

# Duality in ABE:

Converting Attribute Based Encryption  
for Dual Predicate and Dual Policy  
via Computational Encodings

**Nuttapong Attrapadung**, Shota Yamada  
AIST, Japan

@CT-RSA 2015

# Our Main Results in **One Slide**

**Generic dual  
conversion for ABE**

## **Instantiations:**

The first fully secure

- CP-ABE with short key
- CP-ABE all-unbounded

(for boolean formulae,  
span programs)

**1**

**Introduction**

# Attribute Based Encryption (ABE)

ABE for predicate  $R: X \times Y \rightarrow \{0,1\}$



Key for  
 $x \in X$



Ciphertext for  
 $y \in Y$   
(encrypt  $m$ )



$m$  if  $R(x,y)=1$   
 $?$  if  $R(x,y)=0$

# A Predicate

$$R: X \times Y \rightarrow \{0,1\}$$

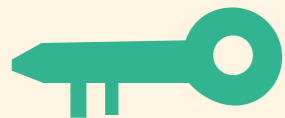
# Its Dual Predicate

$$\bar{R}: Y \times X \rightarrow \{0,1\}$$

$$\bar{R}(y,x) := R(x,y)$$

# Key-Policy ABE

$$R: P \times A \rightarrow \{0,1\}$$



Key for  
policy  $p \in P$



Ciphertext for  
attribute  $a \in A$

$R(p,a)$  iff  $p$  satisfies  $a$

# Ciphertext-Policy ABE

$$\bar{R}: A \times P \rightarrow \{0,1\}$$



Key for  
attribute  $a \in A$




Ciphertext for  
policy  $p \in P$

$\bar{R}(a,p)$  iff  $p$  satisfies  $a$

# Motivation

- KP-ABE, CP-ABE
  - Definitions: directly related.
  - Constructions: NO known relation.
- **Can we generically convert an ABE to its dual?**
  - So that we would only construct KP, and get also CP.
  - Might be difficult? Historically, CP [BSW07, Waters11] was harder to achieve than KP [GPSW06].

# Related Work for Dual Conversion

- Converting KP-ABE for boolean formulae predicate
  - Small classes of predicates
  - Its dual CP: only for bounded-size formulae [GJPS08].
- Converting KP-ABE for all boolean circuits
  - Implies general predicates, but must start with ABE for circuits.
  - Its dual CP: only for bounded-size circuits [GGHSW13].
    - Due to the use of universal circuits.
- Summary: less expressivity, and much less efficient. 



# Our Focus

- Goal: Generic dual conversion for any predicate.
  - Preserving full security, expressivity, and efficiency.
- Tool: Use a generic ABE framework of [A14].
  - An abstraction of dual-system encryption [Waters09] for achieving fully-secure ABE.

[A14] N. Attrapadung, "Dual System Encryption via Doubly Selective Security: Framework, Fully-secure Functional Encryption for Regular Languages, and More", *Eurocrypt 2014*.

# 2 Our Result Overview

# Our Main Result: Dual Conversion

**Fully secure ABE  
for arbitrary  $R$**

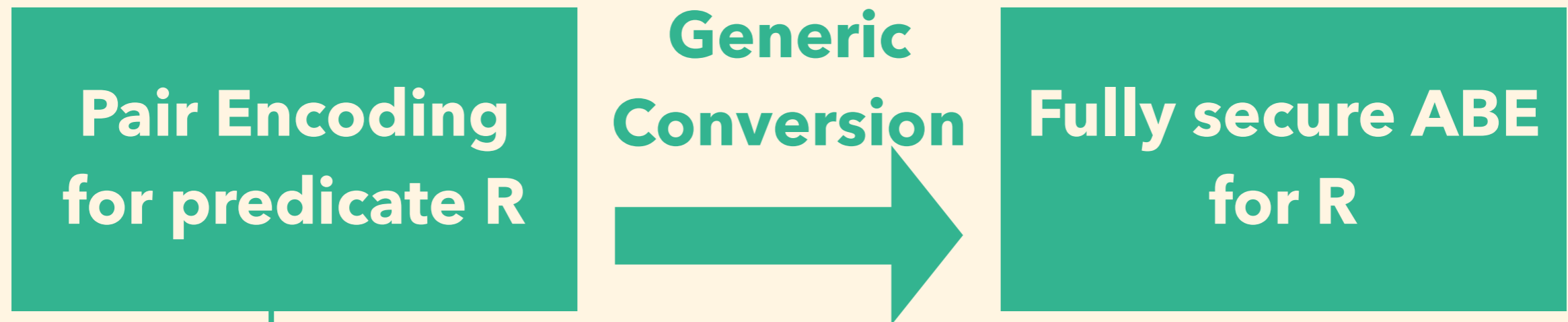
**Generic  
Conversion**

**Fully secure ABE  
for its dual,  $\bar{R}$**

**Restricted to ABE In  
the "pair encoding"  
framework [A14].**

# Recall The “Pair Encoding” Framework

Main Theorem in [A14]



If pair encoding is

- “Perfectly secure” or
- “Doubly selectively secure”.

# Our Main Result: More Precisely

**Doubly selective  
pair encoding for  
arbitrary  $R$**

**Generic  
Conversion**



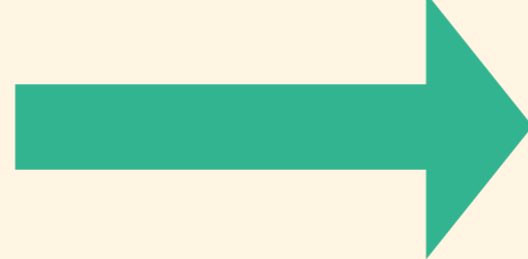
**Doubly selective  
pair encoding for  
its dual,  $\bar{R}$**

# The Only Previous Dual Conversion

A Side Result in [A14]

Perfectly secure  
pair encoding for  
arbitrary  $R$

Generic  
Conversion



Perfectly secure  
pair encoding for  
its dual,  $\bar{R}$

# Implications: Solving Open Problems

No fully-secure ABE known before

Doubly selective encodings known

Perfectly secure encodings known

- KP, CP boolean formula with some bounds  
[LOSTW10, W14, A14]
- spatial, inner-product, ...

- KP unbounded boolean formula
- KP short-ciphertext for boolean formula
- KP over doubly-spatial
- KP regular languages
- CP regular languages

[all in A14]

[NEW! all implied by this work]

- CP unbounded boolean formula
- CP short-key for boolean formula
- CP over doubly-spatial

# 3

## Recall Pair Encoding



# Recall Pair Encoding and ABE [A14]

Pair Encoding for  $R$



ABE for  $R$

Param  $\longrightarrow h$



$PK=(g_1^h, e(g_1, g_1)^a), MSK=a$

Enc1( $x$ )  $\longrightarrow k_x(a, r, h)$



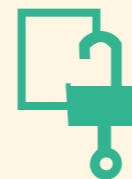
$SK=g_1^{k_x(a, r, h)}$

Enc2( $y$ )  $\longrightarrow c_y(s, h)$



$CT=(g_1^{c_y(s, h)}, e(g_1, g_1)^{as_0}M)$

Pair( $k_x, c_y$ )  $\longrightarrow as_0$



Dec  $\longrightarrow e(g_1, g_1)^{as_0}$

if  $R(x, y)=1$

if  $R(x, y)=1$

- $s_0$  = first entry in  $s$ .
- Require some linearity properties.

- Use composite-order bilinear groups.
- (Neglect details here).

# Security Definitions of Pair Encoding

## Perfect security







Identical (info-theoretic)  $\left\{ \begin{array}{l} k_x(0, r, h) \\ k_x(a, r, h) \end{array} \right.$   $c_y(s, h)$  for  $R(x, y) = 0$

## Doubly selective security



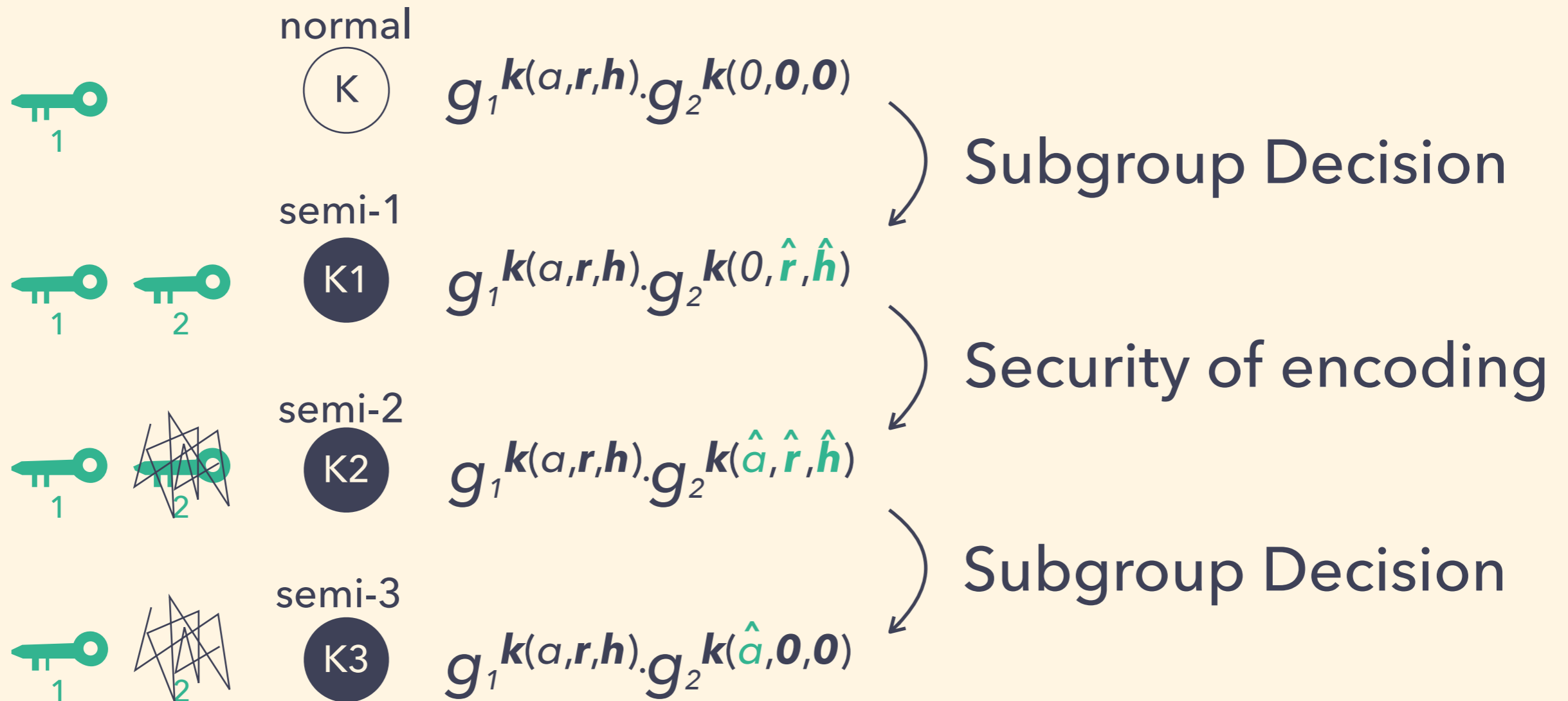
$\left\{ \begin{array}{l} g_2^{k_x(0, r, h)} \\ g_2^{k_x(a, r, h)} \end{array} \right.$   $g_2^{c_y(s, h)}$  for  $R(x, y) = 0$

Cannot distinguish

- **Selective notion:**  queries  $\mathbf{c}$  before  $\mathbf{k}$ . (  then  )
- **Co-selective notion:**  queries  $\mathbf{k}$  before  $\mathbf{c}$ . (  then  )

# Intuition Behind Pair Encoding Security

## Switch Keys from Normal to Semi-functional [A14]



- Only for self-containment, will not use here.

