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CHANGE

Challenge today's security thinking

SESSION ID: CRYP-R01

Finding Shortest Lattice Vectors in the Presence of Gaps

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April 23, 2015

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Outline

- Motivation
- Revisit SVP Algorithms on Lattices with Gaps
- Search SVP for Some Lattice-based Cryptosystems
- Summary

Motivation

Shortest Vector Problem

- SVP: NP-Hard
  - Given a basis of a lattice, find a nonzero shortest lattice vector.
- uSVP$_\gamma$: unique-Shortest Vector Problem
  - $\lambda_2(L) > \gamma\lambda_1(L)$, find a nonzero shortest lattice vector.
- SVP algorithms
  - Deterministic: enumeration, Voronoi cell computation based…
  - Probabilistic: heuristic & **provable sieve**…

Previous Work

- Probabilistic Sieve algorithms:
  - **Heuristic**:

Algorithm	Time	Space
Nguyen, Vidick (2008)	$2^{0.415n}$	$2^{0.2075n}$
Wang, et al. (2011)	$2^{0.3836n}$	$2^{0.2557n}$
Zhang, et. al. (2013)	$2^{0.3778n}$	$2^{0.2833n}$
Becker, et. al. (2013)	$2^{0.3774n}$	$2^{0.2925n}$

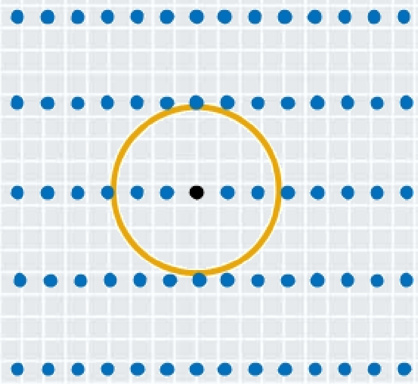
Previous Work

- Probabilistic Sieve algorithms:
  - **Provable**:

Algorithm	Time	Space	Reference
AKS	$2^{O(n)}$	$2^{O(n)}$	[Ajtai,et al. 2001]
Regev	2^{16n}	2^{8n}	[Regev 2004]
NV	$2^{5.9n}$	2^{3n}	[Nguyen, Vidick 2008]
ListSieve	$2^{3.199n}$	$2^{1.325n}$	[Micciancio, Voulgaris 2009]
ListSieve-Birthday	$2^{2.465n}$	$2^{1.233n}$	[Pujol, Stehlé 2009]

Motivation

- What about lattices with **gaps**?
  - Successive minima $\lambda_2(L) > \gamma\lambda_1(L)$
  - Sparse distribution
  - Complexity decreases obviously as the increase of gap
  - Common in cryptographic instances



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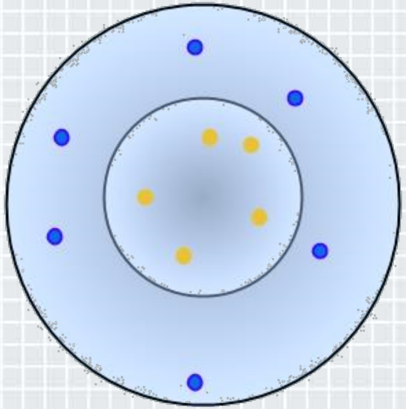
SVP Algorithms on Lattices with Gaps

List-Sieve [MV09]

- Creat a set of short vectors by subtractions.
- All previous vectors are used to reduce a new one.
- Random perturbation technique.
- Several lattice vectors might correspond to one perturbed vector.
- A collision happens with a high probability when there are enough sieved vectors.

ListSieve-Birthday[PS09]

- Apply List-Sieve, sample lattice points that fall inside of the corona which consist of the first list.
- Sample **small** and **independent** points by reducing random points with respect to the first list.
- A collision occurs with high probability.

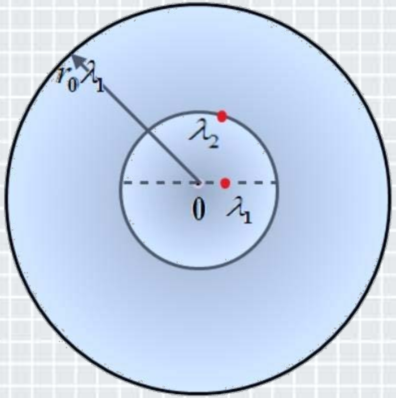


Revisit Sieve Algorithms on Lattices with Gaps

- Two cases
  - λ_2-gap: $\lambda_2(L) > \alpha\lambda_1(L)$
  - λ_{i+1}-gap: $\lambda_{i+1}(L) > \alpha\lambda_1(L)$
- Concretely
  - **Packing density** of lattices with gaps
  - ListSieve-Birthday

Packing density of lattices with λ_2-gap

What is the maximum number of lattice points inside a sphere with radius $r_0\lambda_1$?



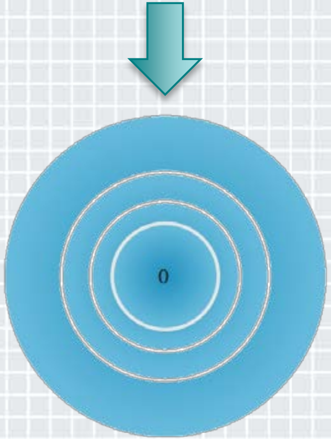
- Our result: If $\lambda_2(L) > \alpha\lambda_1(L)$, then

$$|\mathcal{B}_n(\mathbf{0}, r_0\lambda_1) \cap L| \leq 2^{c_b n + o(n)},$$

where $c_b = \log_2 r_0 - \log_2 \alpha + 0.401$ and $1 \leq \alpha \leq r_0$.

