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Communication Optimal Tardos-based Asymmetric Fingerprinting

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joint work with Aggelos Kiayias, Nikos Leonardos, Helger Lipmaa, and Kateryna Pavlyk

A Motivational Example

A Movie Producer

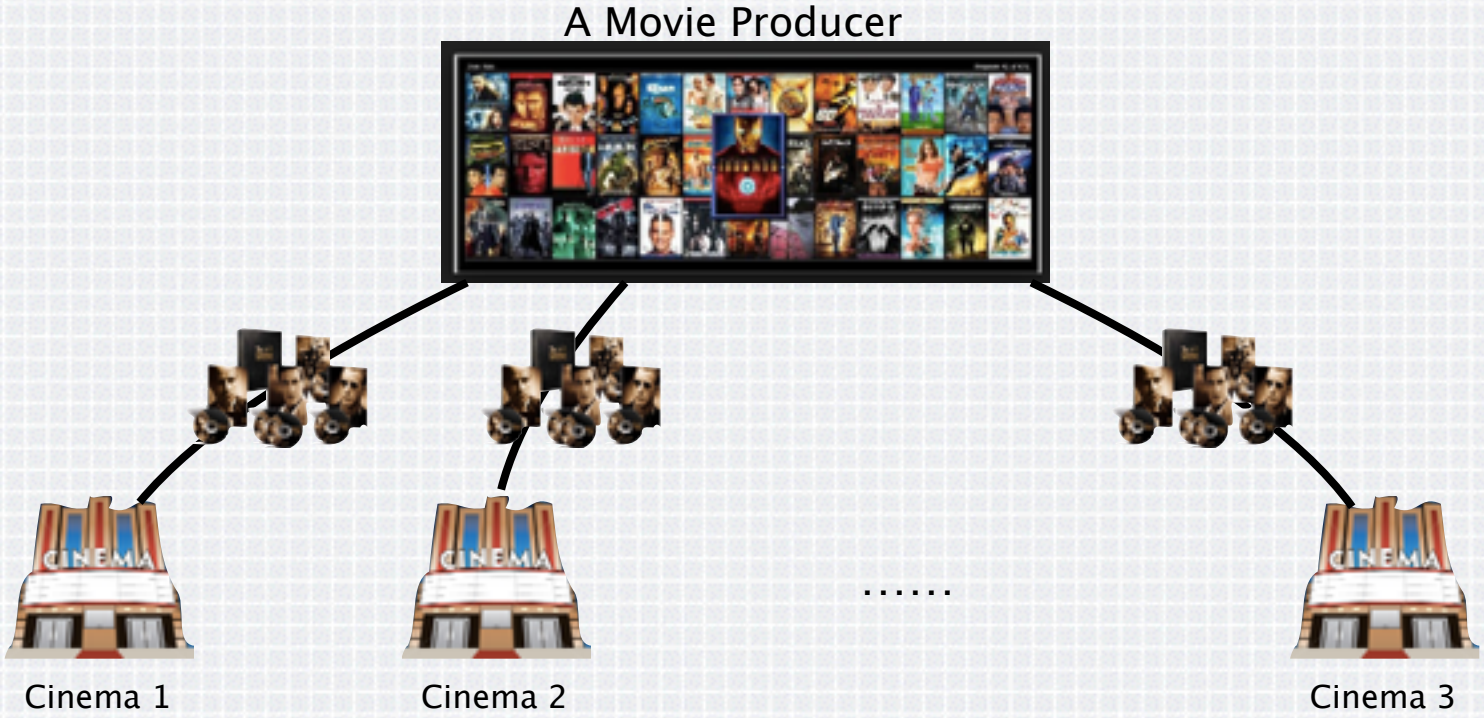
Cinema 1

Cinema 2

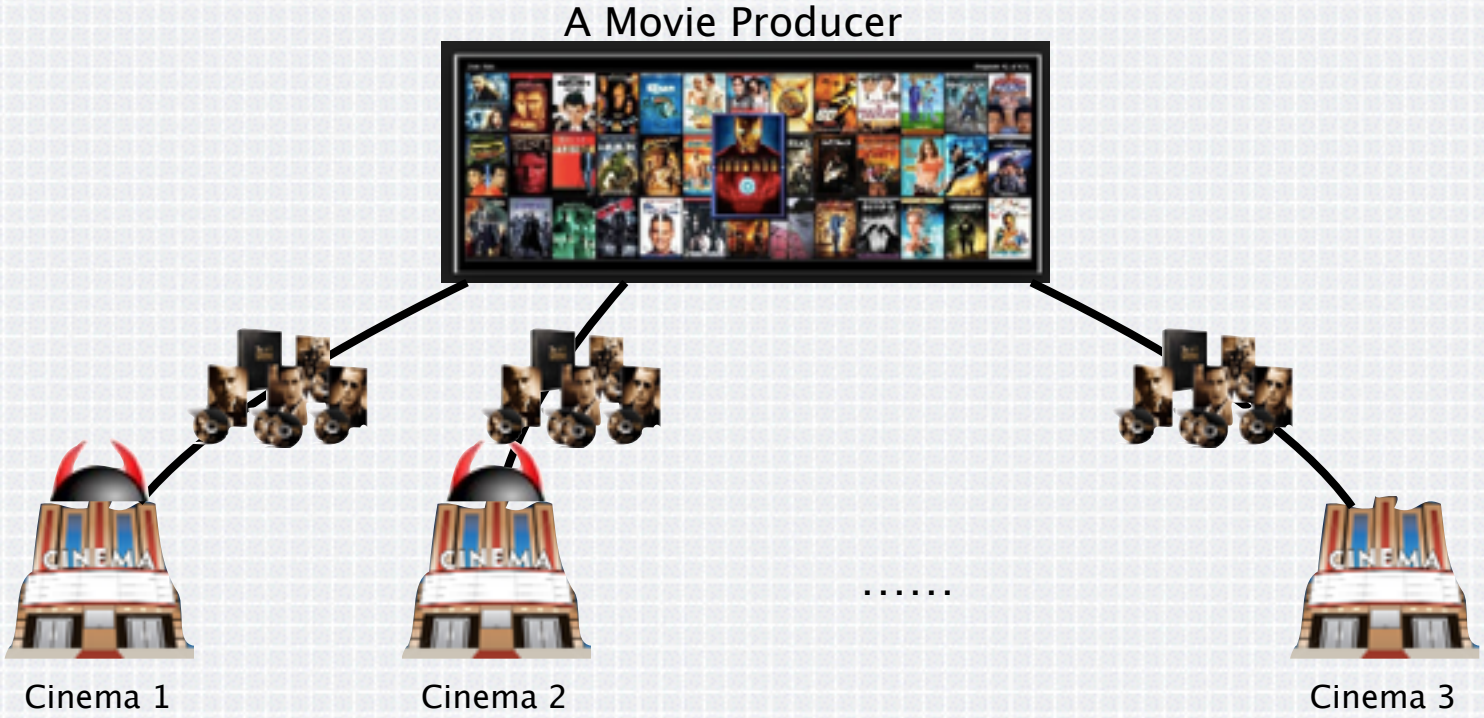
.....