# 4 Our Conversion

# Basic Idea for Dual Conversion

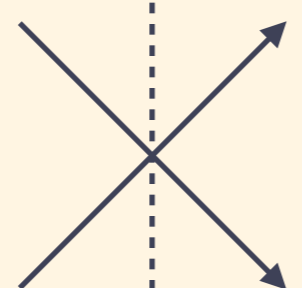
Encoding for  $R$

Encoding for  $\bar{R}$

Enc1 maps  $x \in X$

Enc1 maps  $y \in Y$

defined using Enc2



Enc2 maps  $y \in Y$

Enc2 maps  $x \in X$

defined using Enc1

# Our Dual Conversion

## Encoding for $R$

Param  $\rightarrow \mathbf{h}$

Enc1  $\rightarrow \mathbf{k}_x(\mathbf{a}, \mathbf{r}, \mathbf{h})$

Enc2  $\rightarrow \mathbf{c}_y(\mathbf{s}, \mathbf{h})$

## Encoding for $\bar{R}$

Param  $\rightarrow \bar{\mathbf{h}} = (\mathbf{h}, \bar{\mathbf{b}})$

Enc1  $\rightarrow \bar{\mathbf{k}}_y(\bar{\mathbf{a}}, \bar{\mathbf{s}}, \bar{\mathbf{h}}) = (\mathbf{c}_y(\mathbf{s}, \mathbf{h}), \bar{\mathbf{a}} + \bar{\mathbf{b}}s_0)$

Enc2  $\rightarrow \bar{\mathbf{c}}_x(\bar{\mathbf{r}}, \bar{\mathbf{h}}) = (\mathbf{k}_x(\bar{\mathbf{b}}s_0, \mathbf{r}, \mathbf{h}), \bar{s}_0)$

where  $\bar{\mathbf{s}} = \mathbf{s}$   $\bar{\mathbf{r}} = (\bar{s}_0, \mathbf{r})$

# Our Dual Conversion

## Encoding for $R$

Param  $\rightarrow \mathbf{h}$

Enc1  $\rightarrow \mathbf{k}_x(\mathbf{a}, \mathbf{r}, \mathbf{h})$

Enc2  $\rightarrow \mathbf{c}_y(\mathbf{s}, \mathbf{h})$

Pair( $\mathbf{k}_x, \mathbf{c}_y$ ) =  $\mathbf{a} \mathbf{s}_0$

## Encoding for $\bar{R}$

Param  $\rightarrow \bar{\mathbf{h}} = (\mathbf{h}, \bar{\mathbf{b}})$

Enc1  $\rightarrow \bar{\mathbf{k}}_y(\bar{\mathbf{a}}, \bar{\mathbf{s}}, \bar{\mathbf{h}}) = (\mathbf{c}_y(\mathbf{s}, \mathbf{h}), \bar{\mathbf{a}} + \bar{\mathbf{b}} \mathbf{s}_0)$

Enc2  $\rightarrow \bar{\mathbf{c}}_x(\bar{\mathbf{r}}, \bar{\mathbf{h}}) = (\mathbf{k}_x(\bar{\mathbf{b}} \mathbf{s}_0, \mathbf{r}, \mathbf{h}), \bar{\mathbf{s}}_0)$

Pair: Pair( $\mathbf{k}_x, \mathbf{c}_y$ ) =  $\bar{\mathbf{b}} \bar{\mathbf{s}}_0 \mathbf{s}_0$   
 $(\bar{\mathbf{a}} + \bar{\mathbf{b}} \mathbf{s}_0)(\bar{\mathbf{s}}_0) - \bar{\mathbf{b}} \bar{\mathbf{s}}_0 \mathbf{s}_0 = \bar{\mathbf{a}} \bar{\mathbf{s}}_0$

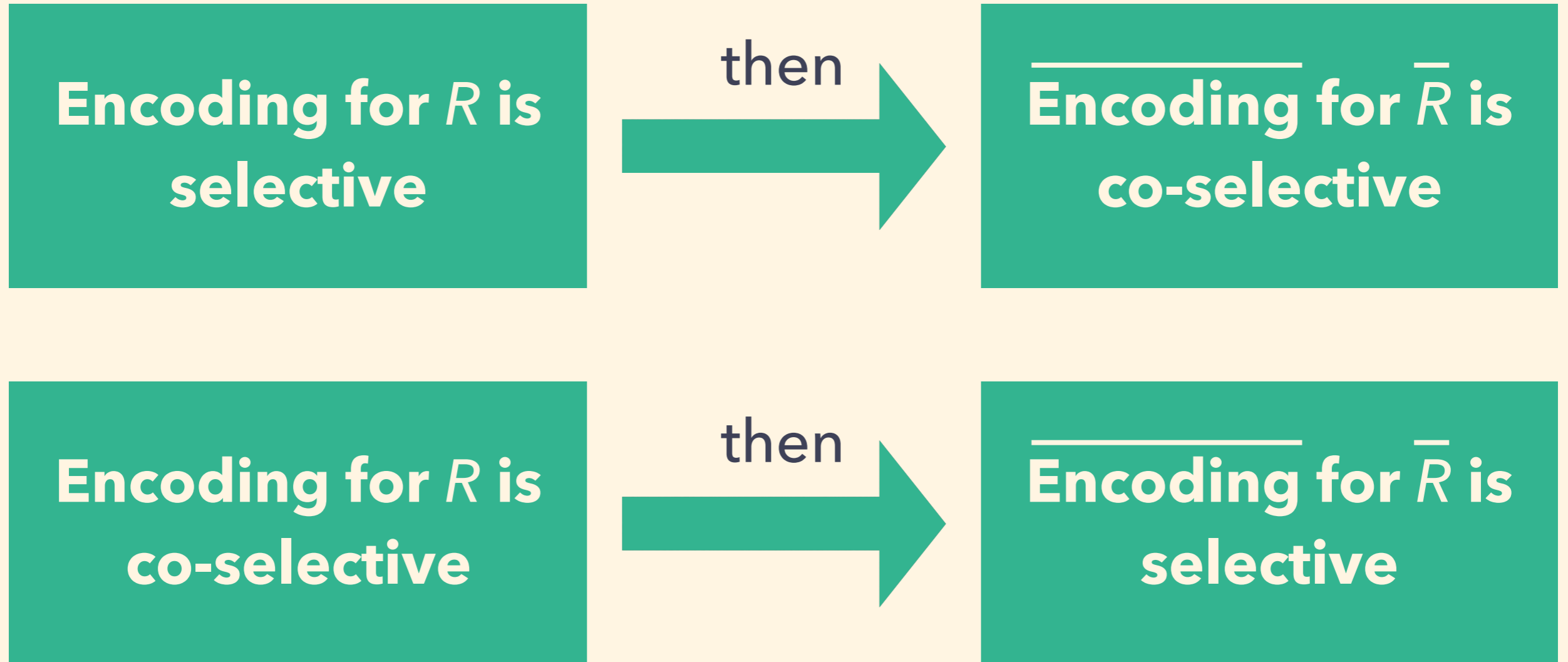
where  $\bar{\mathbf{s}} = \mathbf{s}$     $\bar{\mathbf{r}} = (\bar{\mathbf{s}}_0, \mathbf{r})$

# Our Dual Conversion

- The same conversion as in [A14].
- [A14] only proved for the perfectly secure encodings.
- We make it work also for doubly secure encodings.



# Our New Theorems



Intuition:

- Swap key/cipher encodings  $\rightarrow$  Query order is reversed.
- Hence selective becomes co-selective (and vice versa).

# Difficulty in Proving Theorems

Encoding for  $R$

Enc1:  $\mathbf{k}_x(a, r, h)$

Enc2:  $\mathbf{c}_y(s, h)$

Encoding for  $\bar{R}$

Enc1:  $\bar{\mathbf{k}}_y(\bar{a}, \bar{s}, \bar{h}) = (\mathbf{c}_y(s, h), \bar{a} + \bar{b}s_0)$

Enc2:  $\bar{\mathbf{c}}_x(\bar{r}, \bar{h}) = (\mathbf{k}_x(\bar{b}s_0, r, h), \bar{s}_0)$

(all terms over the exponent)

# Difficulty in Proving Theorems

## Encoding for $R$

$$\text{Enc1: } \begin{cases} \mathbf{k}_x(0, r, h) \\ \mathbf{k}_x(a, r, h) \end{cases}$$

Given IND here

$$\text{Enc2: } \mathbf{c}_y(\mathbf{s}, h)$$

## $\overline{\text{Encoding for } \bar{R}}$

$$\overline{\text{Enc1:}} \quad \overline{\mathbf{k}}_y(\overline{a}, \overline{s}, \overline{h}) = (\mathbf{c}_y(\mathbf{s}, h), \overline{a} + \overline{b}s_0)$$

$$\overline{\text{Enc2:}} \quad \overline{\mathbf{c}}_x(\overline{r}, \overline{h}) = (\mathbf{k}_x(\overline{b}s_0, r, h), \overline{s}_0)$$

(all terms over the exponent)

# Difficulty in Proving Theorems

## Encoding for $R$

$$\text{Enc1: } \begin{cases} \mathbf{k}_x(0, r, h) \\ \mathbf{k}_x(a, r, h) \end{cases}$$

Given IND here

$$\text{Enc2: } \mathbf{c}_y(\mathbf{s}, h)$$

## $\overline{\text{Encoding for } \bar{R}}$

$$\overline{\text{Enc1:}} \begin{cases} \bar{\mathbf{k}}_y(0, \bar{\mathbf{s}}, \bar{h}) \\ \bar{\mathbf{k}}_y(\bar{a}, \bar{\mathbf{s}}, \bar{h}) \end{cases} = (\mathbf{c}_y(\mathbf{s}, h), \bar{a} + \bar{b}s_0)$$

Goal: to prove IND here.

But it totally differs from  $\mathbf{k}_x$ .

$$\overline{\text{Enc2:}} \quad \bar{\mathbf{c}}_x(\bar{r}, \bar{h}) = (\mathbf{k}_x(\bar{b}s_0, r, h), \bar{s}_0)$$

(all terms over the exponent)

# Proof Idea

## Encoding for $R$

Enc1:  $\mathbf{k}_x(a, r, h)$

Enc2:  $\mathbf{c}_y(s, h)$

## Encoding for $\bar{R}$

Enc1:  $\bar{\mathbf{k}}_y(\bar{a}, \bar{s}, \bar{h}) = (\mathbf{c}_y(s, h), \bar{a} + \bar{b}s_0)$

Enc2:  $\bar{\mathbf{c}}_x(\bar{r}, \bar{h}) = (\mathbf{k}_x(\bar{b}s_0, r, h), \bar{s}_0)$

(all terms over the exponent)

# Proof Idea

Encoding for  $R$

Enc1:  $k_x(a, r, h)$

Enc2:  $c_y(s, h)$



Sim

attacks Enc

Encoding for  $\bar{R}$

Enc1:  $\bar{k}_y(\bar{a}, \bar{s}, \bar{h}) = (c_y(s, h), \bar{a} + \bar{b}s_0)$

Enc2:  $\bar{c}_x(\bar{r}, \bar{h}) = (k_x(\bar{b}s_0, r, h), \bar{s}_0)$



Adv

attacks Enc

(all terms over the exponent)

# Proof Idea

Encoding for  $R$

Encoding for  $\bar{R}$



define  $\bar{a}$  on unknown  $a$

Enc1:  $k_x(a, r, h)$

Enc1:  $\bar{k}_y(\bar{a}, \bar{s}, \bar{h}) = (c_y(s, h), \bar{a} + \bar{b}s_0)$

Enc2:  $c_y(s, h)$

Enc2:  $\bar{c}_x(\bar{r}, \bar{h}) = (k_x(\bar{b}s_0, r, h), \bar{s}_0)$



Sim

attacks Enc



Adv

attacks Enc

(all terms over the exponent)

# Proof Idea

Encoding for  $R$

Encoding for  $\bar{R}$



define  $\bar{a}$  on unknown  $a$

Enc1:  $k_x(a, r, h)$

Enc1:  $\bar{k}_y(\bar{a}, \bar{s}, \bar{h}) = (\mathbf{c}_y(\mathbf{s}, h), \bar{a} + \bar{b}s_0)$

But this becomes unknown, too.

Enc2:  $\mathbf{c}_y(\mathbf{s}, h)$

Enc2:  $\bar{\mathbf{c}}_x(\bar{r}, \bar{h}) = (\mathbf{k}_x(\bar{b}s_0, r, h), \bar{s}_0)$



Sim

attacks Enc



Adv

attacks Enc

(all terms over the exponent)



# Proof Idea: Cancellation Trick

Encoding for  $R$

Encoding for  $\bar{R}$



define  $\bar{a}$  on unknown  $a$

Enc1:  $k_x(a, r, h)$

Enc1:  $\bar{k}_y(\bar{a}, \bar{s}, \bar{h}) = (c_y(s, h), \bar{a} + \bar{b}s_0)$

Enc2:  $c_y(s, h)$

Enc2:  $\bar{c}_x(\bar{r}, \bar{h}) = (k_x(\bar{b}s_0, r, h), \bar{s}_0)$



Sim

attacks Enc



Adv

attacks Enc

(all terms over the exponent)

# Proof Idea: Cancellation Trick

Encoding for  $R$

Encoding for  $\bar{R}$



define  $\bar{a}$  on unknown  $a$

Enc1:  $k_x(a, r, h)$

Enc1:  $\bar{k}_y(\bar{a}, \bar{s}, \bar{h}) = (c_y(s, h), \bar{a} + \bar{b}s_0)$



define  $\bar{b}$  on unknown  $a$

Enc2:  $c_y(s, h)$

Enc2:  $\bar{c}_x(\bar{r}, \bar{h}) = (k_x(\bar{b}s_0, r, h), \bar{s}_0)$



Sim

attacks Enc



Adv

attacks Enc

(all terms over the exponent)

# Proof Idea: Cancellation Trick

Encoding for  $R$

Encoding for  $\bar{R}$

Enc1:

$$k_x(a, r, h)$$

Enc1:

$$\bar{k}_y(\bar{a}, \bar{s}, \bar{h}) = (\mathbf{c}_y(\mathbf{s}, h), \bar{a} + \bar{b}s_0)$$

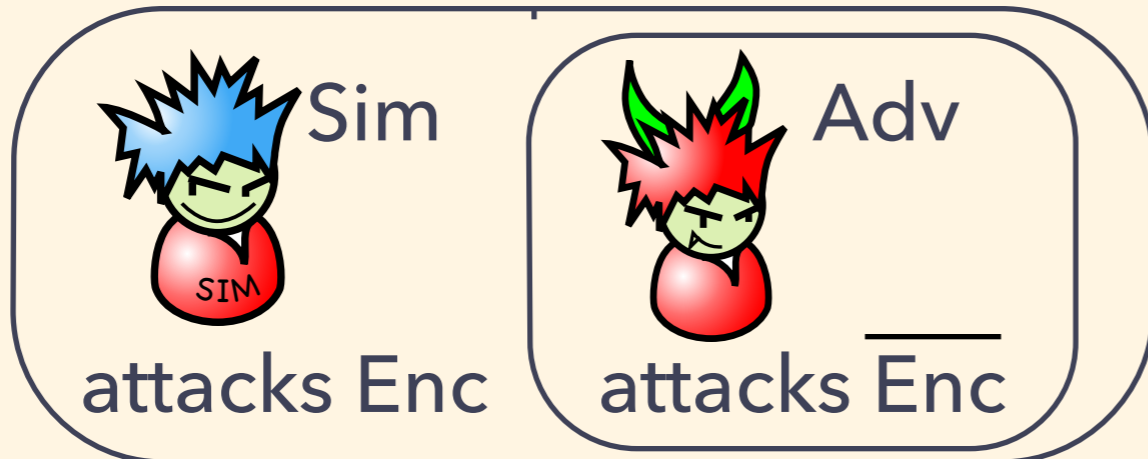
cancellation of two unknowns!