Count the Number of Points

Partition into coronas



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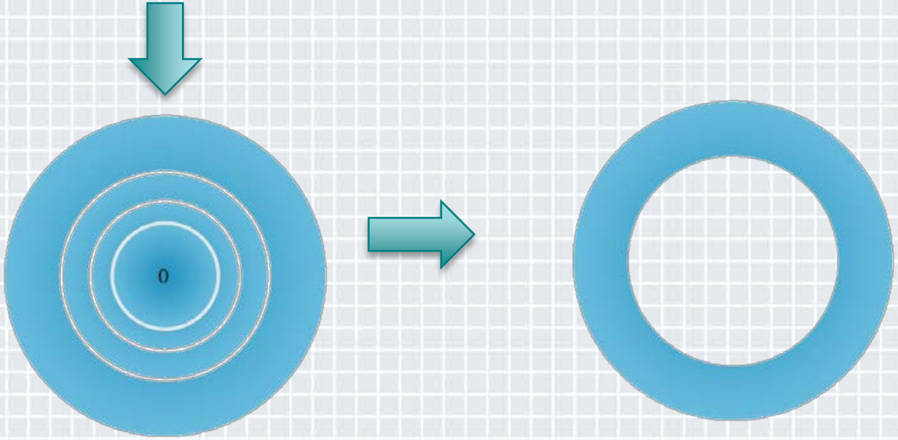

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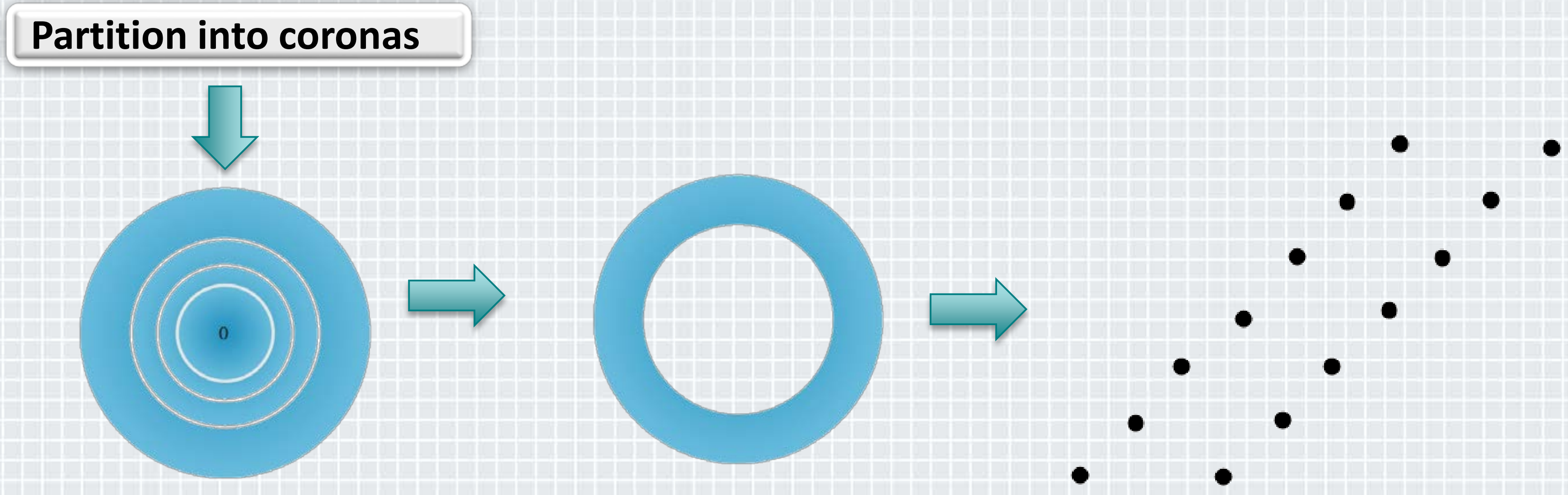

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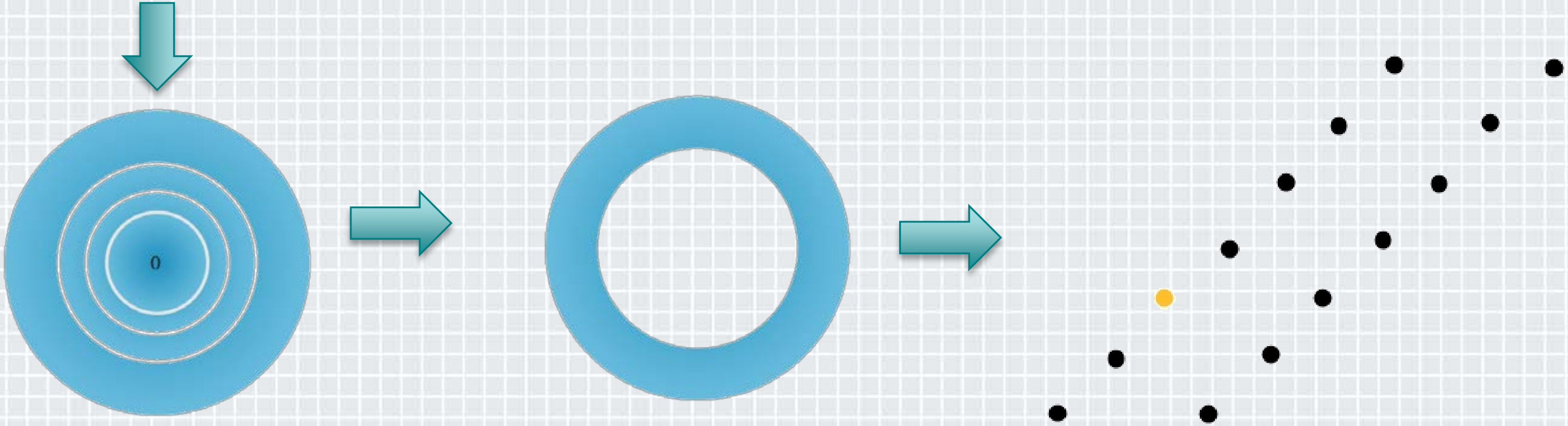

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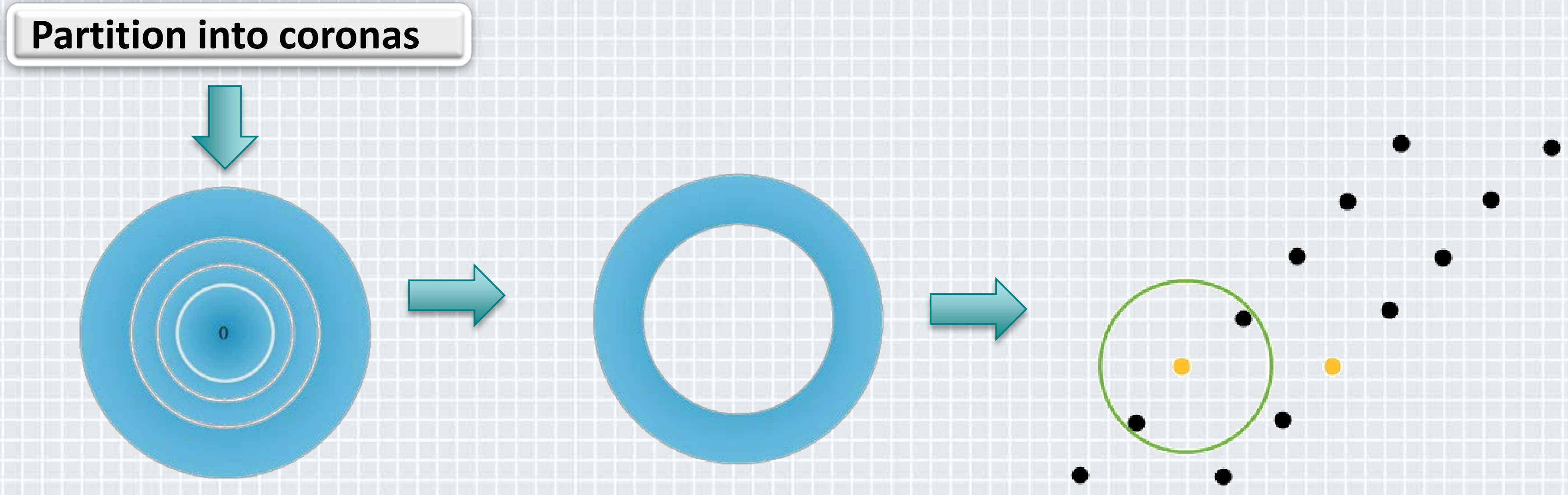