Cinema 3

A Motivational Example



A Motivational Example

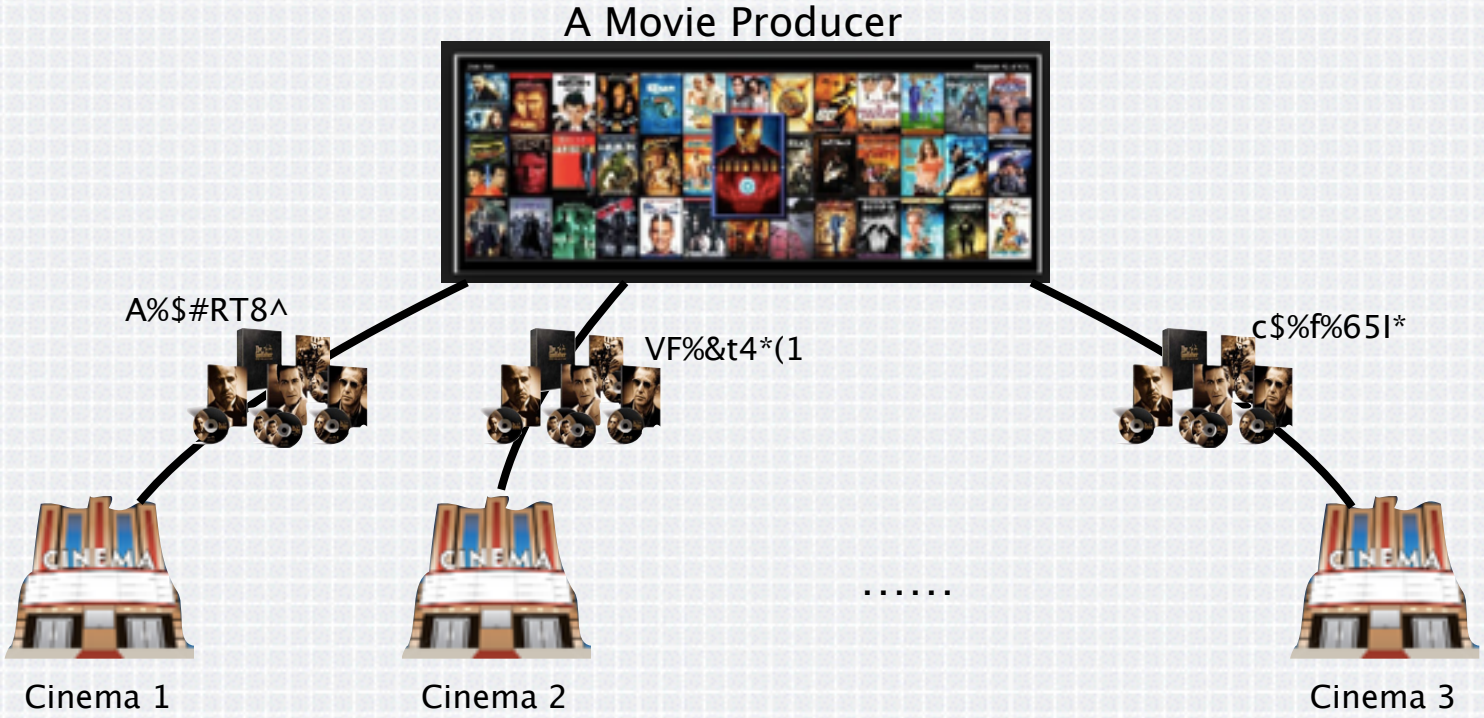


A Motivational Example

A Motivational Example

How to identify the source of the pirate?

