow (\texttt{randNonZero()})^n$;
for $i = 1$ **to** n **do**
  $A_i \leftarrow L_i \oplus L_i^\star$;

$X \leftarrow \langle \mathbf{A}, \mathbf{R} \rangle$;
$\mathbf{T} \leftarrow \texttt{IPHalfMask}(X, \mathbf{L}^\star)$;
$\mathbf{R}^\star \leftarrow \mathbf{R} \oplus \mathbf{T}$;
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For $n = 2$,

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A 1^{st}-order Flaw for any d

$$\Pr[X = x \mid Z = 0] = \begin{cases} \frac{1}{2^\ell} + \frac{1}{2^\ell(2^\ell - 1)^{n-2}} & \text{if } x = 0 \\ \frac{1}{2^\ell} - \frac{1}{2^\ell(2^\ell - 1)^{n-1}} & \text{if } x \neq 0 \end{cases}$$

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$$\mathcal{I}(\mathcal{O}(X); Z) \neq 0$$

Information Leaked by the 1st-order Flaw

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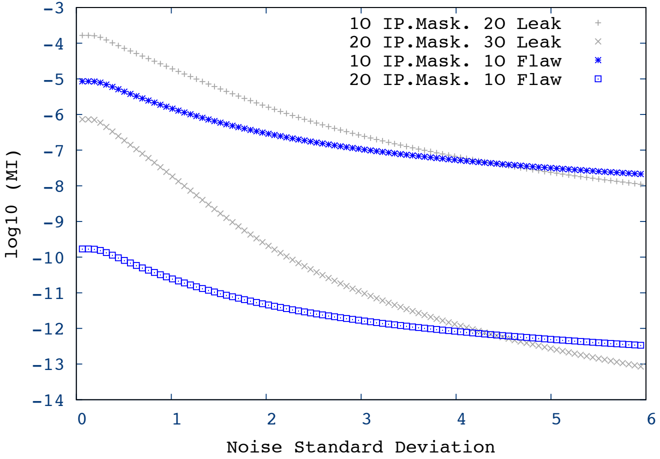

Information Leaked by the 1st-order Flaw

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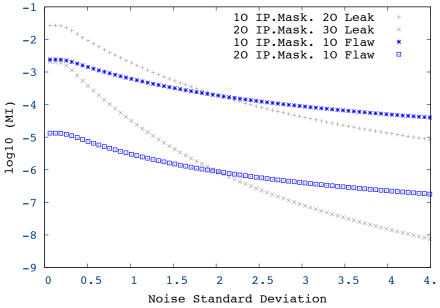

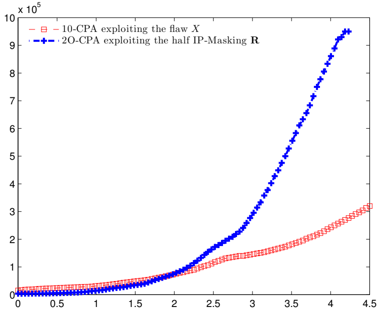

A security flaw in Balasch *et al.* scheme

- 1st-order flaw (exponential decay *w.r.t.* the mask order)
  - ↪ in practice much easier to mount than a dth-order attack.
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  - ↪ ways of improving the $n \geq 130$ bound ?

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IP-Masking Scheme *w.r.t.* to recent results in leakage resilience proofs

- ProufRivain, EUROCRYPT 2013
- security proofs in continuous leakage model

  practical noisy leakage models
- Boolean masking (Ishai *et al.* scheme)
- improvements and link with probing security

  DucDziembowskiFaust, to appear EUROCRYPT 2014

THE MYTH OF GENERIC DPA... AND THE MAGIC OF LEARNING

Carolyn Whitnall[1], Elisabeth Oswald[1], François-Xavier Standaert[2]

[1]Department of Computer Science, University of Bristol
[2]UCL Crypto Group, Université catholique de Louvain

carolyn.whitnall@bris.ac.uk

26th February 2014

Outline

The 'myth'…

- What is 'generic' DPA? – rethinking the role of the power model
- Does 'generic' DPA work? – only in special cases, it turns out

The 'magic'…

- Where do we go from here? – linear regression-based methods as an interesting avenue for generic-emulating DPA
- Does our proposed technique work? – some experimental results

Outline

The 'myth'…

- What is 'generic' DPA? – rethinking the role of the power model
- Does 'generic' DPA work? – only in special cases, it turns out

The 'magic'…

- Where do we go from here? – linear regression-based methods as an interesting avenue for generic-emulating DPA
- Does our proposed technique work? – some experimental results

What is 'generic' DPA?

Intuitive idea

A strategy to exploit the data-dependent leakage of a device without any prior knowledge of the functional form of that leakage.

Typical approach

Use distinguishing statistics which require *few distributional assumptions*:

- Mutual information [Gierlichs et al. CHES '08];
- Kolmogorov–Smirnov test statistic [Veyrat-Charvillon et al. CHES '09];
- Cramér–von Mises [Veyrat-Charvillon et al. CHES '09];
- Copulas [Veyrat-Charvillon et al. CRYPTO '11] ...

But this approach does *not* automatically constitute'generic' DPA:

- Often paired with a power model such as Hamming weight;
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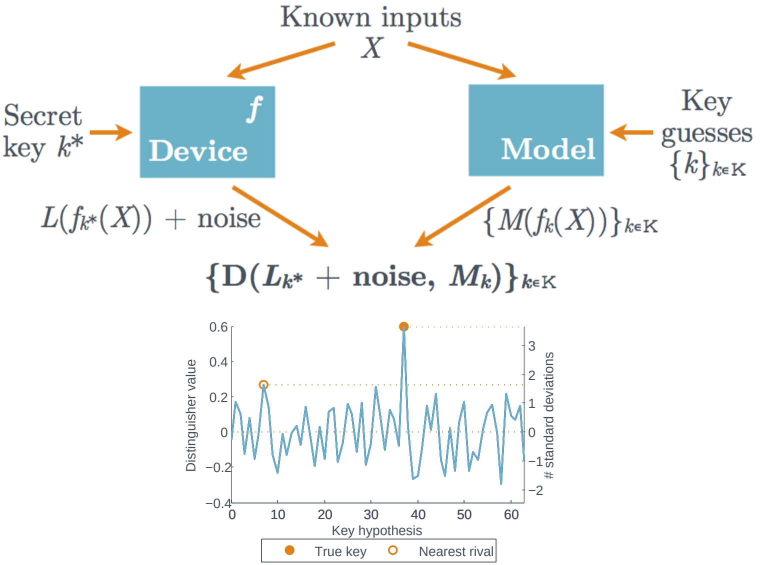

Known inputs
X
Secret key k*
f
Device
Model
Key guesses {k}k∈K
L(fk*(X)) + noise
{M(fk(X))}k∈K
{D(Lk* + noise, Mk)}k∈K
0.6
0.4
0.2
0
−0.2
−0.4
Distinguisher value
3
2
1
0
−1
−2
standard deviations
0
10
20
30
40
50
60
Key hypothesis
True key
Nearest rival

What is 'generic' DPA?