Enc2:

$$c_y(\mathbf{s}, h)$$

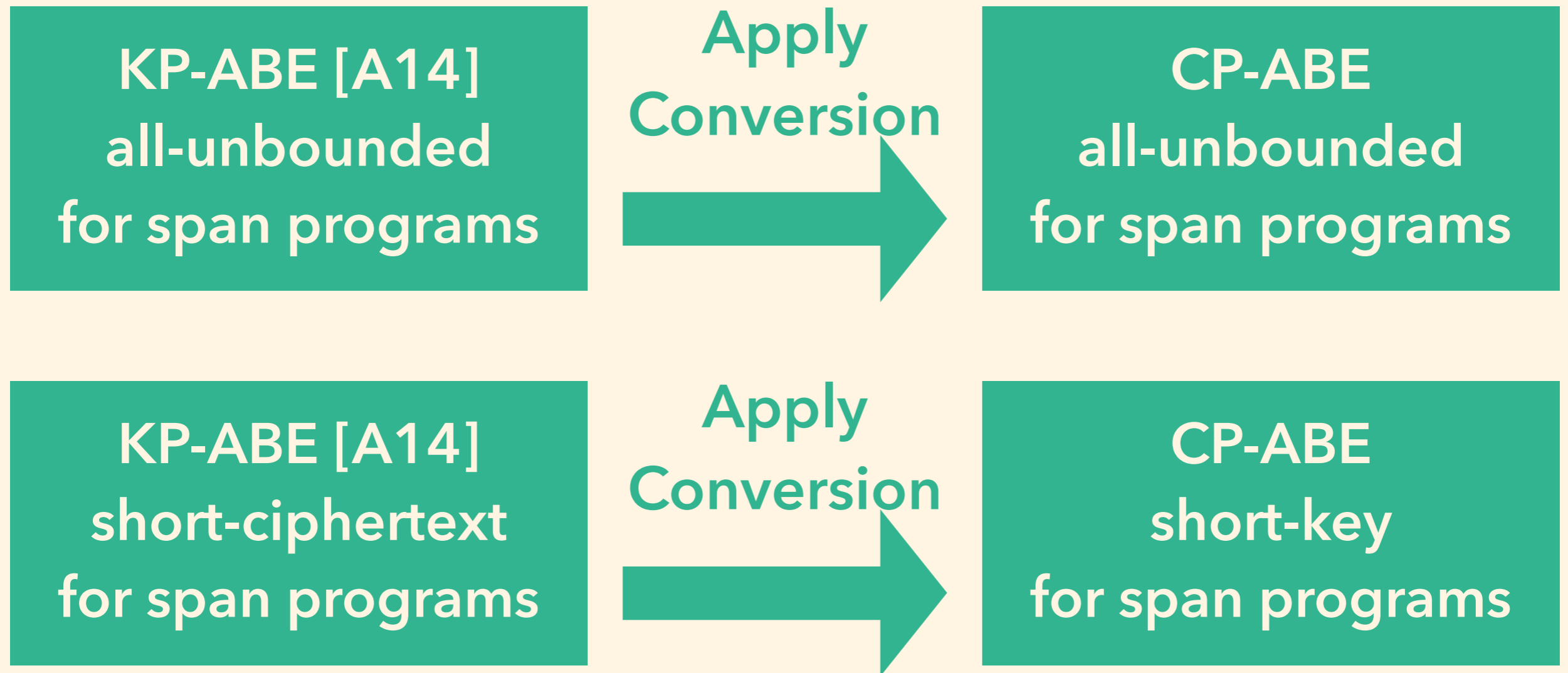
Enc2:

$$\bar{c}_x(\bar{r}, \bar{h}) = (k_x(\bar{b}s_0, r, h), \bar{s}_0)$$



(all terms over the exponent)

# New Instantiations



Doubly selective secure under some Extended DH Exponent assumptions [A14].

**5**

**Concluding Remarks**

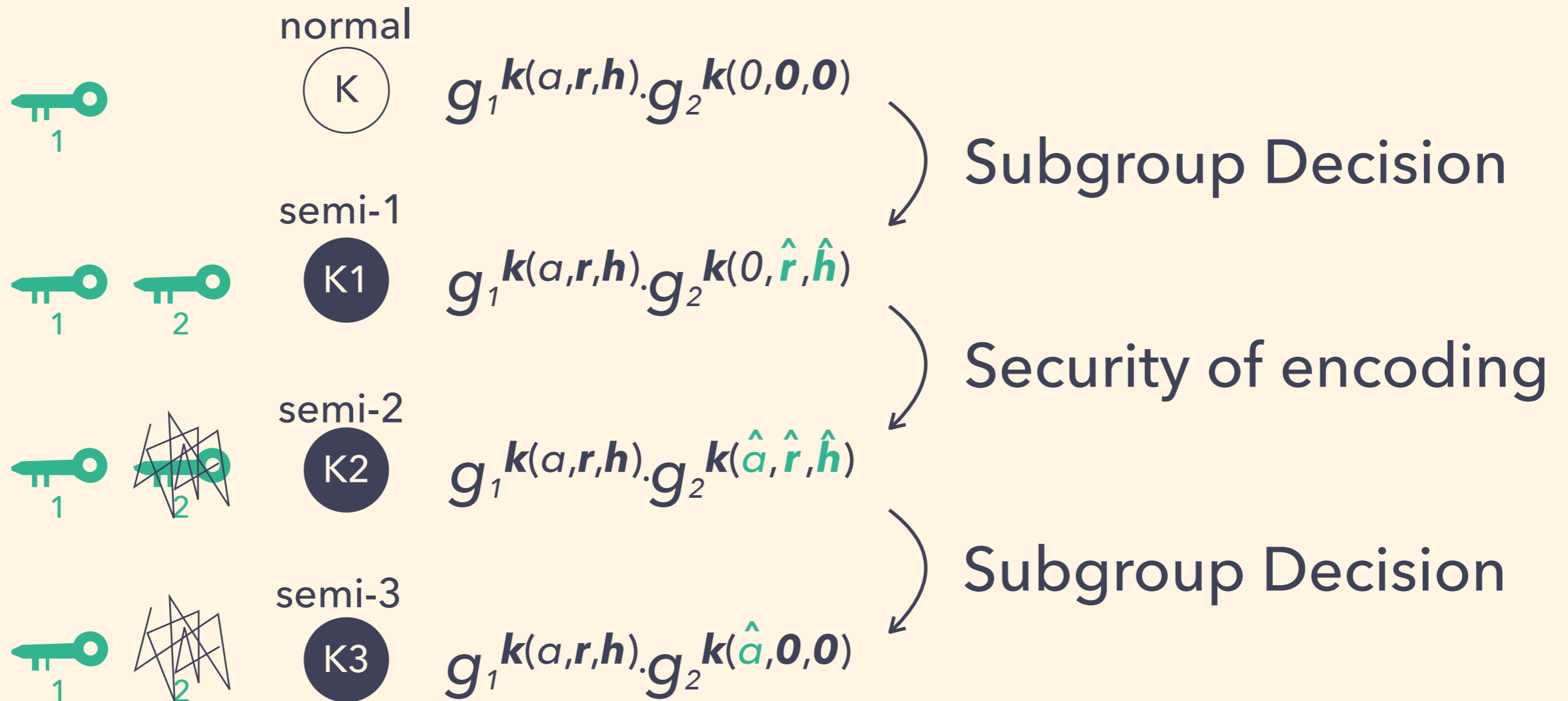
# More Results

- Dual-Policy ABE
  - Conjunctively combine ABE and its dual [AI09].
  - We also provide a conversion from ABE to DP-ABE.
- More refinement:
  - New specific CP-ABE with tighter reduction.
- Full version at <http://eprint.iacr.org/2015/157>.

**Thank you**

# Intuition Behind Pair Encoding Security

## Switch Keys from Normal to Semi-functional [A14]



- Only for self-containment, will not use here.



**RSA**®Conference2015

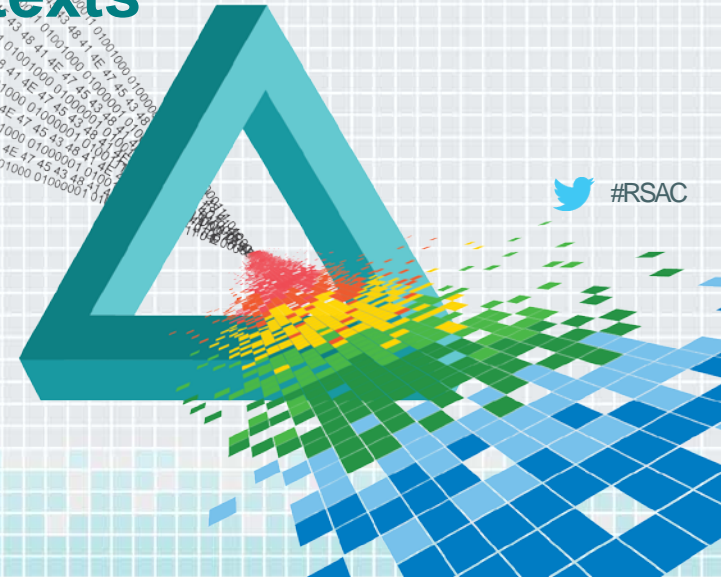
San Francisco | April 20-24 | Moscone Center



# Revocable Hierarchical Identity-Based Encryption: History-Free Update, Security Against Insiders, and Short Ciphertexts

Jae Hong Seo<sup>1</sup> and Keita Emura<sup>2</sup>

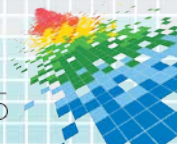
1. Myongji University, Korea
2. NICT, Japan



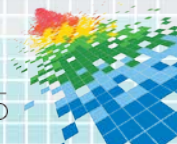
#RSAC

# Contents

- ◆ Identity-based encryption with **revocation** (RIBE)
  - ◆ Trivial Way (by Boneh and Franklin 2001)
  - ◆ Scalable construction (by Boldyreva, Goyal, and Kumar, 2008)
- ◆ Revocable Hierarchical IBE (RHIBE): CT-RSA 2013, Seo and Emura
  - ◆ **History-preserving** updates approach
  - ◆ Security against **outsider**
  - ◆ **Long-size ciphertext** (ciphertext size depends on the level of hierarchy)
- ◆ Our RHIBE Constructions
  - ◆ **History-free** updates approach
  - ◆ Security against **insider**
  - ◆ **Constant-size ciphertext** (in terms of the hierarchy level)



# Identity-Based Encryption and Revocation



# Identity-Based Encryption (IBE)



KGC

Bob@rsa.com



Issue sk



Publish mpk

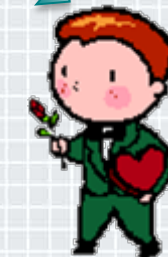


1 time  
download

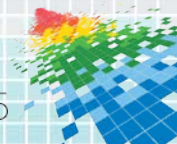


Sender

$\text{Enc}(\text{mpk}, \text{@}, M)$



Receiver



# Identity-Based Encryption (IBE)



KGC

Bob@rsa.com



Publish mpk



1 time  
download

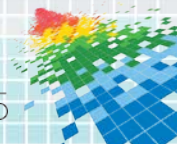


Sender

How to revoke Bob's  
secret key?



Receiver



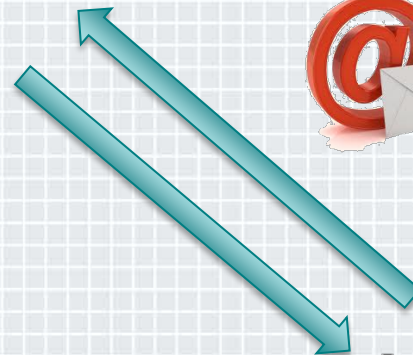
# Revocation Capability in IBE: Boneh-Franklin



KGC

Publish mpk ←

Bob@rsa.com



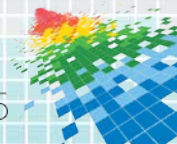
T is also regarded as a part of user's identity



Sender



Receiver



# Revocation Capability in IBE: Boneh-Franklin



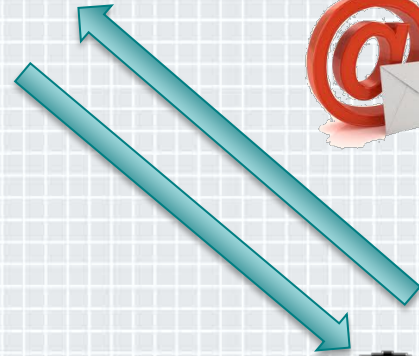
KGC

Publish mpk



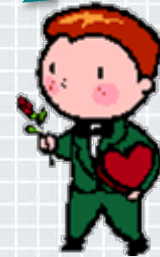
T is also regarded as a part of user's identity

Bob@rsa.com

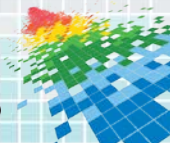


Sender

$\text{Enc}(\text{mpk}, @ || T, M)$



Receiver



# Revocation Capability in IBE: Boneh-Franklin

Publish mpk



KGC

Bob@rsa.com



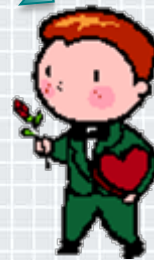
T is also regarded as a part of user's identity

Issue  $sk_{@||T}$   
if @ is not revoked  
on time T.