Fingerprinting



A Motivational Example

Fingerprinting

Fingerprinting

VF%&#RT8^

Fingerprinting

pirate codeword

VF%&#RT8^

Fingerprinting

pirate codeword

VF%&#RT8^

tracing algorithm

Fingerprinting

pirate codeword

VF%&#RT8^

VF%&t4*(1

A%$#RT8^

tracing algorithm

Fingerprinting

pirate codeword

VF%&#RT8^

compare to the DB

VF%&t4*(1

A%$#RT8^

tracing algorithm

Fingerprinting

pirate codeword

VF%&#RT8^

compare to the DB

VF%&t4*(1 → Cinema 1

A%$#RT8^ → Cinema 5

tracing algorithm

The Goals of Fingerprinting

- Individualize contents

The Goals of Fingerprinting

- Individualize contents
- Trace back to the sources

A Catch

Does fingerprinting really de-incentivize illegal content re-distribution?

A Catch

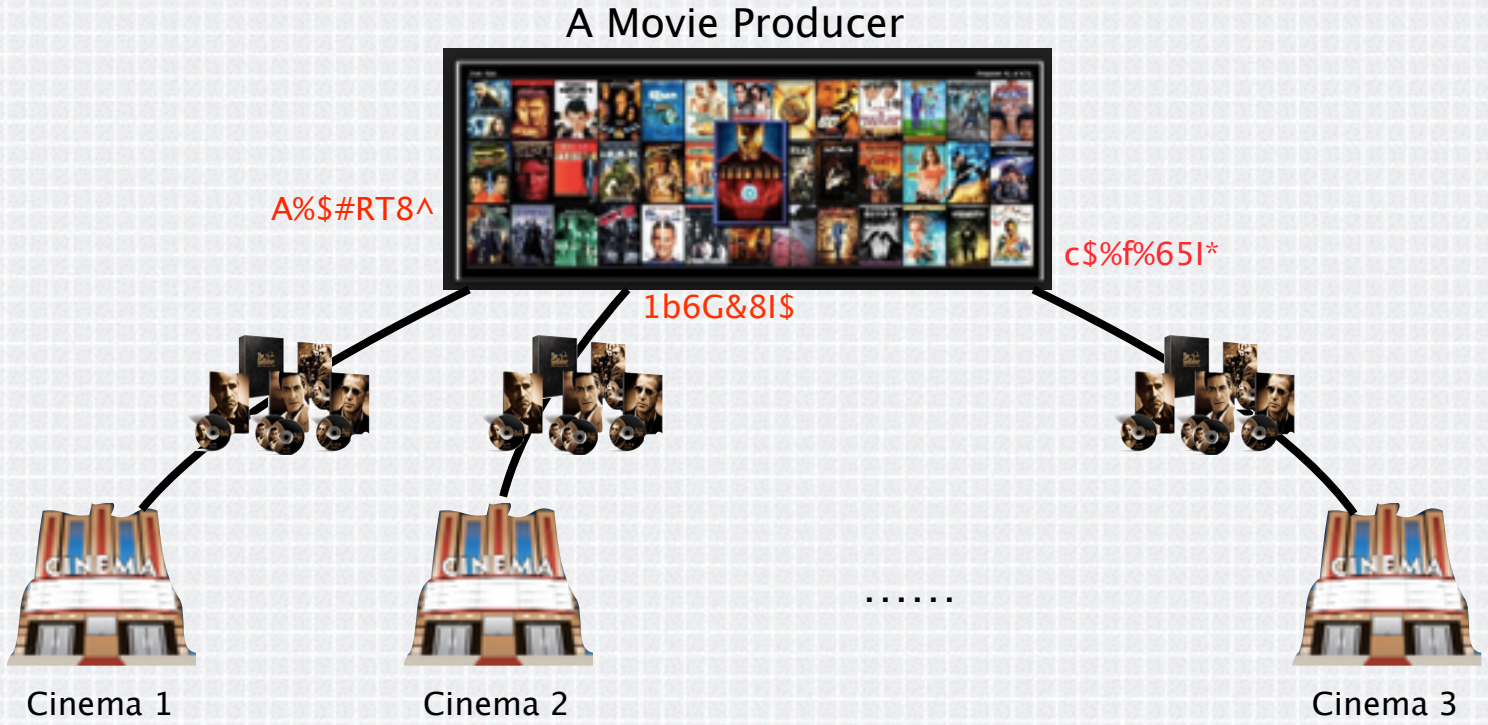
- Both the content provider and the content receiver can leak a copy

