## Analyzing Permutations for AES-like Ciphers: Understanding ShiftRows

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## Outline

1. Motivation
2. Notation and definitions
3. Equivalence results
4. Experiments
5. Conclusion

## Motivation

## Current status

- AES-like designs are very frequent in practice: LED, mCrypton, PRINCE, ECHO, Grøstl, LANE, PHOTON, PAEQ, PRIMATEs, Prøst, STRIBOB, ...


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- Well understood: Pick sufficiently high branch number


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- Crucial: Understanding properties of diffusion and resistance to differential/linear attacks
- MixColumns-like step
- Well understood: Pick sufficiently high branch number
- ShiftRows-like step:
- Unclear; no structured approach
- Choice remains ad-hoc


## Motivation

## Our goal

Contribute to the understanding of picking optimal ShiftRows-like operations for generalized AES-like ciphers

Notation and definitions

## AES-like cipher

- State of size $M \times N$ of $m$-bit words

- Round $t$ equals

$$
R_{t}=\text { AddRoundKey }_{t} \circ \text { Permute }_{\pi_{t}} \circ \text { MixColumns }_{t} \circ \text { SubBytes }_{t}
$$

## AES-like cipher: SubBytes ${ }_{t}$



- Substitutes each state word according to one or more S-boxes

$$
S_{i, j}^{t}: \mathbb{F}_{2}^{m} \rightarrow \mathbb{F}_{2}^{m}
$$

- Allow independent S-boxes for each word $x_{i, j}$ in each round


## AES-like cipher: MixColumns ${ }_{t}$



- Left-multiplies column $j$ in round $t$ by an $M \times M$ matrix $M_{j}^{t}$ over GF( $2^{m}$ )
- Allow independent $M_{j}^{t}$ for each column in each round


## AES-like cipher: Permute $\pi_{\pi_{t}}$



- Shuffles state words according to a permutation $\pi_{t}$ on $\mathbb{Z}_{M} \times \mathbb{Z}_{N}$
- Assume independent permutations $\pi_{t}$ in each round
- We say $\pi=\left(\pi_{0}, \ldots, \pi_{T-1}\right)$ is a permutation sequence for the $T$-round AES-like cipher


## AES-like cipher: Permute $\pi_{\pi_{t}}$



- Shuffles
- Assume optimize the choice of $\pi$
$Z_{M} \times \mathbb{Z}_{N}$
- We say
$T$-round AES-like cipher


## AES-like cipher: AddRoundKey ${ }_{t}$



- A round key is added to the state using $\oplus$ in each round
- Does not affect the properties we investigate, thus not considered further!


## Difference and activity pattern

## Difference

A (non-zero) difference is a value $X \in\left(\mathbb{F}_{2}^{m}\right)^{M \times N} \backslash\{0\}$

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## Activity pattern

For a difference $X$, let the activity pattern $\tilde{X}$ be defined s.t.

$$
\tilde{X}_{i, j}= \begin{cases}1 & X_{i, j} \neq 0 \\ 0 & X_{i, j}=0\end{cases}
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$$

for example

| $X$ |  |  |
| :---: | :---: | :---: |
| 00 00 CA <br> F2 00 24 |  |  |


| $\tilde{X}$ |  |  |
| :--- | :--- | :--- |
| 0 | 0 | 1 |
| 1 | 0 | 1 |

## Trails

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For an AES-like cipher, a $T$-round trail is a $(T+1)$-tuple of differences

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For an AES-like cipher, a $T$-round trail is a $(T+1)$-tuple of differences
Trail weight
The trail weight of $X=\left(X^{0}, \ldots, X^{T}\right)$ is defined as

$$
\sum_{t=0}^{T-1} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \tilde{X}_{i, j}^{t}
$$

## Branch number

## Differential branch number

For a linear automorphism $\theta:\left(\mathbb{F}_{2}^{m}\right)^{M} \rightarrow\left(\mathbb{F}_{2}^{m}\right)^{M}$, we define the (differential) branch number $B_{\theta}$ as the minimum number of non-zero words, in the input- and output differences $(X \oplus Y)$ respectively $(\theta(X) \oplus \theta(Y))$, when taken across all pairs of inputs $X, Y \in \mathbb{F}_{2}^{m}$.