Determined by the power model, not the distinguishing statistic!

Classifying power models according to Stevens' levels of measurement

Suppose M is the power model for leakage function L. Then...

- Direct approximation $M \approx L$ (c.f. the 'ratio scale'), as exploited by profiled attacks (e.g. Bayesian templates and stochastic profiling).
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CLASSIFYING POWER MODELS ACCORDING TO STEVENS' LEVELS OF MEASUREMENT

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(STANDARD, UNIVARIATE) GENERIC DPA STRATEGY

=

GENERIC POWER MODEL

The nominal mapping to the equivalence classes induced by the target function F_k.

+

GENERIC-COMPATIBLE DISTINGUISHER

Any distinguishing statistic which operates on nominal scale measurements.

Does 'generic' DPA work?

A strategy 'works' (given enough data and a compatible distinguisher) if the power model approximation under the correct hypothesis is *strictly more accurate* than the approximation under any incorrect alternative.

1. For F injective: generic power model predictions under all hypotheses are *equally accurate*—no generic strategy works.
2. For F balanced and non-injective; k introduced by (XOR) key addition:
   1. If F is *affine* then no generic strategy is able to distinguish the correct key from any other.
   2. If $a \in \mathbb{F}_2^n$ is a linear structure of F then no generic strategy is able to distinguish between k^* and $k^* \oplus a$.
   3. If, for some $a \in \mathbb{F}_2^n$ we have that $D_aF(x)$ (the differential of F wrt a) depends on x *only via $F(x)$*, then no generic strategy is able to distinguish between k^* and $k^* \oplus a$

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Does 'generic' DPA work?

Suppose F is a balanced, noninjective (n-m) function, with k introduced by (XOR) key-addition.

A *necessary condition* for a generic strategy to distinguish k^* from k is: $\exists x \in \mathbb{F}_2^n$ such that $\# D_{k^* \oplus k} F(F^{-1}[F(x)]) \neq 1$.

If L is injective then this becomes a *sufficient condition*.

S-box design goals of differential uniformity increase the chances of this condition being met for a given XOR difference from the correct key.

CRYPTANALYTIC RESILIENCE $\stackrel{\sim}{\Longleftrightarrow}$ SIDE-CHANNEL VULNERABILITY

OBSERVATION: Leakage function $L : \mathbb{F}_2^m \to \mathbb{R}$ can be expressed as a polynomial in function of the target bits.

- $L(z) = \sum_{u \in \mathbb{F}_2^m} \alpha_u z^u, \ \forall z \in \mathbb{F}_2^m$, where z^u denotes the monomial $\prod_{i=1}^m z_i^{u_i}$, with z_i the i^{th} bit of z.

ATTACK STRATEGY: Using prior knowledge about the contributing terms, estimate the model according to each key guess and pick the one which produces the 'best fit'.

- $\forall k \in \mathcal{K}$ compute the OLS coefficients for $L_{k^*}(X) + \varepsilon = \alpha_{\mathbf{0}} + \sum_{u \in \mathcal{U}} F_k(X)^u \alpha_u$, where $\mathcal{U} \subseteq \mathbb{F}_2^m \setminus \{\mathbf{0}\}$.
- If the R^2 'goodness-of-fit' measure is largest under the correct key guess then the attack has succeeded.

Linear regression-based DPA

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LR-based DPA as a 'generic strategy'

Including all polynomial terms (i.e. selecting $\mathcal{U} = \mathbb{F}_2^m \setminus \{\mathbf{0}\}$) equates to a 'generic strategy' (see paper).

Case 1 – noninjective (cryptographic) target: System of equations is over-determined and…

- Consistent (bar noise) under the correct guess $\longrightarrow$ good model fit;
- Inconsistent under any incorrect guess $\longrightarrow$ poor model fit.

I.e. the true key is distinguished.

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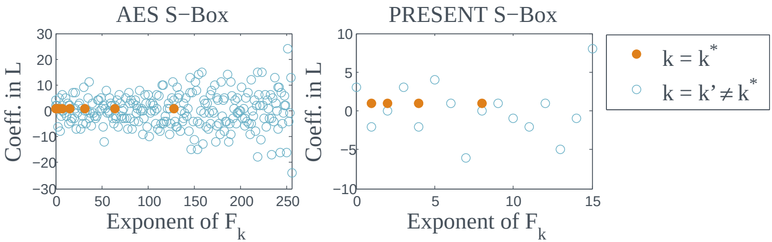
Coefficients from fitted LR models



- Under the correct key guess, coefficients on the fitted terms represent an expression for the leakage function L.
- Under an incorrect guess, the coefficients represent an expression for $L \circ f_k \circ f_{k^*}^{-1}$ – highly nonlinear by design of f.
- Assuming L is always 'simpler' than $L \circ f_k \circ f_{k^*}^{-1}$ this suggests a differentiating criteria.

Stepwise regression

- Model building tool to ‘learn’ the correct model specification.
  - Iteratively adds and removes potential explanatory variables.
  - Favours variables with the most explanatory power.
- Our proposal: Provide the stepwise algorithm with the full set of polynomial terms $\mathcal{U} = \mathbb{F}_2^m$ and let it choose which to privilege.
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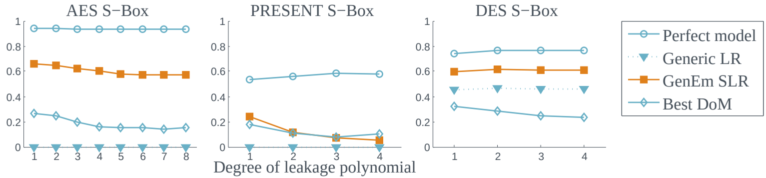

Median asymptotic distinguishing margins for 500 randomly generated leakage functions as leakage degree increases…