A Catch

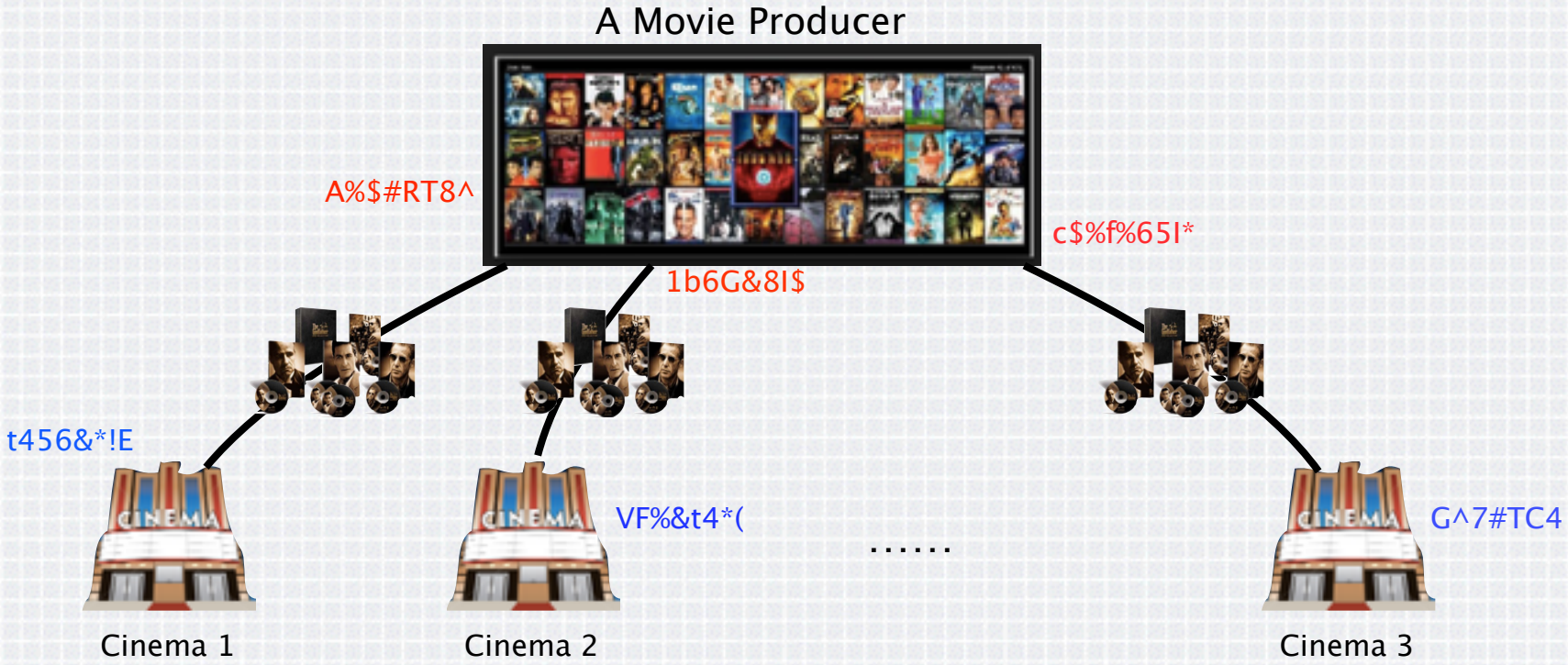
- Both the content provider and the content receiver can leak a copy
- The copy found in the public can not serve as a undeniable proof