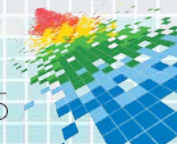


Sender

$Enc(mpk, @ || T, M)$



Receiver





# Revocation Capability in IBE: Boneh-Franklin

Publish mpk ←



KGC

Bob@rsa.com



T is also regarded as a part of user's identity

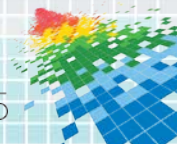
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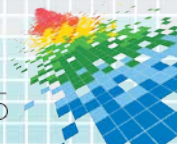
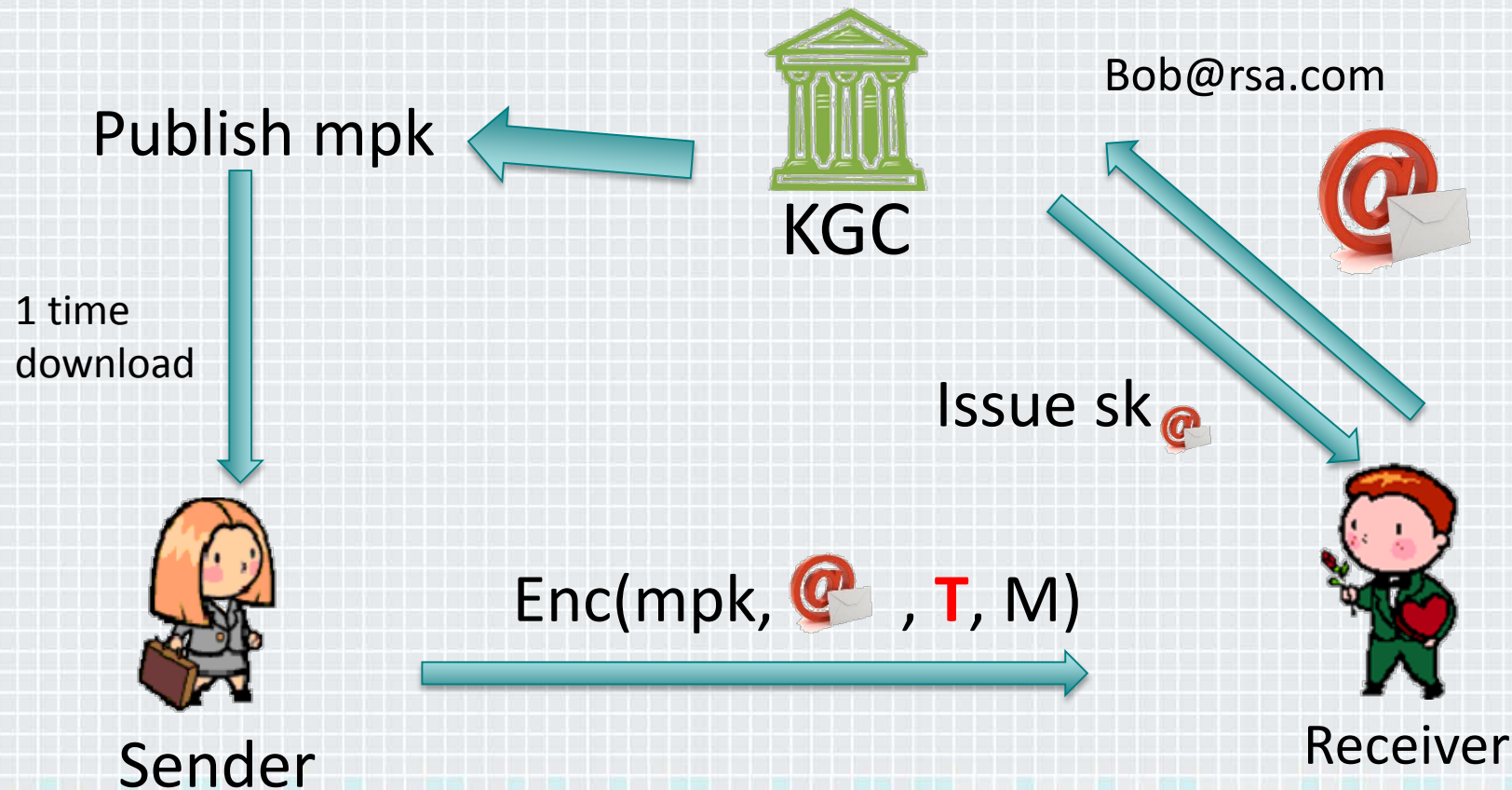
Problem: The overhead on KGC is linearly increased in the number of users ( $O(N-R)$ )

Sender

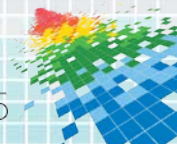
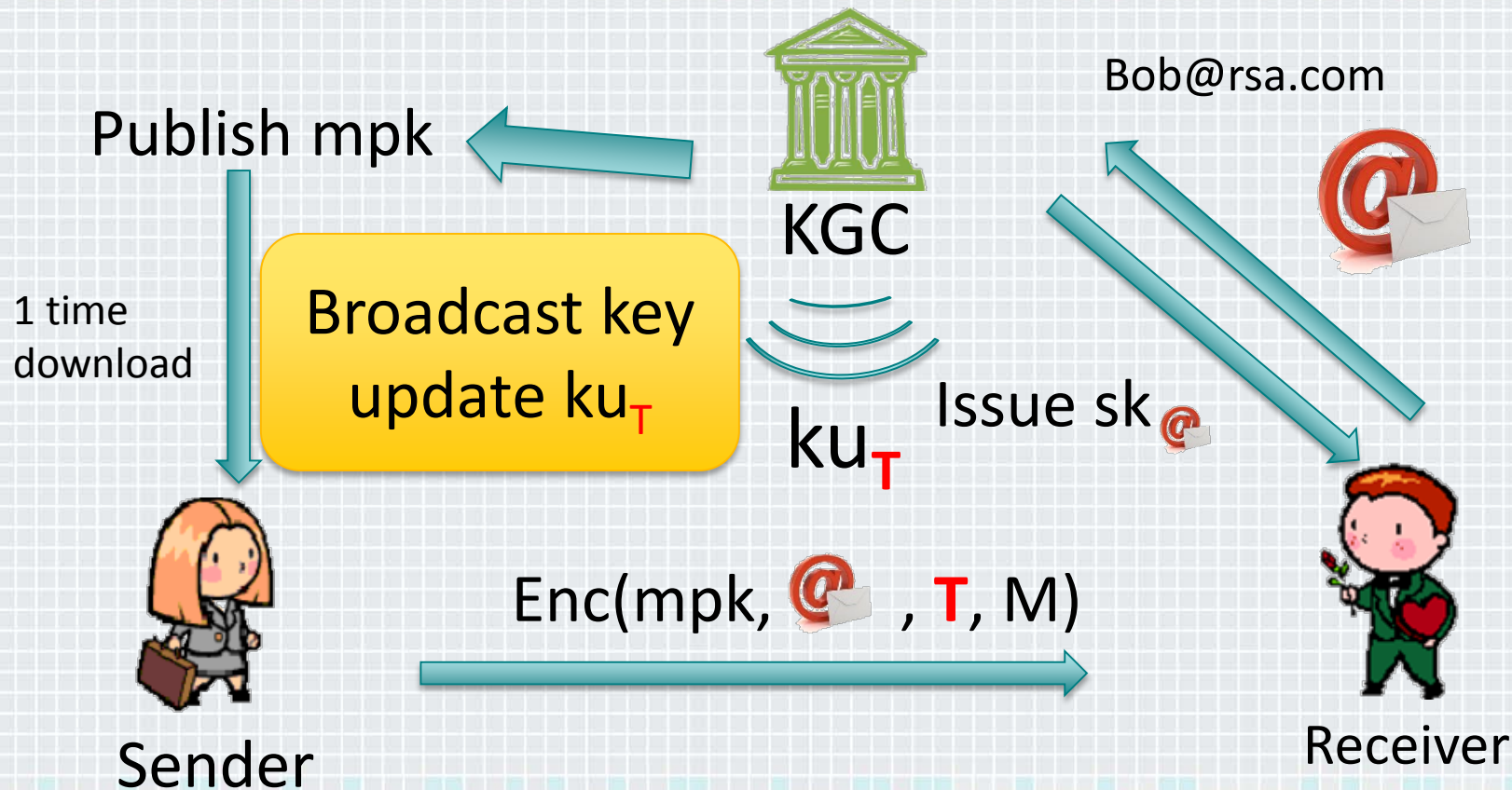
Receiver



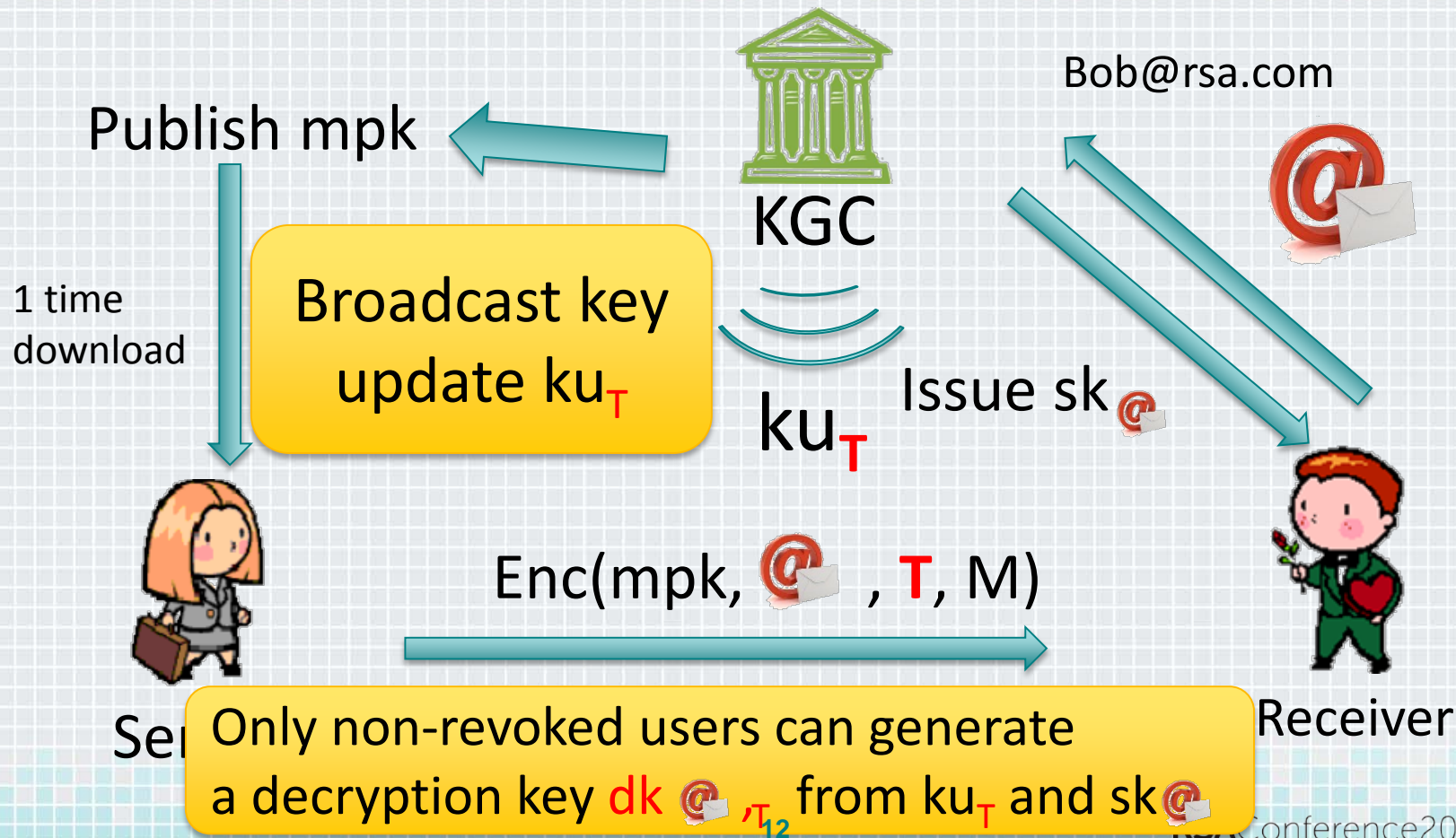
# Revocation Capability in IBE: Boldyreva et al.



# Revocation Capability in IBE: Boldyreva et al.



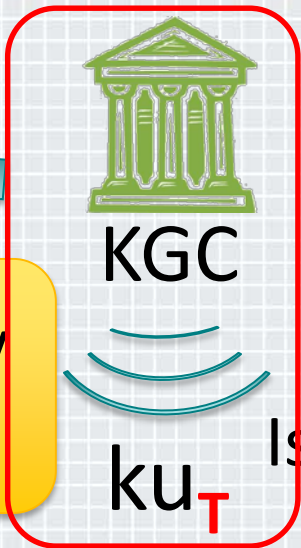
# Revocation Capability in IBE: Boldyreva et al.