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## For an AES-like cipher

The branch number for an AES-like cipher is $B_{\theta}$ if and only if it is the minimum of the branch numbers obtained by left multiplication by any $M_{j}^{t}$.

## Problem modeling

We are interested in determining

$$
\max _{\pi} \min _{\text {trail } X} \text { weight }(X)
$$

for a $T$-round AES-like cipher of dimensions $M \times N$.

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In our modeling of the problem, we consider the following as black-box operations

- SubBytes $_{t}$
- MixColumns ${ }_{t}$, under the requirement of a specific branch number


## Bounds and trail-optimality

## Tightly guaranteed active S-boxes

Consider an AES-like cipher with branch number $B_{\theta}$. We say that a permutation sequence $\pi$ tightly guarantees $k$ active $S$-boxes, denoted $\pi \xrightarrow{B_{\theta}} k$, if and only if, when using $\pi$ for the Permute operation,

- There exists a valid trail of weight $k$ and
- There is no valid trail of positive weight $k^{\prime}<k$.


## Bounds and trail-optimality

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- There exists a valid trail of weight $k$ and
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## Trail-optimality

A permutation sequence $\pi$ is said to be trail-optimal if and only if there exists no $\pi^{\prime} \neq \pi$ such that $\pi^{\prime} \xrightarrow{B_{\theta}} k^{\prime}$ where $k^{\prime}>k$

## Equivalences for

 permutation sequences $\pi$
## Defining equivalence

The goal
Classify permutation sequences $\pi$ incurring the same bound on the trail weight

$$
\Pi(k)=\left\{\pi=\left(\pi_{0}, \ldots, \pi_{T-1}\right) \mid \pi \xrightarrow{B_{\theta}} k\right\},
$$

and thus reducing the search space for a brute-force approach

## Defining equivalence

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## Equivalence of permutation sequences

Informally, we say that two permutation sequences $\pi, \pi^{\prime}$ are equivalent, denoted $\pi \sim \pi^{\prime}$, if and only if $\pi \xrightarrow{B_{\theta}} k \Leftrightarrow \pi^{\prime} \xrightarrow{B_{\theta}} k$

- Note: stronger notion of equivalence in paper


## Structure of a state permutation

## Lemma

Any permutation $\pi_{t}$ on the words of an $M \times N$ state can be written as

$$
\pi_{t}=\gamma^{\prime} \circ \phi \circ \gamma,
$$

where $\gamma, \gamma^{\prime}$ permute inside each column and $\phi$ permutes inside each row

(Thanks to John Steinberger for aiding in this proof)

## Reduction to permuting in the rows

## Lemma

Let $\pi$ be a permutation sequence for an AES-like cipher and let $\gamma, \gamma^{\prime}$ be arbitrary permutations inside the columns of the state. Then

$$
\left(\pi_{0}, \ldots, \pi_{t}, \ldots, \pi_{T-1}\right) \sim\left(\pi_{0}, \ldots, \gamma^{\prime} \circ \pi_{t} \circ \gamma, \ldots, \pi_{T-1}\right)
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holds for all $t=0, \ldots, T-1$.

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## Theorem

Given any permutation sequence $\pi$, one can construct $\pi^{\prime}$ s.t.

- $\pi \sim \pi^{\prime}$ and
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- $\pi \sim \pi^{\prime}$ and
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- Search space (per round) reduced from $(M \cdot N)$ ! to $(N!)^{M}$


## Rotation matrices

In the following, we restrict ourselves to rotation matrices

- Permute $\pi_{\pi_{t}}$ becomes ShiftRows $\sigma_{\sigma_{t}}$
- Much nicer for implementations


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## Rotation matrix

A rotation matrix for an an AES-like cipher is a $\rho \times M$ matrix $\sigma$ over $\mathbb{Z}_{N}$,

$$
\sigma=\left(\begin{array}{ccc}
\sigma_{0,0} & \cdots & \sigma_{0, M-1} \\
\sigma_{1,0} & \cdots & \sigma_{1, M-1} \\
\vdots & \ddots & \vdots \\
\sigma_{\rho-1,0} & \cdots & \sigma_{\rho-1, M-1}
\end{array}\right)
$$

Rotate row $j$ of the state in round $t$ by $\sigma_{i, j}$ where $t \equiv i \bmod \rho$