Asymmetric Fingerprinting

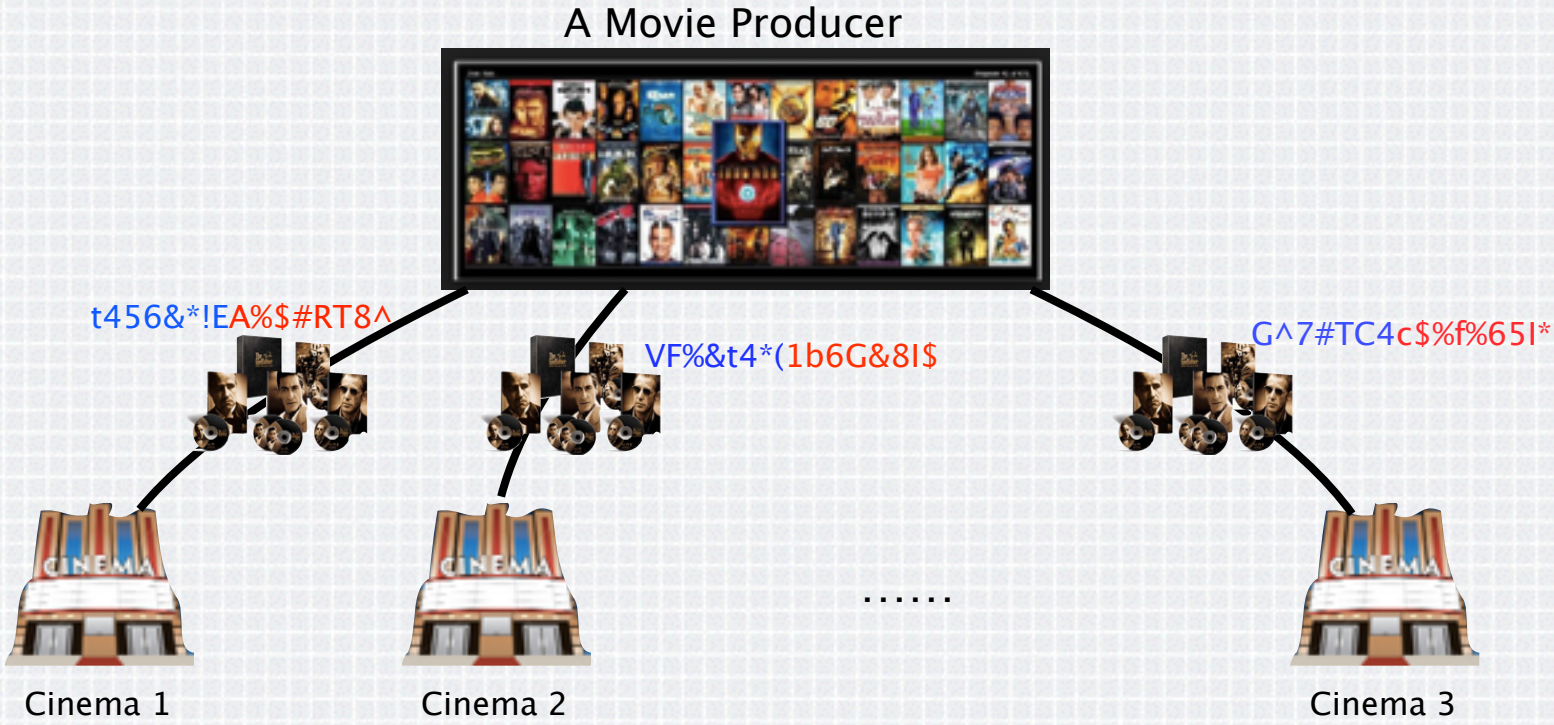
Asymmetric Fingerprinting



Asymmetric Fingerprinting



Asymmetric Fingerprinting



Asymmetric Fingerprinting

Asymmetric Fingerprinting

Asymmetric Fingerprinting

VF%&t4*(1Ab$#&8I$

Asymmetric Fingerprinting

half code

VF%&t4*(1Ab$#&8I$

Asymmetric Fingerprinting

half code

VF%&t4*(1Ab$#&8I$

tracing alg

Asymmetric Fingerprinting

half code

VF%&t4*(1Ab$#&8I$

1b6G&8I$

A%$#RT8^

tracing alg

Asymmetric Fingerprinting

half code

VF%&t4*(1Ab$#&8I$

“half DB”

1b6G&8I$

A%$#RT8^

tracing alg

Asymmetric Fingerprinting

half code

VF%&t4*(1Ab$#&8I$

“half DB”

1b6G&8I$ → Cinema 2

A%$#RT8^ → Cinema 1

tracing alg

Asymmetric Fingerprinting

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

Asymmetric Fingerprinting

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

t456&*!EA%$#RT8^

Cinema 1

VF%&t4*(1b6G&8I$

Cinema 2

Asymmetric Fingerprinting

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

t456&*!EA%$#RT8^

Cinema 1

t456&*!E

VF%&t4*(

VF%&t4*(1b6G&8I$

Cinema 2

Asymmetric Fingerprinting

VF%&t4*(11b6G&8I$ → Cinema 2

G^7#TC4A%$#RT8^ → Cinema 1

t456&*!EA%$#RT8^

t456&*!E

VF%&t4*(

Cinema 1

VF%&t4*(1b6G&8I$

Cinema 2

Cinema 1 ✗

Cinema 2 ✓

Security Considerations

- The content provider can not frame a theater

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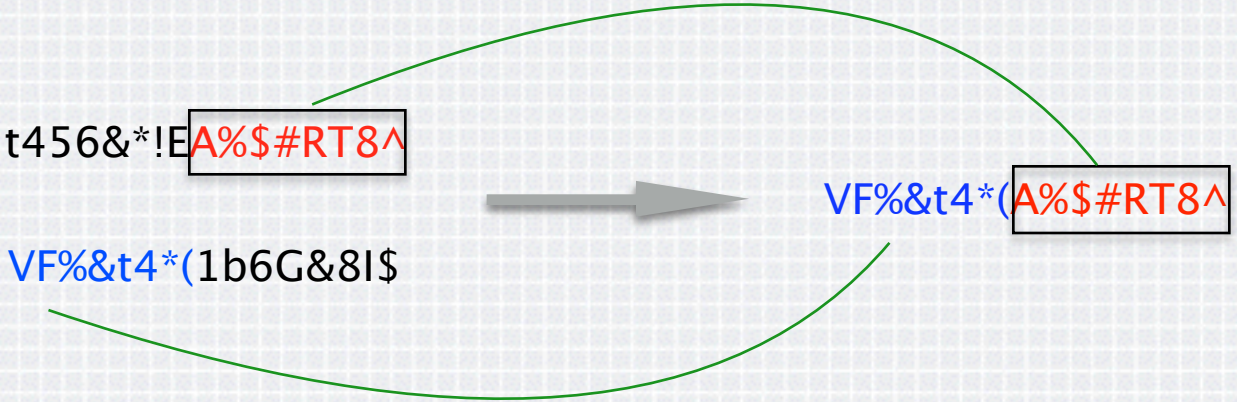
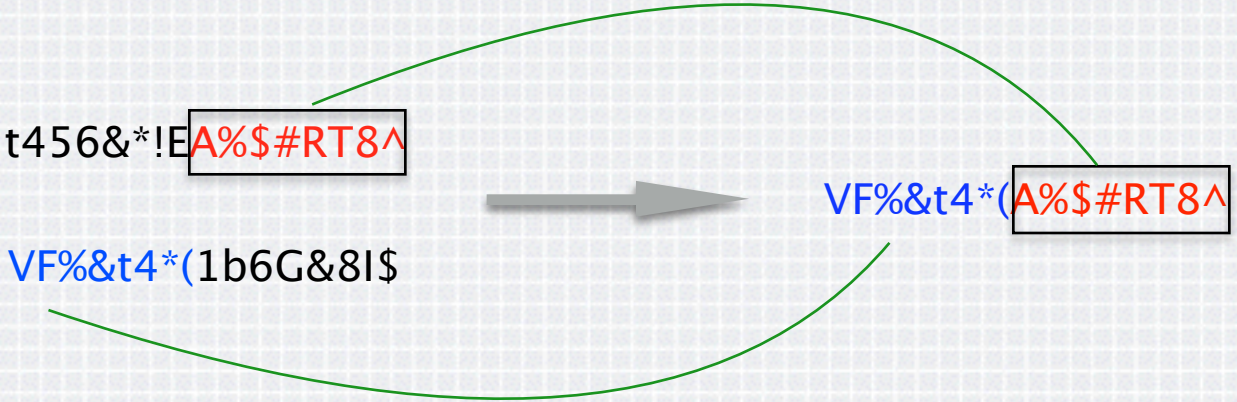
- The content provider can not frame a theater
- All suspects should be accused by the judge