- Stepwise regression is effective against all three targets, even for high degree leakage.
- Stepwise regression succeeds in the scenarios where 'generic' linear regression DPA fails, and achieves larger margins against the (noninjective) DES S-box.
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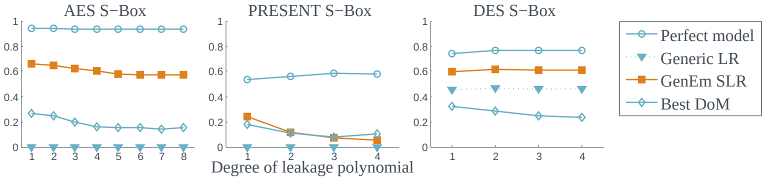

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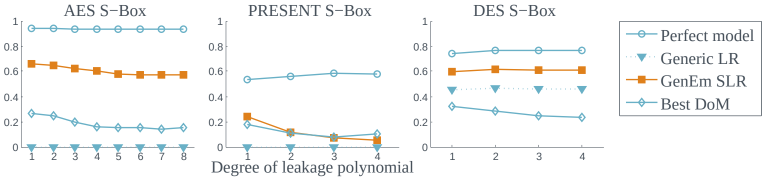

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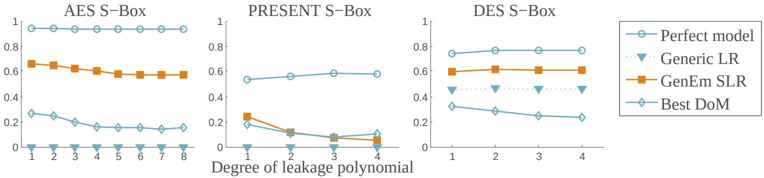

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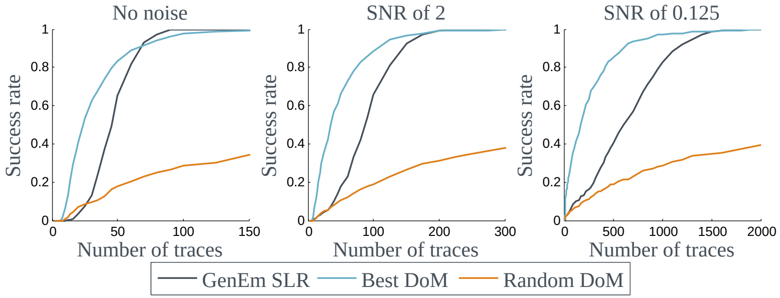

Success rates against high degree leakage of the AES S-box...

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- Lower success rates than the strongest DoM out of all 8 possible bits.
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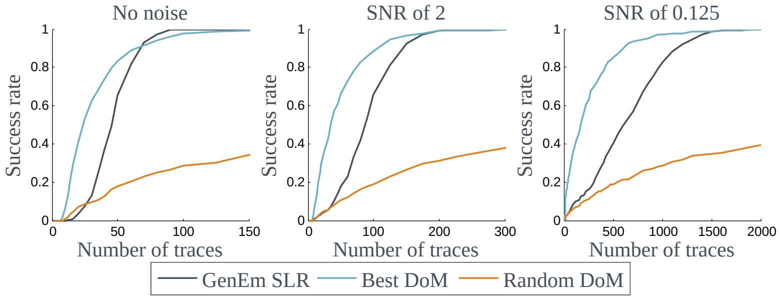

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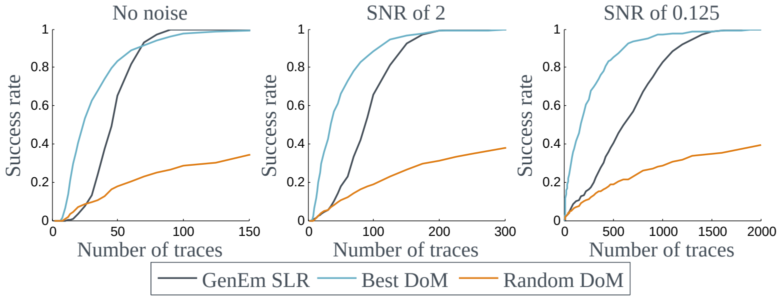

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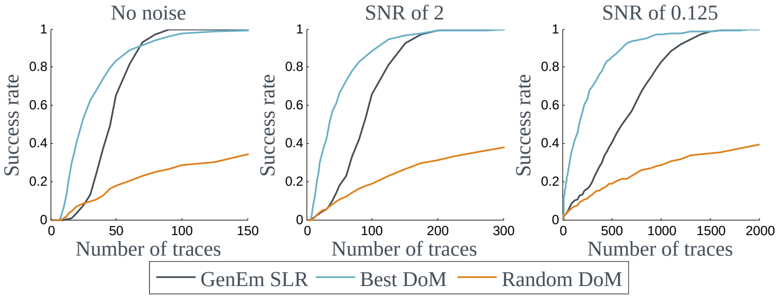

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Conclusion

- The notion of ‘generic DPA’ should follow from the properties of the *power model* used.
- Such a definition facilitates conclusive statements about attack outcomes independent of the distinguishing statistic chosen.
  - Generic strategies *can* succeed against noninjective cryptographic functions.
  - They invariably fail against injective targets – no universally-applicable attacks exist.
- *‘Generic-emulating’* DPA, relying only on ‘non-device-specific intuition’, can succeed against injective targets.
  - E.g. stepwise linear regression – rivals difference-of-means but is more costly to estimate.
  - Can we find other methodologies achieving a similar end? (. . . more efficiently?)

Thank you for listening!

Any questions?

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Capitalizing on
Collective Intelligence

Hardware Implementation and Side-Channel Analysis of Lapin

SESSION ID: CRYP-W02

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Riddle:

Do you know what does Lapin mean?

Do you know what does Lapin mean?

In French: Lapin = Rabbit

Do you know what does Lapin mean?

OR?

Learning **P**arity with **N**oise

L a P i N

Do you know what does Lapin mean?

OR?