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## Rotation matrices: Example

Consider an AES-like cipher of dimension $3 \times 4$ with $\rho=2$ using

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\sigma=\left(\begin{array}{lll}
0 & 3 & 1 \\
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When $t$ is even


When $t$ is odd


## Equivalences for rotation matrices $\sigma$

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Lemma: Re-arranging row entries of $\sigma$
Let $\sigma$ be a rotation matrix and let $\vartheta_{0}, \ldots, \vartheta_{\rho-1}$ denote permutations on each of the $\rho$ rows of $\sigma$. Define $\sigma_{t}^{\prime}=\vartheta_{t}\left(\sigma_{t}\right)$ for all $t$. Then $\sigma \sim \sigma^{\prime}$.

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## Lemma: Row-wise constant addition

Let $\sigma$ be a rotation matrix and let $c_{0}, \ldots, c_{\rho-1} \in \mathbb{Z}_{N}$. Define $\sigma_{t}^{\prime}=\sigma_{t}+c_{t} \bmod N$ for all $t$. Then $\sigma \sim \sigma^{\prime}$.

## Rotation matrix normalized form

## Theorem: Equivalences

Let $\sigma$ be any rotation matrix. Then there exists an equivalent rotation matrix $\sigma^{\prime}$, s.t.

1. Each row $\sigma_{t}^{\prime}$ is ordered lexicographically
2. Each $\sigma_{t, 0}^{\prime}=0$, i.e. $1^{\text {st }}$ element in each row is zero
3. $\sigma_{t, 1}^{\prime} \leq N / 2$ for all $t$, i.e. $2^{\text {nd }}$ element in each row is $\leq N / 2$

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## Normal form

We define the rotation matrix normal form to satisfy 1-3, and heuristically also require that

- When $N$ is even, $\sigma^{\prime}$ should have at least one odd entry
- The elements in each row $\sigma_{t}^{\prime}$ are distinct
- Search space (for full $\sigma$ ) reduced to $\left[\frac{N}{2} \cdot\binom{N / 2}{M-2}\right]^{\rho}$


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Determine optimal rotation matrices $\sigma$ for a range of parameters ( $M, N, T, \rho$ )

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## Approach

Given a fixed rotation matrix $\sigma$, we

- Focus on the MDS case, i.e. $B_{\theta}=M+1$
- Formulate a MIP problem of determining $k$ s.t. $\sigma \xrightarrow{M+1} k$
- Combine brute-forcing the normal forms with solving the MIP model using CPLEX


## Findings

| Rounds | Rijndael-192 | Rijndael-256 | PRIMATEs-80 | Prøst-256 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | - | - | $54 / 56$ | - |  |
| 6 | $42 / 45$ | $50 / 55$ | - | $85 / 90^{\dagger}$ |  |
| 7 | $46 / 48$ | - | - | $96 / 111^{\dagger}$ |  |
| 8 | $50 / 57$ | - | - | - |  |
| 10 | - | $85 / 90$ | - | - |  |
| 12 | $87 / 90$ | $105 / 111$ | - | - |  |
| 4. |  |  |  |  |  |
| Increased $\rho$ from 1 to 2 |  |  |  |  |  |

- Many more results in paper
- † Searched only among diffusion-optimal solutions


## Conclusion and open problems

## What we did

- Took steps to analyze the problem of picking the best permutation for AES-like ciphers
- Focus on rotations as in ShiftRows due to implementation characteristics
- Reduced to normal form and combined with optimization using MIP
- Improve parameters for some existing designs


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## Open problems

- Formulating optimization problem with trail-optimal $\sigma$ as decision variable (bi-level optimization)
- Analysis w.r.t. combining diffusion-optimality with trail-optimality


# Thanks for you attention 

Questions?