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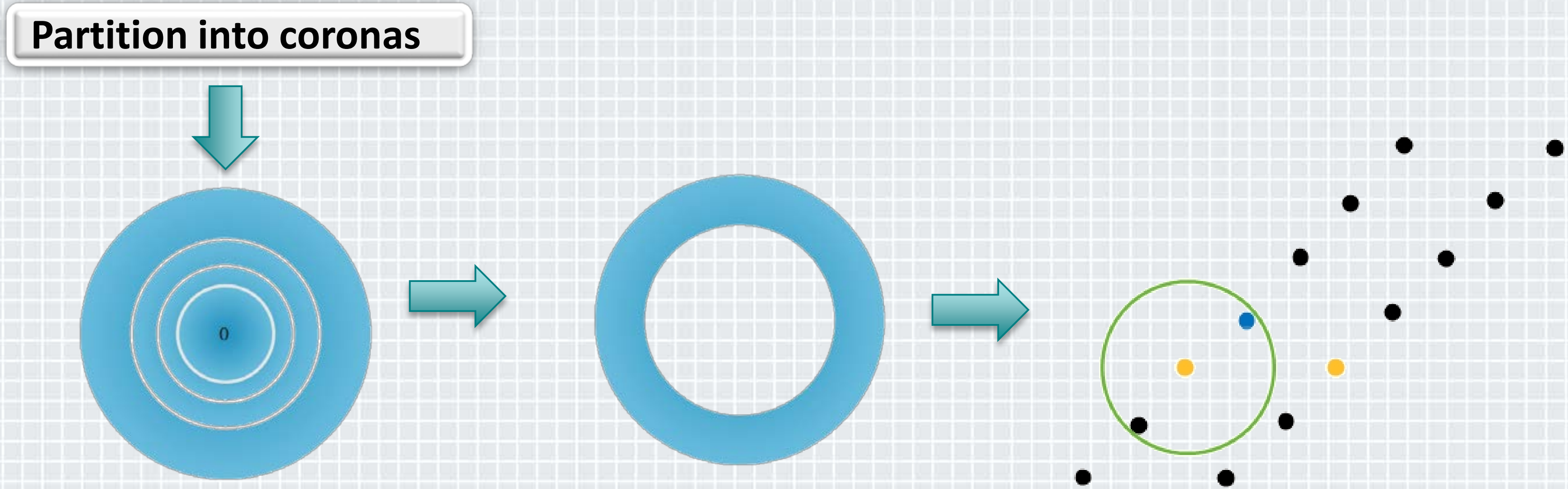

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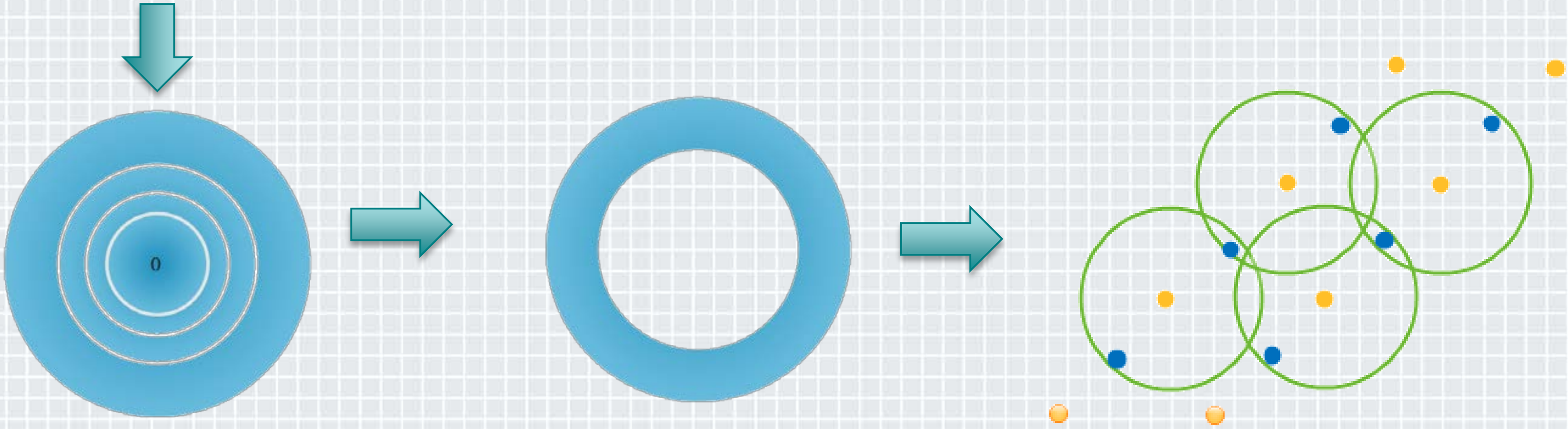

