TU Graz

Revisiting Cryptographic Accumulators, Additional Properties and Relations to other Primitives

David Derler, Christian Hanser, and Daniel Slamanig,
IAIK, Graz University of Technology

April 21, 2015

Outline

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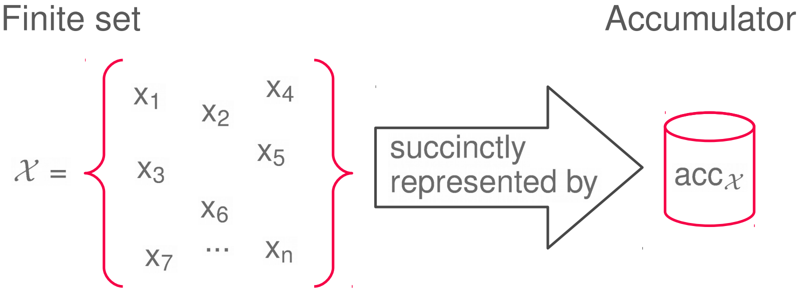
1. Introduction
2. A Unified Formal Model
3. Accumulators from Zero-Knowledge Sets
4. Black-Box Construction of Commitments

Static Accumulators



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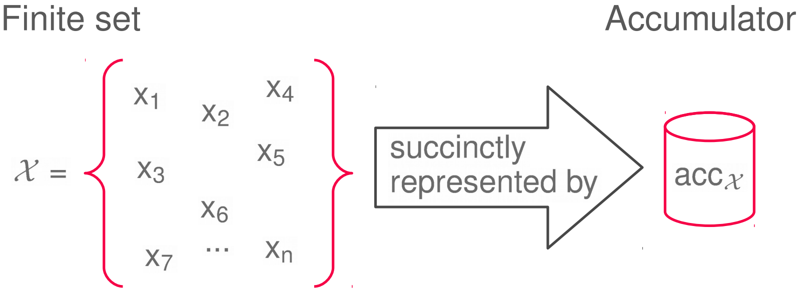

Witnesses wit_x certifying membership of x in $\mathrm{acc}_{\mathcal{X}}$

- Efficiently computable $\forall\, x \in \mathcal{X}$
- Intractable to compute $\forall\, x \notin \mathcal{X}$

A Simple Example - RSA Accumulator

- RSA modulus N
- Accumulator for $\mathcal{X} = \{x_1, \ldots, x_n\}$
  - $\mathsf{acc}_{\mathcal{X}} \leftarrow g^{x_1 \cdot \ \cdots \ \cdot x_{i-1} \cdot x_i \cdot x_{i+1} \cdot \ \cdots \ \cdot x_n} \mod N$

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- Witness for $y \notin \mathcal{X}$
  - Would imply breaking strong RSA
  - ...unless factorization of N is known.

Dynamic and Universal Features

Dynamically add/delete elements

- ...to/from accumulator $\mathsf{acc}_{\mathcal{X}}$
- Update witnesses accordingly
- All updates independent of $|\mathcal{X}|$

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Universal features

- Demonstrate non-membership
- Non-membership witness $\underline{\mathrm{wit}}_x$
  - Efficiently computable $\forall\ x \notin \mathrm{acc}_{\mathcal{X}}$
  - Intractable to compute $\forall\ x \in \mathrm{acc}_{\mathcal{X}}$

Motivation

Accumulators widely used in various applications

- e.g., credential revocation, malleable signatures, ...
- Previous models tailored to specific constructions
  - Different features
  - Private/public updatability

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Thus, accumulators not usable as black-boxes

- Limited exchangeability when used in other constructions
- Relations to other primitives hard to study

Contribution

- Unified formal model for
  - **Static/dynamic/universal** accumulators
  - Introduces **randomized** and **bounded** accumulators
  - Introduces **indistinguishability**
  - Includes **undeniability**

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- **Black-box relations** to commitments and ZK-sets
- **Exhaustive classification** of existing schemes (see Paper)

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Algorithms

Static Accumulators - Algorithms

- *Gen*
- *Eval*
- *WitCreate*
- *Verify*

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We call accumulators

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Dynamic Accumulators additionally provide

- *Add*
- *Delete*
- *WitUpdate*

Algorithms - Universal Accumulators

Static or dynamic accumulator, but in addition

- *WitCreate* and *Verify* take additional parameter *type*

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Static or dynamic accumulator, but in addition