# Revocation Capability in IBE: Boldyreva et al.

Only log-size Overhead!!  
(NNL: Naor-Naor-Lotspiech,  $O(R \log(N/R))$ )

Publish mpk



Bob@rsa.com



1 time  
download

Broadcast key  
update  $ku_T$



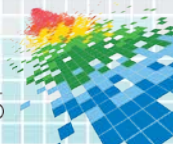
$Enc(mpk, @, T, M)$



Sender

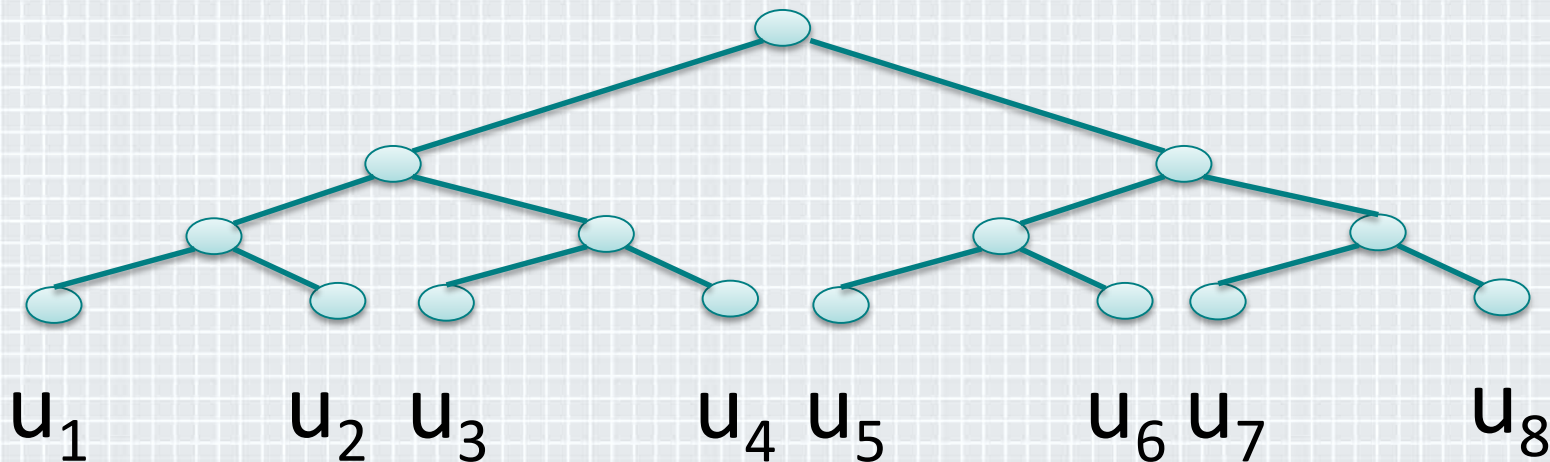
Only non-revoked users can generate  
a decryption key  $dk_{@, T}$  from  $ku_T$  and  $sk_{@}$

Receiver

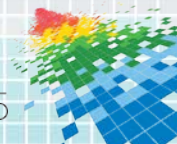


# Broadcast Encryption (BE) Technique Complete Subtree (CS)

- ◆ Each user is assigned to a node

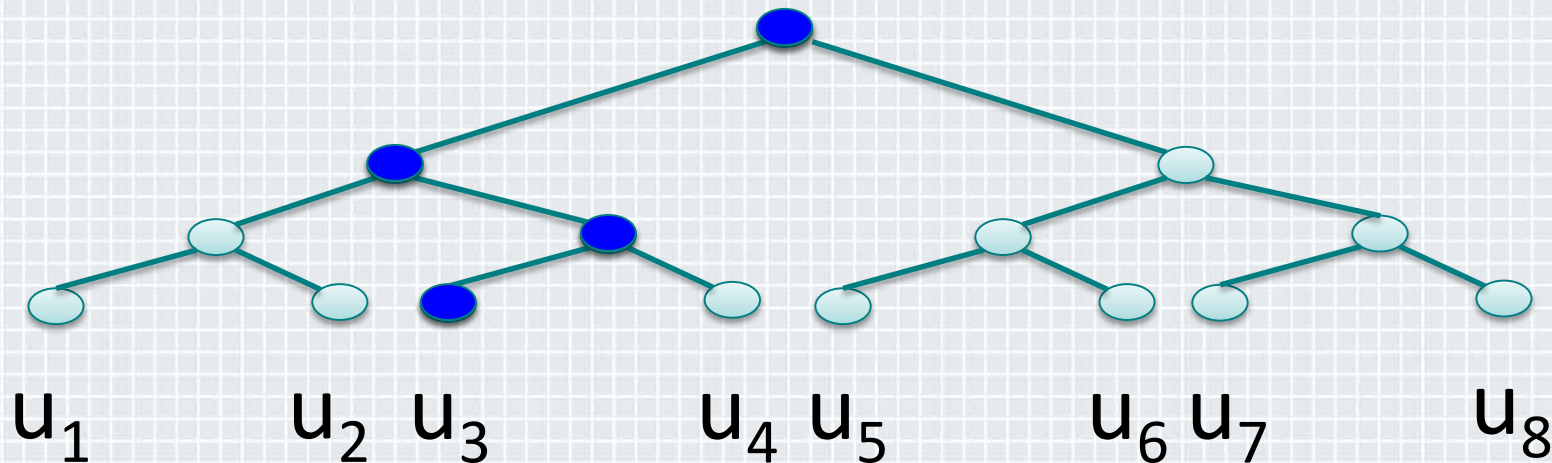


We consider a binary tree kept by KGC

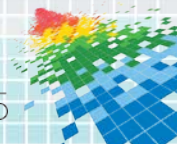


# Broadcast Encryption (BE) Technique Complete Subtree (CS)

- ◆ Each user is issued secret keys on the path to the root node by KGC (sk 📧)



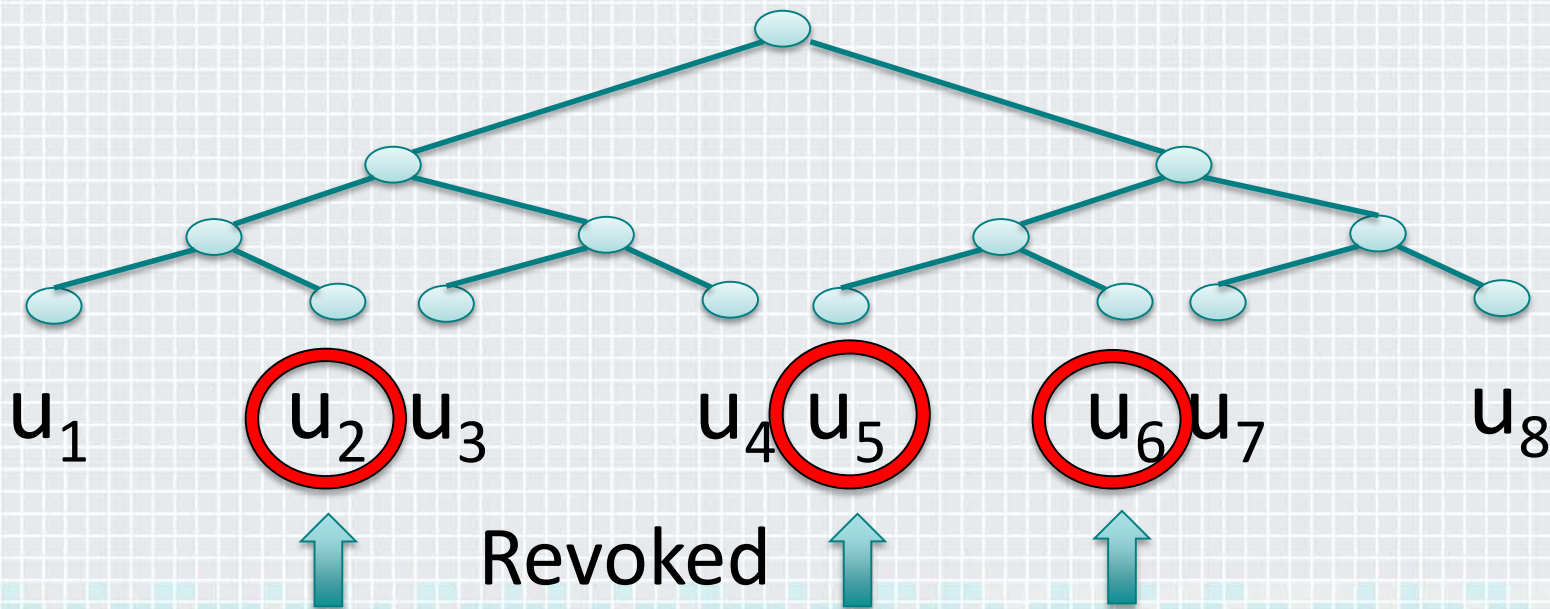
$u_3$  has secret keys on the path to the root node



# Broadcast Encryption (BE) Technique

## Complete Subtree (CS)

- ◆  $ku_T$  is computed for nodes which do not have intersection against paths (to the root node) of revoked users

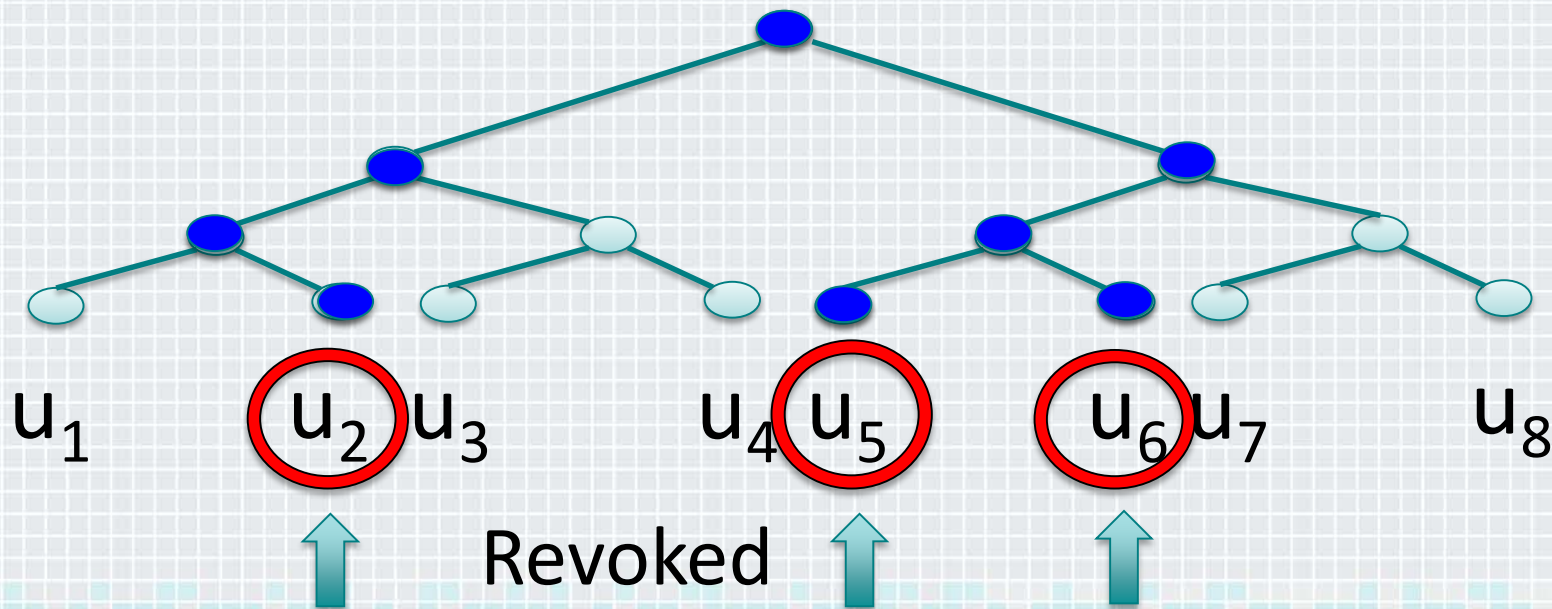




# Broadcast Encryption (BE) Technique

## Complete Subtree (CS)

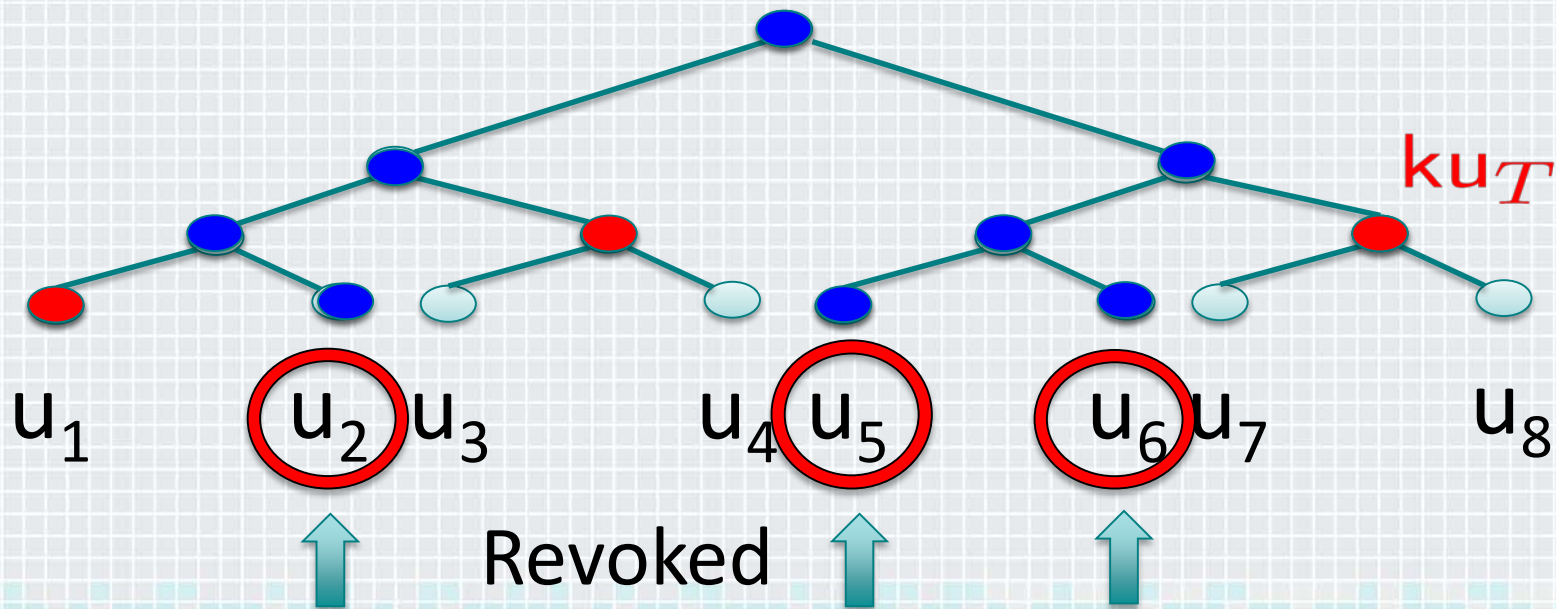
- ◆  $ku_T$  is computed for nodes which do not have intersection against paths (to the root node) of revoked users