## CHANGE

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## Improved Attacks on ReducedRound Camellia-128/192/256

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## Outline

- Introduction
- Description of Camellia
- Key-Dependent Attacks
- Key-dependent 8-round differentials
- Key-dependent multiple differential attack for 10-round Camellia-128
- Meet-in-the-Middle Attacks
- New 7-round property and 12-round attack for Camellia-192

New 8 -round property and 13 -round attack for Camellia-256

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## Description of Camellia

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- In 2000, Proposed by NTT and Mitsubishi.
- Adopted as an international standard ISO/IEC 18033-3, NESSIE block cipher portfolio, as well as an e-Government recommended cipher by CRYPTREC project
- Basic Information
- Block Size: 128
- Key Sizes: 128/192/256(denoted as Camellia-128/192/256)
- Number of Rounds: 18/24/24 for Camellia-128/192/256 Structure: Feistel structure with key-dependent FL layers


## Description of Camellia



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## Key-Dependent Attacks

## Key-dependent truncated differentials

4-Round Truncated Differentials with probability 2-56

| Active S-boxes: $0 \rightarrow 2 \rightarrow 7 \rightarrow 2$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Case-1 | Case-2 | Case-3 | Case-4 |
| $(00000000, * * 000000)$ | $(00000000,0 * * 00000)$ | $(00000000, * 00 * 0000)$ | $(00000000,00 * * 0000)$ |
| $(* * 000000,00000000)$ | $(0 * * 00000,00000000)$ | $(* 00 * 0000,00000000)$ | $(00 * * 0000,00000000)$ |
| $(* * * * * * * 0 *, * * 000000)$ | $(* * * * * * 0,0 * * 00000)$ | $(* * * * * 0 * *, * 00 * 0000)$ | $(* * * * 0 * * *, 00 * * 0000)$ |
| $(* * 000000, * * * * * * 0 *)$ | $(0 * * 00000, * * * * * * * 0)$ | $(* 00 * 0000, * * * * * 0 * *)$ | $(00 * * 0000, * * * * 0 * * *)$ |
| $(00000000, * * 000000)$ | $(00000000,0 * * 00000)$ | $(00000000, * 00 * 0000)$ | $(00000000,00 * * 0000)$ |

## Key-dependent truncated differentials



## Key subsets

|  | ( $c_{1}^{i}$ | $i$ |  | $i$ |  |  |  |  |  | here are 56 such ( $\mathrm{c}_{1}, \mathrm{c}_{2}$ ). |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 01, 02 | 8 | 02,01 | 15 | 04,01 | 22 | 08, 01 | 29 |  |  |  |  |  |  |  |
| 2 | 01,0 | 9 | 02,04 | 16 | 04,02 | 23 | 08, 02 | 30 |  |  |  |  |  |  |  |
| 3 | 01,08 | 10 | 02,08 | 17 | 04,08 | 24 | 08, 04 | 31 | 10 |  |  |  |  |  |  |
|  | 01, 10 | 11 | 02, 10 | 18 | 04, 10 | 25 | 08, 10 | 32 | 10, |  |  | 46 | 40, 08 | 53 | 80,08 |
|  | 01, 20 | 12 | 02, 20 | 19 | 04, 20 | 26 | 08, 20 | 33 | 10,20 | 40 | 20,10 | 47 | 40, 10 | 54 | 80,10 |
|  | 01, 40 | 13 | 02,40 | 20 | 04,40 | 27 | 08, 40 | 34 | 10,40 | 41 | 20,40 | 48 | 40,20 | 55 | 80,20 |
|  | 01, 80 | 14 | 02,80 | 21 | 04, 80 | 8 | 08, 80 |  | 10,80 | 42 | 20,80 |  | 40, 80 | 56 | 80, |

$K$ Dset ${ }_{i}^{1}=\left\{K \mid k f_{2 L}=\left(\neg c_{1}^{i} \wedge *, \neg c_{2}^{i} \wedge *, *, *\right), * \in F_{2}^{8}\right\}$, $K D s e t_{i}^{2}=\left\{K \mid k f_{2 L}=\left(*, \neg c_{1}^{i} \wedge *, \neg c_{2}^{i} \wedge *, *\right), * \in F_{2}^{8}\right\}$, $K D s e t_{i}^{3}=\left\{K \mid k f_{2 L}=\left(\neg c_{1}^{i} \wedge *, *, *, \neg c_{2}^{i} \wedge *\right), * \in F_{2}^{8}\right\}$, $D_{\operatorname{sect}}^{4}=\left\{K \mid k f_{2 L}=\left(*, *, \neg c_{1}^{i} \wedge *, \neg c_{2}^{i} \wedge *\right), * \in F_{2}^{8}\right\}$.