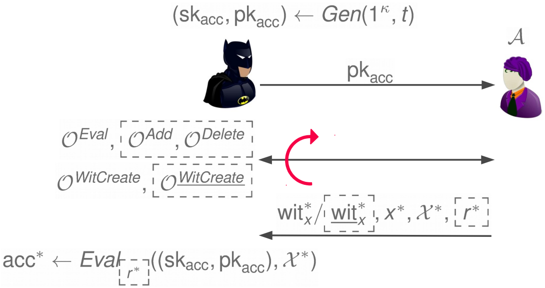
- *WitCreate* and *Verify* take additional parameter *type*
  - Membership ($type = 0$) vs. non-membership mode ($type = 1$)
- For dynamic accumulator schemes
  - The same additionally applies to *WitUpdate*

Security

- Correctness
- Collision freeness
- Undeniability
- Indistinguishability

Security - Collision Freeness

Experiment $\mathbf{Exp}^{cf}_{\kappa}(\cdot)$:



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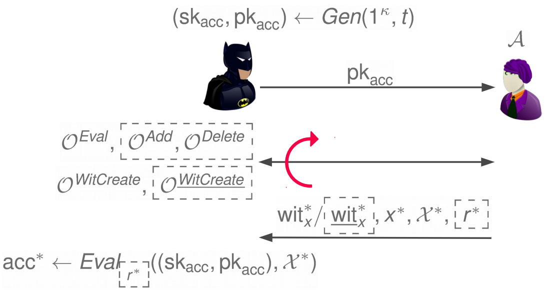
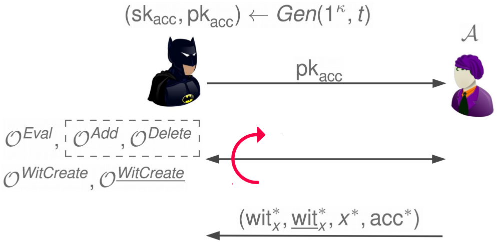


- $\mathcal{A}$ wins if
  - wit^*_x is membership witness for non-member, or
  - $\underline{\mathsf{wit}}^*_x$ is non-membership witness for member

Security - Undeniability

Defined for universal accumulators

Experiment $\textbf{Exp}_{\kappa}^{ud}(\cdot)$:



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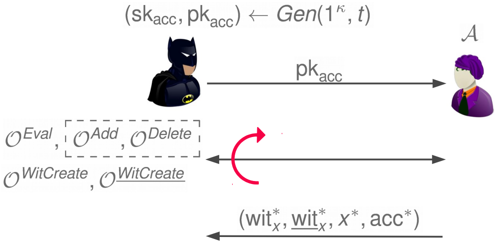

- $\mathcal{A}$ wins if verification succeeds for both wit_x^* and $\underline{\mathsf{wit}}_x^*$

Undeniability $\overset{\not\Leftarrow}{\Rightarrow}$ Collision Freeness

We show that

- Efficient $\mathcal{A}^{cf}$ can be turned into efficient $\mathcal{A}^{ud}$

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Other direction does not hold [BLL02]

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So far, no meaningful formalization

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We provide formalization

- not suffering from shortcomings above

Security - Indistinguishability II

Experiment $\mathbf{Exp}_{\kappa}^{ind}(\cdot)$:

$(\mathsf{sk}_{\mathsf{acc}}, \mathsf{pk}_{\mathsf{acc}}) \leftarrow \mathit{Gen}(1^{\kappa}, t)$,
$b \xleftarrow{R} \{0, 1\}$

$\mathcal{A}$

$\xrightarrow{\mathsf{pk}_{\mathsf{acc}}}$

$\xleftarrow{\mathcal{X}_0, \mathcal{X}_1}$

$(\mathsf{acc}_{\mathcal{X}_b}, \mathsf{aux}) \leftarrow \mathit{Eval}((\mathsf{sk}_{\mathsf{acc}}, \mathsf{pk}_{\mathsf{acc}}), \mathcal{X}_b)$

$\xrightarrow{\mathsf{acc}_{\mathcal{X}_b}}$

$\mathcal{O}^{\mathit{Eval}}, \boxed{\mathcal{O}^{\mathit{Add}_{\not\subseteq}}, \mathcal{O}^{\mathit{Delete}_{\cap}}}$
$\mathcal{O}^{\mathit{WitCreate}_{\cap}}, \boxed{\mathcal{O}^{\underline{\mathit{WitCreate}}_{\not\subseteq}}}$ $\longleftrightarrow$