# Broadcast Encryption (BE) Technique

## Complete Subtree (CS)

- ◆  $ku_T$  is computed for nodes which do not have intersection against paths (to the root node) of revoked users

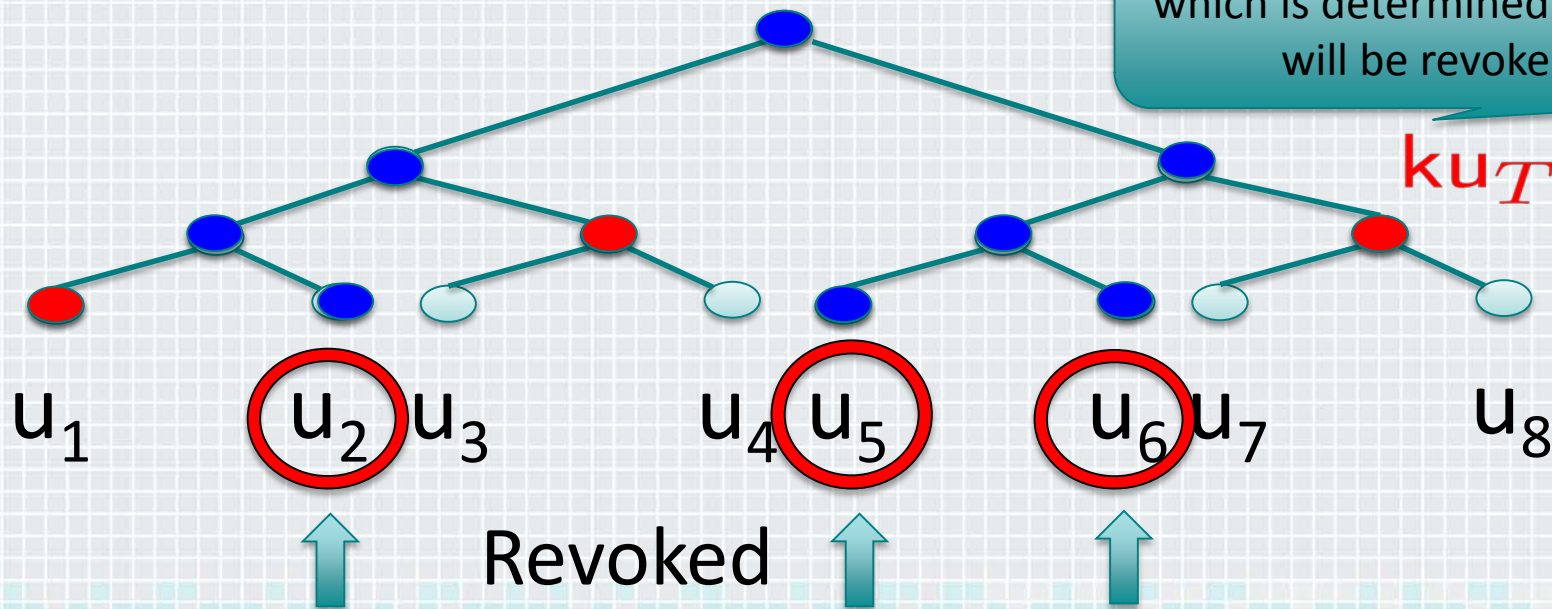


# Broadcast Encryption (BE) Technique

## Complete Subtree (CS)

- ◆  $ku_T$  is computed for nodes which do not have intersection against paths (to the root node) of revoked users

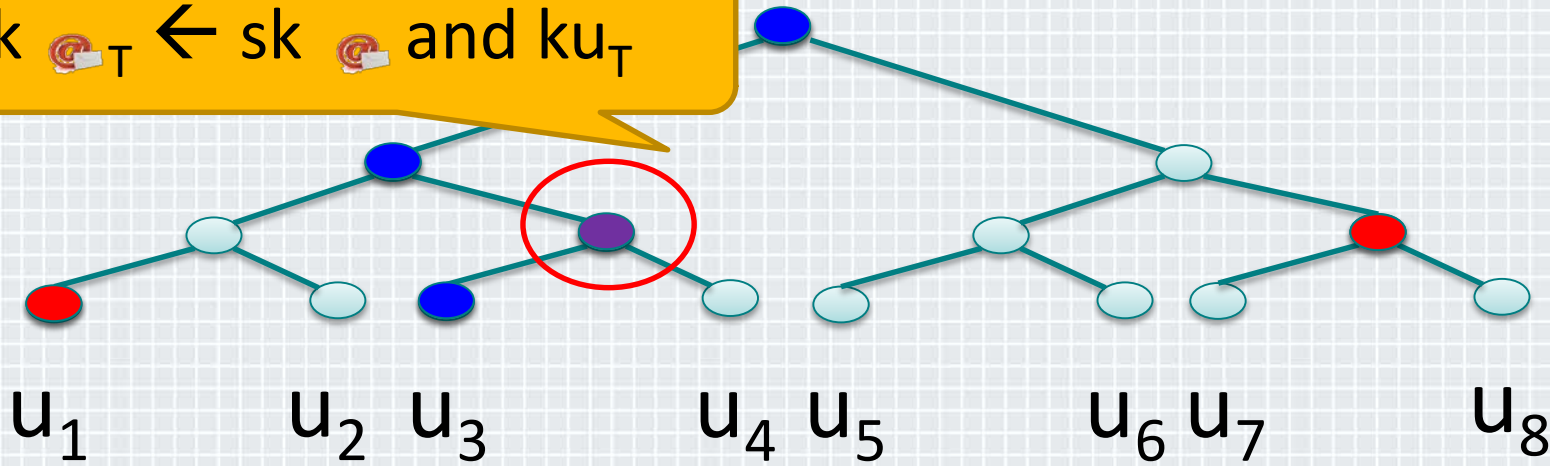
Contain **node information** which is determined by who will be revoked



# Broadcast Encryption (BE) Technique Complete Subtree (CS)

- ◆ From  $\log N$  size public information  $ku_T$ , only non-revoked users can extract useful information.

$dk_{\text{Ⓢ}} \leftarrow sk_{\text{Ⓢ}}$  and  $ku_T$



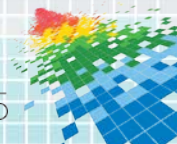
U<sub>3</sub> has secret keys on the path to the root node



# Scalable Revocable IBE

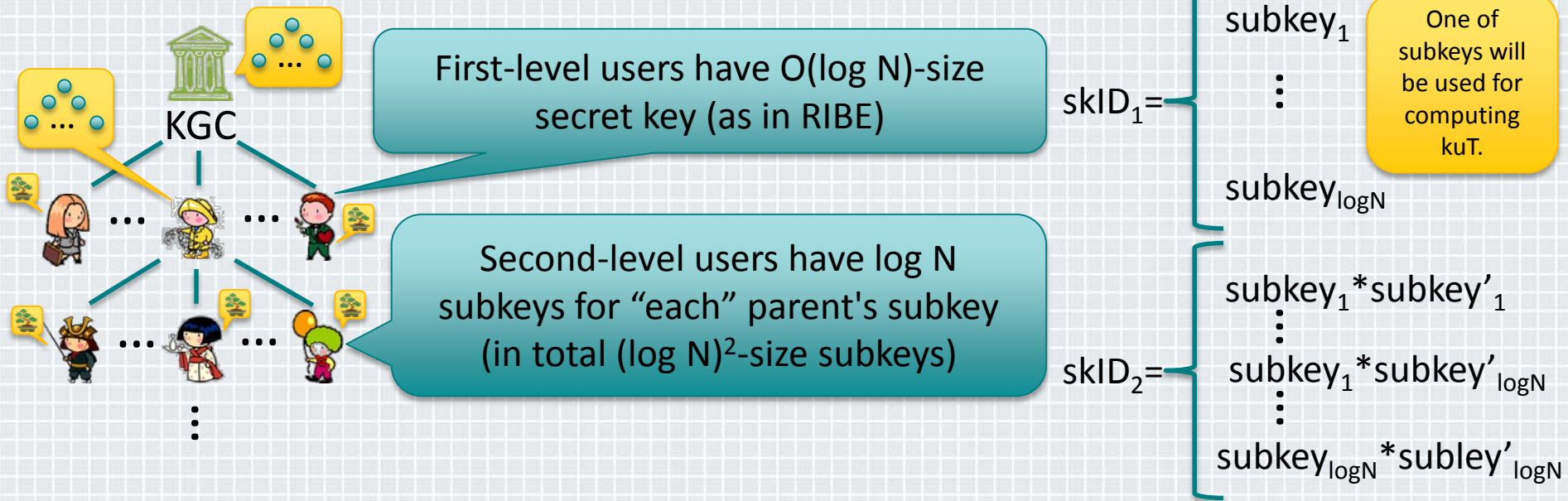
- ◆ First construction
  - ◆ A. Boldyreva, V. Goyal, and V. Kumar. Identity-based encryption with efficient revocation. In ACM CCS 2008
- ◆ First adaptive secure scheme
  - ◆ B. Libert and D. Vergnaud. Adaptive-ID secure revocable identity-based encryption. In CT-RSA 2009.
- ◆ Considering decryption key exposure resistance
  - ◆ J. H. Seo and K. Emura. Revocable identity-based encryption revisited: Security model and construction. In PKC 2013.
  - ◆ An adversary is allowed to obtain
 
$$dk_{ID,T} \text{ if } (ID, T) \neq (ID^*, T^*)$$
- ◆ SD-based construction
  - ◆ K. Lee, D. H. Lee, and J. H. Park. Efficient revocable identity-based encryption via subset difference methods, [eprint.iacr.org/2014/132](http://eprint.iacr.org/2014/132), 2014.

# Revocable Hierarchical IBE (RHIBE)



# Revocable Hierarchical IBE (RHIBE)

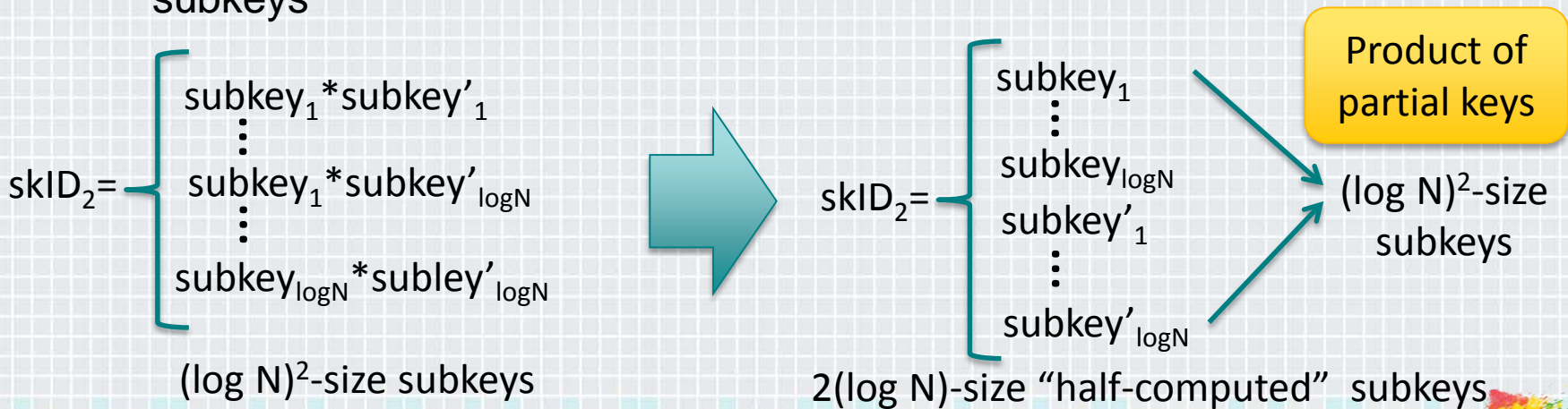
- ◆ A low-level user can stay in the system only if her parent also stays in the current time period.