Count the Number of Points

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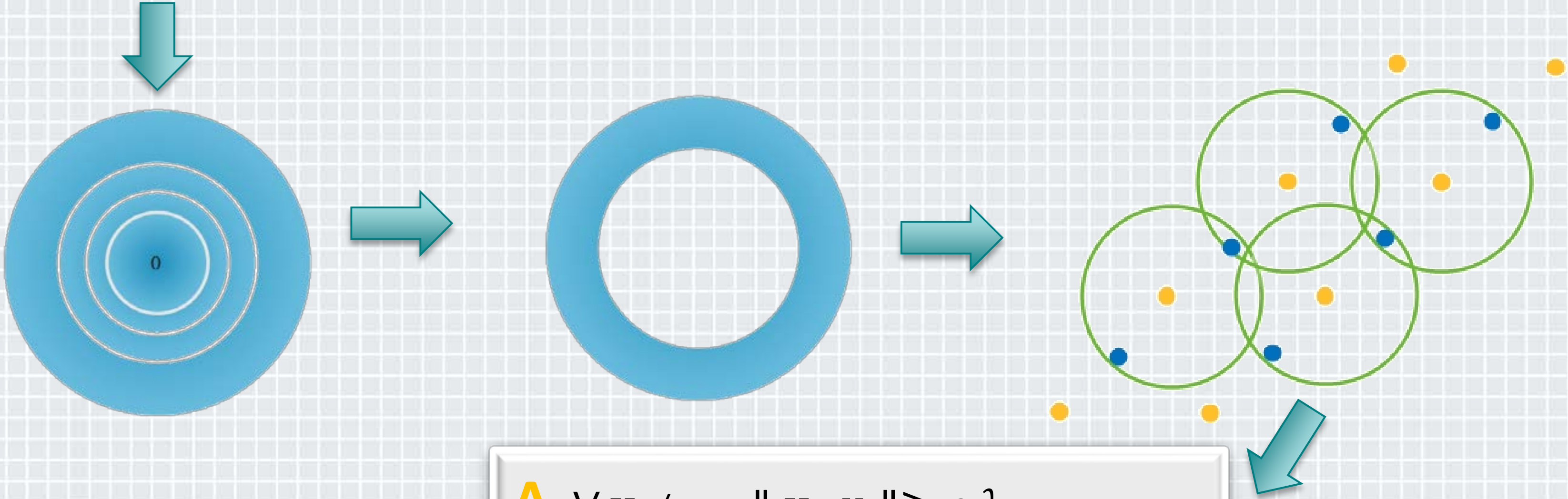
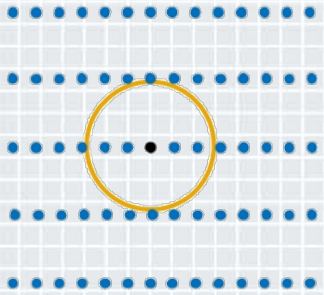


$\mathbf{A}$: $\forall\ \mathbf{v} \neq \mathrm{u},\ \| \mathbf{v}-\mathbf{u} \| \geq \alpha\lambda_1$

$\mathbf{B} = \{\mathbf{w}\!:\ \exists\ \mathbf{t} \in \mathbf{A},\ \text{s.t.}\ \| \mathbf{w}-\mathbf{t} \| < \alpha\lambda_1\}$

Estimate |A|, |B|

- $|\mathbf{B}| \le (1 + 2\lfloor \alpha \rfloor)|\mathbf{A}|$



Estimate |A|, |B|

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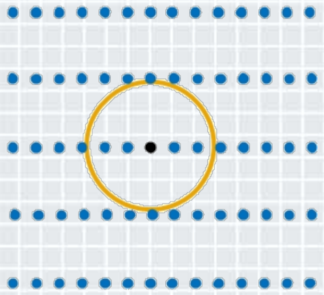
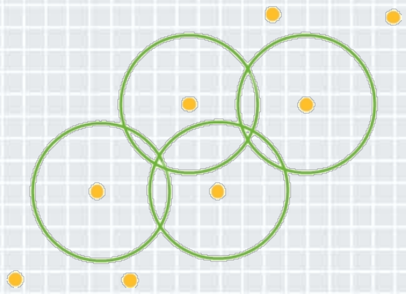


- $|\mathbf{A}| \leq 2^{(\log_2 r_0 - \log_2 \alpha + 0.401)n + o(n)}$



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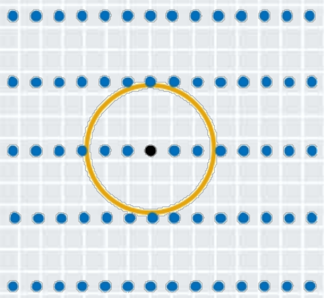
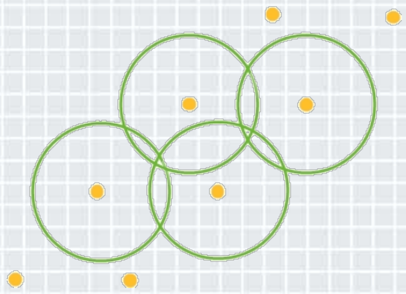


- $|\mathbf{A}| \leq 2^{(\log_2 r_0 - \log_2 \alpha + 0.401)n + o(n)}$



- Finally, $|\mathcal{B}_n(\mathbf{0}, r_0\lambda_1) \cap L| \leq \mathrm{poly}(n) \cdot (|\mathbf{A}| + |\mathbf{B}|) \leq 2^{c_b n + o(n)}$

Complexity Analysis of ListSieve-Birthday

- Time: $2^{c_{time}n+o(n)}$, Space: $2^{c_{space}n+o(n)}$
- Minimize the time complexity,

$$c_{time} = 0.802 + \log_2\left(\frac{1}{\sqrt{1-\frac{1}{4\xi^2}}} + \frac{2\xi}{\alpha \cdot 2^{0.401}\left(1-\frac{1}{4\xi^2}\right)}\right).$$

- When λ_2-gap >1.78, $c_{time} < 2$, $c_{space} < 1$ by selecting $\xi = 1.0015$.

c_{time}s corresponding to different λ_2-gap

α	ξ	r_0	c_{time} gap
1.78	1.0020	4.0409	1.9969
5	1.1768	8.3301	1.4246
8	1.2992	12.3483	1.2585
12	1.4308	17.7075	1.1502
28	1.7952	39.0991	0.9992
100	2.6293	134.8910	0.8859
500	4.4019	664.7420	0.8306

Sieve for SVP with λ_{i+1}-gap

- **λ_{i+1}-gap**

  $\lambda_{i+1}(L) > \alpha\lambda_1(L), 1 \le i \le n-1$

- **NTRU lattice**

  λ_{N+1}-gap ([HPS98], heuristic)

- **Packing density**

$$|\boldsymbol{\mathcal{B}}_n(\mathbf{0}, r_0\lambda_1) \cap L| \le 2^{(\log_2 r_0 - \log_2 \alpha + 0.401)n + (\log_2 \alpha + 0.401)i + o(n)}.$$