Learning **P**arity with **N**oise

L a P i N

With something **random** in between

Outline

- Introduction
- Lapin protocol
- Implementation
- Performance evaluation
- Side-channel analysis
- Conclusion

Introduction to Lapin

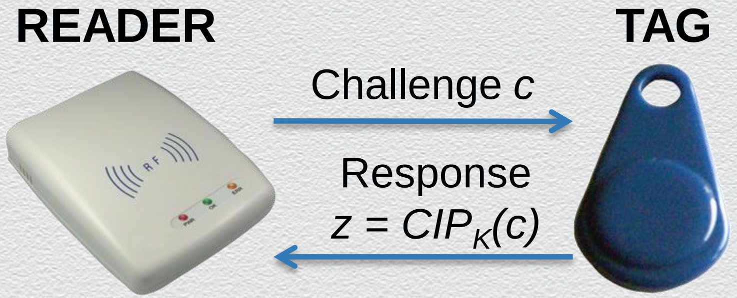
Light-weight Shared-key Authentication Protocols

- Lightweight shared-key authentication protocols are widely used

Example – wireless tags

Light-weight Shared-key Authentication Protocols

- Typical settings:
  1. Reader generates a challenge c
  2. Tag computes response $z = F_K(c)$
  3. Reader computes $z' = F_K(c)$
  4. Reader accepts the Tag if $z = z'$



Ideal Authentication Protocol

Considered conditions:

- Protocol properties:
  1. Provably secure
  2. Small amount of transferred data
  3. Minimum of rounds (i.e. 2)
  4. Fast response (low latency)
- Tag properties:
  1. Small footprint (in hardware)
  2. Small code size (in software)
  3. Low power consumption
  4. Low cost

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Lapin protocol

Lapin[1]

- Based on the Learning Parity with Noise problem (LPN)
- Authentication scheme
- Non-deterministic (because of random errors)
- Defined on the ring $\mathrm{R} = \mathbb{F}_2[X]/f(X)$, deg($f$) = n

- **Lapin is provably secure based on the Ring-LPN problem**

[1] Lapin: an efficient authentication protocol based on Ring-LPN, S. Heyse, E. Kiltz, V. Lyubashevsky, Ch. Paar, K. Pietrzak, p. 346-365, FSE 2012

Lapin Protocol Description

Public parameters: R, $\pi:\{0,1\}^\lambda \to \mathrm{R}, \tau, \tau', \lambda$

Secret key: $K = (s, s') \in \mathrm{R}^2$

	Tag		**Reader**
①		$\xleftarrow{c}$	$c \xleftarrow{\$} \{0,1\}^\lambda$
②	$r \xleftarrow{\$} \mathrm{R}^*;\ e \xleftarrow{\$} \mathrm{Ber}_\tau^{\mathrm{R}} \in \mathrm{R}$		
③	$z := r \cdot (s \cdot \pi(c) + s') + e$	$\xrightarrow{(r,z)}$	
④			if $r \notin \mathrm{R}^*$ reject
⑤			$e' := z - r \cdot (s \cdot \pi(c) + s')$
⑥			if $HW(e') > n \cdot \tau'$ reject else accept

Masking Countermeasure

- **Objective:** decrease the correlation between the consumed power and the processed sensitive data
- **Implementation:** all sensitive variables must be split to shares and computations should be performed on each share separately (if possible)
- **Conditions** for effective masking:
  - The leakage of each share is independent from the others
  - Sufficient noise is present in the device
- **Example:**

$$\begin{aligned} h_1 &= q_1 \\ \dots \\ h_{d-1} &= q_{d-1} \\ h_d &= h \bigoplus \bigoplus_{i=1}^{d-1} q_i \end{aligned}$$

Masking of Lapin

1. Split sensitive variable *s*, *s'* and *e* into *d* shares

$$
\begin{aligned}
s &= s_1 \oplus s_2 \oplus \cdots \oplus s_d\,, \\
s' &= s'_1 \oplus s'_2 \oplus \cdots \oplus s'_d\,, \\
e &= e_1 \oplus e_2 \oplus \cdots \oplus e_d
\end{aligned}
$$

2. Derive a formula allowing to demask the output

$$
\begin{aligned}
z &= (\pi(c) \cdot s \oplus s') \cdot r \oplus e \\
&= [\pi(c) \cdot (s_1 \oplus \cdots \oplus s_d) \oplus (s'_1 \oplus \cdots \oplus s'_d)] \cdot r \oplus (e_1 \oplus \cdots \oplus e_d) \\
&= [(\pi(c) \cdot s_1 \oplus s'_1) \cdot r \oplus e_1] \oplus \cdots \oplus [(\pi(c) \cdot s_d \oplus s'_d) \cdot r \oplus e_d] \\
&= z_1 \oplus \cdots \oplus z_d
\end{aligned}
$$

- **Lapin is linear** = each share is computed **separately**

Implementation

Definition of constants

Constants are chosen as in the Lapin paper (CRT impl.):

- *n = deg(f(X)) = 621*
- *m = 5*
- *M factors of f(X) are:*
- $\tau = {}^1\!/_6$
- $\tau' = 0.29$
- $\lambda = 80$ bits

$$\begin{aligned}
f_1(X) &= X^{127} + X^8 + X^7 + X^3 + 1\\
f_2(X) &= X^{126} + X^9 + X^6 + X^5 + 1\\
f_3(X) &= X^{125} + X^9 + X^7 + X^4 + 1\\
f_4(X) &= X^{122} + X^7 + X^4 + X^3 + 1\\
f_5(X) &= X^{121} + X^8 + X^5 + X + 1
\end{aligned}$$

$\Rightarrow$ 128-bit datapath is suitable, since $deg(f_j(X)) < 128$

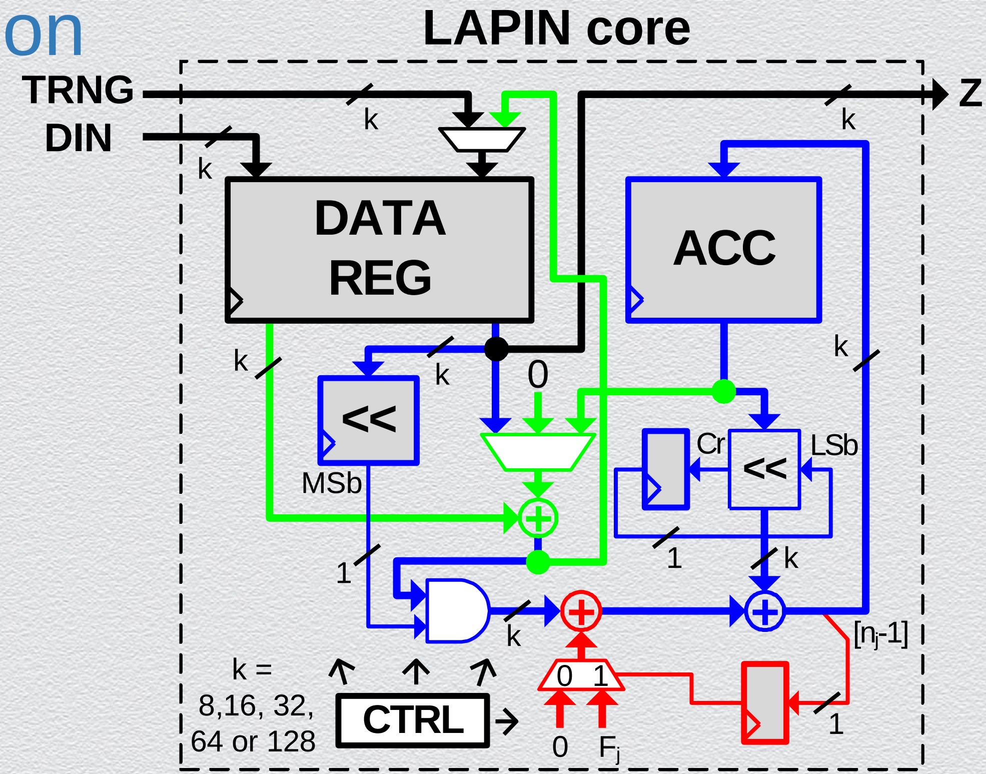
Polynomial multiplication & reduction

- We have implemented a **128-bit "school-book" polynomial multiplication unit** because:
  - It can be performed **in parallel with 1-bit reduction**
  - Its hardware implementation is **very small**
  - Its implementation can operate on **high frequencies**