56 pairs of ( $c_{1}, c_{2}$ ); 4 cases differentials. Produce 224 key Dependent differentials.

Produce 224 key subsets as well.
And denote the other keys as RKset

## Key-dependent 8-round differentials



## Key-dependent attack on10-round Camellia-128

- We launch an example attack
- Choose 8-round differentials that
- $\mathrm{C}_{1}=0 \times 08, \mathrm{c}_{2}=0 \times 10$

ㅁ Cover KDset ${ }_{32}$

- Append 2 rounds on the bottom



## Key-dependent attack on10-round Camellia-128

- Data collection:
$>$ Structure: $L_{0}=\left(\alpha_{1}, x_{1}, x_{2}, \alpha_{1}, x_{3}, \alpha_{1}, x_{4}, \alpha_{1}\right)$ and
$R_{0}=P\left(\alpha_{2}, x_{5}, x_{6}, \alpha_{3}, x_{7}, \alpha_{4}, x_{8}, \alpha_{5}\right) \oplus\left(\alpha_{6}, \alpha_{7}, x_{9}, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}\right)$, where
$x_{i}$ are constant, $\alpha_{j}$ take all values.
$>$ Encrypt to get $2^{56}$ ciphertext $C$, store them indexed by $P^{-1}\left(C_{L}\right)[1,4,6,8]$,
> Construct pairs by choosing $C$ indexed by $P^{-1}\left(C_{L}\right)[1,4,6,8]$ and $C^{\prime}$ indexed by $P^{-1}\left(C_{L}\right)[1,4,6,8] \oplus P^{-1}(0 x 08,0 \times 10,0,0,0,0,0,0)[1,4,6,8]$.
> Choose $2^{33}$ structures, $2^{33+111-32}=2^{112}$ pairs constructed.
> Delete the pairs whose input difference do not belong to $\Delta I N s e t$, and about $2{ }^{93}$ pairs left


## Key-dependent attack on10-round Camellia-128

- Key Guessing
- For each pair and each possible $\Delta R_{9}$, do
A. Deduce 64-bit key $k w_{3} \oplus k_{10}$
B. Deduce 32-bit key $k w_{4} \oplus k_{9}[1,4,6,8]$
C. Increase the counter of 96 -bit subkey " $k w_{3} \oplus k_{10}$, $\left(k w_{4} \oplus k_{9}\right)[1,4,6,8]$ "
- If the right key recovered, then terminate the attack;
- Else replace the attack by choosing other 8-round differentials.
search the Rkset to find the right key.


## Key-dependent attack on10-round Camellia-128

99.99\% of Key Space

- Data: $2^{91} \mathrm{CP}$
- Time: $2^{104.5}$ ENC
- Memory: $2^{96}$ Bytes


## Full Key Space

- Data: $2^{91} \mathrm{CP}$
- Time: $2^{113}$ ENC

Memory: $2^{96}$ Bytes


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## Meet-in-the-Middle Attacks

## Meet-in-the-Middle Attacks

- Idea borrowed from Dunkelman etc.and Derbez-Selçuk's attacks on AES
- $\delta$-set
- Multiset


## New 7-round property for Camellia-192

## 7-round property

- $\mathrm{R}_{2}[1]$ is the active byte
- Multiset of $\left(P^{-1}\left(\Delta L_{8}\right)\right)[6]$ only takes about $2^{128}$ values
- $\Delta X_{4}[1]\left\|\Delta Y_{4}[1]\right\| \Delta Y_{5}[1,2,3,5,8]$ || $\Delta X_{8}[1]\left\|\Delta Y_{8}[1]\right\| X_{7}[7]\left\|X_{8}[6]\right\| k f_{1}$ where...