Sieve for SVP with λ_{i+1}-gap

- $c_{time} = 0.802 + \log_2\left(\frac{1}{\sqrt{1-\frac{1}{4\xi^2}}} + \frac{2\xi}{(\alpha\cdot 2^{0.401})^{(1-\frac{i}{n})}(1-\frac{1}{4\xi^2})}\right).$

i \ α	1.78	5	8	12	28	100	500
$\frac{n}{16}$	1.9225	1.4282	1.2767	1.1744	1.0244	0.9035	0.8393
$\frac{n}{8}$	1.9574	1.4757	1.3231	1.2180	1.0597	0.9261	0.8508
$\frac{n}{4}$	2.0297	1.5805	1.4287	1.3200	1.1473	0.9875	0.8857
$\frac{n}{2}$	2.1848	1.8337	1.7000	1.5968	1.4145	1.2116	1.0455
$\frac{3n}{4}$	2.3541	2.1513	2.0658	1.9956	1.8587	1.6777	1.4876

Sieve for SVP with λ_{i+1}-gap

- $c_{time} = 0.802 + \log_2\left(\frac{1}{\sqrt{1-\frac{1}{4\xi^2}}} + \frac{2\xi}{(\alpha\cdot 2^{0.401})^{(1-\frac{i}{n})}(1-\frac{1}{4\xi^2})}\right).$

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Complexity depends on the **value** and **location** of gap!

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Search SVP for Some Lattice-based Systems

Search SVP for Some Lattice-based Systems

- LWE (Learning with Errors)-based cryptosystem
  - A BDD instance in the q-ary lattice
    $$\Lambda_q(\mathbf{A}^T) = \{\mathbf{y} \in \mathbb{Z}^m : \mathbf{y} = \mathbf{As} \bmod q \text{ for } s \in \mathbb{Z}_q^n\}.$$
  - [LW11] gave its λ_2-gap of the embedding lattice.
  - Our result: For the parameter $n = 128$ in the scheme[Gentry et. al.'08],
    - **λ_2-gap $\approx$ 1288**.
    - Time: $2^{0.8172m+o(m)}$.
    - Space: $2^{0.4086m+o(m)}$.
    - Approximately to $2^{0.802m+o(m)}$ ($2^{0.401m+o(m)}$).

Search SVP for Some Lattice-based Systems

- GGH encryption cryptosystem [Goldreich, Goldwasser, Halevi'97]
  - A BDD-based cryptosystem
  - five challenges: n=200, 250, 300, 350, 400.
  - [Nguyen’99] Four of them are solved and it is indicated the excepted **λ_2-gap> 9.4**.
  - Our result: The time complexity of ListSieve-Birthday is $2^{1.212n+o(n)}$.

Search SVP for Some Lattice-based Systems

- Worst-case/average-case equivalent cryptosystems
  - uSVP_{n^c} based: [Ajtai, Dwork'97, Regev'04].
  - GapSVP_{n^c} based: [Regev'09, Peikert'09].

    Then can be equivalently based on $\mathrm{uSVP}_{\tilde{O}(n^c)}$ since the reduction from uSVP_{γ} to GapSVP_{γ}.
  - Our result: Time complexity is approximately to $2^{0.802n+o(n)}$.

Search SVP for some lattice-based systems

- NTRU encryption cryptosystem [Hoffstein, Pipher, Silverman'98]
  - Adopted to standard of IEEE Std 1363.1 in 2008.
  - [HPS98] Heuristically, the NTRU lattice (dimension=2N) has a λ_{N+1}-gap approximately $\sqrt{\frac{Nq}{4\pi e(d_f \cdot d_g)^{1/2}}}\lambda_1$.
  - For $N = 503, q = 256, d_f = 216, d_g = 72,$ the time to solve this SVP of NTRU lattice is $2^{1.8054n+o(n)}$.

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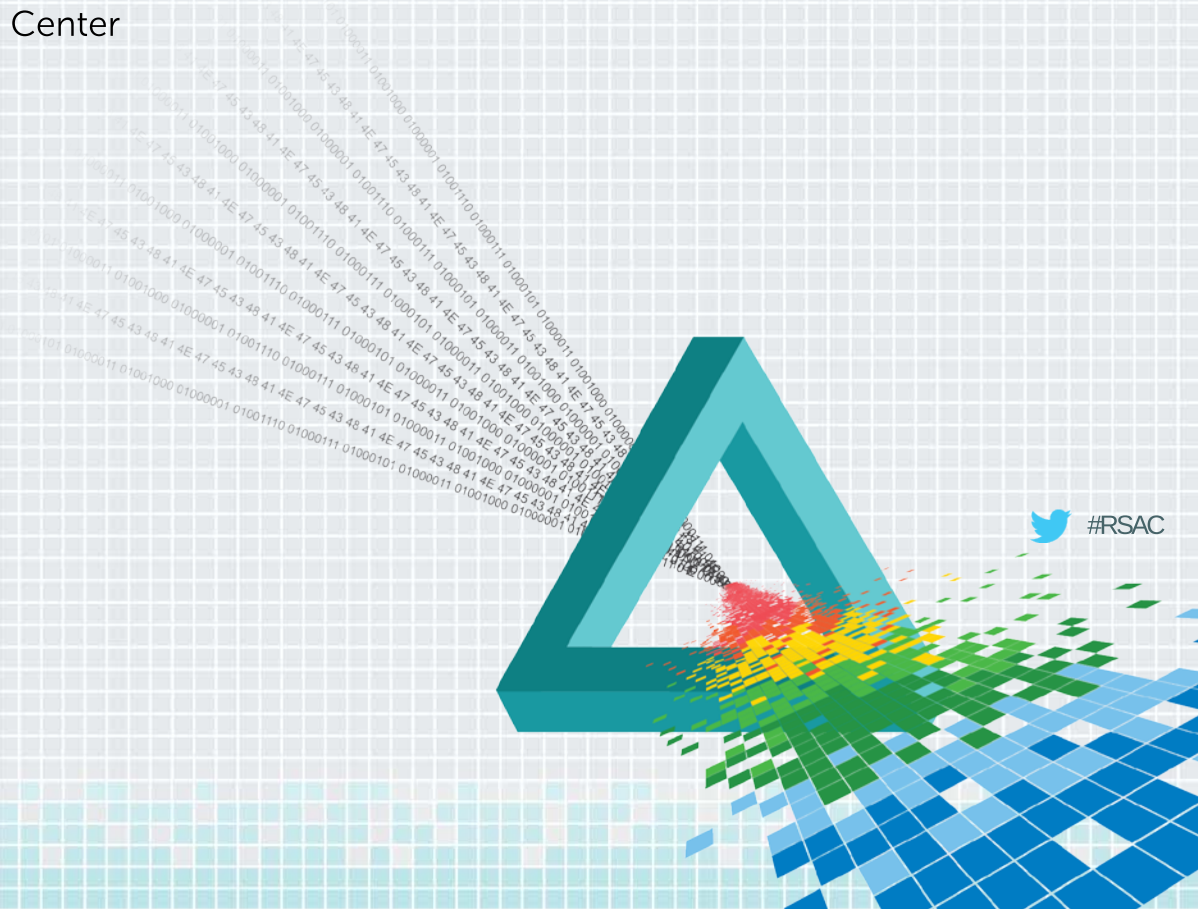

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Summary

Summary

- Study SVP on a lattices possessing gaps
  - New upper bounds for the packing density of lattices with λ_i-gap.
  - Renew the complexity of the ListSieve-Birthday
- Discussions on SVP search for some lattice-based cryptosystems
  - LWE-based, GGH, NTRU…
  - Cryptographic lattices should avoid large gaps.