- ◆ Trivial combination of RIBE and HIBE will result in an impractical scheme with an exponential number of secret keys

# Revocable Hierarchical IBE (RHIBE)

- ◆ The first RHIBE scheme with polynomial size secret keys (Seo-Emura, CT-RSA 2013)
  - ◆ Asymmetric trade between secret key size and generating time for secret key
    - ◆ A parent gives “half-computed” subkeys, and children generate suitable subkeys





# Revocable Hierarchical IBE (RHIBE)

- ◆ The first RHIBE scheme with polynomial size secret keys (Seo-Emura, CT-RSA 2013)

## History-preserving key updates

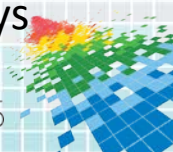
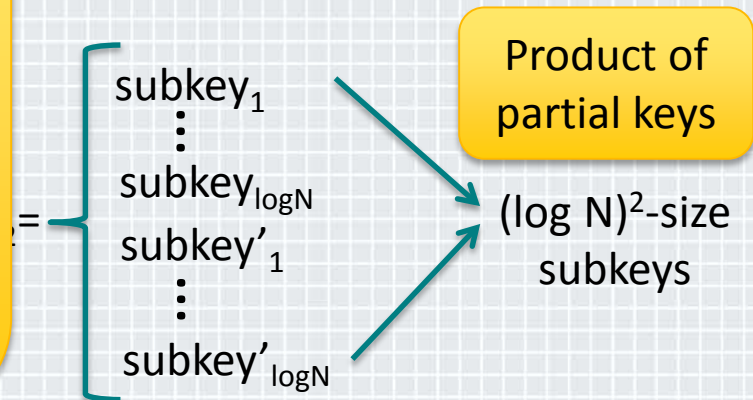
- For the calculation, a child needs to know which partial key of the ancestor was used in each time period.
- Such information is also announced in the key updates

$$ku_{ID|_{l-1}, T} := \left\{ \left\{ L_{v_i} \right\}_{i \in [1, l-1]}, \vec{f}_{ID|_{l-1}, \theta} \right\}$$

(log N)-size subkeys

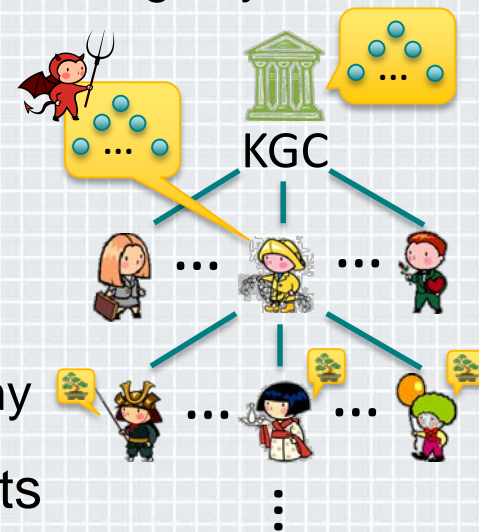
2(log N)-size “half-computed” subkeys

and children generate suitable



# Our Contribution

- ◆ **History-Free Update**
  - ◆ Low-level users do not need to know what ancestors did during key updates.
- ◆ **Security Against Insiders**
  - ◆ An adversary is allowed to obtain state information
- ◆ **Short Ciphertexts**
  - ◆ Constant-size ciphertext in terms of the level of hierarchy
- ◆ Two constructions: Shorter secret keys and ciphertexts
  - ◆ Complete Subtree (CS)
  - ◆ Subset Difference (SD)



# Main Idea for History-Free Update

- ◆ R(H)IBE:
  - ◆ KGC (or a parent user) issues a long-term secret key  $sk_{ID}$  using  $msk$  (or  $sk_{parent-ID}$ ).
  - ◆ KGC (or a parent user) broadcasts key update information  $ku_T$  which is computed by  $msk$  (or  $sk_{parent-ID}$ ).
  - ◆ A (child) user can generate the decryption key  $dk_{ID,T}$  from  $sk_{ID}$  and  $ku_T$  if he/she is not revoked at time  $T$ .
- ◆ Two situations are equivalent:
  - ◆ A user ID is not revoked at time  $T$
  - ◆ The user can generate the decryption key  $dk_{ID,T}$
- ◆ Re-define the key update algorithm

# Main Idea for History-Free Update

## ◆ Previous syntax

$$(\text{mpk}, \text{msk}) \leftarrow \text{Setup}(1^\lambda, N, \ell)$$

$$\text{sk}_{\text{ID}|_i} \leftarrow \text{SKGen}(\text{sk}_{\text{ID}|_{i-1}}, \text{st}_{\text{ID}|_{i-1}}, \text{ID}|_i)$$

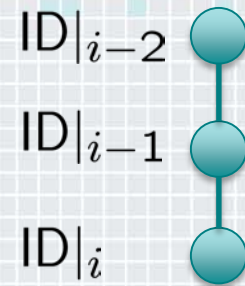
$$\text{ku}_{\text{ID}|_{i-1}, T} \leftarrow \text{KeyUp}(\text{sk}_{\text{ID}|_{i-1}}, \text{ku}_{\text{ID}|_{i-2}, T}, \text{st}_{\text{ID}|_{i-1}}, \text{RL}_{\text{ID}|_{i-1}})$$

$$\text{dk}_{\text{ID}|_i, T} \leftarrow \text{DKGen}(\text{sk}_{\text{ID}|_i}, \text{ku}_{\text{ID}|_{i-1}, T})$$

## ◆ Our modification

$$\text{sk}_{\text{ID}|_i} \leftarrow \text{SKGen}(\text{st}_{\text{ID}|_{i-1}}, \text{ID}|_i)$$

$$\text{ku}_{\text{ID}|_{i-1}, T} \leftarrow \text{KeyUp}(\text{dk}_{\text{ID}|_{i-1}, T}, \text{st}_{\text{ID}|_{i-1}}, \text{RL}_{\text{ID}|_{i-1}})$$



No parent secret key is required  
(for history-free approach)  
State information takes a role of the  
delegation key

dk is used instead of sk and ku

# Main Idea for History-Free Update

- ◆ Previous syntax

$(\text{mpk}, \text{msk}) \leftarrow \text{Setup}(1^\lambda, N, \ell)$

$\text{sk}_{\text{ID}|_i} \leftarrow \text{SKeyGen}(\text{msk}, \text{ID}|_i)$

$\text{ku}_{\text{ID}|_{i-1}, T} \leftarrow \text{KeyUpdate}(\text{sk}_{\text{ID}|_{i-1}}, \text{ID}|_i, T)$

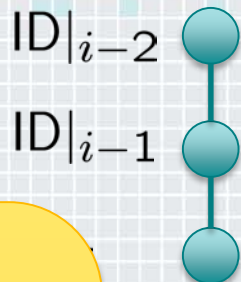
$\text{dk}_{\text{ID}|_i, T} \leftarrow \text{DecKeyGen}(\text{sk}_{\text{ID}|_i}, T)$

- ◆ Our model

$\text{sk}_{\text{ID}|_i} \leftarrow \text{SKeyGen}(\text{mpk}, \text{ID}|_i)$

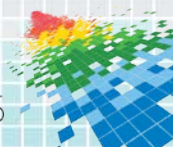
$\text{ku}_{\text{ID}|_{i-1}, T} \leftarrow \text{KeyUpdate}(\text{sk}_{\text{ID}|_{i-1}}, \text{ID}|_i, T)$

The secret key is used only for generating the decryption key dk.  
 Low-level users do not need to know what ancestors did during key updates.



required  
)  
of the

dk is used instead of sk and ku



# Proposed RHIBE Scheme (CS)

- ◆ Based on the BBG HIBE scheme
  - ◆ BBG HIBE (ID) + Boneh-Boyen IBE (Time)
  - ◆ Give a reduction to the BBG HIBE scheme.
    - ◆ [BBG05] Boneh, D., Boyen, X., Goh, E.-J.: Hierarchical identity based encryption with constant size ciphertext. EUROCRYPT 2005.

$$\text{mpk} = \{N, g, h, u_1, \dots, u_\ell, g_1, g_2, \underline{u', h'}\}$$

$$\text{msk} = \{g_2^\alpha\} \quad \text{(Boneh-Boyen hash)}$$

$$\text{sk}_{\text{ID}|_k} = \left\{ P_\theta(u_1^{l_1} \dots u_k^{l_k} h)^{r_\theta}, g^{r_\theta}, u_{k+1}^{r_\theta}, \dots, u_\ell^{r_\theta} \right\}_{\theta \in \text{Path}(\text{ID}|_k)}$$

$$\text{ku}_{\text{ID}|_{k-1}, T} = \left\{ P_\theta^{-1} \cdot g_2^\alpha (u_1^{l_1} \dots u_k^{l_k} h)^{r_\theta} (u'^T h')^{t_\theta}, g^{r_\theta}, g^{t_\theta}, u_k^{r_\theta}, \dots, u_\ell^{r_\theta} \right\}_{\theta \in \text{KUNode}(\text{BT}_{\text{ID}|_{k-1}}, \text{RL}_{\text{ID}|_{k-1}}, T)}$$

$$\text{dk}_{\text{ID}|_k, T} = (g_2^\alpha (u_1^{l_1} \dots u_k^{l_k} h)^r (u'^T h')^t, g^r, g^t, u_{k+1}^r, \dots, u_\ell^r)$$

$$\text{CT} = (\underline{M \cdot e(g_1, g_2)^s}, g^s, \underline{(u_1^{l_1} \dots u_k^{l_k} h)^s}, \underline{(u'^T h')^s})$$

BBG HIBE (ID)

BB IBE (Time)

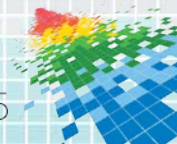
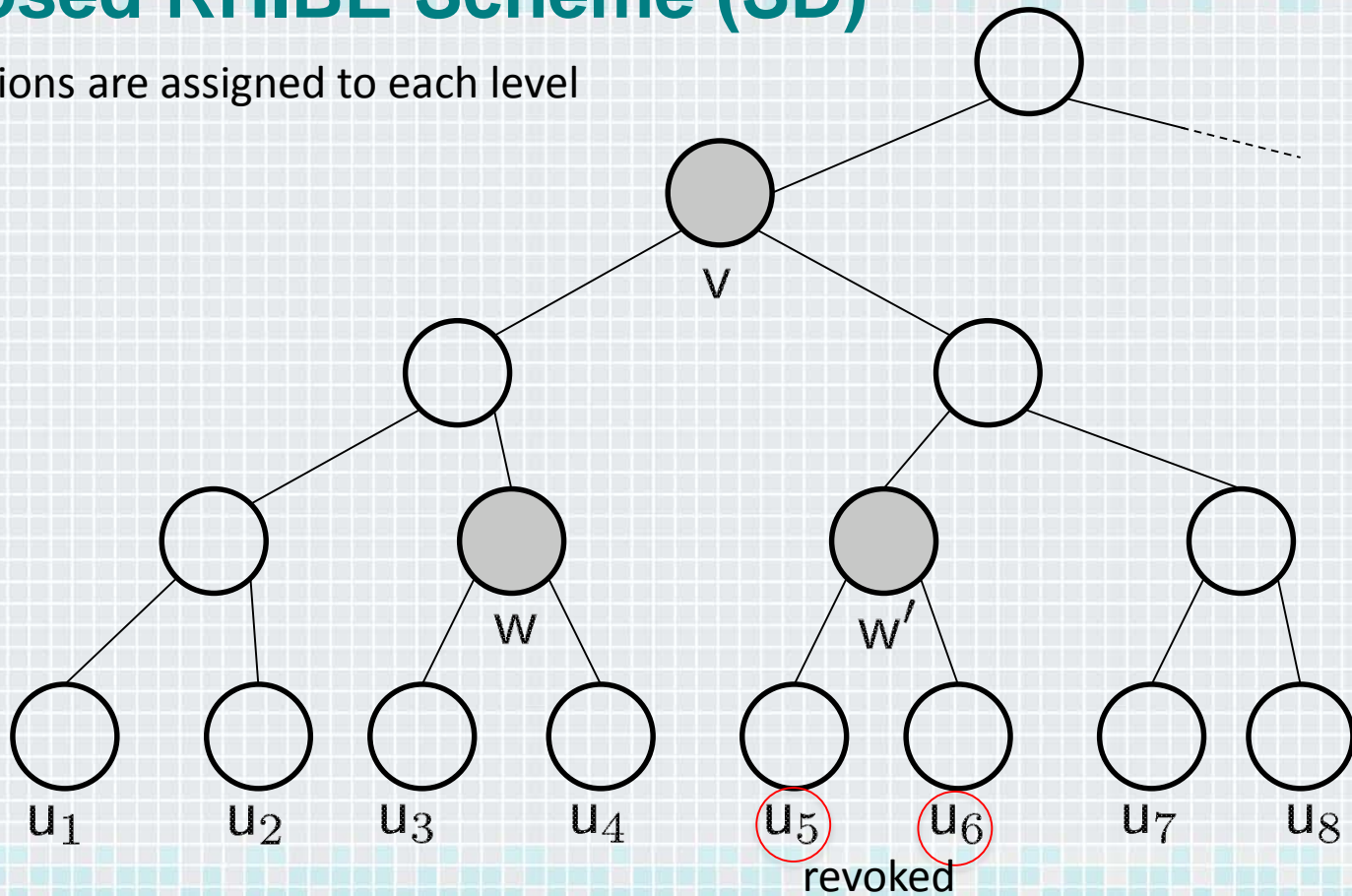
If  $\text{ID}|_k$  is not revoked, then there exists the same  $\theta$  (CS method)