## Proof of 7-round property

- Obviously, Multiset of $\left(P^{-1}\left(\Delta L_{8}\right)\right)[6]$ is determinted by 36 -byte value
- $\mathrm{X}_{4}[1]\left\|\mathrm{X}_{5}[1,2,3,5,8]\right\| \mathrm{X}_{6}\left\|\mathrm{kf}_{1}\right\| \mathrm{Kf}_{2}\left\|\mathrm{X}_{7}[2,3,5,7,8]\right\| \mathrm{X}_{8}[6]$
- If a pair comforms to the truncated differential, then
- 18-byte " $\mathrm{X}_{4}[1]| | \mathrm{X}_{5}[1,2,3,5,8]| | \mathrm{X}_{6}| | \mathrm{X}_{7}[2,3,5,7,8]$ " determined by 9 -byte " $\Delta \mathrm{X}_{4}[1]| | \Delta \mathrm{Y}_{4}[1]| | \Delta \mathrm{Y}_{5}[1,2,3,5,8]| | \Delta \mathrm{X}_{8}[1]| | \Delta \mathrm{Y}_{8}[1]$ " and 128 -bit "kf $\left|\mid k f_{2}\right.$ "
- $\operatorname{Pr}\left(\Delta Y_{7}[4,6,7]=0\right)=2^{-24}$ and

ㅁ $\Delta Y_{7}=\mathrm{P}^{-1}\left(\mathrm{FL}^{-1}\left(\mathrm{P}\left(\Delta \mathrm{Y}_{5}\right) \oplus \Delta \mathrm{L}_{3}\right)\right) \oplus \mathrm{P}^{-1}\left(\Delta \mathrm{~L}_{7}\right)$
Only has 64-bit information for Camellia-192

So there are $2^{128}$ values for multiset of $\left(P^{-1}\left(\Delta L_{8}\right)\right)[6]$

## 12-round attack for Camellia-192

- Add two round on the top

- Three round on the bottom
$\Delta L_{1}=(a 0000000)$





$$
\Delta L_{12}=\left(r_{1} r_{2} r_{3} r_{4} r_{5} r_{6} r_{7} r_{8}\right)
$$

## 12-round attack for Camellia-192

- Precomputation Phase
- Get $2^{128}$ possible values for multiset of $\left(P^{-1}\left(\Delta L_{8}\right)\right)[6]$ stored in $\mathcal{H}$
- Online Phase
- Data collection
> Structure: each contains ${ }^{256}$ plaintexts
$L 0=\left(\alpha, \alpha \oplus x_{1}, \alpha \oplus x_{2}, x_{3}, \alpha \oplus x_{4}, x_{5}, x_{6}, \alpha \oplus x_{7}\right)$ and
$R 0=P\left(\beta_{1}, \beta_{2}, \beta_{3}, \beta_{4}, \beta_{5}, y_{1}, y_{2}, \beta_{6}\right)$, where $x i(i=1, \ldots, 7), y_{1}, y_{2}$ are constant, $\alpha, \beta j(j=1, \ldots, 6)$ take all values.

Choose $2^{57}$ structure, $2^{57+111-16}=2^{152}$ pairs satisfy $P^{-1}\left(\Delta R_{12}\right)[6,7]=0$

## 12-round attack for Camellia-192

- Key Guessing to find a pair conforming to the truncated differential
A. For $l=2,3,4,5,6,7,8$, guess $k_{12}^{\prime}[l]$ one by one, paritially decrypt, use $\Delta Y_{11}[l]=P\left(\Delta L_{12}[l]\right)$ to filter and $2^{152-7 \times 8}=2^{96}$ pairs left.
B. For $l=2,3,5,8$, guess $k_{11}^{\prime}[l]$, paritially decrypt, use $\Delta Y_{11}[l]=$ $P^{-1}\left(\Delta R_{12}\right)[l] \oplus P^{-1}\left(\Delta R_{12}\right)[4]$ to filter. Then guess $k_{11}^{\prime}[1]$ and keep the $\Delta Y_{11}[1]=P^{-1}\left(\Delta R_{12}\right)[1]$ hold. $2^{96-40}=2^{56}$ pairs remain.
c. For $l=1,2,3,5,8$, guess $k_{1}^{\prime}[l]$, make $\Delta Y_{1}[1]=P^{-1}\left(\Delta R_{0}\right)[1]$ and $\Delta Y_{1}[l]=P^{-1}\left(\Delta R_{0}\right)[l] \oplus P^{-1}\left(\Delta R_{0}\right)[4]$ hold. $2^{16}$ pairs remain.