Thank you for your attention!

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Challenge today's security thinking

SESSION ID: CRYP-R01

A Simple and Improved Algorithm for Integer Factorization with Implicit Hints

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Contents

- Introduction: Integer factoring with implicit hints for LSBs of factors
- Our results
  - Algorithm: Better bound, simpler proof
  - (Potential) application to “(batch) FHE over integers” etc.
- Details and computer experiments

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Introduction

Background: Cryptography and Factoring

- Computational hardness of integer factoring is:
  - Necessary (and sometimes sufficient) for security of many cryptosystems
    - Including the RSA cryptosystem
  - Therefore, important to analyze

Background: Factoring with Hints

- Factoring with **explicit** hints
  - E.g., [Coppersmith 1996], where some bits of the factors are known
  - Related to: Side-channel attacks
- Factoring with ***implicit*** hints (**this work**)
  - E.g., [May-Ritzenhofen 2009], where only some ***relations*** of bits of the factors are known
  - Related to: Attacks on implementation with weak randomness

Factoring with Implicit Hints

- Simplest case ([MR09], [Kurosawa-Ueda 2013]):
  For two integers $N_1 = p_1 q_1, N_2 = p_2 q_2$, assume

  (t LSBs of p_1) = (t LSBs of p_2)

  - Or equivalently, $\boldsymbol{p_1 \equiv p_2 \ (mod\ 2^t)}$

- Generalizations (*not considered in this work*):
  - More integers ([MR09], [Sarkar-Maitra 2011], …)
  - MSBs, or combination of LSBs and MSBs ([SM11], …)

Previous Results

- Assume $N_1 = p_1 q_1, N_2 = p_2 q_2$ and $\boldsymbol{p_1 \equiv p_2 \ (mod\ 2^t)}$
- Also assume $\boldsymbol{q_1, q_2 < 2^\alpha}$ (i.e., q_1, q_2 are α-bit primes)
- Polynomial-time factoring of N_1, N_2, if
  - [MR09] $t \geq 2\alpha + 3$
  - [KU13] $t \geq 2\alpha + 1$

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Our Result: Summary

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Our Result

- Assume $N_1 = p_1 q_1, N_2 = p_2 q_2$ and $\boldsymbol{p_1 \equiv p_2 \ (mod\ 2^t)}$
- Also assume $\boldsymbol{q_1, q_2 < 2^\alpha}$ (i.e., q_1, q_2 are α-bit primes)
- Polynomial-time factoring of N_1, N_2, if
  - [MR09] $t \geq 2\alpha + 3$
  - [KU13] $t \geq 2\alpha + 1$
  - **Our result**: $\boldsymbol{t = 2\alpha - O(\log \kappa)}$, where κ is a parameter (e.g., security parameter of a factoring-based cryptosystem)

Non-constant improvement!

Advantage: Simplicity and Generality

- Our result (as well as [KU13]) extends to $p_1 \equiv p_2 \ (mod\ \boldsymbol{T})$ and $q_1, q_2 \leq \boldsymbol{Q}$ for integers T, Q
  - Originally $T = 2^t,\ Q = \ 2^\alpha$
- We do ***NOT*** assume that p_1, p_2, q_1, q_2 are primes
  - Only assume that N_1, N_2, T are mutually coprime (almost automatic)
- Very simple, easy-to-follow proof
  - No lattice inequalities (Minkowski bound, Hadamard's inequality, …)

Related Work

- Factors p_1, p_2, q_1, q_2 in [SM11] (and some others)
  - Prime
  - Balanced (i.e., $|p_i|_2 \approx |q_i|_2$)
    - In fact, their result requires $|p_i|_2$ to be bounded above
- Factors p_1, p_2, q_1, q_2 in our result
  - **Not necessarily prime** — **Good**
  - **Unbalanced** (i.e., $|p_i|_2 \gg |q_i|_2$) — **Sometimes good (next slide)**
    - $|p_i|_2$ is bounded below only by the condition $t = 2\alpha - O(\log \kappa)$