- This **unit can be shared** for Lapin computations as well as error *e* transformation

Implementation description

- 8b to 128b datapath width
- Data registers in RAM
- Accumulator in RAM
- Carry register if k<128
- Shift register must not load sensitive data



Multiplication, reduction and addition

Performance evaluation

Cost evaluation & Timing results

- Lapin was synthesized for Xilinx Virtex 5 FPGA

Datapath (k)	Slices	BRAM		f_{max} (MHz)	Clock cycles		
		18kb	36kb		d = 1	d = 2	d = 3
8	213	2	0	125.3	20,977	41,969	62,961
16	232	2	0	127.5	10,489	20,985	31,481
32	311	1	1	127.2	5,245	10,493	15,741
64	330	0	3	130.2	2,623	5,247	7,871
128	451	0	6	140.3	1,332	2,664	3,996

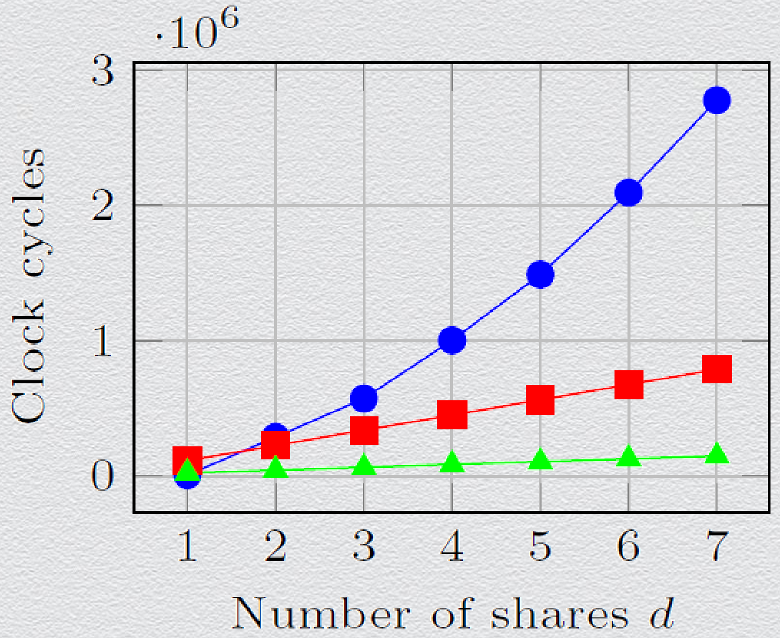
- d = 1: Lapin without masking

  d > 1: Masked Lapin – secure to (*d-1*) – order attacks

Comparison

d	AES softw.	Lapin[a] softw.	Lapin 8b hardw.
1	5,100	112,500	20,977
2	286,844	225,016	41,969
3	572,069	337,532	62,961
4	1,003,154	450,048	83,953
5	1,489,539	562,564	104,945
6	2,095,756	675,080	125,937
7	2,779,561	787,596	146,929

[a] For d>1 values are estimated



- By increasing *d*, number of clk. cycles grows **linearly for Lapin** and **quadratically for AES**

  ⇒ **much cheaper to increase Lapin security to higher-order SCA than that of AES**

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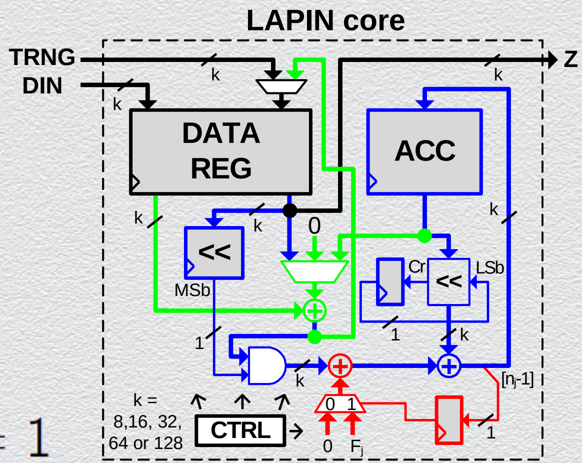
Resistance to side-channel attacks

Leakage Model



- **Target operation:** $s \cdot \pi(c)$, where π is zero padding
- **Assumption:** Accumulator leaks Hamming weight
- Accumulator is updated during the multiplication loop:

$$a_0 = 0 \qquad a_{i+1} \leftarrow \begin{cases} 2 \cdot a_i + s & \text{if } c[80-i] = 1 \\ 2 \cdot a_i & \text{otherwise} \end{cases}$$

- The value of *a* after few clock cycles of computation is a small multiple of the secret:

$$a_{80} = s \cdot c \qquad a_i = s \cdot \underbrace{\sum_{j=1}^{i} c[80-j]X^{i-j}}_{m_i(c)}$$