## 12-round attack for Camellia-192

- Construct $\delta$-set for every remaining pair
A. Deduce $X_{2}[1]| | Y_{2}[1]$ by difference distribution table of $s_{1}$
B. For pair $\left(L_{0}\left\|R_{0}, L_{0}^{\prime}\right\| R_{0}^{\prime}\right)$, corresponding to ( $\left.X_{2}[1], X^{\prime} 2[1]\right)$, change $X^{\prime} 2$ [1] to a different $X^{\prime \prime}{ }_{2}[1]$, compute $\Delta Y^{\prime}{ }_{2}$ [1], get the difference $\Delta L_{0}^{\prime}[1,2,3,5,8]$. Get $L^{\prime \prime}{ }_{0}=L_{0} \oplus \Delta L_{0}^{\prime}$.
c. Compute $\Delta Y^{\prime}{ }_{1}[1,2,3,5,8]$ by the guessed key $k^{\prime}{ }_{1}[1,2,3,5,8]$, obtain $\Delta R^{\prime} 0$ then get $R^{\prime \prime}{ }_{0}=R_{0} \oplus \Delta R_{0}^{\prime}$
D. Get $\delta$-set.


## 12-round attack for Camellia-192

- For each $\delta$-set under 144-bit key guesses
- Compute $Y_{11}[2,3,5,8], P^{-1}\left(L_{10}\right)[6]$ for every (P, C) pair.
- Guess $k_{11}^{\prime}[7]$ to compute $X_{10}[6]$.
- Guess $k_{10}^{\prime}[6]$ to compute the multiset of $P^{-1}\left(\Delta L_{8}\right)[6]$
- Check the multiset belongs to $\mathcal{H}$ or not, the wrong value could pass the check with $\operatorname{Pr}=2^{128} \times 2^{-467.6}=2^{-339.6}$.


## 12-round attack for Camellia-192

## Full Key Space

- Data: $2^{113} \mathrm{CP}$
- Time: $2^{180}$ ENC
- Memory: $2^{158}$ Bytes





## New 8-round property for Camellia-256

-8-round property

- $L_{12}[5]$ is active
- Multiset of bytes $\left(P^{-1}\left(\Delta L_{4}\right)\right)[1]$ Only take $2^{225}$ values
- Determined by the 225-bit:
$\Delta X_{11}[5]| | \Delta Y_{11}[5]| | \Delta Y_{10}[2,3,4,6,7,8]| |$
$\Delta Y_{9}\left\|\Delta X_{6}[1]\right\| \Delta Y_{6}[1]| | k f_{1}\left\|k f_{2 L}[1]\right\|$
$k f_{2 R}[1]| | k f_{2 L}\{9\}$


## Proof of 8-round property

- Multiset of $\left(P^{-1}\left(\Delta L_{8}\right)\right)[6]$ is determinted by 321-bit value
- $\mathrm{X}_{11}[5]\left\|\mathrm{X}_{10}[2,3,4,6,7,8]| | \mathrm{X}_{9}\right\| \mathrm{X}_{8} \| \mathrm{X}_{7}| | k f_{1}\{9-33,42-64\}| | \mathrm{kf}_{2}[1]| | \mathrm{kf}_{2 \mathrm{R}}[1]| | \mathrm{kf} \mathrm{f}_{2}[9]$ ||X $\mathrm{X}_{6}$ [1]
- If a pair comforms to the 8-round truncated differential, then
- 312-bit " $\Delta \mathrm{X}_{11}[5]| | \Delta \mathrm{X}_{10}[2,3,4,6,7,8]| | \mathrm{X}_{9}| | \mathrm{X}_{8}| | \mathrm{X}_{7} \| \mathrm{X}_{6}[1]| | k f_{1}\{9-33,42$ $64\}\left|\mid k f_{2 L}[1]\right.$ " determined by 216 -bit " $\left.\Delta \mathrm{X}_{11}[5]\right|\left|\Delta \mathrm{Y}_{11}[5]\right|\left|\Delta \mathrm{Y}_{10}[2,3,4,6,7,8]\right| \mid$ $\Delta Y_{g}\left\|\Delta X_{6}[1]\right\| \Delta Y_{6}[1]\left\|\left|k f_{1} \|\right| k f_{2}[1] "\right.$
- $\mathrm{kf2R}[1]\left|\mid \mathrm{kf} 2 \mathrm{~L}\{9\}\right.$ are also needed to compute Multiset of $\left(P^{-1}\left(\Delta L_{8}\right)\right)[6]$