$\xleftarrow{b^*}$

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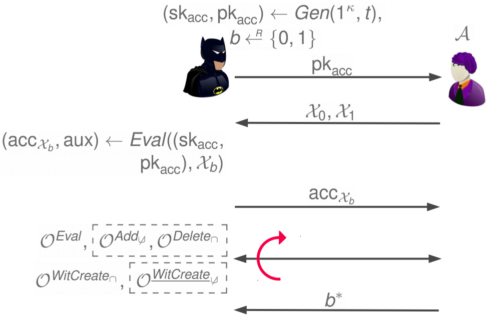

- $\mathcal{A}$ wins if guess correct

Security - Indistinguishability III

Ad-hoc solution in literature

- Insert a (secret) random value z into acc.

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We modify [Ngu05] to provide indistinguishability

- First **indistinguishable** ***t*-bounded dynamic** accumulator

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Zero-Knowledge Sets

Commit to a set $\mathcal{X}$

- Prove predicates of the form
  - $x \in \mathcal{X}$
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  - While not revealing anything else about $\mathcal{X}$

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Observation

- Similar to undeniable indistinguishable accumulators
- Algorithms compatible
- Security notions similar

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- Perfect completeness $\equiv$ correctness
- Soundness $\equiv$ undeniability

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- Zero-knowledge
  - Simulation-based notion
  - $\exists$ simulator $\mathcal{S}$, negl. ϵ, s.t. $\forall$ PPT distinguishers:
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First undeniable, unbounded, indistinguishable acc

- Nearly ZK sets $\rightarrow$ t-bounded

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- Compute commitment $\mathcal{C}$ to message m
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  - Hiding: For $\mathcal{C}$ to either m_0 or m_1. Intractable to decide whether $\mathcal{C}$ opens to m_0 or m_1

Commitments from Accumulators I

Use 1-bounded indistinguishable accumulators

- $\mathcal{C} \leftarrow \mathsf{acc}_{\{m\}}$
- $\mathcal{O} \leftarrow (m, r, \mathsf{wit}_m, aux)$ such that
  - $\mathsf{acc}_{\{m\}} = \mathit{Eval}_r((\emptyset, \mathsf{pk}_{\mathsf{acc}}), \{m\})$
  - $\mathit{Verify}(\mathsf{pk}_{\mathsf{acc}}, \mathsf{acc}_{\{m\}}, \mathsf{wit}_m, m) = \mathtt{true}$

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Observe: cfw-indistinguishability not useful

Commitments from Accumulators II

Straight forward extension to set-commitments

- Use t-bounded accumulators
- Opening w.r.t. entire set

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Trapdoor commitments

- Use sk_{acc} as trapdoor

Conclusion

Unified model for accumulators

- Covering all features existing to date

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Introduce indistinguishability notion

- Provide first indistinguishable dynamic scheme

Show relations to other primitives

- Commitments
- Zero-knowledge sets
  - Yields first undeniable, unbounded, indistinguishable, universal accumulator
- Inspiration for new constructions

Thank you.

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Extended version: `http://eprint.iacr.org/2015/087`

References I

[BLL02] Ahto Buldas, Peeter Laud, and Helger Lipmaa. Eliminating counterevidence with applications to accountable certificate management. *Journal of Computer Security*, 10(3):273–296, 2002.

[Ngu05] Lan Nguyen. Accumulators from bilinear pairings and applications. In *Topics in Cryptology - CT-RSA 2005, The Cryptographers' Track at the RSA Conference 2005, San Francisco, CA, USA, February 14-18, 2005, Proceedings*, pages 275–292, 2005.

Non-Interactive Zero-Knowledge Proofs of Non-Membership

O. Blazy, C. Chevalier, D. Vergnaud

XLim / Université Paris II / ENS

Proof of Knowledge

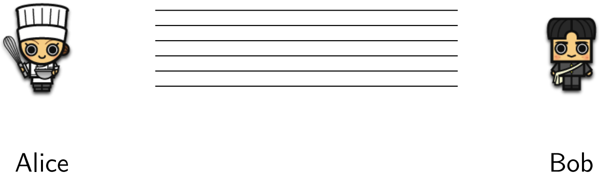
Alice

Bob

- Interactive method for one party to prove to another the knowledge of a secret $\mathcal{S}$.

Classical Instantiations : Schnorr proofs, Sigma Protocols . . .

Proving that a statement is not satisfied



- Interactive method for one party to prove to another the knowledge of a secret $\mathcal{S}$ that does not belong to a language $\mathcal{L}$.