With re-randomization for decryption key exposure resistance

# Proposed RHIBE Scheme (SD)

Linear functions are assigned to each level

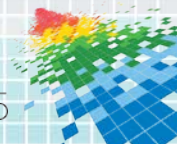
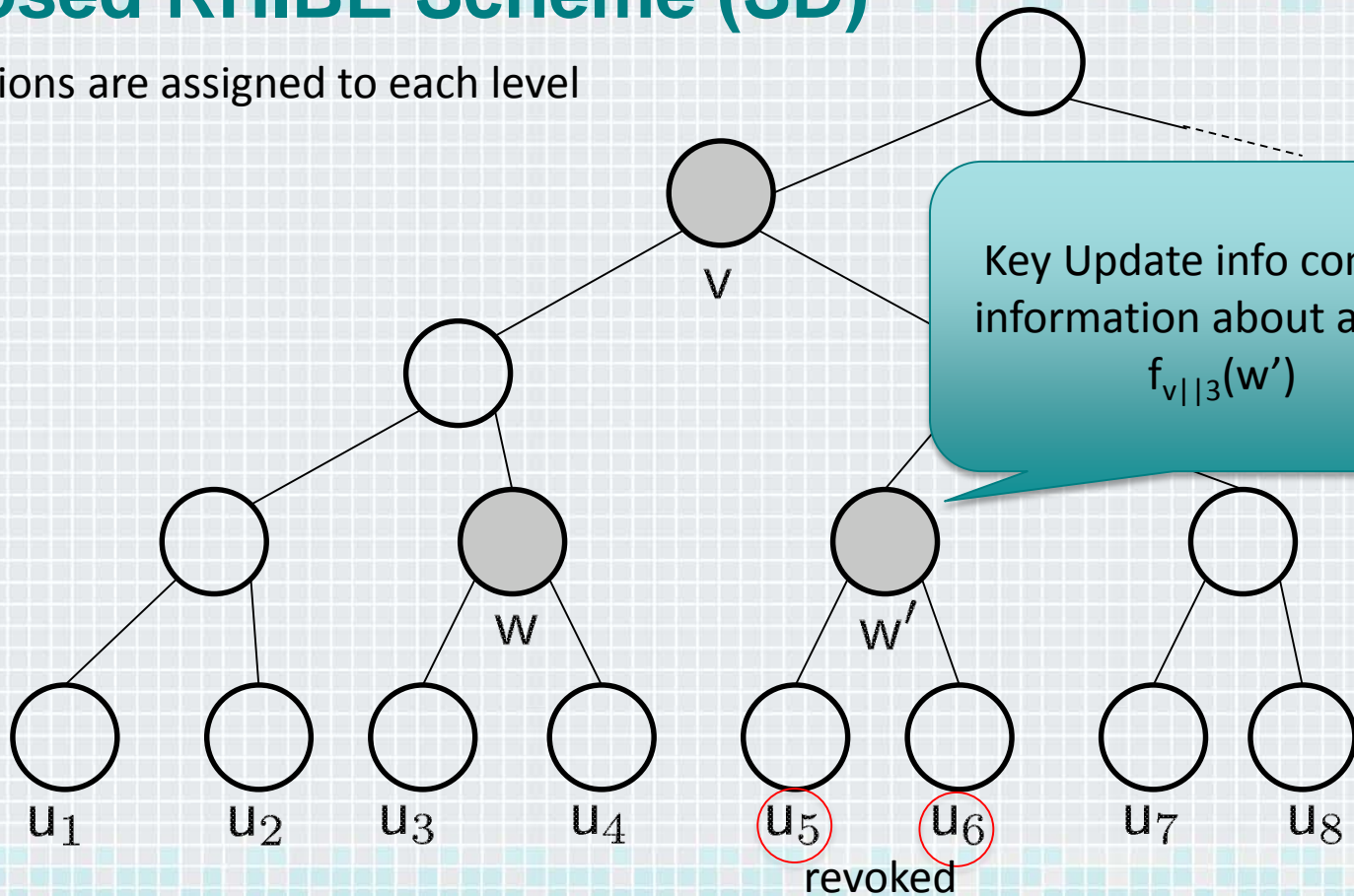
- $f_{v||1}$
- $f_{v||2}$
- $f_{v||3}$
- $f_{v||4}$



# Proposed RHIBE Scheme (SD)

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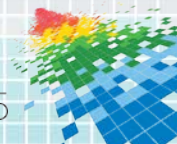
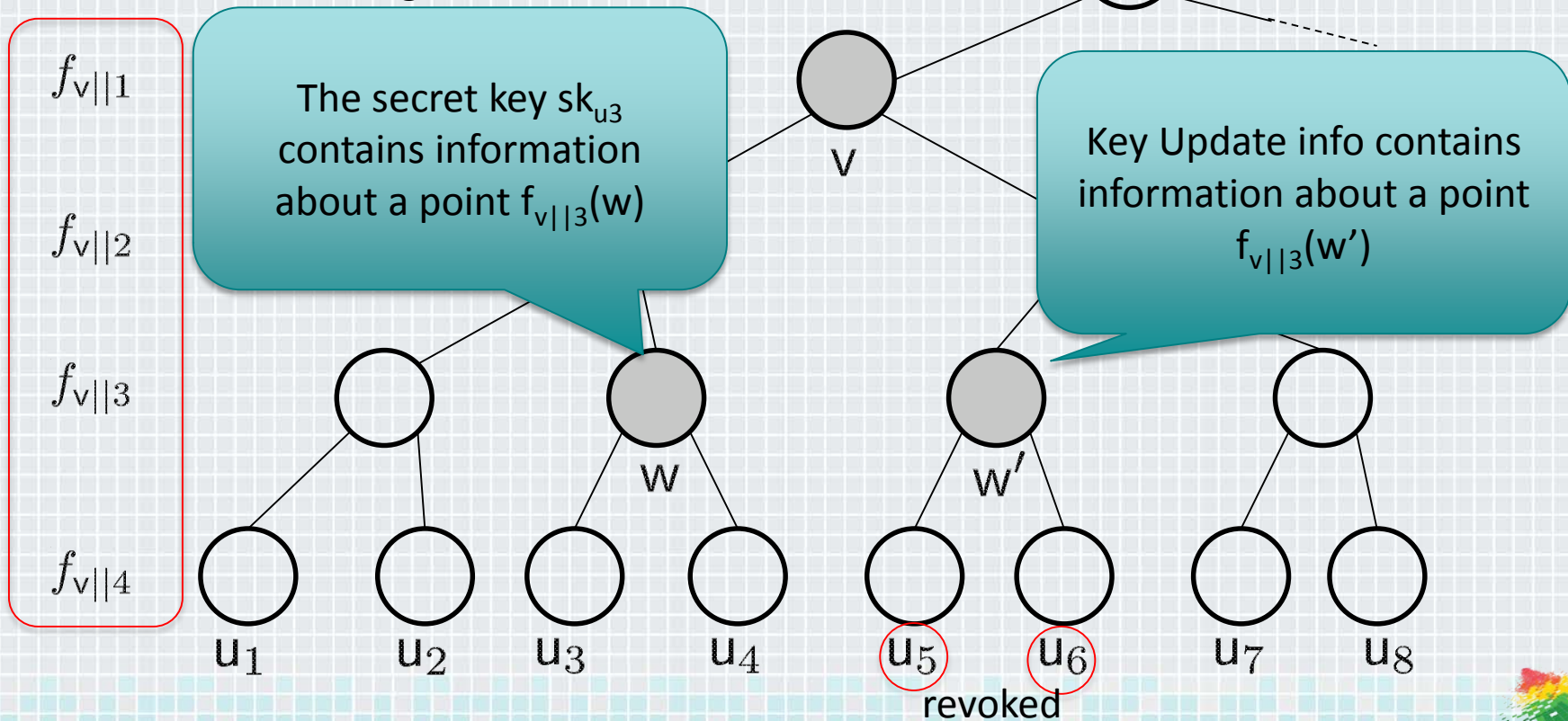
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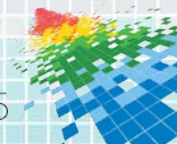
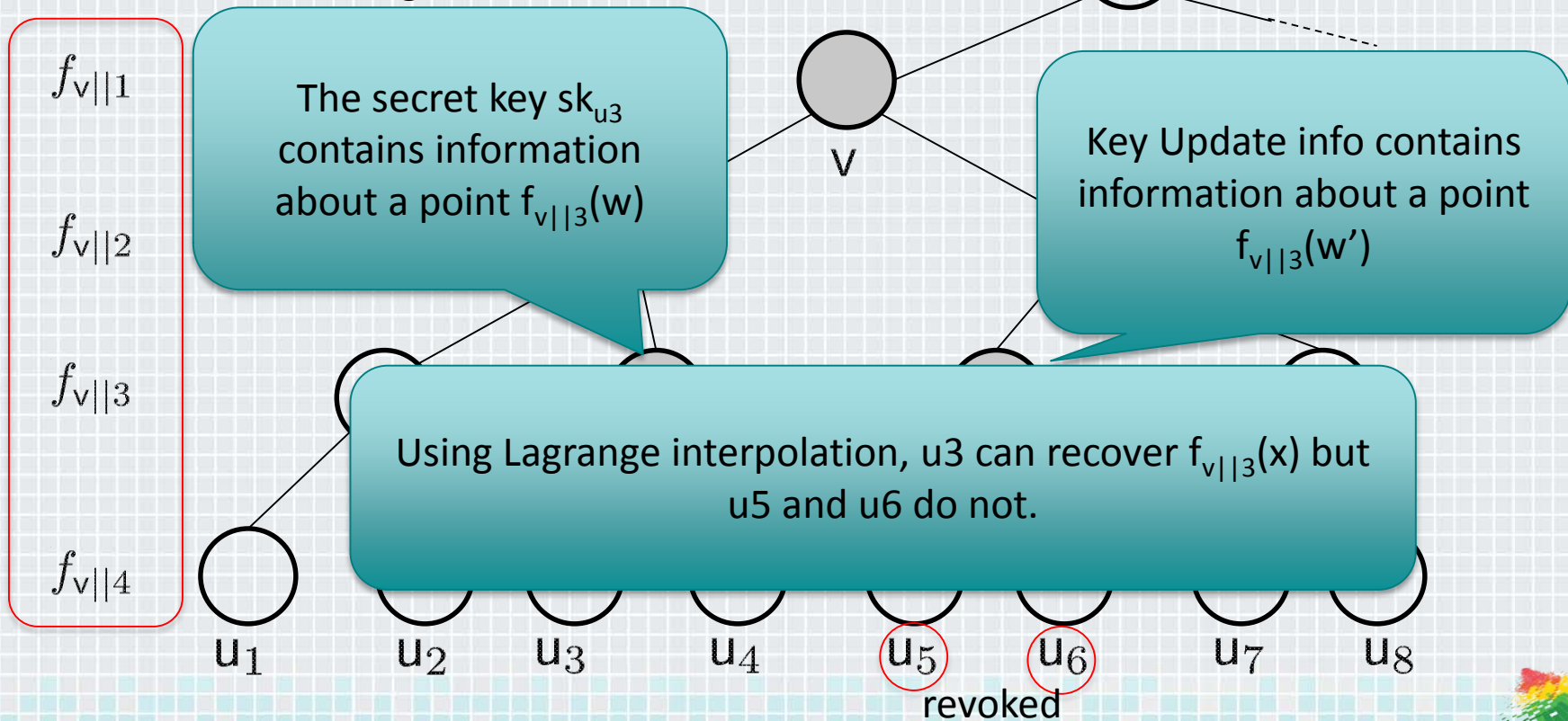
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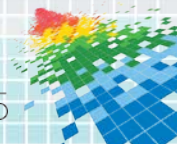


# Proposed RHIBE Scheme (SD)

- ◆ The main part is the same as that of the LLP RIBE scheme.
  - ◆ K. Lee, D. H. Lee, and J. H. Park. Efficient revocable identity-based encryption via subset difference methods, [eprint.iacr.org/2014/132](http://eprint.iacr.org/2014/132), 2014.
- ◆ One difference is: we introduce the **false master key** for history-free construction so that  $sk$  does not contain the master key  $\alpha$

$$f_y(x) := \text{PRF}_k(y)x + \beta$$

See the paper for details



# Comparison

Table 1: Revocable Hierarchical Identity-Based Encryption schemes

	SK size	CT size	KU size	Model	Sec. ag. insiders	DKE resist.	Assum.
Trivial	$\omega(2^\ell)$						
SE13	$O(\ell^2 \log N)$	$O(\ell)$	$O(r \log \frac{N}{r})$	Std., Sel.	✗	✗	static
CS const.	$O(\ell \log N)$	$O(1)$	$O(lr \log \frac{N}{r})$	Std., Sel.	✓	✓	$q$ -type
SD const.	$O(\ell(\log N)^2)$	$O(1)$	$O(lr)$	Std., Sel., SRL	✓	✓	$q$ -type

Std.: standard model, Sel.: selective security, SRL: selective revocation list [BGK08,LLP14]

$\ell$ : maximum hierarchical level,  $N$ : maximum number of users in the system,  $r$ : number of revoked users.

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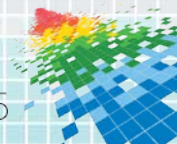
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DBDH

$q$ -weak Bilinear Diffie-Hellman Inversion



# Conclusion and Future work

Table 1: Revocable Hierarchical Identity-Based Encryption schemes

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Std.: standard model, Sel.: selective security, SRL: selective revocation list [BGK08,LLP14]

$\ell$ : maximum hierarchical level,  $N$ : maximum number of users in the system,  $r$ : number of revoked users.

- ◆ RHIBE:
  - ◆ History-free update, insider security, short ciphertext, and DKER
  - ◆ The reduction to the underlying HIBE requires the challenge identity for the security proof.
  - ◆ Adaptive-ID secure RHIBE under a static assumption with these desirable properties