A Subtle Security Consideration

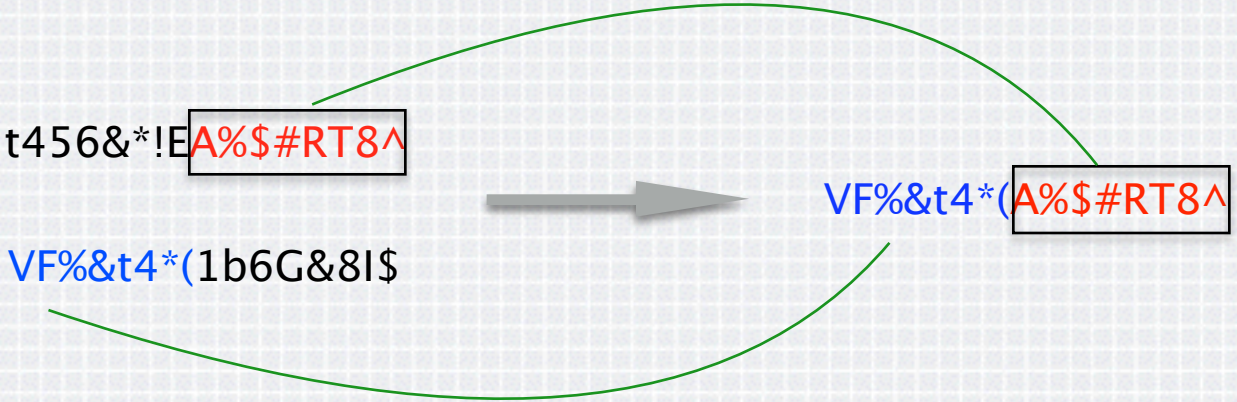


A Subtle Security Consideration



Cinema 1 will be the suspect, but the judge will not accuse him

A Subtle Security Consideration



Accusation Withdraw

A Subtle Security Consideration

1. Cinema should not know how the two halves are mixed

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2. Lower down the tracing parameter at the judge side

Important Efficiency Consideration

Building secure protocols brings some overhead

Important Efficiency Consideration

Even transmitting 2* movie size kills the bandwidth

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Even transmitting 2* movie size kills the bandwidth

And will hinder the adoption of this technique

Important Efficiency Consideration

Rate= $\dfrac{\text{Size of } \quad}{\text{Size of actual transmission}}$

Important Efficiency Consideration

Size of 

Rate= ―――――――――――――――――――― → 1

Size of actual transmission

Important Efficiency Consideration

Fight Piracy without Extra Bandwidth Cost !

High-level Construction Idea

High-level Construction Idea

Careful protocol design to meet both security and rate efficiency

High-level Construction Idea

- Setup phase
- Fingerprint phase
- Identify phase
- Dispute phase

Fingerprint Phase

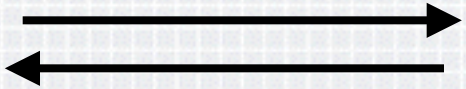
Cinema 1

A Movie Producer

Fingerprint Phase

Cinema 1

Oblivious Sampling



A Movie Producer

Fingerprint Phase

Cinema 1

Oblivious Sampling

A Movie Producer

tA4%$5#6&RT*!8E^

Fingerprint Phase

Cinema 1

Oblivious Sampling

Conditional OT

A Movie Producer

tA4%$5#6&RT*!8E^

Fingerprint Phase

Cinema 1

Oblivious Sampling

Conditional OT

A Movie Producer

tA4%$5#6&RT*!8E^

Fingerprint Phase

Cinema 1

tA4%$5#6&RT*!8E^

Oblivious Sampling

Conditional OT

OT

A Movie Producer

Fingerprint Phase

Cinema 1

tA4%$5#6&RT*!8E^

Oblivious Sampling

Conditional OT

OT

A Movie Producer

A%$#RT8^

Fingerprint Phase

- Content provider only knows half of the codeword

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- Theaters don’t know which part is known to the CP

Fingerprint Phase

- Content provider only knows half of the codeword
- Theaters don’t know which part is known to the CP
- Rate optimal OT and COT are needed

Identify Phase

- Run the tracing algorithm of the underlying fingerprinting code on the half known to the content provider

Dispute Phase

- The accused theaters submit the other halves of the codewords (with proofs of validity)

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- The judge also runs the tracing algorithm with a less restrict parameter on these halves

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- The judge also runs the tracing algorithm with a less restrict parameter on these halves

Weaker judge side parameter is to avoid accusation withdraw

Communication Optimal Tardos-Based Asymmetric Fingerprinting

Linearly Homomorphic Encryption from DDH

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CT-RSA 2015

Outline

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Linearly Homomorphic Encryption ?