  - Device leaks **HW(a_i)**

Attack time points

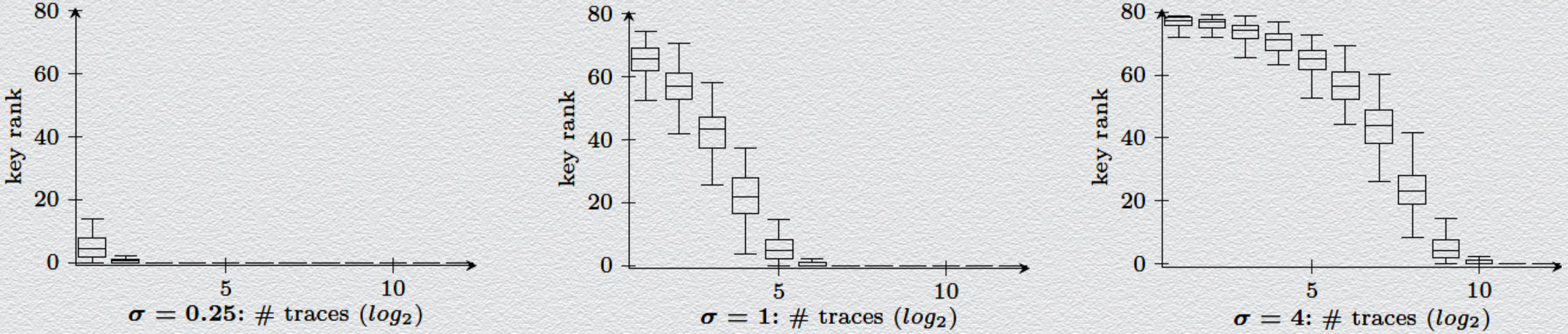
- Two equally efficient attack options:
  - Attack can target **several clock cycles in a single trace** with the **same challenge *c***

$\mathrm{HW}(a_i) = \mathrm{HW}(s \cdot m_i(c))$, for the same secret *c* and different values of *i*

  - Attack can target the **same clock cycle ι in several traces**, while **challenges are chosen** appropriately

$$m_\iota(c_j) = m_j(c)$$

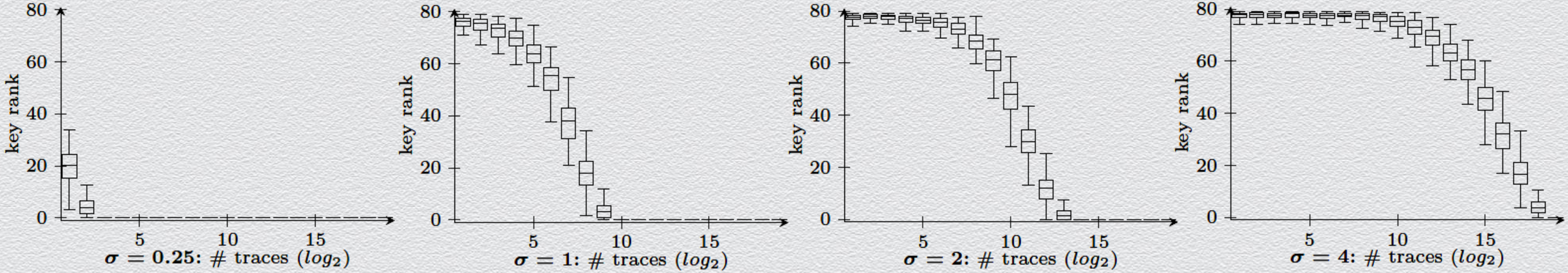
Collision-like Attack on Unprotected Lapin (d = 1)



- **Graphs:** Rank of the full key for *k = 128* using all clock cycles
- We can recover 80 key bits using about $2^6.\sigma^2$ traces for *k = 128*
- For k < 128 about $2^6.\sigma^2.128/k$ traces (128/k measurements are combined to get *HW(a)*)
- **Attack order:** 1st order bivariate (difference of 2 measures, information in average)

Collision-like Attack on Masked Lapin (e.g. d = 2)

- Distributions were used to mount a template attack for k = *128* using all clock cycles



- Data complexity increases roughly by σ^4 $\leftarrow$ typical for second order attacks
- **Attack order:** 2nd order 4-variate (4 measures combined pair-wise using difference, distributions are distinguished using covariance)

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Conclusions & perspectives

Conclusions

- **Lapin is linear → straightforward to mask**
- **First hardware implementation of Lapin**
  - Compact and very fast
  - Flexible datapath size (8-, 16-, 32-, 64- and 128-bit)
- **Advantages of Lapin over AES**
  - Smaller for large datapaths
  - High-order masking overhead increases linearly (quadratically for AES)
  - Shares are manipulated independently (independent leakage property)

Conclusions

- **Leakage model:** Hamming weight of accumulator
- Side-channel attacks against unprotected Lapin ($d = 1$)
  - Collision-like attack – 1st order bivariate attack
- Side-channel attack against masked Lapin ($d \geq 2$)
  - Collision-like attack – 2nd order 4-variate attack

Perspectives

- SCA using **Hamming distance model**
- Further study of the data-dependent algorithmic noise
- On-chip randomness generation is a problem => could it be solved using Learning With Rounding assumption?

Thank you for attention!

Protocol Classification

- Block-cipher based schemes
  - AES-based – may be too heavy for some appl.
  - Present-based - more suitable
- Schemes based on hardness of a mathematical problem
  - **L**earning **P**arity with **N**oise problem (LPN)
    - Hopper-Blum protocol (HB) and its variants (HB+, HB-MP, etc.)
    - **Lapin protocol**[1]
  - Others

[1] Lapin: an efficient authentication protocol based on Ring-LPN, S. Heyse, E. Kiltz, V. Lyubashevsky, Ch. Paar, K. Pietrzak, p. 346-365, FSE 2012

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Learning Parity with Noise Problem (LPN)

- Given a set of samples *(A, t = A . s + e)* with a random error *e*, where $t, e \in \mathbb{F}_2^n$ and $A \in \mathbb{F}_2^{n \times n}$
- Find the secret $s \in \mathbb{F}_2^n$
- Solution:
  - if ***e = 0*** then Gaussian elimination can solve it → **no security!**
  - if ***e > 0*** then it may become an NP-Hard problem
    → **suitable for cryptography!**

Note: The error *e* is generated with the Bernoulli distribution with parameter τ.

$$\mathrm{HW}(e) \approx n\tau$$

Lapin Protocol Parameters

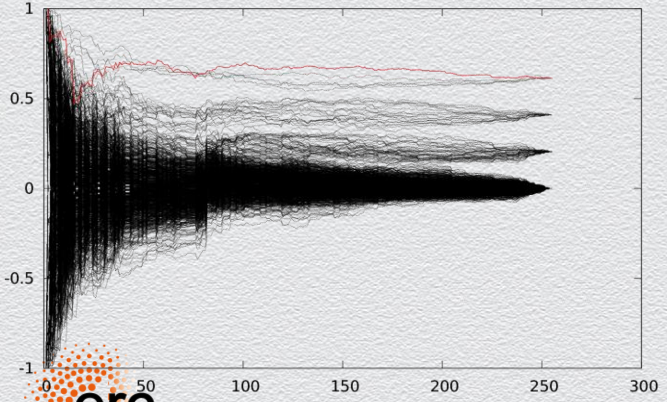
- 2-round protocol
- Public parameters:
  - R, *n* — ring $\mathsf{R} = \mathbb{F}_2[X]/f(X)$, deg(*f*) = *n*
  - λ — security level parameter (in bits)
  - π — mapping $\{0,1\}^\lambda \to \mathsf{R}$
  - $\tau \in (0, {}^1\!/\!_2)$ — parameter of Bernoulli distribution
  - $\tau' \in (\tau, {}^1\!/\!_2)$ — reader acceptance threshold
- Secret parameters:
  - $K = (s, s')$ — shared secret key, while $(s, s') \xleftarrow{\$} \mathsf{R}$