Applications

- Credentials
- Enhanced Authenticated Key Exchange

Additional properties

- Non-Interactive
- Zero-Knowledge
- Implicit

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C

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- Introduced in 1985 by Goldwasser, Micali and Rackoff.

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- Used in many cryptographic protocols
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  - **Anonymous credentials**
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Zero-Knowledge Interactive Proof

Alice   Bob

- interactive method for one party to prove to another that a statement $\mathcal{S}$ is true, without revealing anything other than the veracity of $\mathcal{S}$.

1. **Completeness:** if $\mathcal{S}$ is true, the honest verifier will be convinced of this fact
2. **Soundness:** if $\mathcal{S}$ is false, no cheating prover can convince the honest verifier that it is true
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Certification of Public Keys: SPHF [ACP09]

A user can ask for the certification of pk, but if he knows the associated sk only:

With a Smooth Projective Hash Function

$\mathcal{L}$: pk and $C = \mathcal{C}(\mathsf{sk}; r)$ are associated to the same sk

- U sends his pk, and an encryption C of sk;
- A generates the certificate Cert for pk, and sends it,
  masked by $\mathsf{Hash} = \mathsf{Hash}(\mathsf{hk}; (\mathsf{pk}, C))$;
- U computes $\mathsf{Hash} = \mathsf{ProjHash}(\mathsf{hp}; (\mathsf{pk}, C), r))$, and gets Cert.

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Implicit proof of knowledge of sk

Smooth Projective Hash Functions [CS02]

Definition [CS02,GL03]

Let $\{H\}$ be a family of functions:

- X, domain of these functions
- L, subset (a language) of this domain

such that, for any point x in L, $H(x)$ can be computed by using

- either a *secret* hashing key hk: $H(x) = \mathsf{Hash}_L(\mathsf{hk}; x)$;
- or a *public* projected key hp: $H'(x) = \mathsf{ProjHash}_L(\mathsf{hp}; x, w)$

Public mapping $\mathsf{hk} \mapsto \mathsf{hp} = \mathsf{ProjKG}_L(\mathsf{hk}, x)$

SPHF Properties

For any $x \in X$, $H(x) = \mathsf{Hash}_L(\mathsf{hk}; x)$
For any $x \in L$, $H(x) = \mathsf{ProjHash}_L(\mathsf{hp}; x, w)$

w witness that $x \in L$, $\mathsf{hp} = \mathsf{ProjKG}_L(\mathsf{hk}, x)$

Smoothness

For any $x \notin L$, $H(x)$ and hp are independent

Pseudo-Randomness

For any $x \in L$, $H(x)$ is pseudo-random, without a witness w

The latter property requires L to be a hard-partitioned subset of X.

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The latter property requires L to be a hard-partitioned subset of X.

SPHF Properties

For any $x \in X$, $H(x) = \mathsf{Hash}_L(\mathsf{hk}; x)$
For any $x \in L$, $H(x) = \mathsf{ProjHash}_L(\mathsf{hp}; x, w)$
w witness that $x \in L, \mathsf{hp} = \mathsf{ProjKG}_L(\mathsf{hk}, x)$

Smoothness

For any $x \notin L$, $H(x)$ and hp are independent

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Global Idea

- π: Proof that $W \in \mathcal{L}$
- π: Randomizable, Indistinguishability of Proof
- π': Proof that π was computed honestly

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- Try to prove that $W \in \mathcal{L}$ which will output a π
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From a very high level

- If an adversary forges a proof, this means that both π and π' are valid
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Possible Instantiations

Proof π	Proof π'	Interactive	Properties
Groth Sahai	Groth Sahai	No	Zero-Knowledge
SPHF	SPHF	Yes	Implicit
Groth Sahai	SPHF	Depends	ZK, Implicit

Anonymous Credentials

Allows user to authenticate while protecting their privacy.

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Language Authenticated Key Exchange

Alice

$\rightarrow \mathcal{C}(M_B)$

$\mathcal{C}(M_A), \mathsf{hp}_B \leftarrow$

$\rightarrow \mathsf{hp}_A$

Bob

$H_B \cdot H'_A$

$H'_B \cdot H_A$

Same value iff languages are as expected, and users know witnesses.

Summing up

- Proposed a generic framework to prove negative statement *
- Gives several instantiation of this framework, allowing some modularity
- Works outside pairing environment

Open Problems

- Be compatible with post-quantum cryptography
- Weaken the requirements, on the building blocks

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