- Public key encryption scheme with the following properties:
- Suppose that the set of plaintexts $\mathscr{M}$ is a ring
- $c \leftarrow \mathsf{Encrypt}(pk, m)$, $c' \leftarrow \mathsf{Encrypt}(pk, m')$
- $c_1 \leftarrow \mathsf{EvalSum}(pk, c, c')$ s.t.

$$\mathsf{Decrypt}(sk, c_1) = m + m'$$

- For $\alpha \in \mathscr{M}$, $c_2 \leftarrow \mathsf{EvalScal}(pk, c, \alpha)$ s.t.

$$\mathsf{Decrypt}(sk, c_2) = \alpha m$$

- Applications: Electronic Voting, Private Information Retrieval, Mix-Net, Oblivious Transfer, Fingerprinting...

Examples from Factoring

- Goldwasser Micali (84)
  - Plaintext space $\mathscr{M} = \mathrm{Z}/2\mathrm{Z}$
  - Ciphertext space : Z/NZ where $\mathrm{N} = pq$ is an RSA integer
- Paillier (99)
  - Plaintext space $\mathscr{M} = \mathrm{Z}/\mathrm{NZ}$
  - Ciphertext space : $\mathrm{Z}/\mathrm{N}^2\mathrm{Z}$ where $\mathrm{N} = pq$ is an RSA integer
  - Plaintext encoding :

$$m \in \mathrm{Z}/\mathrm{NZ} \mapsto (1+\mathrm{N})^m \equiv 1 + m\mathrm{N} \pmod{\mathrm{N}^2}$$

From DDH: ElGamal "in the exponent"

- Folklore message encoding: $m \in \mathbf{N} \mapsto g^m$
- $(c_1, c_2) = (g^r, h^r g^m) \leftarrow \mathsf{Encrypt}(pk, m)$
- $\mathsf{Decrypt}(pk, c) : c_2/c_1^x = g^m \rightsquigarrow m$
- m must be small. Can only do a bounded number of homomorphic operations:
  - $(c_1, c_2) = (g^r, h^r g^m) \leftarrow \mathsf{Encrypt}(pk, m)$,
  - $(c_1', c_2') = (g^{r'}, h^{r'} g^{m'}) \leftarrow \mathsf{Encrypt}(pk, m')$,

$$(c_1 c_1', c_2 c_2') = (g^{r+r'}, h^{r+r'} g^{m+m'})$$

$$(c_1^\alpha, c_2^\alpha) = (g^{r\alpha}, h^{r\alpha} g^{m\alpha})$$

DDH group with an easy DL subgroup

- $(\mathrm{G}, \times) = \langle g \rangle$ a cyclic group of order n
- $n = ps$, $\gcd(p, s) = 1$
- $\langle f \rangle = \mathrm{F} \subset \mathrm{G}$ subgroup of G of order p
- The DL problem is easy in F: There exists, Solve, a deterministic polynomial time algorithm s.t.

$$\mathsf{Solve}(p, f, f^x) \rightsquigarrow x$$

- The DDH problem is hard in G even with access to the Solve algorithm

A Generic Linearly Homomorphic Encryption Scheme

- $\mathscr{M} = \mathbf{Z}/p\mathbf{Z}$
- $pk : h = g^x$, $sk : x$, where g has order $n = ps$ for an unknown s
- Encrypt : $c = (c_1, c_2) = (g^r, f^m h^r)$, where $f \in \langle g \rangle$ has order p
- Decrypt : $\mathrm{A} \leftarrow c_2/c_1^x$, $\mathsf{Solve}(p, f, \mathrm{A}) \rightsquigarrow m$
- EvalSum :

$$(c_1 c_1', c_2 c_2') = (g^{r+r'}, h^{r+r'} f^{m+m'})$$

- EvalScal :

$$(c_1^\alpha, c_2^\alpha) = (g^{r\alpha}, h^{r\alpha} f^{m\alpha})$$

An Unsecure Instantiation

- p a prime and $G = \langle g \rangle = (\mathbf{Z}/p^2\mathbf{Z})^\times$ of order $n = p(p-1)$
- $f = 1 + p \in G$, $F = \langle f \rangle = \{1 + kp, k \in \mathbf{Z}/p\mathbf{Z}\}$
- $f^m = 1 + mp$.
- There exist a unique $(\alpha, r) \in (\mathbf{Z}/p\mathbf{Z}, (\mathbf{Z}/p\mathbf{Z})^\times)$ such that $g = f^\alpha r^p$

$$g^{p-1} = f^{\alpha(p-1)} = f^{-\alpha}$$

- Public key : $h = g^x$,

$$h^{p-1} = f^{-\alpha x} \rightsquigarrow x \mod p$$

- $(c_1, c_2) = (g^r, h^r f^m)$

$$c_1^{p-1} = f^{-\alpha r} \rightsquigarrow r \mod p$$

$$c_2^{p-1} = f^{-\alpha x r - m} \rightsquigarrow m \mod p$$

Partial Discrete Logarithm Problem

- $(\mathrm{G}, \times) = \langle g \rangle$ a cyclic group of order n
- $n = ps$, $\gcd(p, s) = 1$
- $\langle f \rangle = \mathrm{F} \subset \mathrm{G}$ subgroup of G of order p
- Partial Discrete Logarithm (PDL) Problem:

$$\text{Given } \mathrm{X} = g^x \text{ compute } x \mod p.$$

- The knowledge of s makes the PDL problem easy.

s must be hidden or unknown !