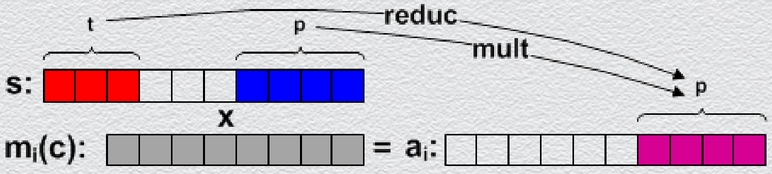
Ring-LPN Problem

- Ring Learning Parity with Noise (Ring-LPN) is an extension of LPN to rings
- The matrix A has a special structure. This way *A . s* is equivalent to the multiplication in the ring $\mathrm{R} = \mathbb{F}_2[X]/f(X)$
- **Lapin is provably secure based on the Ring-LPN problem**

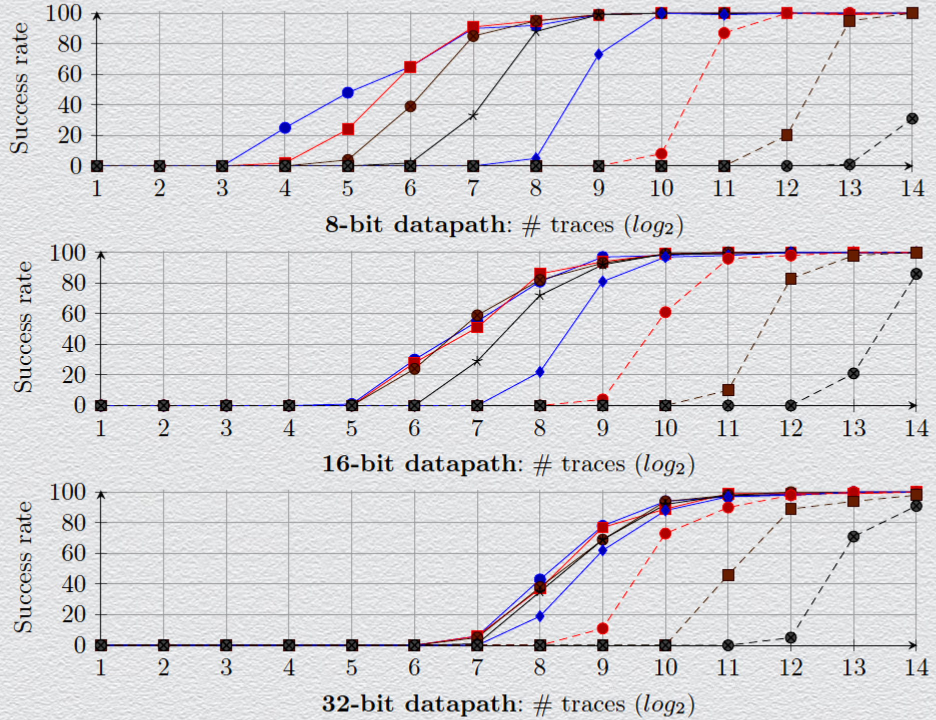
DPA-like Attack Against Unprotected Lapin (d = 1)

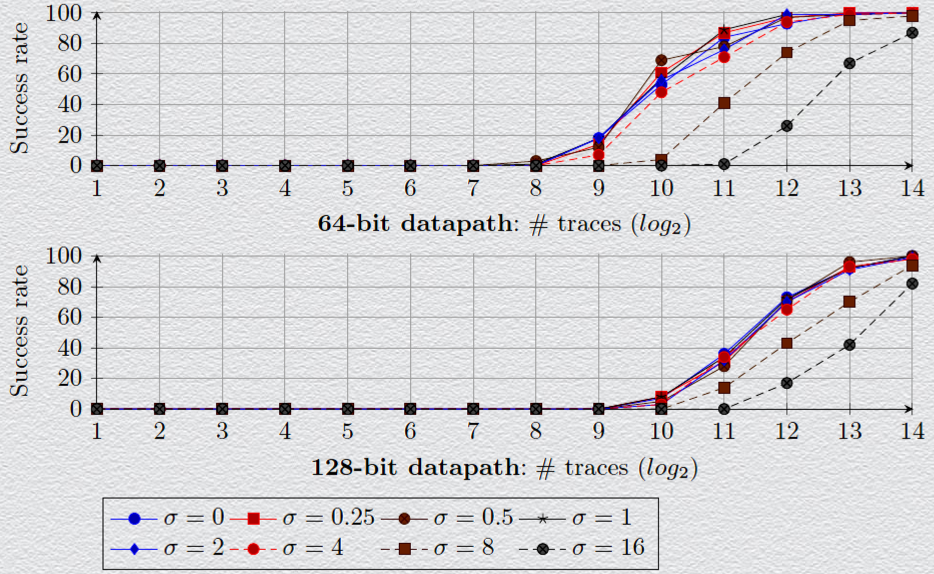
- Attack:
  - Predict some bits of $a_i = s \cdot m_i(c)$
  - If $deg(a_i) \leq t$ we can compute ***p* least significant bits** of a_i from the *p* least significant and *t* most significant bits of *s*.
  - Ex.: Correlation for *t* = *7* and *p* = *3*:





DPA-like Attack Against Unprotected Lapin (d = 1)





- Success rate for full-key recovery

Collision-like Attack on Unprotected Lapin (d = 1)

- **Approach:** Prediction of modular reduction impact on HW (i.e $\alpha \mapsto \alpha \cdot X$)
- **Assumption:** Accumulator contains a value α that will be rotated and reduced in the next clock cycle

$$\alpha \cdot X \bmod f = \begin{cases} (\alpha \lll 1) & \text{if } \mathrm{MSB}(\alpha) = 0 \\ (\alpha \lll 1) \oplus \bar{f} & \text{if } \mathrm{MSB}(\alpha) = 1 \end{cases}$$, where $\bar{f} = f \oplus X^{\deg(f)} \oplus 1$