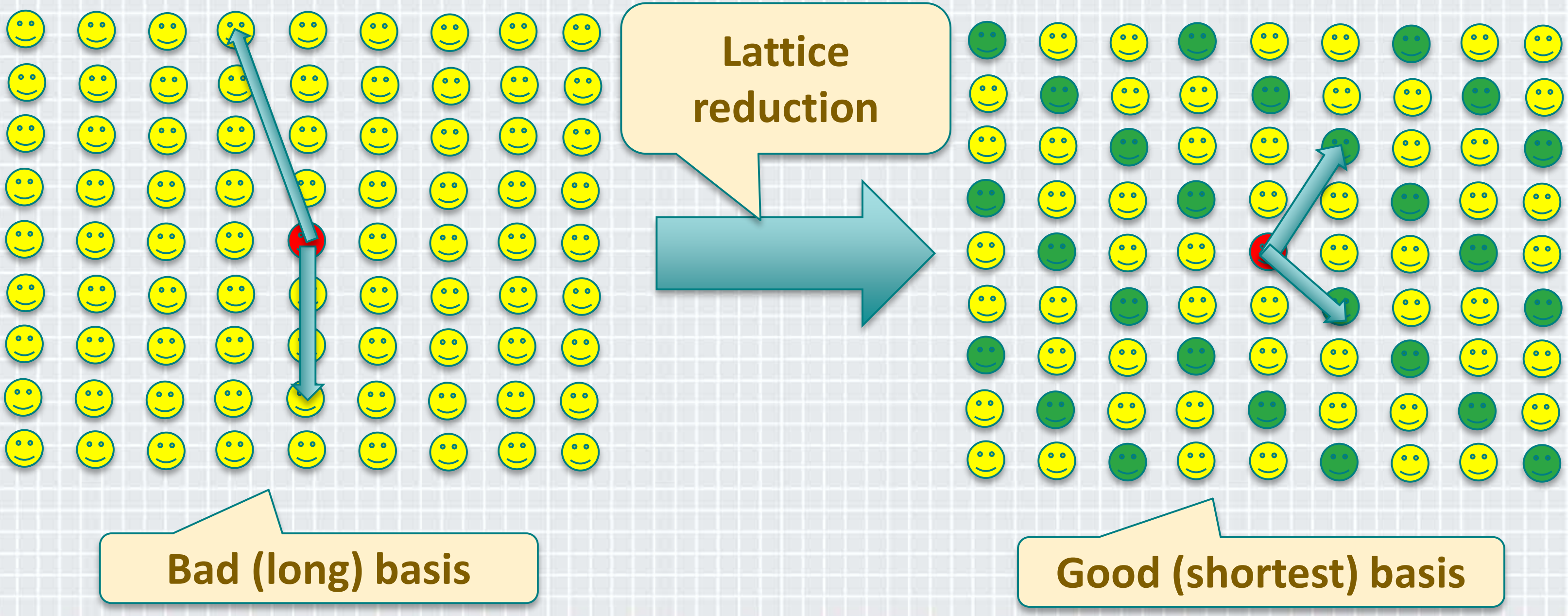
(Potential) Applications

- Variants of (batch) "fully homomorphic encryption over integers" with error-free approximate GCD assumptions [Cheon et al. 2013], [N.-Kurosawa, EUROCRYPT 2015]
  - Ciphertexts are integers modulo $N = qp_1p_2 \cdots p_k$, where $|q|_2 \gg |p_i|_2$
  - Apply our result to factors p_i and $\boldsymbol{N/p_i}$ (**unbalanced**, **non-prime**)
- Okamoto-Uchiyama cryptosystem, Takagi's variant of RSA
  - $N = \boldsymbol{p^r}q, r \geq 2$ (**unbalanced**, **non-prime**)

Our Result: Details

(Integer) Lattice and Basis Reduction

- Lattice in 2-dim. plane



Lattice for Our Problem

- $L = \{(x_1, x_2) \in \mathbb{Z}^2 : N_2 x_1 - N_1 x_2 \equiv 0 \ (mod\ T)\}$

  (recall $N_1 = p_1 q_1, N_2 = p_2 q_2, p_1 \equiv p_2 \ (mod\ T)$)

  - Same as previous work
  - L and initial basis $(1, N_2/N_1\ mod\ T), (0, T)$ are publicly known
  - Involves secret vector $\vec{q} = (q_1, q_2)$

Find this!

Previous Results

- Outline of [KU13]:
  - Find the shortest vector $\vec{v}$ in 2-dim. lattice L by Gaussian reduction
  - If **(*) the second shortest basis vector of L is longer than $\vec{q}$**, then $\vec{\boldsymbol{q}} \propto \vec{v}$, in particular $\vec{\boldsymbol{q}} = (\boldsymbol{q_1}, \boldsymbol{q_2}) = \pm\vec{\boldsymbol{v}}$ (since q_1, q_2 are coprime)
  - (*) is guaranteed when $t \geq 2\alpha + 1$ (by Hadamard's inequality)
    - And not guaranteed if $t < 2\alpha + 1$

Our Idea

- Use not only the shortest vector $\vec{v}$, but **also the second shortest basis vector $\vec{u}$** (both obtained by Gaussian reduction at once)
  - $\vec{q} = (q_1, q_2)$ can be written as $\vec{q} = a\vec{v} + b\vec{u}$, $a, b \in \mathbb{Z}$
  - If **(**)** $|\boldsymbol{a}|, |\boldsymbol{b}| \leq \boldsymbol{poly}(\boldsymbol{\kappa})$, then a, b (hence $\vec{q}$) are found in time $poly(\kappa)$
  - $\vec{q} = a\vec{v} + b\vec{u}$ implies $|a|, |b| = \frac{|(\text{quadratic in } q_i, v_i, u_i)|}{|\det(\vec{v}, \vec{u})|} \leq (\text{const}) \cdot Q^2/T$
    - Where we used $|\det(\vec{v}, \vec{u})| = |\det((1, N_2/N_1 \; mod \; T), (0, T))| = T$ (property of Gaussian reduction) and $||\vec{v}|| \leq ||\vec{u}|| \leq ||\vec{q}|| \leq Q$
      - The other case $||\vec{q}|| < ||\vec{u}||$ is as in the previous work
  - Hence (**) is guaranteed when $Q^2/T = poly(\kappa)$ (or $2\alpha - t = O(\log \kappa)$)

Our Proposed Algorithm

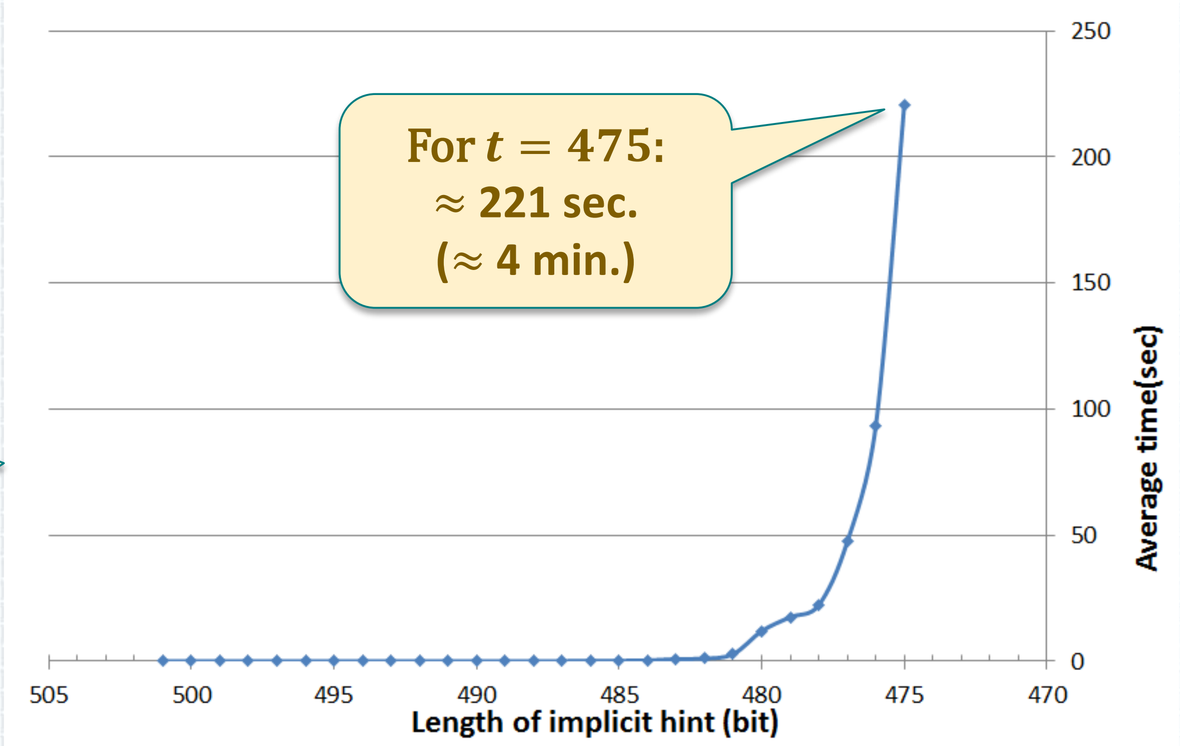
1. Compute $\vec{v}, \vec{u}$ from $(1, N_2/N_1 \; mod \; T), (0, T)$ by Gaussian reduction
2. Output common factors of N_i and v_j (or u_j), if exists
3. For $A = 2, 3, \ldots$, do the following
    1. For integers a, b satisfying $|a| + |b| = A$, do the following
        1. If $|av_1 + bu_1|$ is a non-trivial factor of N_1, output it