A Secure Instantiation

- Bresson, Catalano, Pointcheval (03)
- Let N be an RSA integer, $G = \langle g \rangle \subset (\mathbf{Z}/N^2\mathbf{Z})^\times$
- $n = \mathrm{Card}(G) = Ns$ with $s \mid \varphi(N)$,
- $f = 1 + N \in G$, $F = \langle f \rangle = \{1 + kN, k \in Z/NZ\}$, of order N
- Public key : $h = g^x$, x secret key
- $(c_1, c_2) = (g^r, h^r f^m)$
- Based on DDH in $(\mathbf{Z}/N^2\mathbf{Z})^\times$ and the Factorisation problem.
- The factorisation of N acts as a second trapdoor.

Outline

Definitions

Imaginary Quadratic Fields

- $K = \mathbf{Q}(\sqrt{\Delta_K})$, $\Delta_K < 0$
- Fundamental Discriminant:
  - $\Delta_K \equiv 1 \pmod 4$ square-free
  - $\Delta_K \equiv 0 \pmod 4$ and $\Delta_K/4 \equiv 2, 3 \pmod 4$ square-free
- Non Fundamental Discriminant:
  - $\Delta_\ell = \ell^2 \Delta_K$
  - ℓ is the conductor

Class Group of Discriminant Δ

- Finite Group denoted $C(\Delta)$
- Elements: Equivalence classes of Ideals
- Class Number: $h(\Delta) \approx \sqrt{|\Delta|}$

ElGamal in Class Group

- Buchmann and Williams (88): Diffie-Hellman key exchange and ElGamal
- Düllmann, Hamdy, Möller, Pohst, Schielzeth, Vollmer (90-07): Implementation
- Size of Δ_K? Index calculus algorithm to compute $h(\Delta_K)$ and Discrete Logarithm in $C(\Delta_K)$
- Security Estimates from Biasse, Jacobson and Silvester (10):
  - Complexity conjectured $L_{|\Delta_K|}(1/2, o(1))$
  - Δ_k : 1348 bits as hard as factoring a 2048 bits RSA integer
  - Δ_k : 1828 bits as hard as factoring a 3072 bits RSA integer

Map between two Class Groups

- Let Δ_K be a fondamental negative discriminant, $\Delta_K \neq -3, -4$, ℓ a conductor, and $\Delta_\ell = \ell^2 \Delta_K$
- There exists a surjective morphism, denoted $\bar{\varphi}_\ell$, between $C(\Delta_\ell)$ and $C(\Delta_K)$
- $\bar{\varphi}_\ell$ is effective, can be computed if ℓ is known
- Used by the NICE cryptosystem by Paulus and Takagi (00), $\Delta_K = -q$, $\Delta_p = -qp^2$, p, q primes, p is the trapdoor
- C., Laguillaumie (09) :

In each non trivial class of $\ker \bar{\varphi}_p$, there exists an ideal of the form $\left[p^2\mathbf{Z} + \frac{bp+\sqrt{\Delta_p}}{2}\mathbf{Z}\right]$

Outline

A Subgroup with an Easy DL Problem

- $\Delta_K = -pq$, $\Delta_p = -qp^3$, p, q primes and $pq \equiv 3 \pmod 4$

$$h(\Delta_p) = p \times h(\Delta_K)$$

- Let $f = \left[p^2\mathbf{Z} + \frac{p+\sqrt{\Delta_p}}{2}\mathbf{Z}\right] \in \mathrm{C}(\Delta_p)$

- $\mathrm{F} = \ker \bar{\varphi}_p = \langle f \rangle$ is of order p, and

$$f^m = \left[p^2\mathbf{Z} + \frac{[m^{-1} \mod p]p + \sqrt{\Delta_p}}{2}\mathbf{Z}\right]$$

A New Linearly Homomorphic Encryption Scheme

- $\Delta_K = -pq$, $\Delta_p = -qp^3$, p, q primes and $pq \equiv 3 \pmod 4$ and $(p/q) = -1$, $q > 4p$
- Let g be an element of $C(\Delta_p)$, $h = g^x$ where x secret key
- g has order ps for an unknown $s | h(\Delta_K)$
- $(c_1, c_2) = (g^r, h^r f^m)$ where f has order p
- Based on DDH in $C(\Delta_p)$ (and the Class number problem).
- Linearly homomorphic over $\mathbf{Z}/p\mathbf{Z}$ where p can be chosen (almost) independently from the security parameter

Some Variants

- Faster Variant: most of the work in $C(\Delta_K)$ (based on a non standard problem)
- More general message spaces:
  - $\mathbf{Z}/N\mathbf{Z}$ with $N = \prod_{i=1}^{n} p_i$, with a discriminant of the form $\Delta_K = -Nq$
  - $\mathbf{Z}/p^t\mathbf{Z}$ for $t > 1$, with discriminants of the form $\Delta_{p^t} = p^{2t}\Delta_K$, and $\Delta_K = -pq$

Performance comparison

Cryptosystem	Parameter	Message Space	Encryption (ms)	Decryption (ms)
Paillier	2048 bits modulus	2048 bits	**28**	**28**
BCP03	2048 bits modulus	2048 bits	107	54
New Proposal	1348 bits Δ_K	80 bits	93	49
Fast Variant	1348 bits Δ_K	80 bits	82	45
Fast Variant	1348 bits Δ_K	256 bits	105	68
Paillier	3072 bits modulus	3072 bits	**109**	109
BCP03	3072 bits modulus	3072 bits	427	214
New Proposal	1828 bits Δ_K	80 bits	179	91
Fast Variant	1828 bits Δ_K	80 bits	145	**78**
Fast Variant	1828 bits Δ_K	512 bits	226	159
Fast Variant	1828 bits Δ_K	912 bits	340	271

Timings performed with Sage and PARI/GP.

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