- Since $\mathrm{HW}(\bar{f}) = 3$ the relations between HW of α and $\alpha \cdot X \bmod f$ is as follows:

$$\mathrm{HW}(\alpha \cdot X \bmod f) = \begin{cases} \mathrm{HW}(\alpha) & \text{if } \mathrm{MSB}(\alpha) = 0 \\ \mathrm{HW}(\alpha) + 3 & \text{if } \mathrm{MSB}(\alpha) = 1 \text{ and } \mathrm{HW}(\alpha \lll 1 \wedge \bar{f}) = 0 \\ \mathrm{HW}(\alpha) + 1 & \text{if } \mathrm{MSB}(\alpha) = 1 \text{ and } \mathrm{HW}(\alpha \lll 1 \wedge \bar{f}) = 1 \\ \mathrm{HW}(\alpha) - 1 & \text{if } \mathrm{MSB}(\alpha) = 1 \text{ and } \mathrm{HW}(\alpha \lll 1 \wedge \bar{f}) = 2 \\ \mathrm{HW}(\alpha) - 3 & \text{if } \mathrm{MSB}(\alpha) = 1 \text{ and } \mathrm{HW}(\alpha \lll 1 \wedge \bar{f}) = 3 \end{cases}$$

Collision-like Attack on Unprotected Lapin (d = 1)

- Therefore the distribution for of $\mathrm{HW}(\alpha \cdot X) - \mathrm{HW}(\alpha)$ for a random α is as follows:

$$\textbf{if } \mathbf{MSB}(\boldsymbol{\alpha}) = \mathbf{0}\textbf{:}\ \mathrm{HW}(\alpha \cdot X) - \mathrm{HW}(\alpha) = 0,$$

$$\textbf{if } \mathbf{MSB}(\boldsymbol{\alpha}) = \mathbf{1}\textbf{:}\ \mathrm{HW}(\alpha \cdot X) - \mathrm{HW}(\alpha) = \begin{cases} +3 & \text{with probability } 1/8 \\ +1 & \text{with probability } 3/8 \\ -1 & \text{with probability } 3/8 \\ -3 & \text{with probability } 1/8 \end{cases}$$

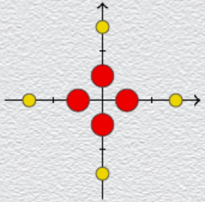
- This can be exploited using two chosen challenges $m_i(c) = m$ and $m_{i'}(c') = m \cdot X$
- Then we can recover $\mathrm{MSB}(m \cdot s)$ by comparing $\mathrm{HW}(m \cdot s)$ and $\mathrm{HW}(m \cdot X \cdot s)$
- **Result:** without noise 2 measures are sufficient to recover 1 key bit with probability 1
- **Advantage:** analysis of the full multiplier state and avoids algorithmic noise due to HW

Collision-like Attack on Masked Lapin (d > 1)

- We must combine leakages from all shares to get the key $s = \bigoplus_{j=1}^{d} s_j$
- We need to choose two challenges such that $m_i(c) = m$ and $m_{i'}(c') = m \cdot X$
- Then we can recover $\mathrm{MSB}(m \cdot s_j)$ by comparing $\mathrm{HW}(m \cdot s_j)$ and $\mathrm{HW}(m \cdot X \cdot s_j)$
- We study 2D distribution: $\left(\mathrm{HW}(\alpha_j \cdot X) - \mathrm{HW}(\alpha_j)\right)_{j=1}^{d},$ $\quad \text{with } \alpha = \bigoplus_{j=1}^{d} \alpha_j$

Collision-like Attack on Masked Lapin (e.g. d = 2)

$$\mathbf{MSB(\alpha)=1:}\quad
\begin{cases}
\mathrm{MSB}(\alpha_1)=0 \rightarrow \mathrm{HW}(\alpha_1\cdot X)-\mathrm{HW}(\alpha_1)=0\\
\mathrm{MSB}(\alpha_2)=1 \rightarrow \mathrm{HW}(\alpha_2\cdot X)-\mathrm{HW}(\alpha_2)\in\{-3,-1,+1,+3\}\\
\mathrm{MSB}(\alpha_1)=1 \rightarrow \mathrm{HW}(\alpha_1\cdot X)-\mathrm{HW}(\alpha_1)\in\{-3,-1,+1,+3\}\\
\mathrm{MSB}(\alpha_2)=0 \rightarrow \mathrm{HW}(\alpha_2\cdot X)-\mathrm{HW}(\alpha_2)=0
\end{cases}$$



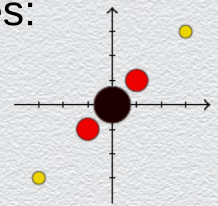
$\sigma(x,y)=0$

$$\mathbf{MSB(\alpha)=0:}\quad
\begin{cases}
\mathrm{MSB}(\alpha_1)=0 \rightarrow \mathrm{HW}(\alpha_1\cdot X)-\mathrm{HW}(\alpha_1)=0\\
\mathrm{MSB}(\alpha_2)=0 \rightarrow \mathrm{HW}(\alpha_2\cdot X)-\mathrm{HW}(\alpha_2)=0\\
\mathrm{MSB}(\alpha_1)=1 \rightarrow \mathrm{HW}(\alpha_1\cdot X)-\mathrm{HW}(\alpha_1)\in\{-3,-1,+1,+3\}\\
\mathrm{MSB}(\alpha_2)=1 \rightarrow \mathrm{HW}(\alpha_2\cdot X)-\mathrm{HW}(\alpha_2)\in\{-3,-1,+1,+3\}
\end{cases}$$

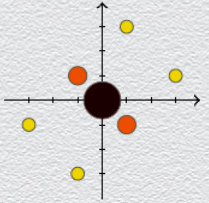
$\mathrm{HW}(\alpha \lll 1 \wedge \bar{f})\in\{0,1,2,3\}$

- Probabilities:

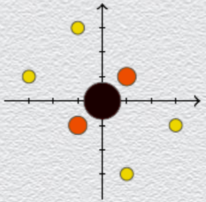
1/16
2/16
3/16
8/16



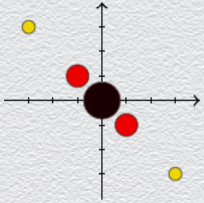
$\sigma(x,y)=6$
$\mathrm{HW}(\alpha\lll 1\wedge\bar{f})=0$



$\sigma(x,y)=2.25$
$\mathrm{HW}(\alpha\lll 1\wedge\bar{f})=1$



$\sigma(x,y)=-2.25$
$\mathrm{HW}(\alpha\lll 1\wedge\bar{f})=2$



$\sigma(x,y)=-6$
$\mathrm{HW}(\alpha\lll 1\wedge\bar{f})=3$