## 13-round attack for Camellia-256

- Add 4 rounds on the top

1 rounds on the bottom

$$
\begin{gathered}
\Delta L_{1}= \\
P\left(f_{1} f_{2} f_{3} 0 f_{4} 00 f_{5}\right) \\
\oplus P(0 e e e e 00 e)
\end{gathered} \stackrel{k_{2}^{\prime}}{\oplus} \stackrel{X_{2}}{\oplus}, \mathrm{~S} \stackrel{Y_{2}}{\square} \stackrel{\mathrm{P}}{ }
$$

$$
\begin{aligned}
& \Delta L_{2}=(\text { fff } 0 f 00 f) \stackrel{l^{\prime}}{\oplus}{ }_{-}^{X_{3}}=\mathrm{S} \xrightarrow{Y_{3}} \rightarrow \mathrm{P} \\
& \Delta L_{3}=(e 0000000)
\end{aligned}
$$



## 13-round attack for Camellia-256

- Precomputation Phase
- Compute $2^{225}$ values of multiset store them in hash table $\mathcal{H}$.
- Online Phase
- Data collection
> Structure: each contains $2^{32}$ ciphertexts
$L_{13}=\left(\alpha_{1}, x_{1}, x_{2}, x_{3}, \alpha_{2}, x_{4}, x_{5}, x_{6}\right)$ and
$R_{13}=P\left(\beta_{1}, y_{1}, y_{2}, y_{3}, \beta_{2}, y_{4}, y_{5}, y_{6}\right)$, where $x_{i}, y i(i=1, \ldots, 6)$ are constant, $\alpha_{j}, \beta_{j}(j=1,2)$ take all values. Decrypt to get the plaintexts.

Choose $2^{81}$ structure to get $2^{144}$ pairs

## 13-round attack for Camellia-256

- Key Guessing to find a pair conforming to the truncated differential
A. Guess $k_{1}^{\prime}$, compute $P^{-1}\left(\Delta L_{1}\right)$, eliminate pairs that do not satisy $P^{-1}\left(\Delta L_{1}\right)[6,7]=0,2^{144-16}=2^{128}$ pairs left.
B. For $l=2,3,4,6,7,8$, guess $k_{2}^{\prime}[l]$, paritially encrypt, make $\Delta Y_{2}[l]=$ $P^{-1}\left(\Delta L_{0}\right)[l]$ hold. Then guess $k_{2}^{\prime}[1]$ to compute $L_{2} \cdot 2^{128-7 * 8}=$ $2^{72}$ pairs remain.
c. For $l=2,3,5,8$, guess $k_{3}^{\prime}[l]$, make $\Delta Y_{3}[1]=P^{-1}\left(\Delta L_{1}\right)[l] \oplus P^{-1}\left(\Delta L_{1}\right)[4]$ Then guess $k_{3}{ }^{\prime}$ [1] and keep the pairs satisfy $\Delta Y_{3}[1]=P^{-1}\left(\Delta L_{1}\right)[1]$. $2^{32}$ pairs left for every 168 -bit key guess.


## 13-round attack for Camellia-256

- Key Guessing to find a pair conforming to the truncated differential
D. For $l=1,5$, guess $k_{13}^{\prime}[l]$, partially decrypt, make $\Delta Y_{13}[l]=$ $P^{-1}\left(\Delta L_{13}\right)[l]$ hold. Then guess $k f_{3 R}[1]$ to compute $\Delta L_{12}{ }^{*}$ [1] and delete the pairs when $\Delta L_{12}{ }^{*}[1] \neq 0.2^{8}$ pairs remain.
E. Compute the value $L_{3}$ by guessing $k_{3}^{\prime}[4,6,7]$ and deduce $k_{4}^{\prime}[1]$ for each pair.
- Construct $\delta$-set for each pair, and compute the corresponding multiset to check it whether belongs to $\mathcal{H}$, and recover the right


## 13-round attack for Camellia-256



## Full Key Space

- Data: $2^{113} \mathrm{CC}$
- Time: $2^{232.7}$ ENC
- Memory: $2^{231}$ Bytes

$$
\Delta L_{3}=(e 0000000)
$$



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That is all
Thank you!