Computer Experiments: Average Time

- $\alpha = 250$
- Range of t: $501 = 2\alpha + 1$ to 475
- Ordinary PC
- 100 trials each

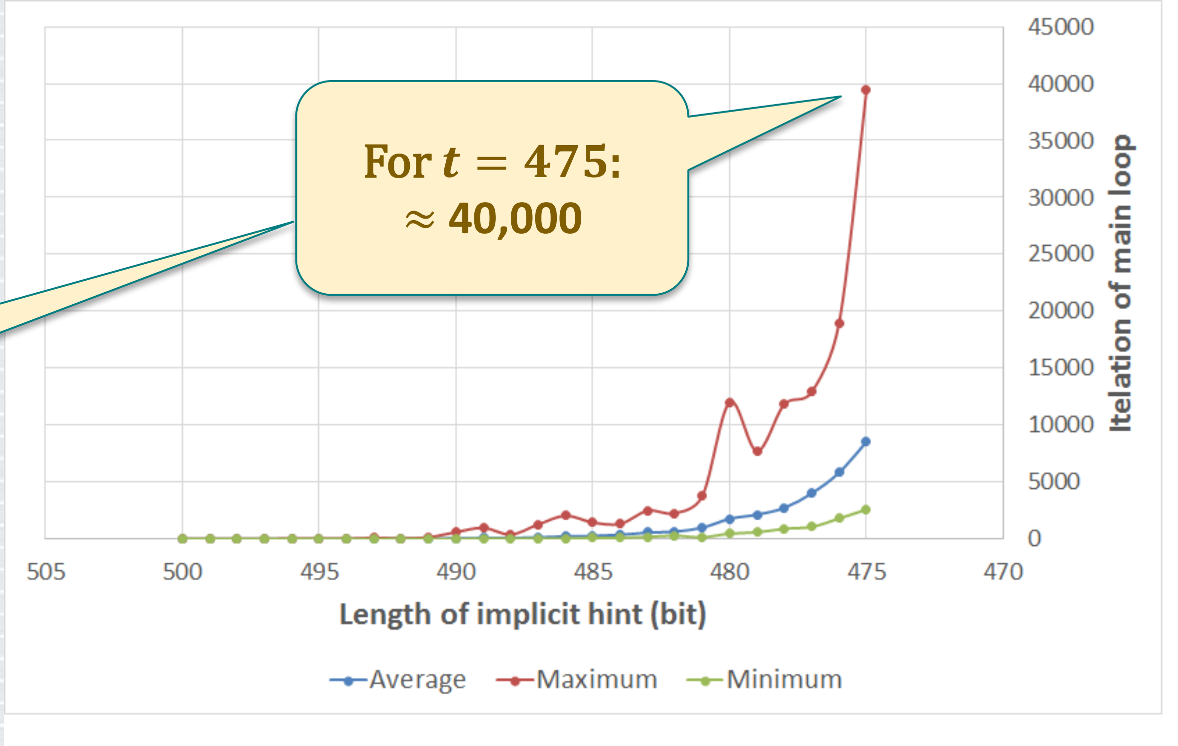
100% Success

(cf., in [KU13]: 40% for $t = 500$ 0% for $t = 499$)



Computer Experiments: # of Iterations

- $\alpha = 250$
- Range of t:
  $501 = 2\alpha + 1$ to 475
- Ordinary PC
- 100 trials each



Bound by our argument:

$\leq 2^{2\alpha+2-t}$

$= 2^{27} \approx 1.34 \times 10^8$

(Probably too loose)

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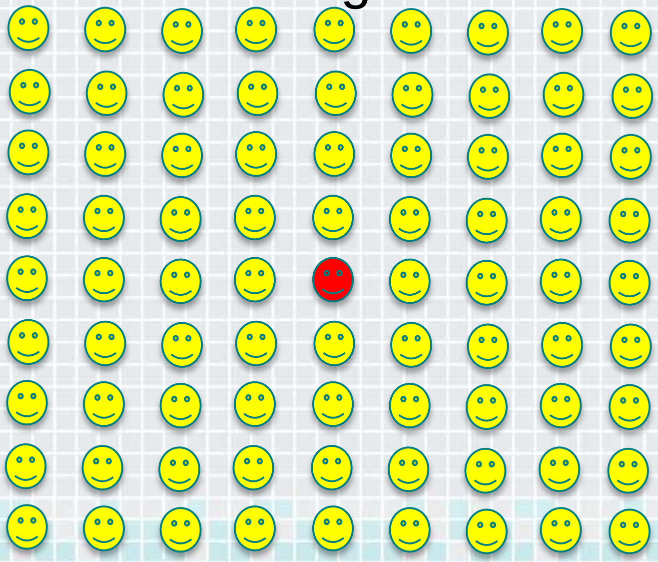
Summary

Summary and Future Work

- Improvement of a known factoring algorithm with implicit hints
  - Better bound, even by a simpler proof
- (Potential) applications; e.g., (batch) FHE over integers
- Future work:
  - Sharper analysis of bounds?
  - More applications?



Thank you
for your attentions!